Fluid kinematics

Introduction :

In general, fluids have a well-known tendency to move or flow. The slight change in shear stress or appropriate imbalance in normal stresses will cause fluid motion. *Fluid kinematics* deals with various aspects of fluid motion without concerning the actual force that causes the fluid motion. In this particular section, we shall consider the 'field' concept to define velocity/ acceleration of fluid by virtue of its motion.

Lagrangian and Eulerian Descriptions:

Lagrangian description: To follow the path of individual objects.
This method requires us to track the position and velocity of each individual fluid parcel (**fluid particle**) and take to be a parcel of fixed identity.

A more common method is **Eulerian description** of fluid motion.
• In the Eulerian description of fluid flow, a finite volume called a **flow domain** or **control volume** is defined, through which fluid flows in and out.
• Instead of tracking individual fluid particles, we define **field variables**, functions of space and time, within the control volume.
• The field variable at a particular location at a particular time is the value of the variable for whichever fluid particle happens to occupy that location at that time.
• For example, the **pressure field** is a **scalar field variable**. We define the **velocity field** as a **vector field variable.**
In the Eulerian description we don't really care what happens to individual fluid particles; rather we are concerned with the pressure, velocity, acceleration, etc., of whichever fluid particle happens to be at the location of interest at the time of interest.
• While there are many occasions in which the Lagrangian description is useful, the Eulerian description is often more convenient for fluid mechanics applications.
• Experimental measurements are generally more suited to the Eulerian description.

Types of fluid flows

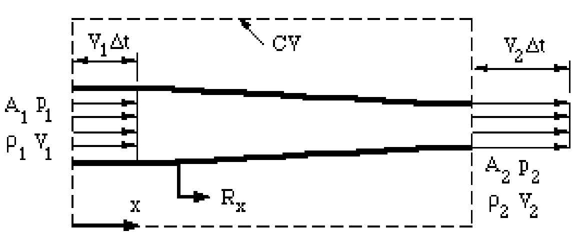
Steady flow is defined as that type of which fluid characteristics velocity, pressure etc at a point do not change with time (dv/dt)(0,0,0)=0	**Unsteady flow** is that type of flow in flow in which the velocity, pressure at a point changes with time. (dv/dt)(0,0,0)=/0
Uniform flow is defined as that type of flow in which velocity at any given with time does not change with respect space. (dv/dt)t=constant=0	**Non- Uniform flow** is defined as that type of flow in which velocity at any given time changes respect to time (dv/dt)t=/0
Laminar flow is defined as that type flow	**Turbulent flow** is defined as that type of flow

in which the fluid particle move well defined path or streamline and all the streamline are straight and parallel Reynolds number<2000 .	in which the fluid particle moves in a zig- zag way Reynolds number>4000
Compressible flow is that type of flow in which the density of the fluid changes from point to point.(eg)Flow of gasses through orifice nozzle and gas turbine.	**Incompressible flow** is that type of flow in which the density is constant for the fluid flow.(eg)Subsonic aerodynamics.
Rotational flow is that type of flow in which in which the fluid particle flowing along streamlines, also rotate about their own axis.	**Irrotational flow** is that type of flow in which the fluid particle while flowing along streamlines; do not rotate about their own axis.
One dimensional flow is that type of flow in which the flow parameter such as velocity is a function of time and one space co-ordinate only, say X. U=F(x),V=0,w=0.	**Two dimensional flow** is that type of flow in which the velocity is a function of time and two rectangular space co-ordinates X and Y. u=F1(X,Y),V=F2(X,Y) and w=0. **Three dimensional flow** is that type of flow in which the velocity is a function of time and three mutually perpendicular directions. U=F1(X,Y,X),v=F2(X,Y,Z),w=F3(X,Y,Z). u,v,w are velocity components in X,Y,Z direction respectively.

Continuity Equation

When a fluid is in motion, it must move in such a way that mass is conserved. Consider the **steady** flow of fluid through a duct (that is, the inlet and outlet flows do not vary with time). The inflow and outflow are **one-dimensional**, so that the velocity V and density are constant over the area A

Now we apply the principle of mass conservation. Since there is no flow through the side walls of the duct, what mass comes in over A_1 goes out of A_2, (the flow is steady so that there is no mass accumulation). Over a short time interval



$$\text{volume flow in over } A_1 = A_1 V_1 \Delta t$$
$$\text{volume flow out over } A_2 = A_2 V_2 \Delta t$$

Therefore

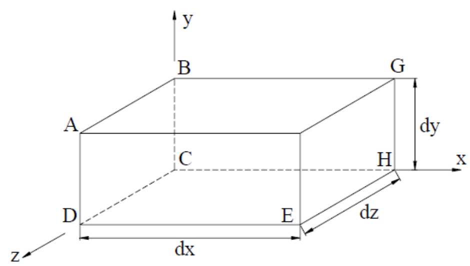
$$\text{mass in over } A = \rho A_1 V_1 \Delta t$$
$$\text{mass out over } A = \rho A_2 V_2 \Delta t$$

$$\text{So:} \quad \boxed{\rho A_1 V_1 = \rho A_2 V_2}$$

This is a statement of the principle of mass conservation for a steady, one-dimensional flow, with one inlet and one outlet. This equation is called the **continuity equation** for steady one-dimensional flow. For a steady flow through a control volume with many inlets and outlets, the net mass flow must be zero, where inflows are negative and outflows are positive

Continuity Equation in 3-D Flow :

The control volume ABCDEFGH in Fig. is taken in the form of a small prism with sides dx, dy and dz in the x, y and z directions, respectively.



The mean values of the component velocities in these directions are u, v, and w.
Considering flow in the x direction,
Mass inflow through ABCD in unit time = $\rho\, udydz$

In the general case, both specific mass ρ and velocity u will change in the x direction and so,

Mass outflow through EFGH in unit time = $\left[\rho u + \dfrac{\partial(\rho u)}{\partial x}dx\right]dydz$

Thus,
Net outflow in unit time in x direction $= \dfrac{\partial(\rho u)}{\partial x}dxdydz$

Similarly,
Net outflow in unit time in y direction $= \dfrac{\partial(\rho v)}{\partial y}dxdydz$

Net outflow in unit time in z direction $= \dfrac{\partial(\rho w)}{\partial z}dxdydz$

Therefore,
Total net outflow in unit time

$$= \left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z}\right]dxdydz$$

Change of mass in control volume in unit time $= -\dfrac{\partial \rho}{\partial t}dxdydz$

(the negative sign indicating that a net outflow has been assumed). Then,
Total net outflow in unit time = Change of mass in control volume in unit time

$$\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]dxdydz=-\frac{\partial\rho}{\partial t}dxdydz$$

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=-\frac{\partial\rho}{\partial t}$$

However, for incompressible flow, the specific mass ρ is constant and the equation simplifies to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

<u>Velocity and Acceleration of fluid flow:</u>

One of the most important fluid variables is the velocity field. It is a vector function of position and time with components as scalar variables i.e.,

$$\vec{V}=u(x,y,z,t)\hat{i}+v(x,y,z,t)\hat{j}+w(x,y,z,t)\hat{k}$$

The magnitude of the velocity vector is, $\left|\vec{V}\right|=\sqrt{\left(u^2+v^2+w^2\right)}$,

The total time derivative of the velocity vector is the acceleration vector field of the flow which is given as,

$$\bar{a}=\frac{d\vec{V}}{dt}=\frac{d\{u(x,y,z,t)\}}{dt}\hat{i}+\frac{d\{v(x,y,z,t)\}}{dt}\hat{j}+\frac{d\{w(x,y,z,t)\}}{dt}\hat{k}$$

For instance, the scalar time derivative of u is expressed as,

$$\begin{aligned}\frac{d\{u(x,y,z,t)\}}{dt}&=\frac{\partial u}{\partial t}+\frac{\partial u}{\partial t}\frac{dx}{dt}+\frac{\partial u}{\partial t}\frac{dy}{dt}+\frac{\partial u}{\partial t}\frac{dz}{dt}\\&=\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial t}+w\frac{\partial u}{\partial t}\\&=\frac{\partial u}{\partial t}+\left(\vec{V}.\nabla\right)u\end{aligned}$$

When u is replaced with v, w in the above equation, then the corresponding expressions would be,

$$\frac{d\{v(x,y,z,t)\}}{dt}=\frac{\partial v}{\partial t}+\left(\vec{V}.\nabla\right)v$$

$$\frac{d\{w(x,y,z,t)\}}{dt}=\frac{\partial w}{\partial t}+\left(\vec{V}.\nabla\right)w$$

Now, summing them into a vector quantity

$$\vec{a} = \frac{d\vec{V}}{dt} = \frac{\partial \vec{V}}{\partial t} + \left(u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \right) = \frac{\partial \vec{V}}{\partial t} + \left(\vec{V}.\nabla \right)\vec{V}$$

where, $\vec{V} = u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$ and $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

In the above equation, first part is called as "local acceleration"

$$\frac{\partial \vec{V}}{\partial t}$$

and the second part $\left(u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z} \right)$ is called a "convective acceleration".

Velocity potential function

An irrotational flow is defined as the flow where the vorticity is zero at every point. It gives rise to a scalar function (φ) which is similar and complementary to the stream function (ψ). Let us consider the equations of irrortional flow and scalar function (φ)

In an irrotational flow, there is no vorticity $\left(\vec{\xi} \right)$

$$\vec{\xi} = \nabla \times \vec{V} = 0$$

Now, take the vector identity of the scalar function (φ)

$$\nabla \times \left(\nabla \phi \right) = 0$$

Where

$$\vec{V} = \nabla \phi$$

So, the velocity components can be written as, $u = \frac{\partial \phi}{\partial x};\ v = \frac{\partial \phi}{\partial y};\ w = \frac{\partial \phi}{\partial z}$

Thus, any irrotational, incompressible flow has a velocity potential and stream function (for two-dimensional flow) that both satisfy *Laplace equation*. Conversely, any solution of *Laplace equation* represents both velocity potential and stream function (two-dimensional) for an irrotational, incompressible flow.

Stream function

The idea of introducing stream function works only if the continuity equation is reduced to two terms. There are 4-terms in the continuity equation that one can get by expanding the vector equation

For a steady, incompressible, plane, two-dimensional flow, this equation reduces to

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Here, the striking idea of stream function works that will eliminate two velocity components u and v into a single variable

So, the *stream function* $\{\psi(x,y)\}$ relates to the velocity components in such

a way that continuity equation is satisfied.

$$u=\frac{\partial \psi}{\partial y};\ v=-\frac{\partial \psi}{\partial x}$$

Visualization of Fluid Flow:

The quantitative and qualitative information of fluid flow can be obtained through sketches, photographs, graphical representation and mathematical analysis. However, the visual representation of flow fields is very important in modeling the flow phenomena. In general, there are four basic types of line patterns used to visualize the flow such as path line, streak line and streamlines.

Path line: It is the actual path traversed by a given fluid particle as it flows from one point to another. Thus, the path line is a Lagrangian concept that can be produced in the laboratory by marking the fluid particle and taking time exposure photograph of its motion.

Streak line: A streak line consists of all particles in a flow that has previously passed through a common point. Here, the attention is focused to a fixed point in space (i.e. Eulerian approach) and identifying all fluid particles passing through that point. These lines are laboratory tool rather than analytical tool. They are obtained by taking instantaneous photographs of selected particles that have passed through a given location in the flow field.

Streamline: These are the lines drawn in the flow field so that at a given instant, they are tangent to the direction of flow at every point in the flow field. Since the streamlines are tangent to the velocity vector at every point in the flow field, there can be no flow across a streamline.

$$\frac{dy}{dx} = \frac{v}{u}$$

This equation can be integrated to obtain the equation of streamlines

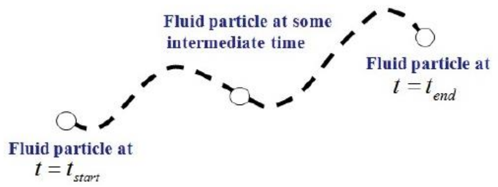


Fig. Path line

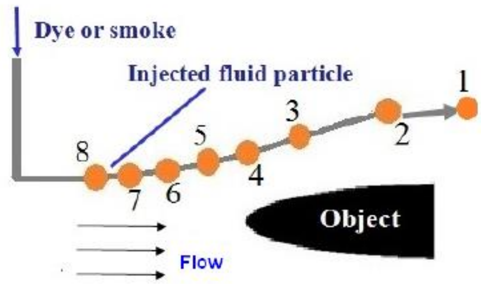


Fig. Streak line

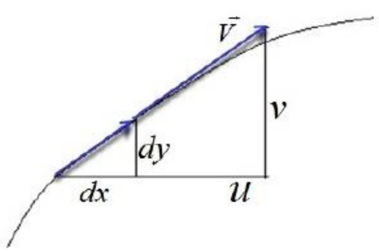


Fig. Stream line