Gauss Theorem

Part I : Find Missing Number in array

> **Gauss theorem** : Sum of n natural number

$$\text{Theorem} = \frac{n * (n+1)}{2}$$

For square number:

$$\frac{n * (n+1) * (2n+1)}{6}$$

Eg: Missing number in arr = [0, 2, 3]

$n = 3$

$Ts = 0 + 1 + 2 + 3 = 6$

$Cs = 0 + 2 + 3 = 5$

$6 - 5 = (1)$

↳ Missing Number

Gauss Theorem - II

Part 2 : Missing & Duplicate Number

Eg: [4, 3, 6, 2, 1, 1]

$n = 6$

Step 1: $\frac{n * (n+1)}{2}$

$Ts = 1 + 2 + 3 + 4 + 5 + 6 = 21$

$Cs = 4 + 3 + 6 + 2 + 1 + 1 = 17$

$Ts - Cs = 21 - 17 = 4$

$x - y = 4$ —— (1)

Step 2: $\frac{n * (n+1) * (2n+1)}{6}$

$Ts = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 = 91$

$Cs = 4^2 + 3^2 + 6^2 + 2^2 + 1^2 + 1^2 = 67$

$Ts - Cs = 24$

$x^2 - y^2 = 24$

$x^2 - y^2 = (x-y)(x+y)$

$(x-y)(x+y) = 24$ —— (2)

Substituting (1) in (2)

$$x - y = 4 \quad \text{—(1)}$$
$$(x-y)(x+y) = 24 \quad \text{—(2)}$$

This can be written as

$$(4)(x+y) = 24$$

$$x+y = \frac{24}{4} = 6$$

$$\therefore x+y = 6 \quad \text{— (3)}$$

Now, we know, from (1) & (3)

$$x - y = 4 \quad \text{— (1)}$$
$$x+y = 6 \quad \text{— (3)}$$

We get

$$\begin{array}{r} x - y = 4 \\ x + y = 6 \\ \hline 2x = 10 \end{array}$$

$$x = 5 \quad \text{— (4)}$$

substituting x in (3)

$$x+y = 6$$
$$5+y = 6$$
$$y = 1 \quad \text{— (6)}$$

x = missing number

y = duplicate number

$\therefore$ $x = 5$ = missing number

$y = 1$ = duplicate number