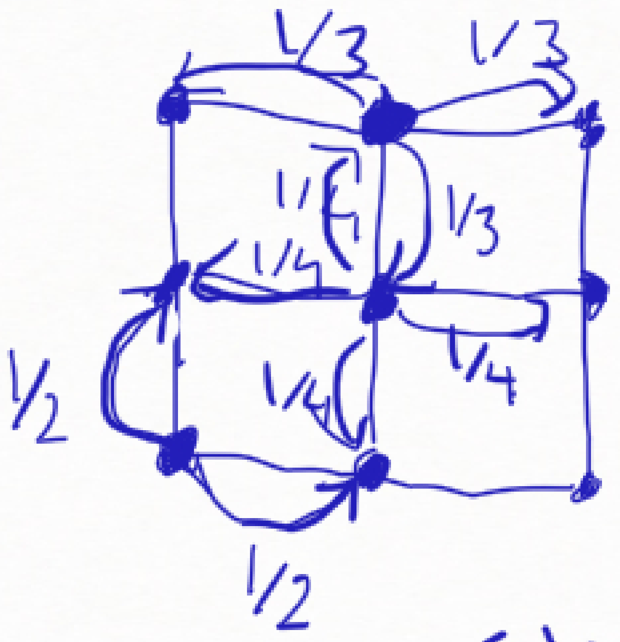
Basic ex. 11



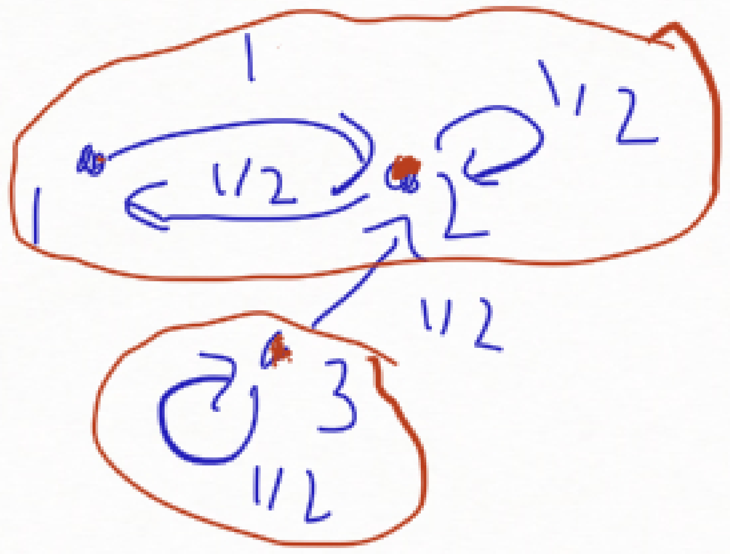
Let X_0 = starting position

$$P(X_0 = \text{"corner"}) = \frac{4}{9}$$
$$P(X_0 = \text{"side"}) = 4/9$$
$$P(X_0 = \text{"mid"}) = 1/9$$

$$\begin{aligned} P(X_3 = \text{"mid"}) &= \overbrace{P(X_3 = \text{"mid"} \mid X_0 = \text{"corner"})}^{=0} P(X_0 = \text{"corn"}) \\ &+ P(X_3 = \text{"mid"} \mid X_0 = \text{"side"}) P(X_0 = \text{"side"}) \\ &+ \underbrace{P(X_3 = \text{"mid"} \mid X_0 = \text{"mid"})}_{=0} P(X_0 = \text{"mid"}) \end{aligned}$$

$$= \underbrace{\left(\frac{1}{3} \cdot 1 \cdot \frac{1}{3} + \frac{2}{3} \cdot 1 \cdot \frac{1}{3}\right)}_{1/3} \cdot \frac{4}{9} = \frac{4}{27}$$

Basic ex. 12



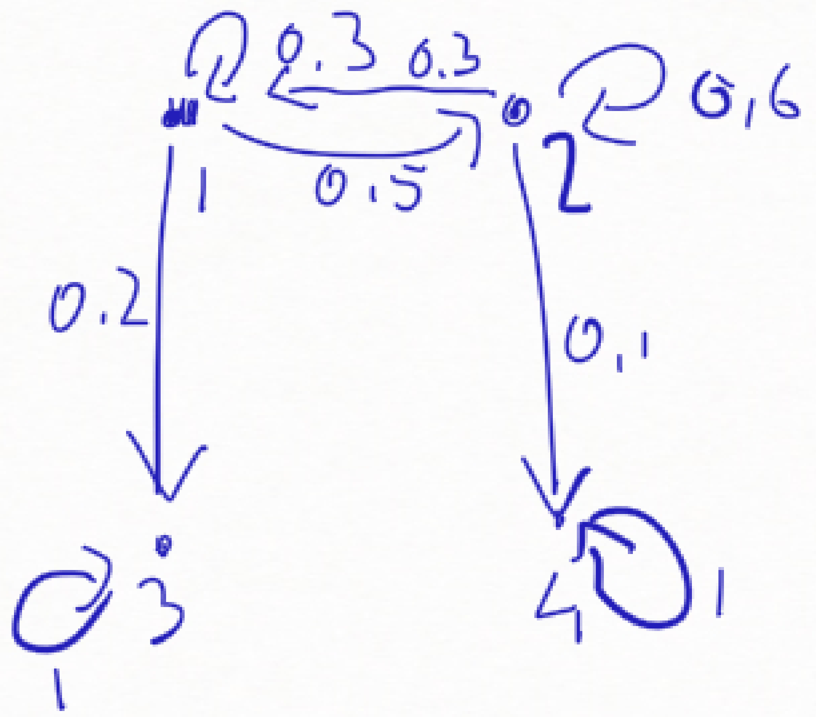
$S = \{1,2,3\} = \underbrace{\{1,2\}}_{C_1} \cup \underbrace{\{3\}}_{T}$

If $\tilde{P} = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ then $\tilde{\pi}\tilde{P} = \tilde{\pi}$ has prob. vector solution $\tilde{\pi} = \left(\frac{1}{3}, \frac{2}{3}\right)$

so $\pi P = \pi$ has prob. vect. solution

$\pi = \left(\frac{1}{3}, \frac{2}{3}, 0\right)$

Basic 14 ex.



By cond. on the outcome of first jump:

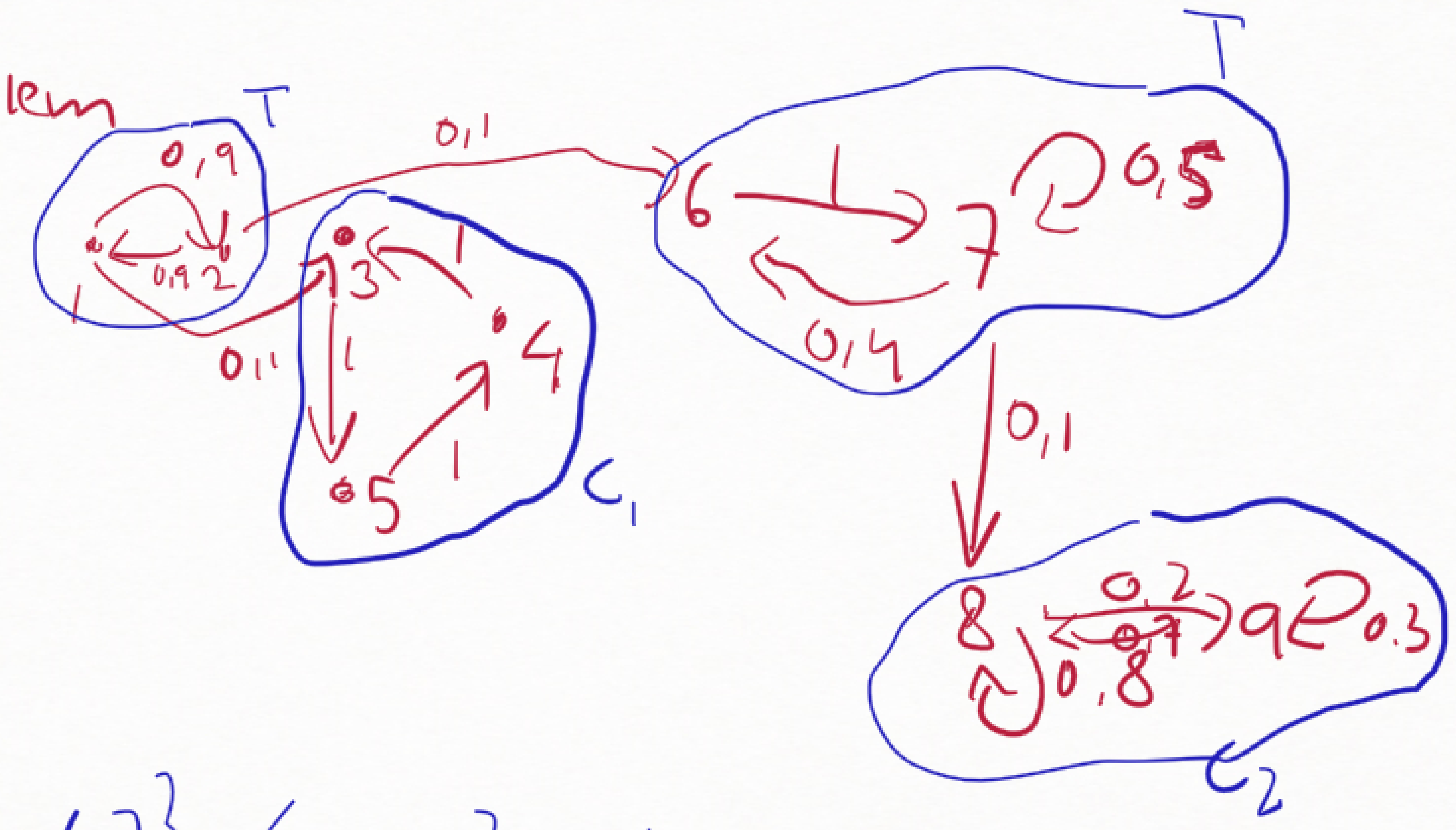
$$\begin{cases} P_{3|1} = \underbrace{0.2 \cdot 1}_{0.2} + 0.3 \cdot P_{3|1} + 0.5 P_{3|2} \\ P_{3|2} = \underbrace{0.1 \cdot 0}_{0} + 0.3 P_{3|1} + 0.6 P_{3|2} \end{cases}$$

Solving this system of linear eq. gives

$$P_{3|1} = \frac{8}{13} \qquad P_{3|2} = \frac{6}{13}$$

Extra problem

A1



$$S = \underbrace{\{1,2,6,7\}}_{T} \cup \underbrace{\{3,4,5\}}_{C_1} \cup \underbrace{\{8,9\}}_{C_2}$$

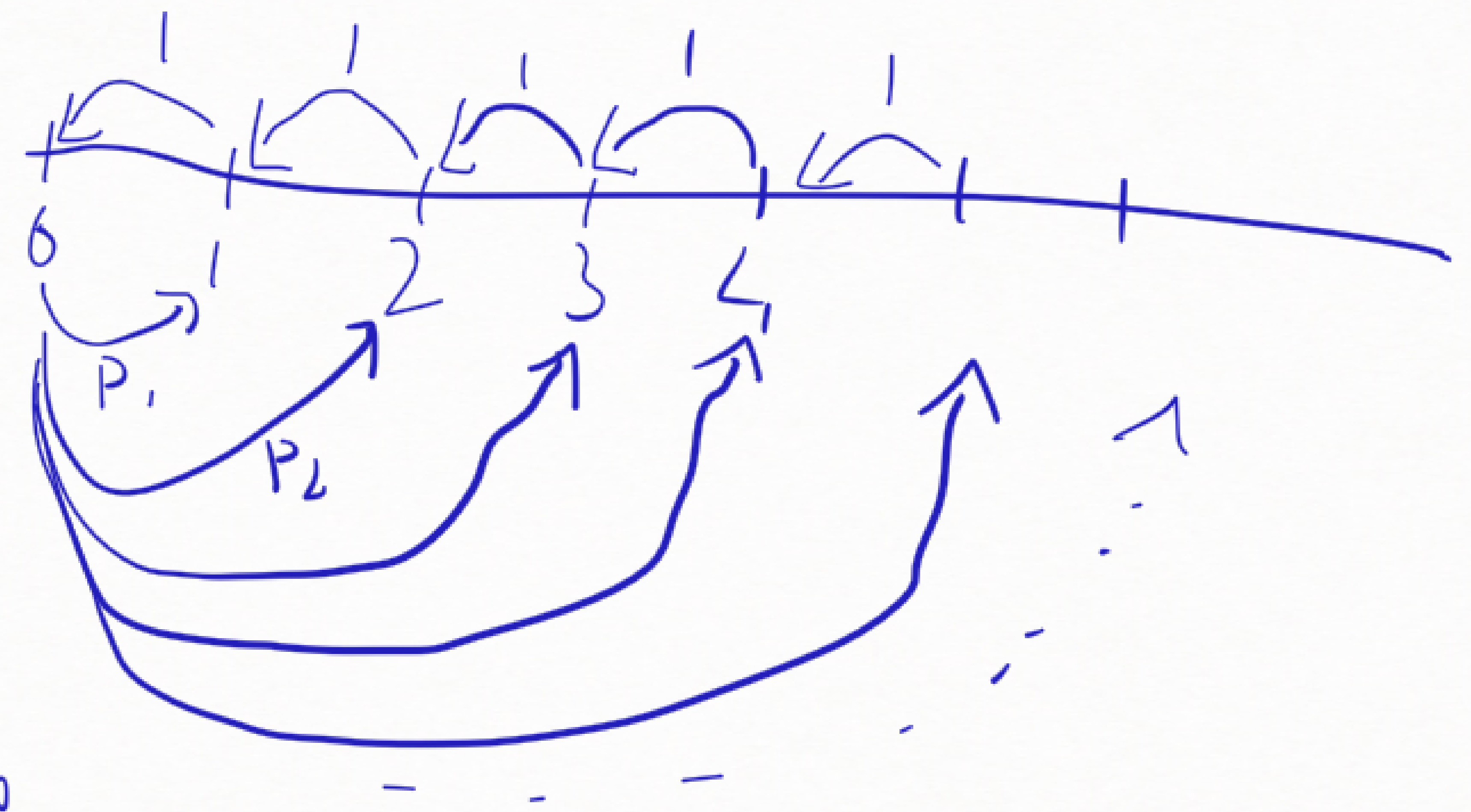
Set of aperiodic states

All states in C_1 have the same period

$d(3) = d(4) = d(5) = \gcd(3,6,9,12,\dots) = 3$

All states in C_2 -||- $d(8) = d(9) = \gcd(1,2,\dots) = 1$

Lawler 2.2



$\left(\sum_{i=1}^{\infty} p_i = 1\right)$

$p_i > 0$

Let T first return time to state 0

Then $P(T=k) = p_{k-1}$ $k \geq 2$ so

$E(T) = \sum k P(T=k) = 2p_1 + 3p_2 + \dots$

$= \underbrace{(p_1 + 2p_2 + 3p_3 + \dots)}_{E(X) \text{ if } P(X=i)=p_i} + \underbrace{(p_1 + p_2 + \dots)}_{1}$

$E(T) < \infty \iff E(X) < \infty$

$$\sum k p_k < \infty \Rightarrow E(T) < \infty$$

and by irreducibility it follows that the MC is pos. rec. under this condition

Thus a stat. dist $\exists$.
where $\pi = (\pi_0, \pi_1, \dots)$ solves

$$\pi_k = \sum_j \pi_j P_{jk} = \pi_0 P_k + \pi_{k+1}$$

so

$$\pi_n = \pi_{n-1} - \pi_0 P_{n-1} = \dots = \underbrace{\pi_0 - \pi_0 \sum_{k=1}^{n-1} p_k}_{\underbrace{\pi_0}_{\frac{1}{E(T)}}\Big(\underbrace{\sum_{k=1}^{\infty} p_k - \sum_{k=1}^{n-1} p_k}_{\sum_{n} p_k}\Big)}$$