A few things that you know from earlier courses in probability theory

- If X is a random variable (r.v.) then

$$\operatorname{Var} X := E\left[(X - E[X])^2\right] = E[X^2] - (E[X])^2 \geq 0$$

(this is the variance of X).

- $\operatorname{Var}(kX) = k^2 \operatorname{Var}(X)$ (if $k \in \mathbb{R}$)
- If X, Y are independent r.v.'s, then

$$E[XY] = E[X]E[Y] \quad \text{and}$$

$$\operatorname{Var}(X+Y) = \operatorname{Var}(X) + \operatorname{Var}(Y)$$

(Thus $\operatorname{Cov}(X,Y) := E[XY] - E[X]E[Y] = 0$ for independent r.v.'s)

- Partial converse: If X, Y are normally distributed and $\operatorname{Cov}(X,Y) = 0$, then X, Y are independent.
- Central limit theorem: If $X_1, X_2, \ldots$ are i.i.d. (independent and identically distributed) with $\operatorname{Var}(X_1) < \infty$, then

$$P\left(\frac{\sum_{k=1}^{n} X_k - n E[X_1]}{\sqrt{n} \sqrt{\operatorname{Var} X_1}} \leq a \right) \longrightarrow \int_{-\infty}^{a} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \, dx$$

as $n \to \infty$.