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(2)

Maxwell Boltzmann =

$$f_{MB} = A * \exp\left(\frac{-\epsilon}{KT}\right)$$

$$x = \frac{-\epsilon}{KT}$$

$$\epsilon = xKT$$

BOSE EINSEIN =

$$f_{BE} = \frac{1}{\left(e^{\left(\frac{\epsilon - \mu}{KT}\right)} - 1\right)}$$

$$\alpha = \frac{-\mu}{KT} \qquad x = \frac{\epsilon}{KT}$$

$$f_{BE} = \frac{1}{\left(e^{(x+\alpha)} - 1\right)}$$

$$\alpha = 0 \quad , \quad f_{BE} = \frac{1}{(e^{x} - 1)}$$

fermi-Dirac =

$$f_{FD} = \frac{1}{e^{\left(\frac{\epsilon - \mu}{KT}\right)} + 1}$$

$$f_{FD} = \frac{1}{\left(e^{(\alpha + x)} + 1\right)}$$

(3) Dulong Petit

$$E = N_A 3 KT$$

$$C_V = \left(\frac{\partial E}{\partial T}\right)_V = 3R$$

Einstein Distribution

$$\bar{E} = \frac{h\nu}{e^{\frac{h\nu}{KT}} - 1}$$

DOS

$$G(\nu)d\nu = f N_A \delta(\nu - \nu_E)\, d\nu$$
$$\int G(\nu)d\nu = f N_A$$

$$U_E = \int \bar{E}(\nu)\, G(\nu)\, d\nu = 3N_A \frac{h\nu_E}{e^{h\nu_E/KT} - 1}$$

Specific heat $\Rightarrow$ $C_V = \left(\frac{\partial U_E}{\partial T}\right)_V$

$$\frac{C_V}{3R} = \left(\frac{h\nu_E}{KT}\right)^2 \frac{e^{h\nu/KT}}{(e^{h\nu/KT}-1)^2} = \left(\frac{\theta_E}{T}\right)^2 \frac{e^{\theta_E/T}}{(e^{\theta_E/T}-1)^2}$$

Debye distribution

DOS

$$G(\nu)\,d\nu = 9N_A\left(\frac{\nu}{\nu_D}\right)^2 \frac{d\nu}{\nu_D}$$

$$U_D = \frac{9N_A}{\nu_D^3}\int_0^{\nu_D} \frac{h\nu^3}{e^{\frac{h\nu}{KT}} - 1}\, d\nu$$

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Date:

$$\frac{C_v}{3R} = \frac{-3(\Theta_D/T)}{e^{\Theta_D/T}-1} + \frac{12}{(\Theta_D/T)^3}\int_0^{\Theta_D/T} \frac{(\Theta_D/T)^3}{e^{\Theta_D/T}-1}\, d\left(\frac{\Theta_D}{T}\right)$$

(4) Rayleigh - Jeans

$$G(\nu)d\nu = \frac{8\pi\nu^2}{c^3} d\nu\, V_0$$

$$u(\nu)d\nu = \bar{E}\,\frac{G(\nu)d\nu}{V_0} = KT\,\frac{8\pi\nu^2}{c^3}\,d\nu$$

DOS

$$G(\nu)d\nu = \pi\left(\frac{8 l_0^3}{c^3}\right)\nu^2 d\nu$$

$$= \pi\left(\frac{\nu}{\nu_0}\right)^2 d\left(\frac{\nu}{\nu_0}\right)$$

$$G(x)dx = \pi x^2 dx$$

$$\lambda_0 = 2l_0$$

$$\nu_0 = c/2l_0$$

$$x = \frac{\nu}{\nu_0}$$

EOS

$$u(\nu)d\nu = \epsilon^*\,\frac{8\pi\epsilon^2}{h^3c^3}\,d\epsilon \rightarrow u(x)dx = \frac{8\pi\epsilon^*}{l^{*3}}\,x^2 dx$$

$$= \frac{8\pi\epsilon^*}{l^{*3}}\,f_{RJ}(x)dx.$$

$$f_{RJ}(x) = x^2$$

Page No.:

Planck's

$$\bar{\epsilon} = \frac{h\nu}{e^{h\nu/KT} - 1}$$

$$u(\nu)d\nu = \frac{\epsilon^* x}{e^x - 1} \frac{8\pi\epsilon^2}{h^3c^3} d\epsilon$$

$$u(x)\,dx = \frac{8\pi\epsilon^*}{e^3} \frac{x^3}{e^x-1} dx = \frac{8\pi\epsilon^*}{e^{*3}} f_p(x)\,dx$$

$$f_p(x) = \frac{x^3}{e^x - 1}$$

5

Stefan-Boltzmann Law

$$u(\nu)d\nu \rightarrow u(x)\,dx = \frac{8\pi\epsilon^*}{e^3}\left(\frac{x^3}{e^x-1}\right)dx$$

$$= \frac{8\pi\epsilon^*}{e^{*3}} f_p(x)\,dx$$

$$f_p(x) = \frac{x^3}{e^x - 1}$$

$\epsilon^* = KT$

$\nu^* = \epsilon^*/h$

$x = \epsilon/\epsilon^*$

Weins Displacement,

$$\lambda T = \frac{hc}{Kx_p} = b$$

$$x = \epsilon/\epsilon^* = \frac{hc}{\lambda K_b T}$$

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Stefan - Boltzmann Law

$$u = \int_0^{\infty} u(\nu)\, d\nu = \frac{8\pi \epsilon^*}{e^{*3}} \int_0^{\infty} \left(\frac{x^3}{(e^x - 1)}\right) dx$$

$$= \frac{8\pi \epsilon^*}{e^{*3}} \int_0^{\infty} f_p(x)\, dx$$

$$= \frac{8\pi \epsilon^*}{e^{*3}} I_p .$$

T.E

$$u = \frac{\pi^4}{15} \frac{8\pi \epsilon^*}{e^{*3}} = \frac{\pi^4}{15} (CT)$$

$$f = \sigma T^4 = \left(\frac{c}{4}\right) \frac{8\pi^5 k^4}{15 c^3 h^3} T^4$$

(7)

Partition function

$$Z(V, T) = \frac{\pi}{2} \int_0^{\infty} n_j^2 \exp\left(-\frac{h^2}{8mV^{2/3} k_B T} n_j^2\right) dn_j$$

Pressure →

$$P = NKT \left(\frac{\partial \ln Z}{\partial V}\right)_T$$

Energy →

$$\langle E \rangle = \frac{U}{N} = KT^2 \left(\frac{\partial \ln Z}{\partial T}\right)_V$$

Entropy

$$S = \frac{U}{T} + NK(\ln Z - \ln N + 1)$$

(5) Internal Energy

Specific heat capacity

$$C_V = \frac{\partial \langle E \rangle}{\partial T}$$

$$\langle \langle \Delta E \rangle^2 \rangle = \frac{\partial^2 \ln Z}{\partial \beta^2} \quad = kT^2 C_V$$

Analytic

$$Z(V,T) = V\left(\frac{2\pi m k}{h^2}\right)^{3/2} T^{3/2}$$

(6) Partition function

(1) $$Z = \sum_j g_j \, e^{-\frac{\epsilon}{kT}}$$

$$Z = 1 + e^{-\epsilon/kT}$$

(2) fractional Population

$$\frac{N_j}{N} = \frac{g_j \, e^{-\epsilon_j/kT}}{Z}$$

Low temp

$$N_1 = \frac{N}{Z} e^{-0/kT}$$

$$= \frac{N}{Z} \cong N$$

$$N = \frac{N}{Z} e^{-\epsilon/kT}$$

$$\sim 0$$

(3) Internal Energy

$$U = NkT^2 \left(\frac{\partial}{\partial T}(\ln Z)\right)_V$$

$$U = \sum_j N_j \epsilon_j$$

$$= N_1 \epsilon_1 + N_2 \epsilon_2$$

$$= 0 + \epsilon \frac{N}{Z} e^{-\epsilon/kT}$$

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Date:

$$\bar{U} = \frac{U}{N} = \frac{\epsilon}{z} e^{-\epsilon/kT}$$

Entropy

$$S = Nk_B \ln \frac{Z}{N} + \frac{U}{T} + Nk_B$$

$$\boxed{S = Nk \ln z + \frac{U}{T}}$$

$$= Nk_B \ln Z + \frac{U}{T} - Nk_B \ln Z + Nk_B$$

$$= Nk_B \ln Z + N\frac{\bar{U}}{T} + Nk_B (1 - \ln N)$$

$$\frac{S}{N} = k_B \ln Z + \frac{\bar{U}}{T} + k_B (1 - \ln N)$$

Free Energy

$$E = -Nk_B T \ln Z$$