DAMT

Final Exam

Q1) Coin flipped 100 times
Heads = 58
tail = 42

Let the probability of getting head be p.

$P = 1/3,\ 1/2,\ 2/3$

Using Maximum Likelihood Estimation.

$$P[H=58 \mid P=1/3] = \binom{100}{58}\left(\frac{1}{3}\right)^{58}\left(1-\frac{1}{3}\right)^{42}$$

$$= \boxed{2.4115 \times 10^{-7}} \simeq \boxed{0.0000}$$

$$P[H=58 \mid P=1/2] = \binom{100}{58}\left(\frac{1}{2}\right)^{58}\left(1-\frac{1}{2}\right)^{42}$$

$$= \boxed{0.02229}$$

$$P[H=58 \mid P=2/3] = \binom{100}{58}\left(2/3\right)^{58}\left(1-2/3\right)^{42}$$

$$= \boxed{0.015804}$$

$\therefore$ Likelihood is maximized for $P = 1/2$

Hence $p = 1/2$ is the maximum Likelihood estimation.

2)

$\mu = 3.7$

$SD = 1.5$

$$z = \frac{x - \mu}{\sigma}$$

$$x = \frac{350}{100} = 3.5$$

$$= \frac{3.5 - 3.7}{1.5} = -0.133$$

from SD, we find 0.4483

$$P(z > -0.133) = 1 - (z < 0.13)$$
$$= 1 - (0.4483)$$
$$= 1 - 0.4483$$
$$= 0.5517$$

The probability that 100 person Get ticket is 0.5517

(3) $Var(x_i) = 9$
mean $= 25.5$

find approximate 99% confidence interval for $\theta = Ex_i$

$1-\alpha = 0.99$

$\alpha = 0.01$ or 1%.

$\sigma^2 = 9$, so $\sigma = 3$

$\bar{x} = 25.5$

Formula $= P\left(\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \leq \theta \leq \bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}\right)$

$= 25.5 - z_{\frac{0.01}{2}} \cdot \frac{3}{\sqrt{100}}$

$z_{0.05} = \phi^{-1}(1-0.005)$

$\phi^{-1}(0.995)$

$z_{0.005} = 2.575$

$\bar{x} - z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 25.5 - 2.575 \times \frac{3}{10}$

$= 24.727$

$\bar{x} + z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 25.5 + 2.575 \times \frac{3}{10}$

$= 26.272$

$\therefore$ we can say that $[24.727, 26.272]$ is a $(1-\alpha)$ 99% confidence interval for θ.

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4) <u>Step 1</u> : State Null Hypothesis

Mean is 450

$H_0 : \mu = 450$

Alternate Hypothesis claim that student's test score are greater than other population.

$H_1 : \mu > 450$

$\therefore$ This is One tailed test.

Sample mean = 457.15

<u>Step 2</u> : Consider 5% as significance level

$$Z_{0.05} = \phi^{-1}(1-0.05)$$
$$= \phi^{-1}(0.95) = 1.645$$

<u>Step 3</u> : probability of random chance

$$Z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{457.15 - 450}{\frac{90}{\sqrt{20}}}$$
$$= 0.35528$$

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Step 4 : Decision

Since $Z <$ threshold value
$(0.35528 < 1.645)$

$\therefore$ The Null Hypothesis is accepted.

Hence, Mean = 450
The students score is not greater.

(5) Let Sample - 1 = high protein
Sample - 2 = Low protein

This is two sample t-test.

H_0 = High protein diet did not have any positive effect on weight gain

H_1 = High protein diet had positive effect on weight gain

T - test for independent groups.

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{Var_1}{n_1} + \frac{Var_2}{n_2}}}$$

DOF (Degree of freedom) $= n_1 + n_2 - 2$
$= 14 + 8 - 2 = 20$

$\bar{x}_1 = \frac{1707}{14} = 121.928$

$\bar{x}_2 = \frac{883}{8} = 110.375$

$\sigma_1^2 = \frac{\Sigma(x_i - \bar{x}_1)^2}{n_1}$

$= \frac{(150 - 121.928)^2 + \cdots (112 - 121.928)^2}{14}$

$\sigma_1^2 = 680.4948$

$\boxed{\sigma = 26.08629}$

$\sigma_2^2 = \frac{\Sigma(x_i - \bar{x}_2)^2}{n_2}$

$(112 - 110.375)^2 + \cdots (134 - 110.375)^2$

$\sigma_2^2 = 356.7343$

$\boxed{\sigma_2 = 18.887413}$

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{121.92857 - 110.375}{\sqrt{\left(\frac{26.08629}{14}\right)^2 + \left(\frac{18.887}{8}\right)}}$

$t = \frac{11.55357}{9.65394} = \boxed{1.196772}$

t_{20} with significance of 5% = 1.725

∴ $1.196 < 1.725$

∴ We accept Null Hypothesis.

∴ We can say that high protein diet has no effect on weight gain

(6) a. $P(e, !b, r, !a, c)$

$P(e) = 0.01$

$P(!b) = 0.8$

$P(r|e) = 0.6$

$P(!a|e, !b) = \cancel{0.01}\ 0.1$

$P(c|!a) = 0.01$

$P(e) * P(!b) * P(r|e) * P(!a|e, !b) * P(c|!a)$

$= 0.01 \times 0.8 \times 0.6 \times \cancel{0.01}\ 0.1 \times 0.01$

$= \cancel{4.8 \times 10^{-7}}\ 4.8 \times 10^{-6}$

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6b) $P(e \mid !r, c)$

(A, B)

<u>Case 1</u> → $P(e, !r, c, a, B)$

$= P(e) * P(!r \mid e) * P(c \mid a)$

$* P(a/e, b) * P(b)$

$= 0.01 \times 0.4 \times 0.7 \times 0.99 \times 0.2$

$= \underline{5.544 \times 10^{-4}}$

<u>Case 2</u> → $!A, B$

$P(e, !r, c, !a, B)$

$P(e) * P(!r/e) * P(c \mid !a) * P(!a/e, b) * P(b)$

$= 0.01 * 0.4 * 0.01 * 0.01 * 0.2$

$= \underline{8 \times 10^{-8}}$

<u>Case 3</u> → $A, !B$

$P(e, !r, c, a, !b)$

$= P(e) * P(!r \mid e) * P(c/a) * P(a/e, !b) * P(!b)$

$= 0.01 * 0.4 * 0.7 * 0.9 * 0.8$

$= 2.016 * 10^{-3}$

<u>Case 4</u> $!A, !B$

$P(e, !r, c, !A, !B)$

$= P(e) * P(!r \mid e) * P(c \mid !a) * P(!a/e, !b) * P(!b)$

$= 0.01 * 0.4 * 0.01 * 0.$[illegible] $\times 0.8$

$= 3.2 \times 10^{-6}$

adding all the four cases

$$5.544 \times 10^{-4} + 8 \times 10^{-8} + 2.016 \times 10^{-3} + 3.2 \times 10^{-6}$$

$$= \underline{2.57368 \times 10^{-3}}$$

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