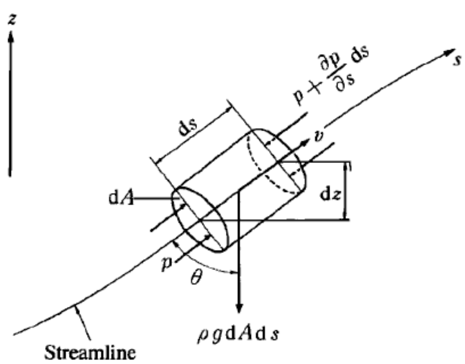
Fluid Dynamics

Introduction:

Fluid dynamics is that branch of science which deals with the study of the motion of fluids, the forces that are responsible for this motion and the interaction of fluid with solids. Fluid dynamics stands central to many branches of science and engineering and touches many aspects of our daily life. Fluid dynamics plays an important role in defense, transportation, manufacturing, environment, energy, industry, medicine and biology, etc. Starting from prediction of the aerodynamic behavior of moving vehicles, to the movement of physiological fluids in human body, the weather prediction, cooling of electronic components, performance of micro–fluidic devices, all demand applications of principles of fluid mechanics.

Euler's equation of motion:

General flows are three dimensional, but many of them may be studied as if they are one dimensional.
For example, whenever a flow in a tube is considered, if it is studied in terms of mean velocity, it is a one-dimensional flow which is studied very simply. Let us cylindrical element of fluid having the cross-sectional area *dA* and length *ds* which lay along the streamline.



Let *p* be the pressure acting on the lower face, and pressure *p*+ d*p* acts on the upper face a distance *ds* away. The gravitational force acting on this element is its weight, *pgdAds*. Applying Newton's second law of motion (1) to this element, the resultant force acting on it, and producing acceleration along the streamline.

The following equation is obtained:

$$\rho\ dA\ ds\frac{\mathrm{dv}}{\mathrm{dt}} = -dA\ \frac{\partial p}{\partial s}\ ds = \rho\ g\ dA\ ds\ \cos(\Theta)$$

Recalling that

$$\frac{\mathrm{dv}}{\mathrm{dt}} = \frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial s}, \qquad \cos(\Theta) = \frac{\partial z}{\partial s}$$

$$\frac{\partial v}{\partial t} + v\,\frac{\partial v}{\partial s} = -\frac{1}{\rho}\frac{\partial p}{\partial s} - g\,\frac{\partial z}{\partial s}$$

The above equation is called **Euler's equation of motion**

The equation can be rewrite as $$\frac{\partial}{\partial s}\left(\frac{1}{2}v^2 + \frac{p}{\rho} + gz\right) = 0$$

Bernoulli’s equation:

The sum of the kinetic, potential, and flow energies of a fluid particle is constant along a streamline during steady flow when the compressibility and frictional effects are negligible

The Bernoulli’s equation can be obtained by integrating Euler’s equation of motion

$$\frac{v^2}{2} + \frac{p}{\rho} + gz = const.$$

We recognize that v^2 as kinetic energy, gz as potential energy, and p/ρ as pressure energy, all per unit mass. Therefore the Bernoulli equation can be viewed as ”**conservation of mechanical energy principle**”.

It is often convenient to represent the level of mechanical energy graphically using *heights* to facilitate visualization of various terms of the Bernoulli equation. This is done by dividing each term of Bernoulli equation by *g* to give

$$\frac{v^2}{2g} + \frac{p}{\rho g} + z = H = const.$$

Each term in this equation has the dimension of length and represent some kind of ”*head*” of a flowing fluid as follows:

H is **total head** of flow

• $p/\rho g$ is **pressure head**; it represents the height of a fluid column that produces the static pressure *p*

• $v^2/2g$ is **velocity head**; it represents the elevation needed for the fluid to reach the velocity *v* during frictionless free fall.

• *z* is the **elevation head**; it represents the potential energy of fluid.

For the real fluids, the Bernoulli’s equation can be written as

$$\mathbf{p1/\rho g + v_1^2/2g + z_1 = p_2/\rho g + v_2^2/2g + z_2 + h_L}$$

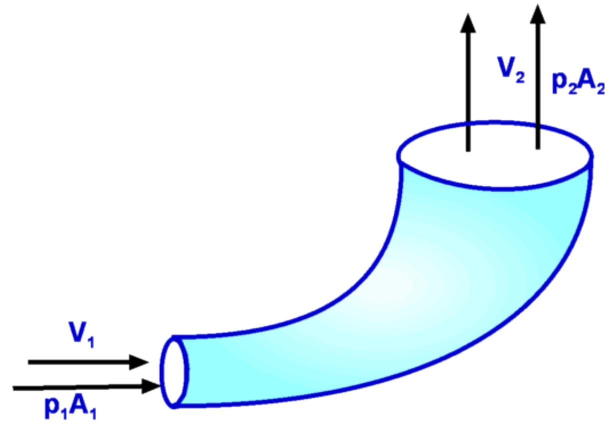
Where the losses are considered.

Forces exerted by flowing fluid on pipe bend

Consider a flow through a pipe bend as shown. The flow enters the bend with a speed V_1 and leaves it a speed V_2, the corresponding areas of cross section being A_1 and A_2 respectively. The velocities have components u and v in x and y directions. As the flow negotiates the bend it exerts a force upon it. This force is readily calculated by the momentum theorem.

The continuity equation yields

$$\rho \, V_1 \, A_1 \; = \; \rho \, V_2 \, A_2 \; = \; \dot{m}$$



Carrying out a force balance in x-direction, we have

$$p_1 \, A_1 \; - \; p_2 \, A_2 \, \cos\theta \; + \; F_x$$

$$= \; \rho V_2 \, A_2 \, V_2 \, \cos\theta \; - \; \rho \, V_1 \, A_1 \, V_1$$

$$= \dot{m} \, V_2 \, \cos\theta \; - \; \dot{m} \, V_1 \; = \; \dot{m} \, (V_2 \, \cos\theta \; - V_1)$$

In the y-direction we have,

$$0 \; - \; p_2 \, A_2 \, \sin\theta \; + \; F_y \; = \; \rho V_2 \, A_2 \, V_2 \, \sin\theta \; - \; 0$$

Thus the force components acting on the bend are

$$F_x \; = \; p_2 \, A_2 \, \cos\theta \; - \; p_1 \, A_1 \; - \; \dot{m} \, (V_2 \, \cos\theta \; - \; V_1)$$

$$F_y \; = -p_2 \, A_2 \, \sin\theta \; + \; \dot{m} \, V_2 \, \sin\theta$$

Moment of momentum equation:

Moment of momentum equation is derived from moment of momentum principle which states as follows:

"The resulting torque acting on a rotating fluid is equal to the rate of change of moment of momentum".

When the moment of momentum of flow leaving a control volume is different from that entering it, the result is a torque acting over the control volume.

Let,

Q = steady rate of flow of fluid

ρ = Density of fluid

V_1 = velocity of fluid at section 1

r_1 = radius of curvature at section 1

V_2 and r_2 = velocity and radius of curvature at section 2

Momentum of fluid at section 1= Mass X velocity = $\rho Q \times V_1$

Moment of momentum per second of fluid at section 1 = $\rho Q \times V_1 \times r_1$

Similarly, moment of momentum per second of fluid at section 2 = $\rho Q \times V_2 \times r_2$

Rate of change of moment of momentum = $\rho Q \times V_2 \times r_2 - \rho Q \times V_1 \times r_1 = \rho Q (V_2 \times r_2 - V_2 \times r_2)$

According to the moment of momentum principle,

Resulting torque = Rate of change of moment of momentum

$$T = \rho Q (V_2 \times r_2 - V_2 \times r_2)$$

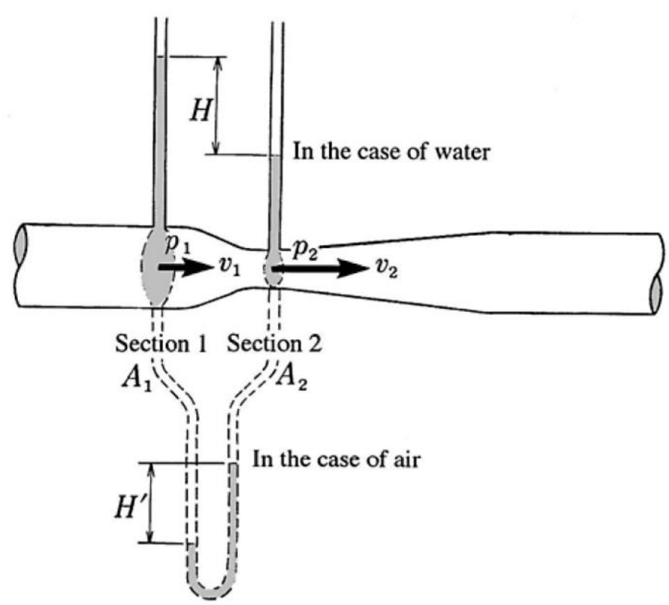
This equation is used:

(i) To find torque exerted by water on sprinkler, and
(ii) To analyse flow problems in turbines and centrifugal pumps.

Fluid flow measurements

1. Venturimeter

It is a device used for measuring the rate of flow of a fluid flowing through a pipe. It consists of three parts:
• A short converging part
• Throat
• Diverging part



Let d1 = diameter at the inlet (section 1)
p1 = pressure at section 1
v1 = velocity at section 1
A1= area at section1
d2, p2, v2, A2 are the corresponding values at the throat (section 2)
Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{w_1^{\ 2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{w_2^{\ 2}}{2g}+z_2.$$

As pipe is horizontal $z_1=z_2$

$$\Rightarrow \frac{p_1-p_2}{\rho g}=\frac{v_2^{\ 2}-v_1^{\ 2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{\ 2}-v_1^{\ 2}}{2g}$$

Where $h\equiv\frac{p_1-p_2}{\rho g},$ difference of pressure heads at sections 1 and 2.

From the continuity equation at sections 1 and 2, we obtain

$$A_1v_1=A_2v_2\Rightarrow v_1=\frac{A_2v_2}{A_1}$$

Hence $h=\frac{v_2^{\ 2}}{2g}\left[\frac{A_1^2-A_2^2}{A_1^2}\right]$

$$\Rightarrow v_2=\frac{A_1}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

Discharge $Q=A_1v_1=A_2h$

$$\Rightarrow Q=\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

Actual discharge will be less than theoretical discharge.

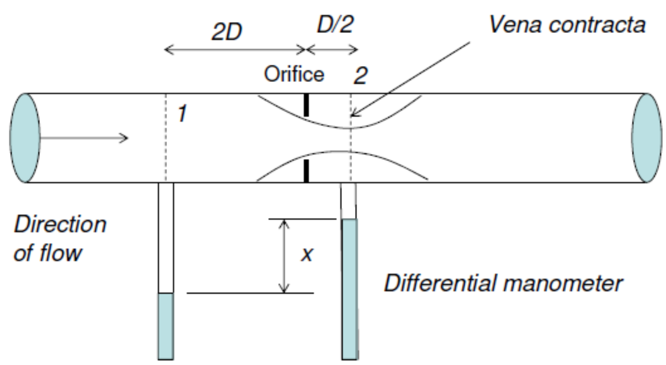
$$Q_{actual}=C_d\,\frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}\sqrt{2gh}$$

C_d is the coefficient of venturimeter and its value is always less than 1.

2. Orifice meter

It is a device used for measuring the rate of flow of a fluid flowing through a pipe.
It is a cheaper device as compared to venturimeter. This also works on the same principle as that of venturimeter.
It consists of flat circular plate which has a circular hole, in concentric with the pipe. This is called orifice.
The diameter of orifice is generally 0.5 times the diameter of the pipe (D), although it may vary from 0.4 to 0.8 times the pipe diameter.

Let d_1 = diameter at section 1
p_1 = pressure at section 1
v_1 = velocity at section 1
A_1= area at section1
d_2, p_2, v_2, A_2 are the corresponding values at section 2.



Applying Bernoulli's equations at sections 1 and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^{2}}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^{2}}{2g}+z_2$$

$$\Rightarrow\left(\frac{p_1}{\rho g}+z_1\right)-\left(\frac{p_2}{\rho g}+z_2\right)=\frac{v_2^{2}-v_1^{2}}{2g}$$

$$\Rightarrow h=\frac{v_2^{2}-v_1^{2}}{2g}$$

$$\Rightarrow v_2=\sqrt{2gh+v_1^2}$$

Where *h* is the differential head.

Let A_0 is the area of the orifice.

Coefficient of contraction, $C_c=\frac{A_2}{A_0}$

By continuity equation, we have

$$A_1 v_1 = A_2 v_2$$

$$\Rightarrow v_1 = \frac{A_0 C_c}{A_1} v_2$$

Hence,

$$v_2 = \sqrt{2gh + \frac{A_0^2 C_c^2 v_2^2}{A_1^2}}$$

$$\Rightarrow v_2 = \frac{\sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

Thus, discharge,

$$Q = A_2 v_2 = v_2 A_0 C_c = \frac{A_0 C_c \sqrt{2gh}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

If C_d is the co-efficient of discharge for orifice meter,

which is defined as

$$C_d = C_c \frac{\sqrt{1 - \frac{A_0^2}{A_1^2}}}{\sqrt{1 - \frac{A_0^2}{A_1^2} C_c^2}}$$

Hence,

$$Q = C_d \frac{A_0 A_1 \sqrt{2gh}}{\sqrt{A_1^2 - A_0^2}}$$

The coefficient of discharge of the orifice meter is much smaller than that of a venturimeter.

3. Pitot Tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel.

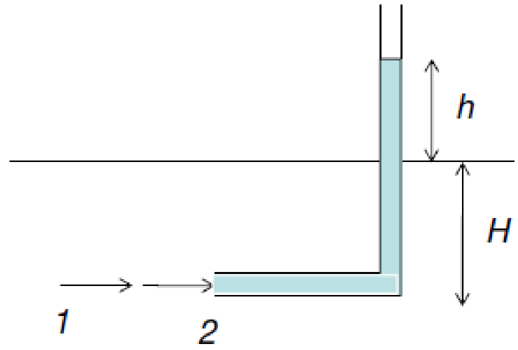
• Principle: If the velocity at any point decreases, the pressure at that point increases due to the conservation of the kinetic energy into pressure energy.

• In simplest form, the Pitot tube consists of a glass tube, bent at right angles.

Let p_1 = pressure at section 1
p_2 = pressure at section 2
v_1 = velocity at section 1
v_2 = velocity at section 2 = 0

H = depth of tube in the liquid

h = rise of liquid in the tube above the free surface



Applying Bernoulli's equations at sections 1and 2, we get

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

But $z_1 = z_2$, and $v_2 = 0.$

$\frac{p_1}{\rho g}=$ Pressure head at 1=H

$\frac{p_2}{\rho g}=$ Pressure head at 2=h+H

Substituting these values, we get $H+\frac{v_1^2}{2g}=h+H$

$$\Rightarrow v_1=\sqrt{2gh}$$

This is theoretical velocity. Actual velocity is given by

$$(v_1)_{act}=C_v\sqrt{2gh}$$

$C_v \equiv$ coefficient of pitot-tube