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Exam - 1

1 a)

$P(C) = 0.3\% = 0.003$

$P(S|C) = 0.85$

$P(S|\sim C) = 0.25$

$P(\sim S|\sim C) = 1 - 0.25 = 0.75$

$P(\sim S|C) = 1 - 0.85 = 0.15$

a) $P(C|\sim S) = \frac{P(\sim S|C) \cdot P(C)}{P(\sim S)}$

$= \frac{P(\sim S|C) \cdot P(C)}{P(\sim S|C) \cdot P(C) + P(\sim S|\sim C) \cdot P(\sim C)}$

$= \frac{0.15 \times 0.003}{0.15 \times 0.003 + 0.75 \times 0.997}$

$= \frac{0.00045}{0.00045 + 0.74475}$

$= \frac{0.00045}{0.7452} = 0.0006039$

b) $P(C|S) = \frac{P(S|C) \cdot P(C)}{P(S)}$

$= \frac{P(S|C) \cdot P(C)}{P(S|C) \cdot P(C) + P(S|\sim C) * P(\sim C)}$

$= \frac{0.85 \times 0.003}{(0.85 \times 0.003) + (0.25)(0.997)}$

$$= \frac{0.00255}{(0.00255) + (0.24925)}$$

$$= \frac{0.00255}{0.2518}$$

$$= 0.010127$$

c) Ratio of $\frac{P(C/S)}{P(C/\sim S)}$

$$= \frac{0.010127}{0.0006039}$$

$$= 16.769$$

2 a) ~~Less than 5 : $\frac{4}{10} \times \frac{4}{10} \times \frac{4}{10} \times \frac{6}{10} = 0.064 \times 0.6$~~

~~$= 0.0384$~~

2 b) pulling 3 marble gretar than equal 8

$$\frac{3}{10} \times \frac{2}{9} \times \frac{1}{8} = 0.00833$$

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2a) $4C_3 \times \left(\frac{4}{10}\right)^3 \times \frac{6}{10}$

$= 4 \times 0.064 \times 0.6$

$= 0.1536$

3) a) Hypergeometric Distribution

b)

	Choose	Not chose	total
Female	4	0	4
Male	1	5	6
Total	5	5	10

$$P(X=K) = \frac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$$

$$= \frac{4C_4 \times 6C_1}{10C_5} = \frac{1 \times 6}{252}$$

$$= \frac{1}{42} = 0.023809$$

c)

	Choose	Notchose	total
Female	2	2	4
Male	3	3	6
Total	5	5	10

$$= \frac{4C_2 \times 6C_3}{10C_5} = \frac{6 \times 20}{252} = \frac{120}{252}$$

$$= \frac{10}{21} = 0.476190$$

4) (a) $P(X=0, Y<4) = ?$

$= P_{X,Y}(0,2) + P_{(X,Y)}(0,3)$

$= 1/6 + 1/9$

$= 5/18$

(b) $P_X(x) = \sum_{y_i \in R_y} P_{XY}(x_i, y_j)$ $x \in R_x$

$P_X(y) = \sum_{x_i \in R_x} P(x_i, y_j)$ for any $y \in R_y$

$\therefore P_X(x) = \begin{cases} 1/3 & x=0 \\ 5/18 & x=1 \\ 7/18 & x=2 \\ 0 & \text{Otherwise} \end{cases}$ Marginal PMF of X

$P_X(y) = \begin{cases} 1/3 & y=2 \\ 7/18 & y=3 \\ 5/18 & y=4 \\ 0 & \text{otherwise} \end{cases}$ Marginal PMF of Y

(c) $P(Y=3 \mid X=1) = \dfrac{P(X=1, Y=3)}{P(X=1)}$

$= \dfrac{P_{XY}(1,3)}{P_X} = \dfrac{1/9}{5/18} = \frac{1}{9} \times \frac{18}{5} = \frac{2}{5}$

$= \underline{0.4}$

5) $f_{xy} = \begin{cases} 3x + \frac{2y}{3} & 0 \le x, y \le 1 \end{cases}$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} 3xy^2 f_{xy}(x,y)\,dx\,dy$$

$$= \int_0^1\int_0^1 3xy^2\left(3x+\frac{2y}{3}\right) dx\,dy$$

$$= \int_0^1\int_0^1 (9x^2y^2 + 2xy^3)\,dx\,dy$$

$$= \int_0^1 9^3x^2\frac{y^3}{3} + 2x\frac{y^2}{4}\Big)_0^1 dx$$

$$= \int_0^1 3x^2(1-0) + x/_2(1-0)\;dx$$

$$= \int_0^1 3x^2 + x/_2\;dx$$

$$= \frac{3x^3}{3} + \frac{x^2}{4}\Big]_0^1$$

$$= 1(1-0) + \frac{1}{4}(1-0)$$

$$= 1 + 1/4 = \frac{5}{4}$$

6a) $1 = \int_0^\infty \int_0^\infty ((cx + 4y^2)\ dx\,dy$

$1 = \int_0^1 \int_0^1 ((cx + 4y^2)\ dx\,dy$

$1 = \int_0^1 cx + y + \frac{4y^3}{3} \Big)_0^1 dx$

$1 = \int_0^1 \Big[cx(1-0) + \frac{4}{3}(1-0)\ dx$

$1 = \int_0^1 cx + \frac{4}{3}\ dx$

$1 = \frac{cx^2}{2} + \frac{4}{3}x \Big)_0^1$

$1 = \frac{cx^2}{2} + \frac{4}{3}x \Big)_0^1 = \frac{c}{2}(1-0) + \frac{4}{3}(1-0)$

$\frac{c}{2} = 1 - \frac{4}{3}$

$c = -2/3$

b) $f_X(x) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dy \quad \forall x$

$$f_X(x) = \int_0^1 \left(-\frac{2}{3}x + 4y^2\right) dy$$

$$f_X(x) = -2/3\,xy + 4y^3/3 \Big|_0^1$$

$$f_X(x) = -\frac{2}{3}x(1-0) + \frac{4}{3}(1-0)$$

$$f_X(x) = -\frac{2}{3}x + \frac{4}{3}$$

$$f_X(x) = \begin{cases} -2/3\,x + 4/3 & 0 \le x \le 1 \\ 0 & \text{Otherwise} \end{cases}$$