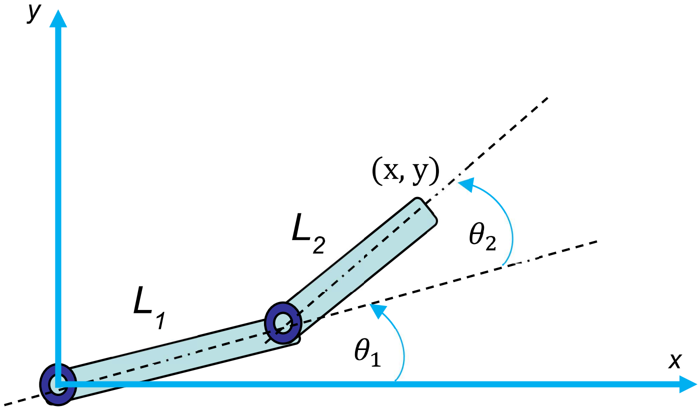
Exercise 4

Find the IGM (Inverse geometric model) of a 2 DOF planar robot (see figure below): given *x* and *y*, what are θ_1 and θ_2?

$x = L_1c_1 + L_2c_{1+2}$

$y = L_1s_1 + L_2s_{1+2}$

Hint: use the trigonometric formulas for the sine and cosine of the sum of two angles, as well as the one of the sum of squares of sine and cosine.



Exercise 4 – Solution

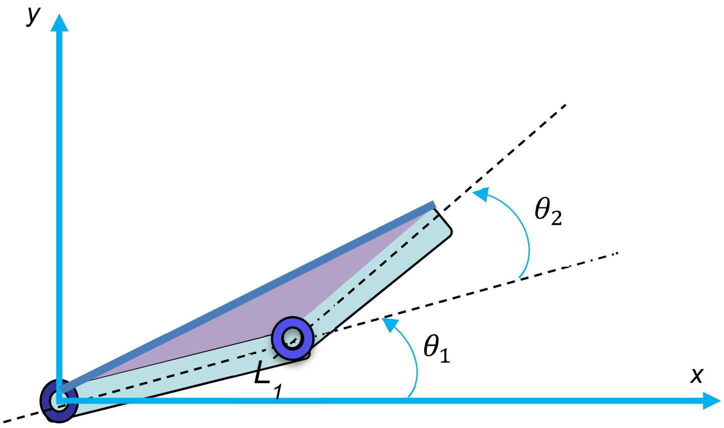
As for the first exercise, we consider the simple planar manipulator with two segments. It is asked to find the angles θ_1 and θ_2 from a given position (*x, y*).

We know the DGM:

$x = L_1c_1 + L_2c_{1+2}$
$y = L_1s_1 + L_2s_{1+2}$

We also know that:

$1 = c^2 + s^2$



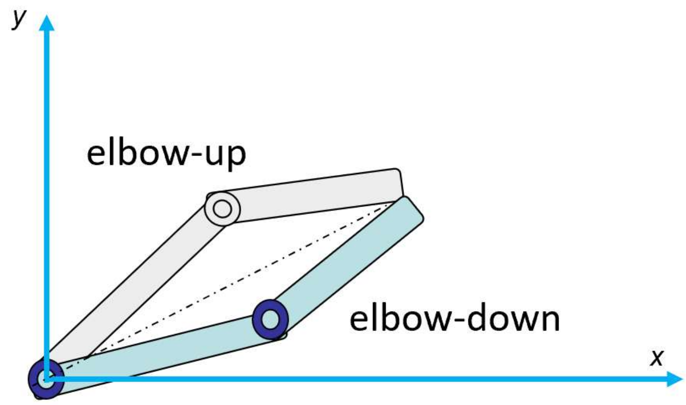
Using the law of cosines we see that the angle θ_2 is given by:

$c_2 = \frac{x^2+y^2-L_1^2-L_2^2}{2L_1L_2}$

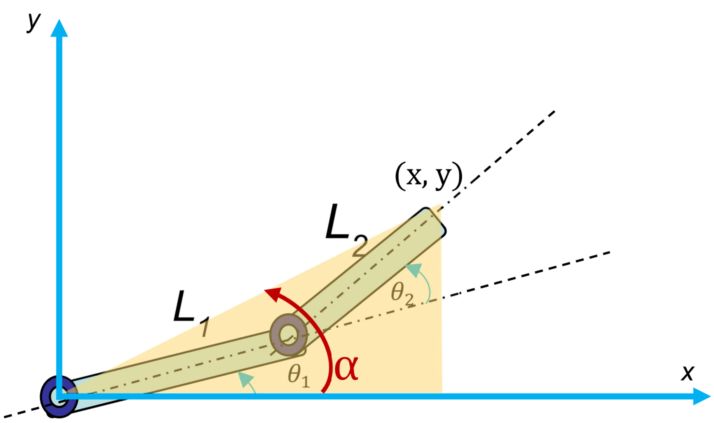
$s_2 = \pm\sqrt{1-c_2^2}$

Hence, θ_2 can be found by:

$$\theta_2 = \arctan\frac{\pm\sqrt{1-c_2^2}}{c_2}$$



The choice of $\pm$ is arbitrary but is important (it must be consistent) for pairs of final solutions. Moreover, finding the angle of θ_2 by using arctan function is advantageous since it is recovering both elbow-up and elbow-down solutions by choosing the positive and negative signs, respectively.



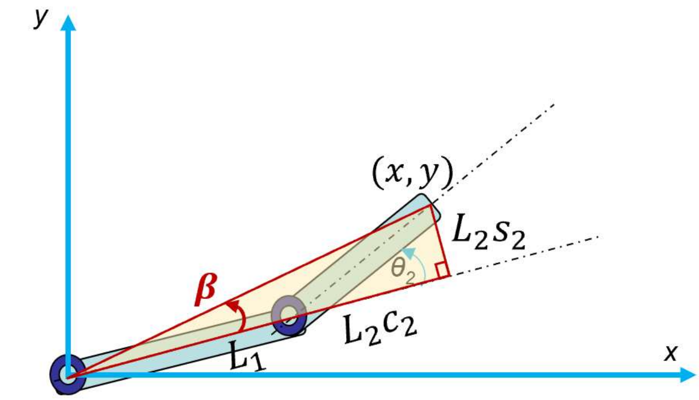
θ_1 can be defined as $\theta_1 = \alpha - \beta$ where

$$\alpha = \arctan\left(\frac{y}{x}\right)$$

$$\beta = \arctan\left(\frac{L_2 s_2}{L_1 + L_2 c_2}\right)$$

Therefore,

$$\theta_1 = \arctan\left(\frac{y}{x}\right) - \arctan\left(\frac{L_2 s_2}{L_1 + L_2 c_2}\right)$$



Although the solution is complete like this, we need to be careful about the quadrant of the (x, y) position since arctan function is used to calculate the angles θ_1 and θ_2.

Hint: While programming you can use atan2() instead of atan() function, which will handle the determination of the quadrant.

$$\operatorname{atan2}(y, x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

Exercise 5

Consider the two sequences of exercises 1 and 2:

$$\mathbf{R}_z(90°) \rightarrow \mathbf{R}_y(90°)$$

$$\mathbf{R}_y(90°) \rightarrow \mathbf{R}_z(90°)$$

For each of these sequences:

1. Determine the resulting corresponding quaternion.
2. Deduce:
   (a) the corresponding angles of rotation.
   (b) the corresponding unit axes of rotation.