How?

The main option is to write your answers in the boxes in Inspera. As an option, if you need to, you can submit hand-written solutions on paper "the normal way". In part A you are hopefully able to answer in the Inspera only, whereas there might a little bit more math to write in part B. If you submit paper-answers, make a note about it in Inspera in the corresponding question.

You can write the answers in swedish or in english.

Grades

The exam is divided into two parts, part A and part B. Part A is related to grade 3 and part B to grade 4 and 5. On Part A there are 2 question related to each goal and maximum 2 points per question. In part B there are maximum 6 point per question.

The grades are

- **grade 3**
  You must at least solve one question on each goal. This corresponds minimum 6 points, distributed among the three goals (minimum 2 points per goal).
- **grade 4**
  You must fullfil grade 3 + at least completely solve one of the two questions in part B. This corresponds to minimum 6 points on part B.
- **grade 5**
  You must fullfil grade 3 + completely or with minor errors solve the two questions in part B. This corresponds to minimum 10-11 points on part B.

Tools available

The tools available in Inspera is

- Online Python
  The idea is that you are able to use Python as a pocket calculator, very much in the same way as on the problem solving sessions. It will be enough to be able to use **numpy**.)
- Numpy Cheat Sheet is available as a link
- Numpy reference manual is available as a link
- Formula sheet

You should be able to find these resources at the bottom part of Inspera.

Tools to take with you

Pocket calculator is allowed, so you can bring it to the exam if you want to. It's not necessary though, you can use Python instead.

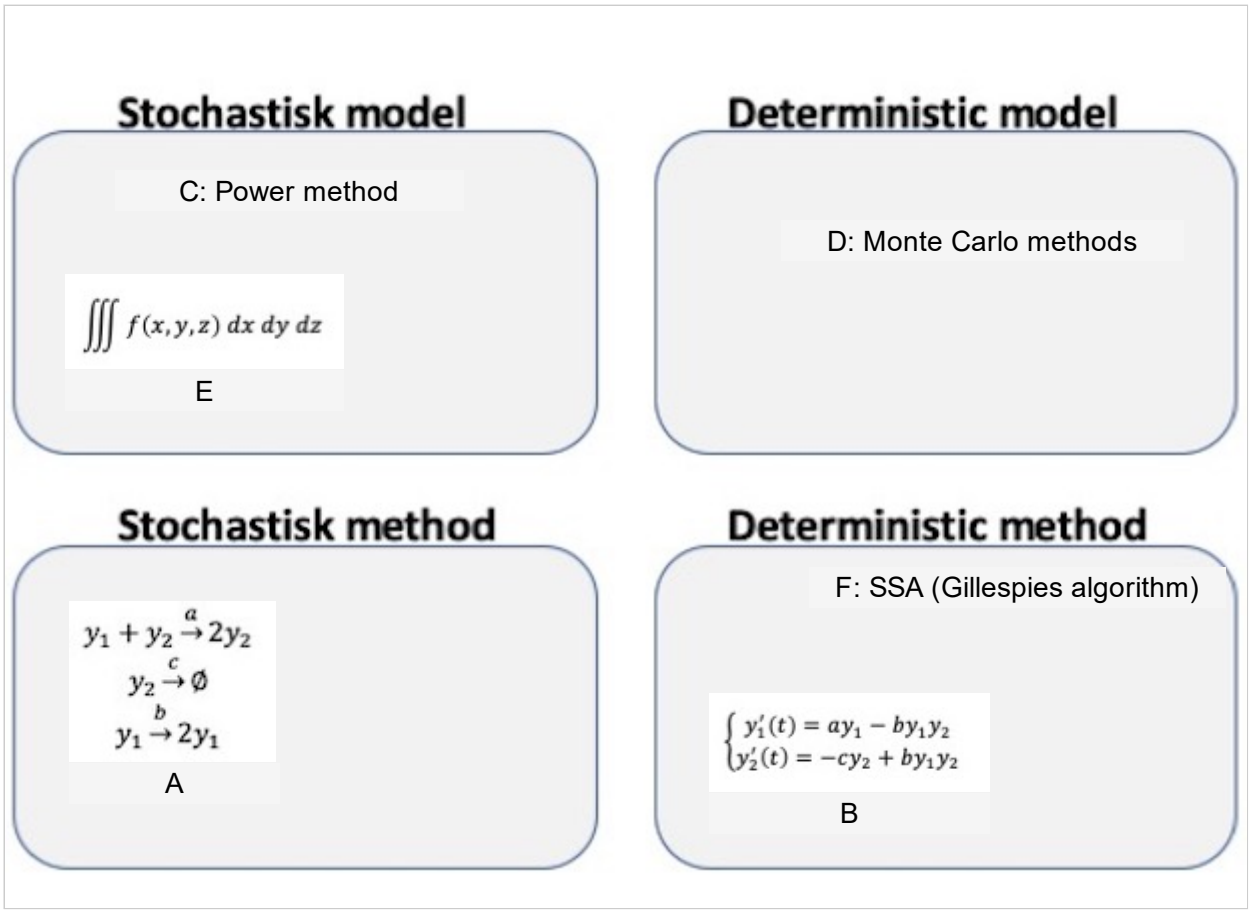
Good Luck!

1 (Concepts)

In the rectangles A-F below, you'll find text/formulas that are examples of either a stochastic model/method or a deterministic model/method.

Unfortunately they have been placed in the areas a bit randomly (!). Drag and drop the rectangles so that they end up in the right area.

Drag and drop to the right place



Maximum marks: 2

2 Below is four different definitions and a number of concepts. Drag and drop the correct concept corresponding to the each definition.

Help

a) _____ describes the orthogonalization of a matrix

b) _____ describes diffusion of particles

c) The calculation $x^{+}=A^{+}b$ result in a _____

d) A method that generates a sequence of approximations that (hopefully) converges to the correct solution is called _____

Singular value decomposition	Brownian motion	Condition number
Iterative method	Similarity transformation	QR-factorization
Least Squares solution	Order of accracy	Rank-1 matrix
Normal equations	Markov process	

Maximum marks: 2

3 (Algorithm)

A matrix *A* has the QR-factorization

$$Q = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad R = \begin{pmatrix} -2 & -3 & -2 \\ 0 & -5 & 2 \\ 0 & 0 & -4 \end{pmatrix}.$$

Given a right-hand-side

$$y = \begin{pmatrix} 2 \\ -2 \\ 8 \\ -4 \end{pmatrix}$$

solve the least squares problem $Ax = y$ using the QR-factorization and without forming the matrix *A* explicitly.

Solve the problem by hand *following the algorithm* that would be used by a computer. It is important that you follow the steps in the algorithm (correct answer but with a less good algorithm will result in no points).

Here, it is possible to fill in your solution in the answer box, without too much advanced math symbols.

Fill in your answer here

Maximum marks: 2

4 (**Algorithm**)

In order to show that you understand the Monte Carlo algorithm, formulate the Monte Carlo-algorithm applied to the problem $I = \int_a^b f(x)\, dx$.

Use the formulation to solve the integral when $f(x) = \frac{1}{1+0.5x^2}$ and the interval $a = 1$ and $b = 2$.

Use five random numbers to solve the problem, and use Python to get the random numbers.

Note, you must formulate the problem as a Monte Carlo-problem (see formula sheet). Solve the integral by hand according to the algorithm formulation. Use Python as a pocket calculator. Write down the numbers you get and the final answer. You can round the numbers to 2 or 3 decimals.

Fill in your answer here

Maximum marks: 2

5 **(Analysis)**

Assume you get the result I in the previous Monte Carlo integration question, and you would like to have the integral value + an error estimate (error range) of the form $I \pm |e|$. Explain how you would be able to achieve this.

Fill in your answer here

Maximum marks: 2

6 (Analysis)
A matrix has the SVD $A = U\Sigma V^T$, where

$$\Sigma = \begin{pmatrix} 420.0 & 0 & 0 & 0 \\ 0 & 45.1 & 0 & 0 \\ 0 & 0 & 9.3 & 0 \\ 0 & 0 & 0 & 1.2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Based on this information, answer the following questions:

What is rank(A)?

- 0
- 1
- 2
- 3
- 4
- Not possible to decide based on the given information

What is rank(A^TA)?

- 0
- 1
- 2
- 3
- 4
- Not possible to decide based on the given information

What is rank(AA^T)?

- 0
- 1
- 2
- 3
- 4
- Not possible to decide based on the given information

If we form $A_2 = \sigma_1 u_1 v_1^T + \sigma_2 u_2 v_2^T$, what is $\|A - A_2\|_2$?

- 420.0
- 45.1
- 9.3
- 1.2
- 10.5
- 55.6
- Not possible to decide based on the given information

Maximum marks: 2

7 (Part B, higher grade)

The table below contains data about blood pressure, age and weight in a population (a small population here for simplicity).

Systolic blood pressure	Age (years)	Weight (kg)
132	52	78
143	59	83
153	67	88
162	73	96
154	64	89
168	74	100
137	54	85
149	61	85
159	65	94
128	46	76
166	72	98

It is well known that high blood pressure is one of the most important risk factors for cardiovascular disease. The question here is how strong the relationship is between high blood pressure and age and weight?

Under the assumption that there is a linear relationship between blood pressure (the dependent variable) the two variables age and weight, set up and solve this problem. This involves

- to choose appropriate ansatz, and form the matrix.
- to discuss what method that is suitable in this case, and to solve the problem with the chosen method. Your discussion should contain pros and cons with the different solution methods.

You can use Python when you do the calculations. You can either give the solution in the answer box or submit a hand written solution.

Fill in your answer here

Maximum marks: 6

8 (Part B, higher grade)

When we compute $A = U\Sigma V^T$ on a computer, so called iterative methods are the only realistic option. The QR-method (QR-iteration) is one method that is commonly used. The method is based on so called similarity transformations $B = VAV^{-1}$and the Schur decomposition $A = QSQ^T$.

Explain how the QR-method can be used in the SVD-computation, and how the method works on a principal level. Your explanation must include

- in what way the QR-method is an iterative method, for example what is the successive approximation we get in this case (for example , what is the approximation at iteration number *k*)
- how the the concepts of Schur decomposition and similarity transformations come in here
- how the iterative method could be used to find the SVD

Here, you can probably present your answer in the answer box (it is not too much math symbols). Optionally, you can of course submit a hand written answer.

Fill in your answer here

Maximum marks: 6