Probability and Martingales, 1MS045

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The purpose of this course is to familiarise the participants with key concepts and results in probability theory, especially the theory of martingales, with applications in finance.

1 Assessment

The grade for this course is divided into two parts:

- **Homework assignments (4hp):** two sets of assignments. First assignment published 30 September, due 18 October. Second assignment published 27 November, due 18 December. At least 45% on each assignment required to pass (grade G). For students who do not pass, there will be an additional assignment at the end of the semester.
- **Final exam (6hp)** on 13 January. The exam consists of eight questions (worth 5 points each); approximately 50% theory (definitions, theorems, proofs) and 50% problem solving (similar to problem sessions and assignments). The exam follows the department's standard grading scale: 0–17 grade U; 18–24 grade 3; 25–31 grade 4; 32–40 grade 5. Allowed tools: pen, paper, non-programmable calculator.

2 Lecture plan

2.1 Period 1

3 Sep Measure Spaces [Wil91, §1]

5 Sep Events [Wil91, §2]

11 Sep Random variables [Wil91, §3]

13 Sep Independence [Wil91, §4]

16 Sep Problem Session

18 Sep Integration [Wil91, §5]

30 Sep Expectation I [Wil91, §6]

2 Oct Expectation II, strong law [Wil91, §6,§7]

9 Oct Product measure, conditional expectation I [Wil91, §8,§9]

11 Oct Problem Session

15 Oct Conditional expectation II [Wil91, §9]

17 Oct Martingales I [Wil91, §10]

21 Oct Martingales II [Wil91, §10]

22 Oct Problem session

28 Oct Convergence theorem, $\mathcal{L}^2$ martingales [Wil91, §11,§12]

2.2 Period 2

Dates to be finalised.

- Uniform integrability [Wil91, §13]
- UI martingales [Wil91, §14]
- Problem session
- Pricing and arbitrage [EK05, §1.1–§1.4]
- Discrete markets [EK05, §1.5–§1.6, §2.1–§2.2]
- Martingale measures I [EK05, §2.3–§2.4]
- Martingale measures II [EK05, §2.5–§2.7]
- The first fundamental theorem [EK05, §3.1–§3.2]
- Complete markets [EK05, §4.1–§4.3]
- Problem session

3 Textbooks

The references above refer to the two main textbooks *Probability with martingales* [Wil91] and *Mathematics of financial markets* [EK05]

References

[Wil91] *D. Williams*, Probability with martingales. Cambridge etc.: Cambridge University Press (1991; Zbl 0722.60001)

[EK05] *R. J. Elliott* and *P. E. Kopp*, Mathematics of financial markets. 2nd ed. New York, NY: Springer (2005; Zbl 1140.91032)