Sample Variance

Sample variance is used to calculate the variability in a given sample. A sample is a set of observations that are pulled from a population and can completely represent it. The sample variance is measured with respect to the mean of the data set. It is also known as the estimated variance.

As data can be of two types, grouped and ungrouped, hence, there are two formulas that are available to calculate the sample variance. Furthermore, the square root of the sample variance results in the sample standard deviation. In this article, we will elaborate on sample variance, its formulas, and various examples.

What is Sample Variance?

Sample variance is used to measure the spread of the data points in a given data set around the mean. All observations of a group are known as the population. When the number of observations start increasing it becomes difficult to calculate the variance of the population. In such a situation, a certain number of observations are picked out that can be used to describe the entire group. This specific set of observations form a sample and the variance so calculated is the sample variance.

Sample Variance Definition

Sample variance can be defined as the expectation of the squared difference of data points from the mean of the data set. It is an absolute measure of dispersion and is used to check the deviation of data points with respect to the data's average.

Sample Variance Example

Suppose a data set is given as 3, 21, 98, 17, and 9. The mean (29.6) of the data set is determined. The mean is subtracted from each data point and the summation of the square of the resulting values is taken. This gives 6043.2. To get the sample variance, this number is divided by one less than the total number of observations. Thus, the sample variance is 1510.8.

Sample Variance Formula

Formulas for Sample Variance cuemath THE MATH EXPERT

	Sample Variance
Ungrouped Data	$\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n - 1}$
Grouped Data	$\frac{\sum_{i=1}^{n} f(m_i - \bar{x})^2}{N - 1}$

There can be two types of data - grouped and ungrouped. When data is in a raw and unorganized form it is known as ungrouped data. When this data is sorted into groups, categories, or tables it is known as grouped data. The sample variance formulas for both types of data are specified below:

- **Ungrouped Data:** $s^2 = \frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}$
- **Grouped data:** $s^2 = \frac{\sum_{i=1}^{n} f(m_i-\bar{x})^2}{N-1}$

n = total number of observations.

$N = \sum_{i}^{n} f_i$

$\sum_{i=1}^{n} f_i$

f = the frequency of occurrence of an observation for grouped data

m_i = Mid-point of the i^{th} interval

Mean for grouped data, $\bar{x} = \frac{\sum_{i=1}^{n} m_i f_i}{\sum_{i=1}^{n} f_i}$

Mean for ungrouped data, $\mu = \frac{\sum_{i=1}^{n} x_i}{n}$

The sample variance, on average, is equal to the population variance.

Let us understand the sample variance formula with the help of an example.

Example: There are 45 students in a class. 5 students were randomly selected from this class and their heights (in cm) were recorded as follows:

131	148	139	142	152

Sample size (n) = 5

Sample Mean = (131 + 148 + 139 + 142 +152) / 5 = 712 / 5 = 142.4 cm

Using the sample variance formula,

Sample Variance = $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1} = \frac{\sum_{i=1}^{5}(x_i-142.4)^2}{5-1}$

$= [(131-142.4)^2+(148-142.4)^2+(139-142.4)^2+(142-142.4)^2+$

$(152-142.4)^2] / 4 = 66.3\ cm^2$

Answer: Sample Mean = 142.4 cm, Sample Variance = 66.3 cm^2.

How to Calculate Sample Variance?

Depending upon the type of data available, there can be different steps that can be used to calculate the sample variance. However, the general algorithm that should be followed is given below:

Suppose the data set is given as {5, 6, 1}

- **Step 1:** Calculate the mean of the data set. The mean can be defined as the sum of all observations divided by the total number of observations. Add all data values and divide by the sample size n. Thus, (5 + 6 + 1) / 3 = 4
- **Step 2:** Subtract the mean from each data point in the data set. This gives (5 - 4), (6 - 4), (1 - 4).
- **Step 3:** Take the square of the values obtained in step 2; $(5 - 4)^2 = 1$, $(6 - 4)^2 = 4$, $(1 - 4)^2 = 9$
- **Step 4:** Add all the squared differences from step 3; 1 + 4 + 9 = 14
- **Step 5:** To get the sample variance, divide this value by one less than the total number of observations; 14 / (3 - 1) = 7. Thus, for the given example the sample variance is 7.

Sample Variance vs Population Variance

Both sample variance and population variance are used to measure how far a data point is from the mean of the data set. However, the value of the sample variance is higher than the population variance. The table given below outlines the difference between sample variance and population variance.

Sample Variance	Population Variance
When the variance is calculated using the sample data it gives the sample variance.	When the variance is calculated using the entire data, also known as the population, it gives the population variance.
The formula for sample variance is given as $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}$	The formula for population variance is equal to $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n}$

Related Articles:

- Standard Deviation
- Summary Statistics
- Variance Calculator

Important Notes on Sample Variance

- The variance that is computed using the sample data is known as the sample variance.
- Sample variance can be defined as the average of the squared differences from the mean.
- There are two formulas to calculate the sample variance: $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}$ (ungrouped data) and $\frac{\sum_{i=1}^{n} f(m_i-\bar{x})^2}{n-1}$ (grouped data)

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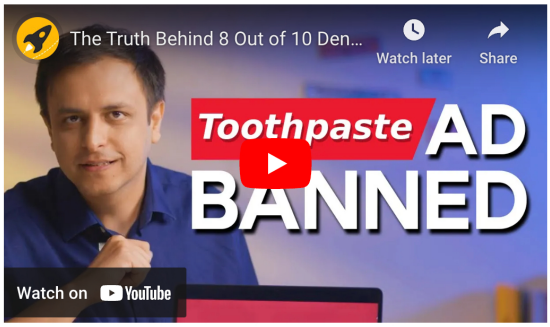
Sample Variance Worksheet
Worksheet On Mean, Median, Mode



Examples on Sample Variance

Example 1: Given the data set {4.5, 9.8, 2.3, 5.3, 8.9}, find the sample variance.

Solution: n = 5, $\bar{x}$ = (4.5 + 9.8 + 2.3 + 5.3 + 8.9) / 5 = 6.16

Sample Variance = $\frac{\sum_{i=1}^{n}(x_i-\mu)^2}{n-1}$

= $\frac{(4.5-6.16)^2+(9.8-6.16)^2+(2.3-6.16)^2+(5.3-6.16)^2+(8.9-6.16)^2}{5-1}$ = 9.788

Answer: Sample variance = 9.788

Example 2: Find the sample variance of the following data:

x_i	f
10 - 20	7
20 - 30	2
30 - 40	10
40 - 50	1

Solution:

x_i	f	m_i	$(m_i - \bar{x})^2$	$f(m_i - \bar{x})^2$
10 - 20	7	15	156.25	1093.75
20 - 30	2	25	6.25	12.5
30 - 40	10	35	56.25	562.5
40 - 50	1	45	306.25	306.25

	$\sum_{i=1}^{n} f_i$ = 20			$\sum_{i=1}^{n} f(m_i - \bar{x})^2$ = 1975

$\bar{x} = \frac{\sum_{i=1}^{n} m_i f_i}{\sum_{i=1}^{n} f_i} = 27.5$

Sample Variance = $\frac{\sum_{i=1}^{n} f(m_i - \bar{x})^2}{N-1}$ = 1975 / (20 - 1) = 103.94

Answer: Sample variance = 103.94

Example 3: There were 105 oak trees in a forest. 6 were randomly selected and their heights were recorded in meters. Find the variance and standard deviation in the heights.

Heights (in m) = {43, 65, 52, 70, 48, 57}

Solution: As the variance of a sample needs to be calculated thus, the formula for sample variance is used.

n = 6, Mean = (43 + 65 + 52 + 70 + 48 + 57) / 6 = 55.833 m.

Sample Variance = $\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}$ = 526.833 / (6 - 1) = 105.367 m^2

Sample Standard Deviation = $\sqrt{105.367}$ = 10.26 m

Answer: Sample Variance = 105.367 m^2, Sample Standard Deviation = 10.26 m

Example 4: If all values in a data set are the same then the sample variance is equal to?

Solution:

Variance is the degree of spread or change in the given data points. The variance is calculated in relation to the mean of the data. The more the spread of the data, the more will be the variance in relation to the mean.

The formula for variance :

$s^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}$,

s^2 = sample variance

x_i = Each data value

μ = mean of the data set

n = total number of values in the data set.

Special case: When all the data set points are the same

In this case, the mean of the data set i.e. μ is the same as each data value i.e. x_i

Thus, $x_i - \mu = 0$

Hence, variance becomes 0.

Practice Questions on Sample Variance

Q.1

What is the formula for ungrouped sample variance?

- $\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1}$
- $\frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n}$
- $\frac{\sum_{i=1}^{n} (x_i - \mu)}{n-1}$
- $\frac{\sum_{i=1}^{n} (x_i - n)^2}{-\mu}$

Check Answer

Q.2

State if true or false: Sample variance is a measure of central tendency.

- True

False

Check Answer

FAQs on Sample Variance

What is the Sample Variance in Statistics?

What is the Symbol of Sample Variance?

What is the Formula for Sample Variance?

How to Find the Sample Variance?

Can Sample Variance be Negative?

Is Sample Variance the Same as Standard Deviation?

What is the Difference Between Sample Variance and Population Variance?

What do Small and Big Variance Mean in Sample Variance Formula?