30.07.2021

Quantum Mechanics Practicals

Lecture 1

Dimensional Analysis

$M \quad L \quad T$

V

$$Q_0 = f(\underbrace{Q_1, Q_2, \ldots, Q_n}_{\downarrow})$$

independent & complete variables

$$[Q_i] = [L]^{l_i} [T]^{t_i} [M]^{m_i}$$

Pick up a complete dimensionally independent subset $Q_1, \ldots, Q_k$ $\quad k \leq n$

$Q_{k+1}, \ldots, Q_n \longrightarrow$

$$[Q_j] = [Q_1]^{N_{j1}} [Q_2]^{N_{j2}} \cdots [Q_k]^{N_{jk}} \qquad j = k+1, \ldots, n$$

$n-k$ no of

Define dimensionless parameters

$$\Pi_i = \frac{Q_{k+i}}{[Q_1]^{N_{k+i,1}} \cdots [Q_k]^{N_{k+i,k}}} \qquad i = 1, \ldots, n-k$$

$$[Q_0] = [Q_1]^{N_{01}} [Q_2]^{N_{02}} \cdots [Q_k]^{N_{0k}}$$

$$\Pi_0 = \frac{Q_0}{[Q_1]^{N_{01}} [Q_2]^{N_{02}} \cdots [Q_k]^{N_{0k}}}$$

$$\Pi_0 = f(\underbrace{Q_1, \ldots, Q_k}_{\text{have} \rightarrow \text{dimension}}, \Pi_1, \ldots \Pi_{n-k})$$

For dimensionally homogeneous eq^n

$Q_1, \ldots Q_k$ should be absent

$$y = \frac{1}{2} g t^2$$

$$9.8\, m s^{-2}$$

$$\Pi_0 = f(\Pi_1, \ldots, \Pi_{n-k})$$

$$y = 4.9\, t^2$$

<u>Buckingham's Π-Theorem</u> (1914)

. .

$$a\frac{dx}{dt} + bx = c f(t)$$

$x = \chi\, x_c$ $\hookrightarrow$ characteristic length

$\chi \hookrightarrow$ dimensionless

$t = \tau\, t_c$

$\tau \hookrightarrow$ dimensionless

$$\frac{d}{dt} = \frac{d\tau}{dt}\frac{d}{d\tau} = \frac{1}{t_c}\frac{d}{d\tau}$$

$$a\,\frac{1}{t_c}\frac{d}{d\tau}(\chi x_c) + b(\chi x_c) = c\, f(\tau t_c) \rightarrow \phi(\tau)$$

$$\left(\frac{a x_c}{t_c}\right)\frac{d\chi}{d\tau} + (b x_c)\chi = c\,\phi(\tau)$$

$$\frac{dx}{d\tau} + \left(\frac{b t_c}{a}\right) x = \left(\frac{c t_c}{a x_c}\right) \phi(\tau)$$

$$\frac{b t_c}{a} = 1 \implies t_c = \frac{a}{b} .$$

$$\frac{dx}{d\tau} + x = \left(\frac{c}{b x_c}\right) \phi(\tau)$$

$$x_c = \frac{c}{b}$$

$$\therefore$$

$$\boxed{\frac{dx}{d\tau} + x = \phi(\tau)}$$

Diffusion eqn

$c(x,t) \rightarrow$ conc.

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2}$$

$D \rightarrow$ Diffusion const.

$$[D] = [L]^2 [T]^{-1} \qquad \frac{L_D^2}{t_D}$$

$L_D \quad , \quad t_D$

$$\boxed{L_D = \sqrt{D t_D}}$$

$$\eta = \frac{x}{L_D}$$

$$t = \tau t_D$$

$$\frac{\partial c}{\partial t} = \frac{\partial c}{\partial \tau} \cdot \frac{d\tau}{dt} = \frac{1}{t_0} \frac{\partial c}{\partial \tau}$$

$$\frac{\partial c}{\partial x} = \frac{\partial c}{\partial \eta} \cdot \frac{\partial \eta}{\partial x} = \frac{1}{L_0} \frac{\partial c}{\partial \eta}$$

Example

simple pendulum

IC.

$\theta(0) = \theta_0$

$\left.\frac{d\theta}{dt}\right|_{t=0} = a$

$$\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$$

$$t = t_c \tau \qquad \frac{d^2\theta}{dt^2} = \frac{1}{t_c^2}\frac{d^2\theta}{d\tau^2}$$

$$\frac{1}{t_c^2}\frac{d^2\theta}{d\tau^2} + \frac{g}{l}\sin\theta = 0$$

$\theta(\tau = 0) = \theta_0$

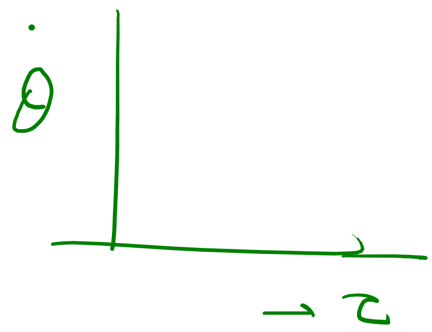
$\left.\frac{d\theta}{d\tau}\right|_{\tau=0} = a\sqrt{\frac{l}{g}}$

$\alpha =$

$$\frac{d^2\theta}{d\tau^2} + \frac{g t_c^2}{l}\sin\theta = 0$$

$$t_c^2 = \frac{l}{g}$$

$$\frac{d^2\theta}{d\tau^2} + \sin\theta = 0$$



$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos(\omega t)$$

Differential Equations

ODE
PDE

nth order ODE
$\downarrow$
<u>n constants</u>

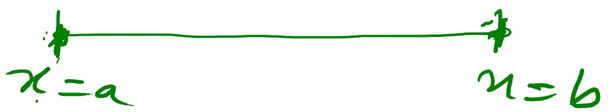
<u>IVP</u> → The values of the dependent variable y & its derivatives are given at <u>one</u> value of the independent variable x

$y(x=\underline{x_0}) = y_0$, $y'(x=\underline{x_0}) = v_0$, $y''(x=\underline{x_0}) = z_0$, ...

BVP → The diff eqn satisfies the certain <u>B.C.</u> at <u>more than</u> one value of the independent variable

$[a, b]$

<u>ODE</u> 2nd order



$a<b$

Two-point BVP

2nd order ODE $\rightarrow$ Two pt BVP

$$y'' = f(x, y, y') \qquad a \leq x \leq b$$

B.C. $\quad \alpha_1 y(a) + \alpha_2 y'(a) = \alpha_3$

and $\quad \beta_1 y(b) + \beta_2 y'(b) = \beta_3$ $\Big]$ linear B.C

3 special cases

1. Dirichlet B.C. $\quad \alpha_2 = \beta_2 = 0 \qquad \alpha_1, \beta_1 \neq 0$

$$y(a) = \alpha = \alpha_3/\alpha_1$$
$$y(b) = \beta = \beta_3/\beta_1$$

2. Neuman B.C. $\quad \alpha_1 = \beta_1 = 0, \quad \alpha_2, \beta_2 \neq 0$

$$y'(a) = \alpha', \quad y'(b) = \beta'$$

3 Robin or mixed B.C

$\alpha_1, \beta_1, \alpha_2, \beta_2 \neq 0$