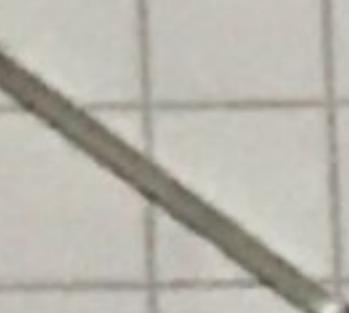


m oDulE
J14 p

num3



CHAPITRE 01

fonction recursive
+ + +

1) les fonction de Base

$$\left\{ \begin{array}{l} \text{succ } n \rightarrow n + 1 \\ U_i^n \text{ projection } (n_1, \dots, n_n) \rightarrow n_i \\ Z_f \text{ null } n \rightarrow 0 \end{array} \right.$$

Remarque

$$U_1^2(x, y) = x$$

$$U_2^2(x, y) = y$$

$$U_2^3(x, y, z) = y$$

a) Multiplication

$$1) 0 \times n = 0$$

$$2) (n + 1) \times y = (n \times y) + y$$

$$\begin{aligned} 5 \times 4 &= 5 + 5 + 5 + 5 \\ &= (5 \times 3) + 5 \end{aligned}$$

fonction معروفة (1)
forme récursive (2)

$$\left\{ \begin{array}{l} * \text{Mult}(x, 0) = U_1^2(x, 0) \\ \quad = 0 \\ * \text{Mult}(n + 1, y) = \\ \quad \text{add}(\text{Mult}(n, y), y) \end{array} \right.$$

b) L'addition

$$1) 0 + y = y$$

$$2) (\text{sc}_x + 1) + y = (\text{sc}_x + y) + 1$$

$$\left\{ \begin{array}{l} \text{plus}(0, y) = \text{U}_2^2(0, y) = y \\ \text{plus}(\text{sc}_x + 1, y) = S(\text{plus}(\text{sc}_x, y)) \end{array} \right.$$

c) Puissance

$$1) \text{sc}^0 = 1 \quad \text{Valeur de base}$$

$$2) \text{sc}^{y+1} = \text{sc}^y \times \text{sc}$$

$$\left\{ \begin{array}{l} \text{puis}(x, 0) = 1 \\ \text{puis}(x, y+1) = \text{Mult}(x, \text{puis}(x, y)) \end{array} \right.$$

d) factorielle

$$1) 0! = 1$$

$$2) (\text{sc}_x + 1)! = (\text{sc}_x + 1) \times !$$

$$(6)! = 6 \times 5!$$

$$(6)! = 6 \times 5 \times 4 \times 3 \dots 1$$

$$\text{fact}(0) = 1$$

$$\text{fact}(x+1) = \text{mul}(S(x), \text{fact}(x))$$

$$\begin{matrix} \downarrow \\ x+1 \end{matrix}$$

$$\text{sc}_x!$$

e) Subtraction $x - (y + 1) = (x - y) - 1$!!!

$\text{pred} \neq \text{succ}$

$$\text{pred}(x) = \begin{cases} \text{pred}(0) = 0 \\ \text{pred}(x+1) = x \end{cases}$$

$$\left\{ \begin{array}{l} \min(x, 0) = x \\ \min(x, y+1) = \text{pred}(\min(x, y)) \end{array} \right.$$

f) Sign(x)

$$\text{Sign}(x) = \begin{cases} \text{Sign}(0) = 0 \\ \text{Sign}(x+1) = \pm 1 \end{cases}$$

$$\text{Sign}(x) = \begin{cases} 1 & \text{Si } x > 0 \\ 0 & x = 0 \end{cases}$$

(+, 0) \rightarrow مسح

g) négation

$$\neg x = \begin{cases} 1 & \text{Si } x = 0 \\ 0 & \text{Sinon} \end{cases}$$

Sign معكوس للـ

$$\begin{aligned} \text{Sign}(3) &= 1 \\ \text{négation}(1) &= 0 \end{aligned}$$

$$\neg x = 1 - \text{Sign}(x) = \min(1, \text{Sign}(x))$$

موجول رفع بجوار في
négation معبر و ساش

b) ou (U)

$$\text{ou}(x, y) = \begin{cases} \text{ou}(x, 0) = \text{Sign}(x) \\ \text{ou}(0, y+1) = 1 \end{cases}$$

لما يكون الناتج في

(0, 0) فالـ

| x | v | y |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

$$\text{ou}(x, y) =$$

$$\text{plus}(\text{Sign}(x), \text{Sign}(y))$$

Sol 2

$$\text{ou}(x, y) = \text{Sign}(\text{plus}(x, y))$$

$$\text{ou}(1, 0) = \text{plus}(1, 0) = 1 = \text{Sign}(1) = 1$$

$$\text{ou}(0, 0) = \text{plus}(0, 0) = 0 = \text{Sign}(0) = 0$$

i) ET (n)

$$\text{ET}(x, y) = \begin{cases} \text{ET}(0, y) = 0 \\ \text{ET}(n+1, y) = \text{Sign}(y) \end{cases}$$

| x | n | y |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 1 |

Sol 2

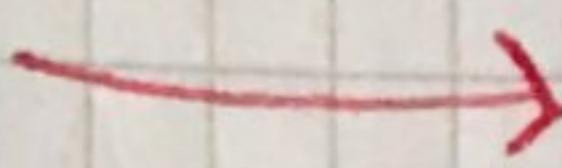
$$\text{ET}(x, y) = \text{Mult}(\text{Sign}(x), \text{Sign}(y))$$

$$x = 1 \quad y = 0$$

$$\text{Sign}(1) = 1 \quad \text{Sign}(0) = 0$$

$$\text{Mult}(1 \cdot 0) = 0 \quad \text{ET}(1, 0) = 0.$$

j)



$$\text{DC} \rightarrow y = \text{True}$$

$$\text{imp}(x,y) = \text{Or}(\neg(x),y)$$

R)



$$x \neq y \left\{ \begin{array}{ll} 1 & \text{Si } x < y \\ 0 & \end{array} \right.$$

إذا كان x أكبر من y
· 0 يكون x يساوي y

$$\text{Sign } x \left\{ \begin{array}{ll} 1 & \text{Si } x > 0 \\ 0 & \text{Sinon} \end{array} \right.$$

$$\text{inf}(x,y) = \text{Sign}(\text{minus}(y,x)).$$

exemple

$$*\text{inf}(5,7) = \text{Sign}(7-5) = \text{Sign}(2) = 1$$

donec $\text{DC}\{y \rightarrow 1 \rightarrow \text{True}$

$$*\text{inf}(8,2) = \text{Sign}(2-8) = \text{Sign}(0) = 0$$

donec $\text{DC}\{y \rightarrow 0 \rightarrow \text{False}$

f)

$$\text{Sup}(x,y) \left\{ \begin{array}{ll} 1 & \text{Si } x > y \\ 0 & \end{array} \right.$$

يكون x يساوي y
negation sign

$$\text{Sup}(x,y) = \text{Sign}(\text{minus}(x,y))$$

$$\text{Sup}(x,y) = \text{negation}(\text{minus}(y,x))$$

m) <

نحوه واحد بالـ

y < x \Leftrightarrow $x \in y + 1$

$$x \in y \left\{ \begin{array}{l} 1 \text{ Si } (y+1) \times > 0 \\ 0 \end{array} \right. \quad \begin{array}{l} x \in y + 1 \\ (y+1) - x > 0 \end{array}$$

$$\text{inf-eg}(x, y) = \text{Sign}(\text{minus}(\text{succ}(y), x))$$

n) >

$$\text{Superg}(x, y) \left\{ \begin{array}{l} 1 \text{ Si } x > y \rightarrow x+1 > y \\ 0 \end{array} \right. \quad \begin{array}{l} (x+1) - y < 0 \end{array}$$

$$\text{Superg}(x, y) = \text{Sign}(\text{minus}(\text{succ}(x), y))$$

o) =

$$\text{eg}(x, y) = \text{Et}(\text{Superg}(x, y), \text{infeg}(x, y))$$

= عددي $\{ , \}$ نحوي

$$(<) \cup (>) = (=)$$

D)

division

$$3 \times y \leq x$$

وكل ما يزيد عن 3 يكتب

$$y \times (3+1) > x$$

$$\text{Div}(x,y) = N3 | y \times (3+1) > x$$

$$\begin{array}{c|c} x & y \\ \hline R & 3 \end{array}$$

$$x = 7 \quad y = 2$$

$$2 + (0 + 1) = 2 \rightarrow f$$

$$2 \times (1 + 1) = 4 \rightarrow f$$

$$2 \times (3 + 1) = 8 \rightarrow v$$

$$2 + 0 + 3 = 3$$

صيغة مقاييس

لي يتحقق $N3$

B)

reste de division

$$R = x - (y \times z)$$

$$z = \text{div}(x,y)$$

$$\text{rem}(x,y) = \text{minus}(x, \text{mult}(y, \text{div}(x,y)))$$

R)

la division parfait

$$\text{rcl}(x,y) = \begin{cases} 1 & \text{si } \text{rem}(x,y) = 0 \\ 0 & \text{autre cas} \end{cases}$$

$$\text{neg}(x) = \begin{cases} 1 & \text{si } x = 0 \\ 0 & \text{autre cas} \end{cases}$$

$$x/y = \text{neg}(\text{rem}(x,y))$$

x/y divisible

reste $R = 0$, j

$$5/2 \quad R = 1$$

$$7/1 = 0$$

donc $x/y = 0$

S)

Paire

y = 2

Paire($x1 = \text{neg}(\text{rem}(x, 2))$)

Remarque

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots + \frac{x^n}{n!}$$

fonction est composé d'autre fonctions qui recursive
factoriel, plus, puissance