5301 - HW4

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HomeWork - 4
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1) $X = 1 \rightarrow 0.13 + 0.15 + 0.05 = 0.33$

$X = 2 \rightarrow 0.15 + 0.2 + 0.12 = 0.47$

$X = 3 \rightarrow 0.07 + 0.1 + 0.03 = 0.2$

$Y = 1 \rightarrow 0.13 + 0.15 + 0.07 = 0.35$

$Y = 2 \rightarrow 0.15 + 0.2 + 0.1 = 0.45$

$Y = 3 \rightarrow 0.05 + 0.12 + 0.03 = 0.2$

a) $H(X) = -[0.33\log(0.33) + 0.47\log(0.47) + 0.2\log(0.2)]$

$= 0.52782 + 0.51195 + 0.46438$

$= 1.50415$

b) $H(Y) = -[0.35\log(0.35) + 0.45\log(0.45) + 0.2\log(0.2)]$

$= [0.53010 + 0.51840 + 0.46438]$

$= 1.51288$

c) $D(X||Y) = \sum_x P(X)\log_b\left[\frac{P(X)}{q(X)}\right]$

$= 0.33\log_2\left[\frac{0.33}{0.35}\right] + 0.47\log_2\left[\frac{0.47}{0.45}\right] + 0.2\log\left[\frac{0.2}{0.2}\right]$

$= -0.02801 + 0.02948 + 0$

$= 1.47 \times 10^{-3}$

$= 0.00147$

d) $D(Y||X)$

$$0.35\log_2\left[\frac{0.35}{0.33}\right] + 0.45\log_2\left[\frac{0.45}{0.47}\right] + 0.2\log\left[\frac{0.2}{0.2}\right]$$

$$= 0.02971 - 0.02823$$

$$= 1.48 \times 10^{-3}$$

$$= 0.00148$$

e) $H(X,Y) = -\sum_x \sum_y P(x,y)\log_2 P(x,y)$

$$-[0.13\log_2(0.13) + 2(0.15\log_2(0.15)) + 0.05\log(0.05) + 0.2\log_2(0.2) + 0.12\log_2(0.12) + 0.07\log_2(0.07) + 0.1\log_2(0.1) + 0.03\log_2(0.03]$$

$$= 0.38264 + 0.82108 + 0.21609 + 0.46438 + 0.36706 + 0.26855 + 0.33219 + 0.15176$$

$$= 3.00375 \text{ bits}$$

f) $H(Y|X) = H(X,Y) - H(X)$

$$= 3.00375 - 1.50415$$

$$= 1.4996$$

$$H(Y) - H(Y|X) = 1.51288 - 1.4996$$

$$= 0.01328$$

9) $I(X,Y) = H(Y) - H(Y|X)$

$= 1.51288 - 1.4996$

$= 0.01328$

2) Given Entropy is 4.3 bits

1 Hartelys = $\log_2(10)$ bits = 3.32193

Convert bits to hartley

$1 \text{ bit} = \frac{1}{3.32193}$ hartley

so $4.3 \text{ bit} = \frac{4.3}{3.32193} = \boxed{1.29443 \text{ hartley}}$

$1 \text{ nat·s} = \frac{1 \text{ hartley}}{2.303}$

so $1 \text{ hartley} = 2.303 \text{ nat-s}$

$1.29443 \text{ hartley} = 2.303 \times 1.29443$

$= \boxed{2.98107 \text{ nat-s}}$

(3) As it is a standard die probability of getting 2, 3, 1, 6, 4 is 1/6.

$I = \log(1/p)$

$$I_A + I_B = \log(1/p_A) + \log(1/p_B)$$

$$= \log(1/1/6) + \log(1/1/6) + \log(1/1/6) + \log(1/1/6) + \log(1/1/6)$$

$$= \log(6) + \log(6) + \log(6) + \log(6) + \log(66)$$

$$= \log(6^5) = 5 \log 6$$

$$= 5 \times 2.58496$$

$$= \boxed{12.9248 \text{ bits}}$$

4) Die 1

probability of getting 1, 2, 3, 4, 5, 6 = 1/6

Entropy = $-\Sigma p(x) \log_b p(x)$

$$= -6 \cdot 1/6 \log_2 (6 1/6)$$

$$= -6 [1/6 \log (1/6)$$

$$= \boxed{2.58496 \text{ bits}}$$

Die 2

$P(1) = 0.35$

$P(6) = 0.25$

$P(2,3,4,5) = [1 - [0.35 + 0.25]] / 4$

$= 0.1$

$-[0.35\log(0.35) + 4(0.1\log(0.1)) + 0.25\log(0.25$

$: -[-0.53010 - 1.32877 - 0.5]$

$= 2.35887$ bits