Statistical Mechanics (32221602): Practical Lab **Sem 6 : Radiation Laws**

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Experiment 4

The Laws of Radiation
Rayleigh-Jeans and Planck's (Energy Density)

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1. METHOD

   (a) **Rayleigh-Jeans (Energy Density)**

   According to Rayleigh-Jeans, em waves confined to a cavity at temperature T with walls as perfect reflectors consist of standing waves filling a certain volume V_0 of the cavity, with density of states given by

   $$G(\nu)d\nu = \frac{8\pi\nu^2}{c^3} d\nu V_0$$

   Rayleigh-Jeans further assumed that, since em waves consists of two degrees of freedom, each of the standing waves shall have energy $\bar{\epsilon} = kT$ according to the Law of Equipartition of energy. Thus, the energy density for a particular mode will be

   $$u(\nu)\ d\nu = \bar{\epsilon}\ \frac{G(\nu)d\nu}{V_0} = kT\ \frac{8\pi\nu^2}{c^3} d\nu$$

   (b) **Planck's (Energy Density)**

   Plancks assumed that the energy of the states must be quantized and each mode can have energy $\epsilon = nh\nu \quad \forall n = 1, 2, 3, ...$ obeying the Maxwell distribution function $f(\epsilon) = e^{-(\epsilon/kT)}$. The average energy for a particular mode is

   $$\bar{\epsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

   and the energy density will be

   $$u(\nu)\ d\nu = \bar{\epsilon}\ \frac{G(\nu)d\nu}{V_0} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}\ \frac{8\pi\nu^2}{c^3} d\nu$$

2. CODING AND PLOTS

   (a) **(Density of states: Rayleigh-Jeans law and Plancks law)**

   A cavity of dimension l_0 can be characterized by a standing em wave of wavelength $\lambda_0 = 2\ l_0$ and so a characteristic frequency $\nu_0 = c/\ 2\ l_0$. Defining a dimensionless quantity $x = \nu/\nu_0$, the density of states can be rewritten as

   $$G(\nu)d\nu = \pi \left(\frac{8\ l_0^3}{c^3}\right) \nu^2\ d\nu = \pi \left(\frac{\nu}{\nu_0}\right)^2 d\left(\frac{\nu}{\nu_0}\right) \rightarrow G(x)\ dx = \pi x^2\ dx$$

   Assuming the cavity to be $1A^o$ across, **Plot the density of states versus frequency**
   (i) for the complete range of frequencies from CMBR to GRBs (10^{10} to 10^{30} Hz),
   (ii) for the visible range and
   (iii) interpret (see whether dos diverges with frequency).
   Hint: For this wide spectrum of frequency plot we will need Log plot

(b) **(Rayleigh Jeans law, Energy of states)**

If the temperature of the cavity is T, then we can ascribe a characteristic energy $\epsilon^* = kT$ to a typical photon. Such photon would have a de Broglie wavelength $l^* = h/p^* = hc/\epsilon^*$ and characteristic frequency $\nu^* = \epsilon^*/h$. Defining a dimensionless quantity $x = \epsilon/\epsilon^*$, the energy density for a particular mode ($\epsilon = h\nu$) can be rewritten as

$$u(\nu)\, d\nu = \epsilon^* \frac{8\pi\epsilon^2}{h^3\, c^3}\, d\epsilon \to u(x)\, dx = \frac{8\pi\epsilon^*}{l^{*3}} x^2\, dx = \frac{8\pi\epsilon^*}{l^{*3}} f_{RJ}(x) dx$$

where $f_{RJ}(x) = x^2$

Plot the $f_{RJ}(x)$ versus x for $x = [0, 12]$

Plot the spectral energy density versus frequency

(1) in complete range

(2) in the visible range for the three different temperatures $T = 1200\ K, 1500\ K, 1800\ K$. Interpret the resulting plots.

Hint: (i) input frequency range ν, (ii) calculate dimensionless variable $x = (h/\epsilon^*)\ \nu$, (iii) calculate RJ function $f_{RJ}(x)$, (iv) get the energy density as $u(x) = \frac{8\pi\epsilon^*}{l^{*3}} f_{RJ}(x)$ and lastly (v) plot u vs ν.

(c) **(Plancks Radiation law, Energy of states)**

With the above characteristic parameters of the cavity, we get the average energy for a particular mode to be

$$\bar{\epsilon} = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

The energy density will be

$$u(\nu)\, d\nu = \frac{\epsilon^* x}{e^x - 1} \frac{8\pi\epsilon^2}{h^3\, c^3}\, d\epsilon \to u(x)\, dx = \frac{8\pi\epsilon^*}{l^{*3}} \frac{x^3}{e^x - 1}\, dx = \frac{8\pi\epsilon^*}{l^{*3}} f_P(x) dx$$

where $f_P(x) = \frac{x^3}{e^x - 1}$

Plot the average energy versus frequency (in the visible range) and interpret

Plot the spectral energy density versus frequency (in the visible range) and interpret.

Plot it at three different temperatures $T = 1200\ K, 1500\ K, 1800\ K$

Hint: (i) input frequency range ν, (ii) calculate dimensionless variable $x = (h/\epsilon^*)\ \nu$, (iii) calculate Planck function $f_P(x)$, (iv) get the energy density as $u(x)\ dx = \frac{8\pi\epsilon^*}{l^{*3}} f_P(x) dx$ and lastly (v) plot u vs ν