UPPSALA UNIVERSITY
Dept of Mathematics
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Matematical statistics
Inference II 1MS037
2023–11–28

Hand-in assignment 2

There are three compulsory home assignments. You may hand in the assignments in groups of two students. To pass the assignment, 10p are needed. Solutions should be well motivated. Hand in your solutions via Studium *as a single pdf file.* If you have worked together with somebody else, it is enough to hand in for one of you as long as you write both names.

The deadline for this assignment is December 5.

1. Suppose we have a random sample $(x_1, x_2, x_3) = (1, 11, 12)$ from a distribution with expectation θ. Estimate θ by using

   (a) the least squares estimate. (0.5p)

   (b) the least absolute value. (0.5p)

   (c) the trimmed mean with $k = 3$ (see example 4.13). (2p)

   (d) the Winzorized mean with $k = 3$ (see example 4.14). (2p)

2. Suppose we have one observation of the continuous random variable X, with density function

   $$f(x) = \beta^{-2} x \exp\left(-\frac{x}{\beta}\right),$$

   for $x \geq 0$ and 0 otherwise, with $\beta > 0$. Consider the estimator $T(X) = X^2/6$ of the parameter $\theta = \beta^2$.

   Hint: Without proof, you may use that $E(X^k) = (k+1)!\beta^k$ for $k = 1, 2, \ldots$.

   (a) Show that $T(X)$ is unbiased for θ. (1p)

   (b) Is $T(X)$ efficient for θ? Motivate your answer. (4p)

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3. Suppose that $X_1, ..., X_n$ are independent Bernoulli variables with parameter p, i.e. $P(X_i = 1) = p = 1 - P(X_i = 0)$ for $i = 1, ..., n$. Suppose we want to estimate

$$\gamma(p) = P(\cap_{i=1}^{n-1} I\{X_i > X_n\}),$$

where $I\{A\} = 1$ if A is true and 0 otherwise. Hence, $\gamma(p)$ is the probability that all $X_1, ..., X_{n-1}$ are strictly greater than X_n.

(a) Show that $U = I\{\cap_{i=1}^{n-1}\{X_i > X_n\}\}$ is an unbiased estimator of $\gamma(p)$.(1p)

(b) Use the Rao-Blackwell theorem to construct an unbiased estimator of $\gamma(p)$ with smaller variance than U. (3p)

(c) Give the variances of U and of the estimator i (b) explicitly, to verify that the latter estimator has the lowest variance among the two. (1p)

4. Consider a random sample $\mathbf{X} = (X_1, ..., X_n)$ where the X_i are Exponentially distributed with intensity β, i.e. with density function

$$f(x) = \beta \exp(-\beta x), \quad x > 0,$$

and 0 otherwise, with $\beta > 0$. The goal is to estimate $\mu = E(X_i) = 1/\beta$.

(a) Let $T = \sum_{i=1}^{n} X_i$. Show that T is complete and sufficient. (3p)

(b) Make use of the fact that $U = X_1$ is unbiased for μ to construct the best unbiased estimator (BUE) of μ. (2p)