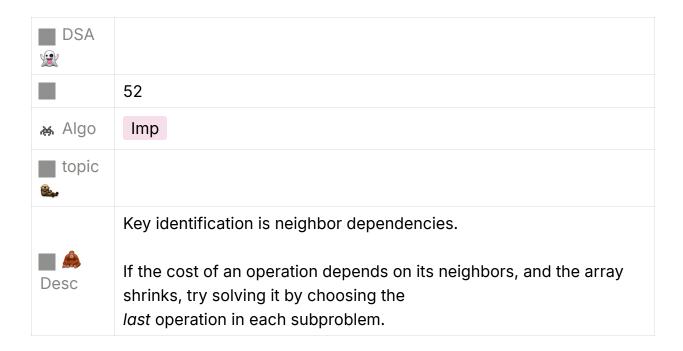


# 312. Burst Balloons



# A classic DP (Dynamic Programming) problem known as "Burst Balloons"

### **Let's explore your confusion:**

You asked:

Why not just remove a number (like 1 from [3,5,1,8]), and then the remaining subproblem [3,5,8] has no dependencies?

At first glance, it **seems** like removing any balloon independently shouldn't affect others — but **that's not the case**.

**6** Key Insight:

The formula for coins when bursting nums[i] is:

```
coins = nums[i-1] * nums[i] * nums[i+1]
```

So when you burst a balloon, the reward depends on its left and right neighbors — meaning the remaining balloons are not independent anymore because the neighbor relationship changes.

That's why we flip the problem:

Instead of asking "Which balloon to burst now?",

we ask: "Which balloon should be the last to burst in this subarray?"

# Why bursting the last balloon in a range creates independent subproblems

Let's say you have balloons between index left and right.

You decide that k will be the **last** balloon to burst in that range.

#### Then:

The coins gained by bursting k last will be:
 nums[left-1] \* nums[k] \* nums[right+1]

```
• Everything left of k (i.e., left to k-1) and right of k (i.e., k+1 to right) can be solved independently.
```

This ensures **no dependency leakage** because left-1 and right+1 are fixed during the subproblem.

That's why we pad the array with 1 on both ends  $\rightarrow$  [1,3,5,1,8,1].

#### So why not go forward?

If you remove a balloon in the middle early on, the neighbors change and thus **you** lose control over the surrounding context.

This violates the principle of **optimal substructure**, which is needed for DP.

#### Final Takeaway:

- Forward approach fails because neighbor context keeps shifting, breaking subproblem independence.
- Reverse (burst last) approach works because:
  - We fix the neighbors: <a href="left-1">left-1</a>, <a href="right+1">right+1</a>.
  - The two sides of the chosen last balloon can be solved **independently**.
  - This allows clean memoization.

# Why would removing a balloon and creating a copy not work?

# ✓ You're Right:

If modifying the array (removing elements) causes issues due to neighbor dependencies, then why not:

- · Copy the array at each recursive step,
- Burst a balloon,
- Pass the new array forward.

That way the original array stays intact.

#### But Here's the Catch:

Yes, this does work logically, but...

! It's incredibly inefficient.

Because every recursive step:

- Creates a new array (copy),
- Does O(n) work to remove a balloon,

And continues this for n balloons.

That makes the time complexity **O(n!)**, just like full brute-force.

So even with copying, you don't get dynamic programming benefits like memoization because:

- You're changing the structure (the array itself),
- So subproblems are hard to identify or reuse.

#### But When We Use "Burst Last" Approach:

Instead of copying arrays, we do this:

- Work on **fixed intervals** [left, right]
- For each interval, try all k in (left, right) as the last balloon
- Memoize dp[left][right] so we never recompute that interval

This makes time complexity:

```
O(n^3)
```

Which is **exponentially faster** than O(n!).

### Summary

Idea	Is it Correct?	Is it Efficient?
Copy array & simulate bursting	✓ Logically correct	X Very slow (O(n!))
DP with "burst last in interval"	✓ Correct	▼ Efficient (O(n³))

#### Visualization (Burst Last Approach):

Suppose nums = [3,1,5,8], padded: [1,3,1,5,8,1]

Let dp[i][i] = max coins you can get by bursting all balloons between i and i (exclusive)

We do:

This solves the problem bottom-up, without modifying arrays.

## Key Observations & Thought Process in an Interview

#### 1. Non-local dependencies (left/right neighbors involved)

You get coins based on **left and right neighbors**:

```
ini
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coins = nums[i - 1] * nums[i] * nums[i + 1]
```

#### Red Flag:

Bursting nums[i] affects the rewards of bursting other balloons due to changing neighbors.

#### Trigger:

This breaks greedy and forward DP. Time to fix the neighbors instead of the balloon!

#### 2. Order of operation affects rewards

Bursting the same balloon at different times gives different coins due to shifting neighbors.

Instead of thinking "Which balloon do I burst now?", flip to:

"Which balloon should be the last one in this subproblem?"

That fixes neighbors and enables **independent subproblems**.

#### 3. Subproblem Structure Exists → DP Time!

If you realize:

- The total problem can be broken into left/right subproblems.
- Subproblems overlap (same [left, right] range may occur again).

Boom: DP with memoization or bottom-up table.

#### 4. Optimal Substructure

The total coins for bursting [left+1 ... right-1] balloons is:

```
sql
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dp[left][right] = max over k in (left+1 to right-1) of:
dp[left][k] + dp[k][right] + nums[left] * nums[k] * nums[right]
```

This clean recurrence is a dead giveaway that **DP on intervals** is the best approach.

#### 5. Avoid modifying the input

If you find yourself **removing elements** or **copying arrays** during recursion:

1 That's a hint you might need to switch to a DP-based indexing approach.

# Interview Strategy Summary (Mental Checklist)

Question Characteristic	Think
Bursting/removal gives score based on neighbors	Fix neighbors → burst last
Changing the order gives different scores	DP, not greedy
Problem has overlapping subproblems	Memoization
Problem asks for <b>max</b> score / coins	Optimization + DP
Removing elements breaks independence	Use index ranges instead

Would you like a one-liner decision rule to help you in interviews? Here it is:

If the cost of an operation depends on its neighbors, and the array shrinks, try solving it by choosing the last operation in each subproblem.

This applies to problems like:

- Burst Balloons
- Matrix Chain Multiplication
- Optimal BST