Experiment 5

The Laws of Radiation - Stefan-Bolztmann Law (Radiant Flux)

METHOD

If the temperature of the cavity is T, then we can ascribe a characteristic energy $\epsilon^* = kT$ to a typical photon. Such photon would have a de Broglie wavelength $l^* = h/p^* = hc/\epsilon^*$ and characteristic frequency $\nu^* = \epsilon^*/h$. Defining a dimensionless quantity $x = \epsilon/\epsilon^*$, the spectral energy density for a particular mode ($\epsilon = h\nu$) can be rewritten as

$$u(\nu)\, d\nu \to u(x)\, dx = \frac{8\pi\epsilon^*}{l^{*3}} \left(\frac{x^3}{e^x - 1} \right) dx = \frac{8\pi\epsilon^*}{l^{*3}} f_P(x) dx$$

where $f_P(x) = \frac{x^3}{e^x-1}$ and

$$u = \int_0^\infty u(\nu)\, d\nu = \int_0^\infty \bar{\epsilon}\ \frac{G(\nu)\, d\nu}{V_0} = \frac{8\pi h}{c^3} \int_0^\infty \left(\frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1} \right) d\nu = \frac{8\pi^2 k^4}{15 c^3 h^3} T^4 = \frac{4}{c}\sigma T^4$$

with density of states given by

$$G(\nu) d\nu = \frac{8\pi\nu^2}{c^3} d\nu V_0$$

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CODING AND PLOTS

1. **(Wiens Displacement law, Energy of states)**
   The plot of $f_P(x)$ versus dimensionless variable x peaks at a certain $x = x_p$. Correspondingly, the spectral energy density will peak at x_p irrespective of the temperature of the cavity.

   (a) Find the value of x_p.

   (b) Since the dimensionless variable x can also be written as $x = \epsilon/\epsilon^* = hc/\lambda k_b T$, deduce the Wien's constant b

   $$\lambda T = \frac{hc}{kx_p} = b$$

   Prove that it is $b = 0.003$ *meter Kelvin* and hence the Weins displacement Law

2. **(Stefan-Bolztmann Law)**
   With the characteristic parameters of the cavity, the total energy density u of the radiation in a cavity at temperature T is given by

   $$u = \int_0^\infty u(\nu)\, d\nu = \frac{8\pi\epsilon^*}{l^{*3}} \int_0^\infty \left(\frac{x^3}{e^x - 1}\right) dx = \frac{8\pi\epsilon^*}{l^{*3}} \int_0^\infty f_P(x)\, dx = \frac{8\pi\epsilon^*}{l^{*3}} I_p$$

   The integral I_P is independent of the parameters of the cavity.
   With numerical integration show that $I_P = \pi^4/15$
   The total energy density u of the radiation in a cavity at temperature T thus becomes

   $$u = \frac{\pi^4}{15}\frac{8\pi\epsilon^*}{l^{*3}} = \frac{\pi^4}{15} C(T)$$

   Calculate the factor $C(T)$ for temperatures $T = [100K, 10000K]$ in steps of 500 K.
   Find the energy density $u(T) = (\pi^4/15)C(T)$ for these temperatures. The total energy density $u(T)$ of the radiation in a cavity at temperature T and the radiant flux $F(T)$ are related by $F = \frac{c}{4}u$. Find the radiant flux $F(T) = (c/4)(\pi^4/15)C(T)$ for these temperatures.
   Plot

   (a) F versus T and also

   (b) plot lnF versus lnT. Find the value of slope and intercept from the graph in part

   (c) interpret.

   Hence, deduce the Stefan Boltzmann law

   $$F = \sigma\, T^4 = \left(\frac{c}{4}\right) \frac{8\pi^{\overset{5}{\cancel{2}}} k^4}{15c^3h^3} T^4$$

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