Uppsala University
Department of Economics
Fredrik Armerin
070 – 949 83 97 (Mikael Bask)

Exam number:

Exam in Master / Financial Theory

General instructions

- No technical aids are allowed.
- All calculations should be clearly motivated.
- Do not skip steps in the formal derivations.
- Answer the questions without providing additional / unrelated information. I deduct points for incorrect statements you make.
- If you cannot solve a question without making additional assumptions, state these assumptions clearly and explain in writing why they are necessary.
- The writing time is 5 hours. Write your examination number in the indicated space and on all papers you hand in.
- The total number of points is 50. For grade E is 25 points required, for grade D is 27.5 points required, for grade C is 32.5 points required, for grade B is 37.5 points required and for grade A is 45 points required.

Good luck!

Problem 1

a) Write down the equation for the capital market line (CML) and define its different parts. (2 p)

b) How is the excess return HML in the Fama-French model defined?(2 p)

c) Determine the coefficient of relative risk aversion for an investor with utility function $u(x) = 3 + \ln x$. (2 p)

d) Three assets has the following mean and standard deviation of their rate of return:

Asset no i	$E[r_i]$	$\mathrm{Std}[r_i]$
1	0.2	0.5
2	0.32	0.4
3	0.15	0.05

Can all three assets be on the mean-variance frontier? Motivate your answer. (2 p)

e) In prospect theory, what is the form of the utility function to the left and to the right of the reference point, respectively? You may answer by drawing and explaining a graph. (2 p)

Problem 2

a) The rate of return of two assets have the following standard deviations:

$$\sigma_1 = 0.2 \text{ and } \sigma_2 = 0.4.$$

Determine the variance of the minimum variance portfolio if the rate of returns are uncorrelated and shortselling is not allowed. (2 p)

b) Consider the factor model

$$r_i = E[r_i] + \beta_{i1}\big(F_1 - E[F_1]\big) + \beta_{i2}\big(F_2 - E[F_2]\big) + \varepsilon_i.$$

Write down the equation for the expected return $E[r_i]$ if there exists a risk-free asset and the conditions of the APT holds. (2 p)

c) What is the difference in the definitions of the Macauly and Fisher-Weil duration? You need not derive any expressions – only explain the difference in how they are defined. (2 p)

d) Explain why we refer to the two expressions

$$r_{t+1} = \frac{P_{t+1} + D_{t+1} - P_t}{P_t} \quad \text{and} \quad r_{t+1} = E_t \left[\frac{P_{t+1} + D_{t+1} - P_t}{P_t} \right]$$

as being ”ex post” and ”ex ante” respectively. (2 p)

e) Assume that the rate of return r of a stock satisfies

$$E[r] = \alpha + r_f + \beta\big(E[r_m] - r_f\big),$$

where r_f is the risk-free rate of return, r_m is the rate of return of the market portfolio and β is the beta value in CAPM. Which sign does α have if the stock is overpriced? Motivate your answer.

Problem 3

There are N risky assets and the rate of return vector $\boldsymbol{r} = (r_1, r_2, \ldots, r_N)^T$ has

$$E[\boldsymbol{r}] = \boldsymbol{\mu} \quad \text{and} \quad \mathrm{Var}[\boldsymbol{r}] = \Sigma.$$

There is also a risk-free asset with rate of return r_f and we assume that the conditions for CAPM to hold are satisfied. Let $\boldsymbol{\pi}$ be a vector of portfolio weights in the risky assets.

a) Show that the excess return $E\left[r_p\right] - r_f$ of the portfolio is given by

$$E\left[r_p\right] - r_f = \beta_p \, SR_m \, \sigma_m,$$

where β_p is the beta of the portfolio, SR_m is the Sharpe ratio of the market portfolio and σ_m is the market portfolio's volatility. (5 p)

Now assume that the rate of return vector is normally distributed.

b) Find the optimal portfolio weights if the utility of having the portfolio with rate of return $\boldsymbol{\pi} \cdot \boldsymbol{r}$ is given by

$$E[\boldsymbol{\pi} \cdot \boldsymbol{r}] - \frac{\gamma}{2}\mathrm{Var}[\boldsymbol{\pi} \cdot \boldsymbol{r}]$$

for a constant $\gamma > 0$, i.e. solve the problem

$$\begin{array}{ll} \max\limits_{\boldsymbol{\pi}} & \boldsymbol{\pi} \cdot \boldsymbol{\mu} - \frac{\gamma}{2}\boldsymbol{\pi} \cdot \Sigma\boldsymbol{\pi} \\ \text{s.t.} & \boldsymbol{\pi} \cdot \mathbf{1} = 1. \end{array}$$

(5 p)

Problem 4

The expected rate of return of a firm's stock is $E[r] = 0.12$. There is a risk-free asset with rate of return $r_f = 0.03$ and the expected return of the market portfolio is $E[r_m] = 0.15$.

a) How large is the beta value of this firm according to the CAPM? (2 p)

The firm uses a plowback ratio of $b = 0.2$ and has return on equity $r_e = 0.40$. The earnings per share today, at time t, is $e_t = 6$ euros.

b) Calculate and give an economic interpretation of the present value of growth opportunities (PVGO). (4 p)

c) How large is the growth rate of the firm's dividends? (2 p)

d) How large is the firm's price-earnings ratio? (2 p)

Problem 5

On a bond market, the following two coupon bond's are trading.

Time to maturity (years)	Coupon rate	Yield to maturity
1	q_1	y_1
2	q_2	y_2

Let y_1^Z and y_2^Z denote the 1 and 2 year zero coupons bond yields implied by the two traded bonds.

a) Determine y_1^Z in terms of q_1, q_2, y_1 and y_2. (3 p).

b) Show that the forward rate $f_{1,2}$ satisfies

$$1 + f_{1,2} = \frac{\left(1 + y_2^Z\right)^2}{1 + y_1^Z}.$$

(2 p)

c) Now assume that there also exists a coupon bond with time to maturity 4 years, coupon rate q_4 and yield to maturity y_4. Given all three bonds, is it possible to uniquely determine zero coupon bond yields for 3 and/or 4 years? Motivate your answer. (5 p)