Formelsamling för kursen Sannolikhetsteori I, 1MS034

Sannolikhetsteori

Nedan betecknar μ väntevärdet och σ standardavvikelsen i fördelningarna. $\psi(t) := E(e^{tX})$ betecknar den momentgenererande funktionen.

Diskreta fördelningar

Bernoullifördelning

$X \sim \mathrm{Be}(p)$ om $p(1) = p$ och $p(0) = q := 1 - p$ för $0 \leq p \leq 1$.
$\mu = p, \quad \sigma^2 = pq, \quad \psi(t) = q + pe^t$.

Binomialfördelning

$X \sim \mathrm{Bin}(n, p)$ om $p(k) = \binom{n}{k} p^k \, q^{n-k}, \quad k = 0, 1, 2, \ldots, n.$
$0 \leq p \leq 1, q = 1 - p, \quad \mu = np, \quad \sigma^2 = npq, \quad \psi(t) = (q + pe^t)^n$.

Hypergeometrisk fördelning

$X \sim \mathrm{Hyp}(N, n, m)$, eller $X \sim \mathrm{Hyp}(N, n, p)$ med $p = m/N$, om

$$p(k) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}} = \frac{\binom{Np}{k}\binom{Nq}{n-k}}{\binom{N}{n}}, \text{ för } k = 0, 1, \ldots, n$$

(för de k som är möjliga; k får t.ex. inte överstiga $m = Np$).
$0 \leq p \leq 1, \quad q = 1 - p, \quad \mu = np = nm/N, \quad \sigma^2 = npq \, \dfrac{N-n}{N-1}$.

Poissonfördelning

$X \sim \mathrm{Po}(\lambda)$ om $p(k) = \dfrac{\lambda^k}{k!}\, e^{-\lambda}, \quad k = 0, 1, 2, \ldots, \quad \lambda \geq 0.$

$\mu = \lambda, \quad \sigma^2 = \lambda, \quad \psi(t) = e^{\lambda(e^t - 1)}.$

Geometrisk fördelning

$X \sim \mathrm{Geo}(p)$ om $p(k) = pq^k, \quad k = 0, 1, 2, \ldots.$

$0 < p \leq 1, \quad q = 1 - p, \quad \mu = \dfrac{q}{p}, \quad \sigma^2 = \dfrac{q}{p^2}, \quad \psi(t) = p/(1 - qe^t).$

För första gången-fördelning

$X \sim \mathrm{ffg}(p)$ om $p(k) = pq^{k-1}, \quad k = 1, 2, 3, \ldots.$

$0 < p \leq 1, \quad q = 1 - p, \quad \mu = \dfrac{1}{p}, \quad \sigma^2 = \dfrac{q}{p^2}, \quad \psi(t) = pe^t/(1 - qe^t).$

Kontinuerliga fördelningar

Rektangelfördelning (Kontinuerlig likformig fördelning)

$X \sim \mathrm{Re}(a, b)$ om $f(x) = \dfrac{1}{b - a}, \quad a \leq x \leq b.$

$\mu = \dfrac{a + b}{2}, \quad \sigma^2 = \dfrac{(b - a)^2}{12}, \quad \psi(t) = \dfrac{e^{bt} - e^{at}}{t(b - a)}.$

Γ*-fördelning*

$X \sim \Gamma(p, \beta)$ om $f(x) = \dfrac{\beta^p\, x^{p-1}}{\Gamma(p)} e^{-\beta x}, \quad x \geq 0,$

där $\Gamma(p) = \displaystyle\int_0^\infty x^{p-1} e^{-x}\, dx \quad (= (p - 1)!$ om p är ett heltal.)

$\mu = p/\beta, \quad \sigma^2 = p/\beta^2, \quad \psi(t) = \big(\beta/(\beta - t)\big)^p.$

Exponentialfördelning

$X \sim \mathrm{Exp}(\beta)$ om $X \sim \Gamma(1, \beta)$, dvs. $f(x) = \beta\, e^{-\beta x}, \quad x \geq 0.$

$\mu = 1/\beta, \quad \sigma^2 = 1/\beta^2, \quad \psi(t) = \beta/(\beta - t).$

Normalfördelning

$X \sim \mathrm{N}(\mu, \sigma^2)$ om $f(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$, för $-\infty < x < \infty, \sigma > 0.$

μ är väntevärdet och σ^2 är variansen.

För N(0, 1)-fördelningen gäller att fördelningsfunktionen betecknas med $\Phi(x)$ och kvantilerna med λ_α.

$$\psi(t) = e^{\mu t + t^2\sigma^2/2}.$$

Flerdimensionella fördelningar

Multinomialfördelning

Den r-dimensionella slumpvariabeln $(X_1, \ldots, X_r)$ är multinomialfördelad, med parametrar n och $p_1, \ldots, p_r$, där $p_i \geq 0$, $i = 1, \ldots, r$, och $\sum_1^r p_i = 1$, om

$$p_{(X_1,\ldots,X_r)}(k_1, \ldots, k_r) = \frac{n!}{k_1! \cdots k_r!}\, p_1^{k_1} \cdots p_r^{k_r},$$

för icke-negativa heltal $k_1, \ldots, k_r$ med $\sum_1^r k_i = n$.
För komponenterna X_i, $i = 1, \ldots, n$ gäller att $X_i \sim \mathrm{Bin}(n, p_i)$ och att $C(X_i, X_j) = -np_ip_j$ för $i \neq j$.

Bivariat normalfördelning

$(X, Y) \sim \mathrm{N}(\mu_x, \mu_y, \sigma_x^2, \sigma_y^2, \rho)$ om

$$f_{(X,Y)}(x, y) = C \cdot \exp\left(-\frac{1}{2(1-\rho^2)}\, Q_\rho\left(\frac{x-\mu_x}{\sigma_x}, \frac{y-\mu_y}{\sigma_y}\right)\right),$$

där konstanten C definieras av

$$C := \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}}$$

och den kvadratiska formen Q_ρ av

$$Q_\rho(u, v) := u^2 + v^2 - 2\rho uv.$$

$X \sim \mathrm{N}(\mu_x, \sigma_x^2)$, $Y \sim \mathrm{N}(\mu_y, \sigma_y^2)$ och $\rho(X, Y) = \rho$.

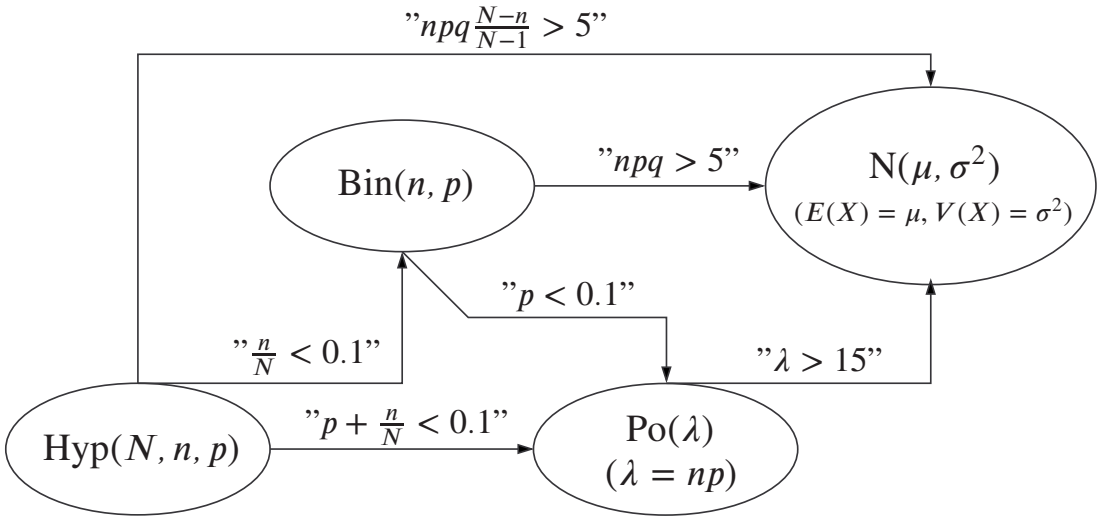
Flerdimensionell likformig fördelning

(X, Y) har en likformig fördelning över området $\Omega \subset \mathcal{R}^2$ om

$$f_{(X,Y)}(x, y) = \frac{1}{|\Omega|}, \quad \text{för } (x, y) \in \Omega,$$

där $|\Omega|$ anger arean av Ω.

Approximationer



Kovarianser

$C(X, Y) = C(Y, X), \quad C(aX + b, Y) = a\,C(X, Y),$
$C(X + Y, Z) = C(X, Z) + C(Y, Z).$

Felfortplantning

$$E(g(X)) \approx g(\mu), \quad V(g(X)) \approx (g'(\mu))^2 V(X).$$

$$\begin{aligned} E(h(X_1, \ldots, X_n)) &\approx h(\mu_1, \ldots, \mu_n), \\ V(h(X_1, \ldots, X_n)) &\approx \sum_i (h_i'(\mu_1, \ldots, \mu_n))^2\, V(X_i) \\ &\quad + 2 \sum_{i<j} h_i'(\mu_1, \ldots, \mu_n)\, h_j'(\mu_1, \ldots, \mu_n)\, C(X_i, X_j), \end{aligned}$$

där $h_i'(\mu_1, \ldots, \mu_n)$, $i = 1, \ldots, n$, betecknar de partiella derivatorna.

Betingade väntevärden

$$\begin{aligned} E(X) &= E(E(X \mid Y)), \\ V(X) &= E(V(X \mid Y)) + V(E(X \mid Y)). \end{aligned}$$

Tabeller

Tabell 1. *Det grekiska alfabetet*

alfa	A	α	iota	I	ι	rho	P	ρ, ϱ
beta	B	β	kappa	K	κ	sigma	Σ	σ, ς
gamma	Γ	γ	lambda	Λ	λ	tau	T	τ
delta	Δ	δ	my	M	μ	ypsilon	Y	υ
epsilon	E	ϵ, ε	ny	N	ν	fi	Φ	ϕ, φ
zeta	Z	ζ	xi	Ξ	ξ	chi	X	χ
eta	H	η	omikron	O	o	psi	Ψ	ψ
theta	Θ	θ, ϑ	pi	Π	π	omega	Ω	ω

Tabell 2. *Binomialfördelningen*

Tabellen ger $F(k) = P(X \leq k)$ då $X \sim \text{Bin}(n, p)$, för $0.05 \leq p \leq 0.5$.
För $p > 0.5$ utnyttjas att $Y := n - X \sim \text{Bin}(n, 1 - p)$.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
2	0	0.9025	0.8100	0.7225	0.6400	0.5625	0.4900	0.3600	0.2500
	1	0.9975	0.9900	0.9775	0.9600	0.9375	0.9100	0.8400	0.7500
3	0	0.8574	0.7290	0.6141	0.5120	0.4219	0.3430	0.2160	0.1250
	1	0.9928	0.9720	0.9392	0.8960	0.8438	0.7840	0.6480	0.5000
	2	0.9999	0.9990	0.9966	0.9920	0.9844	0.9730	0.9360	0.8750
4	0	0.8145	0.6561	0.5220	0.4096	0.3164	0.2401	0.1296	0.0625
	1	0.9860	0.9477	0.8905	0.8192	0.7383	0.6517	0.4752	0.3125
	2	0.9995	0.9963	0.9880	0.9728	0.9492	0.9163	0.8208	0.6875
	3	1.0000	0.9999	0.9995	0.9984	0.9961	0.9919	0.9744	0.9375
5	0	0.7738	0.5905	0.4437	0.3277	0.2373	0.1681	0.0778	0.0312
	1	0.9774	0.9185	0.8352	0.7373	0.6328	0.5282	0.3367	0.1875
	2	0.9988	0.9914	0.9734	0.9421	0.8965	0.8369	0.6826	0.5000
	3	1.0000	0.9995	0.9978	0.9933	0.9844	0.9692	0.9130	0.8125
	4	1.0000	1.0000	0.9999	0.9997	0.9990	0.9976	0.9898	0.9688
6	0	0.7351	0.5314	0.3772	0.2621	0.1780	0.1176	0.0467	0.0156
	1	0.9672	0.8857	0.7765	0.6554	0.5339	0.4202	0.2333	0.1094
	2	0.9978	0.9842	0.9527	0.9011	0.8306	0.7443	0.5443	0.3438
	3	0.9999	0.9987	0.9941	0.9830	0.9624	0.9295	0.8208	0.6562
	4	1.0000	1.0000	0.9996	0.9984	0.9954	0.9891	0.9590	0.8906
	5	1.0000	1.0000	1.0000	0.9999	0.9998	0.9993	0.9959	0.9844
7	0	0.6983	0.4783	0.3206	0.2097	0.1335	0.0824	0.0280	0.0078
	1	0.9556	0.8503	0.7166	0.5767	0.4449	0.3294	0.1586	0.0625
	2	0.9962	0.9743	0.9262	0.8520	0.7564	0.6471	0.4199	0.2266
	3	0.9998	0.9973	0.9879	0.9667	0.9294	0.8740	0.7102	0.5000
	4	1.0000	0.9998	0.9988	0.9953	0.9871	0.9712	0.9037	0.7734
	5	1.0000	1.0000	0.9999	0.9996	0.9987	0.9962	0.9812	0.9375
	6	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9984	0.9922
8	0	0.6634	0.4305	0.2725	0.1678	0.1001	0.0576	0.0168	0.0039
	1	0.9428	0.8131	0.6572	0.5033	0.3671	0.2553	0.1064	0.0352
	2	0.9942	0.9619	0.8948	0.7969	0.6785	0.5518	0.3154	0.1445
	3	0.9996	0.9950	0.9786	0.9437	0.8862	0.8059	0.5941	0.3633
	4	1.0000	0.9996	0.9971	0.9896	0.9727	0.9420	0.8263	0.6367
	5	1.0000	1.0000	0.9998	0.9988	0.9958	0.9887	0.9502	0.8555
	6	1.0000	1.0000	1.0000	0.9999	0.9996	0.9987	0.9915	0.9648
	7	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9961

Binomialfördelningen, forts.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
9	0	0.6302	0.3874	0.2316	0.1342	0.0751	0.0404	0.0101	0.0020
	1	0.9288	0.7748	0.5995	0.4362	0.3003	0.1960	0.0705	0.0195
	2	0.9916	0.9470	0.8591	0.7382	0.6007	0.4628	0.2318	0.0898
	3	0.9994	0.9917	0.9661	0.9144	0.8343	0.7297	0.4826	0.2539
	4	1.0000	0.9991	0.9944	0.9804	0.9511	0.9012	0.7334	0.5000
	5	1.0000	0.9999	0.9994	0.9969	0.9900	0.9747	0.9006	0.7461
	6	1.0000	1.0000	1.0000	0.9997	0.9987	0.9957	0.9750	0.9102
	7	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996	0.9962	0.9805
	8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9980
10	0	0.5987	0.3487	0.1969	0.1074	0.0563	0.0282	0.0060	0.0010
	1	0.9139	0.7361	0.5443	0.3758	0.2440	0.1493	0.0464	0.0107
	2	0.9885	0.9298	0.8202	0.6778	0.5256	0.3828	0.1673	0.0547
	3	0.9990	0.9872	0.9500	0.8791	0.7759	0.6496	0.3823	0.1719
	4	0.9999	0.9984	0.9901	0.9672	0.9219	0.8497	0.6331	0.3770
	5	1.0000	0.9999	0.9986	0.9936	0.9803	0.9527	0.8338	0.6230
	6	1.0000	1.0000	0.9999	0.9991	0.9965	0.9894	0.9452	0.8281
	7	1.0000	1.0000	1.0000	0.9999	0.9996	0.9984	0.9877	0.9453
	8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9983	0.9893
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9990
11	0	0.5688	0.3138	0.1673	0.0859	0.0422	0.0198	0.0036	0.0005
	1	0.8981	0.6974	0.4922	0.3221	0.1971	0.1130	0.0302	0.0059
	2	0.9848	0.9104	0.7788	0.6174	0.4552	0.3127	0.1189	0.0327
	3	0.9984	0.9815	0.9306	0.8389	0.7133	0.5696	0.2963	0.1133
	4	0.9999	0.9972	0.9841	0.9496	0.8854	0.7897	0.5328	0.2744
	5	1.0000	0.9997	0.9973	0.9883	0.9657	0.9218	0.7535	0.5000
	6	1.0000	1.0000	0.9997	0.9980	0.9924	0.9784	0.9006	0.7256
	7	1.0000	1.0000	1.0000	0.9998	0.9988	0.9957	0.9707	0.8867
	8	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9941	0.9673
	9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993	0.9941
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
12	0	0.5404	0.2824	0.1422	0.0687	0.0317	0.0138	0.0022	0.0002
	1	0.8816	0.6590	0.4435	0.2749	0.1584	0.0850	0.0196	0.0032
	2	0.9804	0.8891	0.7358	0.5583	0.3907	0.2528	0.0834	0.0193
	3	0.9978	0.9744	0.9078	0.7946	0.6488	0.4925	0.2253	0.0730
	4	0.9998	0.9957	0.9761	0.9274	0.8424	0.7237	0.4382	0.1938
	5	1.0000	0.9995	0.9954	0.9806	0.9456	0.8822	0.6652	0.3872
	6	1.0000	0.9999	0.9993	0.9961	0.9857	0.9614	0.8418	0.6128
	7	1.0000	1.0000	0.9999	0.9994	0.9972	0.9905	0.9427	0.8062
	8	1.0000	1.0000	1.0000	0.9999	0.9996	0.9983	0.9847	0.9270
	9	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9972	0.9807
	10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9968
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998

Binomialfördelningen, forts.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
13	0	0.5133	0.2542	0.1209	0.0550	0.0238	0.0097	0.0013	0.0001
	1	0.8646	0.6213	0.3983	0.2336	0.1267	0.0637	0.0126	0.0017
	2	0.9755	0.8661	0.6920	0.5017	0.3326	0.2025	0.0579	0.0112
	3	0.9969	0.9658	0.8820	0.7473	0.5843	0.4206	0.1686	0.0461
	4	0.9997	0.9935	0.9658	0.9009	0.7940	0.6543	0.3530	0.1334
	5	1.0000	0.9991	0.9925	0.9700	0.9198	0.8346	0.5744	0.2905
	6	1.0000	0.9999	0.9987	0.9930	0.9757	0.9376	0.7712	0.5000
	7	1.0000	1.0000	0.9998	0.9988	0.9944	0.9818	0.9023	0.7095
	8	1.0000	1.0000	1.0000	0.9998	0.9990	0.9960	0.9679	0.8666
	9	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9922	0.9539
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9987	0.9888
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9983
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
14	0	0.4877	0.2288	0.1028	0.0440	0.0178	0.0068	0.0008	0.0001
	1	0.8470	0.5846	0.3567	0.1979	0.1010	0.0475	0.0081	0.0009
	2	0.9699	0.8416	0.6479	0.4481	0.2811	0.1608	0.0398	0.0065
	3	0.9958	0.9559	0.8535	0.6982	0.5213	0.3552	0.1243	0.0287
	4	0.9996	0.9908	0.9533	0.8702	0.7415	0.5842	0.2793	0.0898
	5	1.0000	0.9985	0.9885	0.9561	0.8883	0.7805	0.4859	0.2120
	6	1.0000	0.9998	0.9978	0.9884	0.9617	0.9067	0.6925	0.3953
	7	1.0000	1.0000	0.9997	0.9976	0.9897	0.9685	0.8499	0.6047
	8	1.0000	1.0000	1.0000	0.9996	0.9978	0.9917	0.9417	0.7880
	9	1.0000	1.0000	1.0000	1.0000	0.9997	0.9983	0.9825	0.9102
	10	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9961	0.9713
	11	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994	0.9935
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9991
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
15	0	0.4633	0.2059	0.0874	0.0352	0.0134	0.0047	0.0005	0.0000
	1	0.8290	0.5490	0.3186	0.1671	0.0802	0.0353	0.0052	0.0005
	2	0.9638	0.8159	0.6042	0.3980	0.2361	0.1268	0.0271	0.0037
	3	0.9945	0.9444	0.8227	0.6482	0.4613	0.2969	0.0905	0.0176
	4	0.9994	0.9873	0.9383	0.8358	0.6865	0.5155	0.2173	0.0592
	5	0.9999	0.9978	0.9832	0.9389	0.8516	0.7216	0.4032	0.1509
	6	1.0000	0.9997	0.9964	0.9819	0.9434	0.8689	0.6098	0.3036
	7	1.0000	1.0000	0.9994	0.9958	0.9827	0.9500	0.7869	0.5000
	8	1.0000	1.0000	0.9999	0.9992	0.9958	0.9848	0.9050	0.6964
	9	1.0000	1.0000	1.0000	0.9999	0.9992	0.9963	0.9662	0.8491
	10	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9907	0.9408
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9981	0.9824
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9963
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Binomialfördelningen, forts.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
16	0	0.4401	0.1853	0.0743	0.0281	0.0100	0.0033	0.0003	0.0000
	1	0.8108	0.5147	0.2839	0.1407	0.0635	0.0261	0.0033	0.0003
	2	0.9571	0.7892	0.5614	0.3518	0.1971	0.0994	0.0183	0.0021
	3	0.9930	0.9316	0.7899	0.5981	0.4050	0.2459	0.0651	0.0106
	4	0.9991	0.9830	0.9209	0.7982	0.6302	0.4499	0.1666	0.0384
	5	0.9999	0.9967	0.9765	0.9183	0.8103	0.6598	0.3288	0.1051
	6	1.0000	0.9995	0.9944	0.9733	0.9204	0.8247	0.5272	0.2272
	7	1.0000	0.9999	0.9989	0.9930	0.9729	0.9256	0.7161	0.4018
	8	1.0000	1.0000	0.9998	0.9985	0.9925	0.9743	0.8577	0.5982
	9	1.0000	1.0000	1.0000	0.9998	0.9984	0.9929	0.9417	0.7728
	10	1.0000	1.0000	1.0000	1.0000	0.9997	0.9984	0.9809	0.8949
	11	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9951	0.9616
	12	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9991	0.9894
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9979
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
17	0	0.4181	0.1668	0.0631	0.0225	0.0075	0.0023	0.0002	0.0000
	1	0.7922	0.4818	0.2525	0.1182	0.0501	0.0193	0.0021	0.0001
	2	0.9497	0.7618	0.5198	0.3096	0.1637	0.0774	0.0123	0.0012
	3	0.9912	0.9174	0.7556	0.5489	0.3530	0.2019	0.0464	0.0064
	4	0.9988	0.9779	0.9013	0.7582	0.5739	0.3887	0.1260	0.0245
	5	0.9999	0.9953	0.9681	0.8943	0.7653	0.5968	0.2639	0.0717
	6	1.0000	0.9992	0.9917	0.9623	0.8929	0.7752	0.4478	0.1662
	7	1.0000	0.9999	0.9983	0.9891	0.9598	0.8954	0.6405	0.3145
	8	1.0000	1.0000	0.9997	0.9974	0.9876	0.9597	0.8011	0.5000
	9	1.0000	1.0000	1.0000	0.9995	0.9969	0.9873	0.9081	0.6855
	10	1.0000	1.0000	1.0000	0.9999	0.9994	0.9968	0.9652	0.8338
	11	1.0000	1.0000	1.0000	1.0000	0.9999	0.9993	0.9894	0.9283
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9975	0.9755
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9995	0.9936
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9988
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Binomialfördelningen, forts.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
18	0	0.3972	0.1501	0.0536	0.0180	0.0056	0.0016	0.0001	0.0000
	1	0.7735	0.4503	0.2241	0.0991	0.0395	0.0142	0.0013	0.0001
	2	0.9419	0.7338	0.4797	0.2713	0.1353	0.0600	0.0082	0.0007
	3	0.9891	0.9018	0.7202	0.5010	0.3057	0.1646	0.0328	0.0038
	4	0.9985	0.9718	0.8794	0.7164	0.5187	0.3327	0.0942	0.0154
	5	0.9998	0.9936	0.9581	0.8671	0.7175	0.5344	0.2088	0.0481
	6	1.0000	0.9988	0.9882	0.9487	0.8610	0.7217	0.3743	0.1189
	7	1.0000	0.9998	0.9973	0.9837	0.9431	0.8593	0.5634	0.2403
	8	1.0000	1.0000	0.9995	0.9957	0.9807	0.9404	0.7368	0.4073
	9	1.0000	1.0000	0.9999	0.9991	0.9946	0.9790	0.8653	0.5927
	10	1.0000	1.0000	1.0000	0.9998	0.9988	0.9939	0.9424	0.7597
	11	1.0000	1.0000	1.0000	1.0000	0.9998	0.9986	0.9797	0.8811
	12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9942	0.9519
	13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987	0.9846
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998	0.9962
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9993
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
19	0	0.3774	0.1351	0.0456	0.0144	0.0042	0.0011	0.0001	0.0000
	1	0.7547	0.4203	0.1985	0.0829	0.0310	0.0104	0.0008	0.0000
	2	0.9335	0.7054	0.4413	0.2369	0.1113	0.0462	0.0055	0.0004
	3	0.9868	0.8850	0.6841	0.4551	0.2631	0.1332	0.0230	0.0022
	4	0.9980	0.9648	0.8556	0.6733	0.4654	0.2822	0.0696	0.0096
	5	0.9998	0.9914	0.9463	0.8369	0.6678	0.4739	0.1629	0.0318
	6	1.0000	0.9983	0.9837	0.9324	0.8251	0.6655	0.3081	0.0835
	7	1.0000	0.9997	0.9959	0.9767	0.9225	0.8180	0.4878	0.1796
	8	1.0000	1.0000	0.9992	0.9933	0.9713	0.9161	0.6675	0.3238
	9	1.0000	1.0000	0.9999	0.9984	0.9911	0.9674	0.8139	0.5000
	10	1.0000	1.0000	1.0000	0.9997	0.9977	0.9895	0.9115	0.6762
	11	1.0000	1.0000	1.0000	1.0000	0.9995	0.9972	0.9648	0.8204
	12	1.0000	1.0000	1.0000	1.0000	0.9999	0.9994	0.9884	0.9165
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9969	0.9682
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9994	0.9904
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9978
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9996
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Binomialfördelningen, forts.

n	$k \backslash p$	0.05	0.1	0.15	0.2	0.25	0.3	0.4	0.5
20	0	0.3585	0.1216	0.0388	0.0115	0.0032	0.0008	0.0000	0.0000
	1	0.7358	0.3917	0.1756	0.0692	0.0243	0.0076	0.0005	0.0000
	2	0.9245	0.6769	0.4049	0.2061	0.0913	0.0355	0.0036	0.0002
	3	0.9841	0.8670	0.6477	0.4114	0.2252	0.1071	0.0160	0.0013
	4	0.9974	0.9568	0.8298	0.6296	0.4148	0.2375	0.0510	0.0059
	5	0.9997	0.9887	0.9327	0.8042	0.6172	0.4164	0.1256	0.0207
	6	1.0000	0.9976	0.9781	0.9133	0.7858	0.6080	0.2500	0.0577
	7	1.0000	0.9996	0.9941	0.9679	0.8982	0.7723	0.4159	0.1316
	8	1.0000	0.9999	0.9987	0.9900	0.9591	0.8867	0.5956	0.2517
	9	1.0000	1.0000	0.9998	0.9974	0.9861	0.9520	0.7553	0.4119
	10	1.0000	1.0000	1.0000	0.9994	0.9961	0.9829	0.8725	0.5881
	11	1.0000	1.0000	1.0000	0.9999	0.9991	0.9949	0.9435	0.7483
	12	1.0000	1.0000	1.0000	1.0000	0.9998	0.9987	0.9790	0.8684
	13	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9935	0.9423
	14	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9984	0.9793
	15	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9997	0.9941
	16	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9987
	17	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9998
	18	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Tabell 3. *Poissonfördelningen*

Tabellen ger $F(k) = P(X \leq k)$ då $X \sim \text{Po}(\lambda)$, för $0.1 \leq \lambda \leq 15$.

$k \backslash \lambda$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
0	0.9048	0.8187	0.7408	0.6703	0.6065	0.5488	0.4966	0.4493
1	0.9953	0.9825	0.9631	0.9384	0.9098	0.8781	0.8442	0.8088
2	0.9998	0.9989	0.9964	0.9921	0.9856	0.9769	0.9659	0.9526
3	1.0000	0.9999	0.9997	0.9992	0.9982	0.9966	0.9942	0.9909
4	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992	0.9986
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998

$k \backslash \lambda$	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.2
0	0.4066	0.3679	0.3012	0.2466	0.2019	0.1653	0.1353	0.1108
1	0.7725	0.7358	0.6626	0.5918	0.5249	0.4628	0.4060	0.3546
2	0.9371	0.9197	0.8795	0.8335	0.7834	0.7306	0.6767	0.6227
3	0.9865	0.9810	0.9662	0.9463	0.9212	0.8913	0.8571	0.8194
4	0.9977	0.9963	0.9923	0.9857	0.9763	0.9636	0.9473	0.9275
5	0.9997	0.9994	0.9985	0.9968	0.9940	0.9896	0.9834	0.9751
6	1.0000	0.9999	0.9997	0.9994	0.9987	0.9974	0.9955	0.9925
7	1.0000	1.0000	1.0000	0.9999	0.9997	0.9994	0.9989	0.9980
8	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9995
9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
10	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

$k \backslash \lambda$	2.4	2.6	2.8	3.0	3.2	3.4	3.6	3.8
0	0.0907	0.0743	0.0608	0.0498	0.0408	0.0334	0.0273	0.0224
1	0.3084	0.2674	0.2311	0.1991	0.1712	0.1468	0.1257	0.1074
2	0.5697	0.5184	0.4695	0.4232	0.3799	0.3397	0.3027	0.2689
3	0.7787	0.7360	0.6919	0.6472	0.6025	0.5584	0.5152	0.4735
4	0.9041	0.8774	0.8477	0.8153	0.7806	0.7442	0.7064	0.6678
5	0.9643	0.9510	0.9349	0.9161	0.8946	0.8705	0.8441	0.8156
6	0.9884	0.9828	0.9756	0.9665	0.9554	0.9421	0.9267	0.9091
7	0.9967	0.9947	0.9919	0.9881	0.9832	0.9769	0.9692	0.9599
8	0.9991	0.9985	0.9976	0.9962	0.9943	0.9917	0.9883	0.9840
9	0.9998	0.9996	0.9993	0.9989	0.9982	0.9973	0.9960	0.9942
10	1.0000	0.9999	0.9998	0.9997	0.9995	0.9992	0.9987	0.9981
11	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9996	0.9994
12	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998
13	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Poissonfördelningen, forts.

$k \backslash \lambda$	4.0	4.5	5.0	5.5	6.0	6.5	7.0	7.5
0	0.0183	0.0111	0.0067	0.0041	0.0025	0.0015	0.0009	0.0006
1	0.0916	0.0611	0.0404	0.0266	0.0174	0.0113	0.0073	0.0047
2	0.2381	0.1736	0.1247	0.0884	0.0620	0.0430	0.0296	0.0203
3	0.4335	0.3423	0.2650	0.2017	0.1512	0.1118	0.0818	0.0591
4	0.6288	0.5321	0.4405	0.3575	0.2851	0.2237	0.1730	0.1321
5	0.7851	0.7029	0.6160	0.5289	0.4457	0.3690	0.3007	0.2414
6	0.8893	0.8311	0.7622	0.6860	0.6063	0.5265	0.4497	0.3782
7	0.9489	0.9134	0.8666	0.8095	0.7440	0.6728	0.5987	0.5246
8	0.9786	0.9597	0.9319	0.8944	0.8472	0.7916	0.7291	0.6620
9	0.9919	0.9829	0.9682	0.9462	0.9161	0.8774	0.8305	0.7764
10	0.9972	0.9933	0.9863	0.9747	0.9574	0.9332	0.9015	0.8622
11	0.9991	0.9976	0.9945	0.9890	0.9799	0.9661	0.9467	0.9208
12	0.9997	0.9992	0.9980	0.9955	0.9912	0.9840	0.9730	0.9573
13	0.9999	0.9997	0.9993	0.9983	0.9964	0.9929	0.9872	0.9784
14	1.0000	0.9999	0.9998	0.9994	0.9986	0.9970	0.9943	0.9897
15	1.0000	1.0000	0.9999	0.9998	0.9995	0.9988	0.9976	0.9954
16	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9990	0.9980
17	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9996	0.9992
18	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9997
19	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Poissonfördelningen, forts.

$k \backslash \lambda$	8.0	9.0	10.0	11.0	12.0	13.0	14.0	15.0
0	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1	0.0030	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
2	0.0138	0.0062	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000
3	0.0424	0.0212	0.0103	0.0049	0.0023	0.0011	0.0005	0.0002
4	0.0996	0.0550	0.0293	0.0151	0.0076	0.0037	0.0018	0.0009
5	0.1912	0.1157	0.0671	0.0375	0.0203	0.0107	0.0055	0.0028
6	0.3134	0.2068	0.1301	0.0786	0.0458	0.0259	0.0142	0.0076
7	0.4530	0.3239	0.2202	0.1432	0.0895	0.0540	0.0316	0.0180
8	0.5925	0.4557	0.3328	0.2320	0.1550	0.0998	0.0621	0.0374
9	0.7166	0.5874	0.4579	0.3405	0.2424	0.1658	0.1094	0.0699
10	0.8159	0.7060	0.5830	0.4599	0.3472	0.2517	0.1757	0.1185
11	0.8881	0.8030	0.6968	0.5793	0.4616	0.3532	0.2600	0.1848
12	0.9362	0.8758	0.7916	0.6887	0.5760	0.4631	0.3585	0.2676
13	0.9658	0.9261	0.8645	0.7813	0.6815	0.5730	0.4644	0.3632
14	0.9827	0.9585	0.9165	0.8540	0.7720	0.6751	0.5704	0.4657
15	0.9918	0.9780	0.9513	0.9074	0.8444	0.7636	0.6694	0.5681
16	0.9963	0.9889	0.9730	0.9441	0.8987	0.8355	0.7559	0.6641
17	0.9984	0.9947	0.9857	0.9678	0.9370	0.8905	0.8272	0.7489
18	0.9993	0.9976	0.9928	0.9823	0.9626	0.9302	0.8826	0.8195
19	0.9997	0.9989	0.9965	0.9907	0.9787	0.9573	0.9235	0.8752
20	0.9999	0.9996	0.9984	0.9953	0.9884	0.9750	0.9521	0.9170
21	1.0000	0.9998	0.9993	0.9977	0.9939	0.9859	0.9712	0.9469
22	1.0000	0.9999	0.9997	0.9990	0.9970	0.9924	0.9833	0.9673
23	1.0000	1.0000	0.9999	0.9995	0.9985	0.9960	0.9907	0.9805
24	1.0000	1.0000	1.0000	0.9998	0.9993	0.9980	0.9950	0.9888
25	1.0000	1.0000	1.0000	0.9999	0.9997	0.9990	0.9974	0.9938
26	1.0000	1.0000	1.0000	1.0000	0.9999	0.9995	0.9987	0.9967
27	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998	0.9994	0.9983
28	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9997	0.9991
29	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9996
30	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9998
31	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999
32	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Tabell 4. *Normalfördelningens fördelningsfunktion,* $\Phi(t)$

Tabellen ger $\Phi(t) = P(X \leq t)$ då $X \sim \mathrm{N}(0, 1)$, för $0 \leq t \leq 3.9$.
För $t < 0$ utnyttjas att $\Phi(-t) = 1 - \Phi(t)$.
För stora t kan man utnyttja approximationen $1 - \Phi(t) \approx \varphi(t)/t$,
där $\varphi(t) = \frac{1}{\sqrt{2\pi}} e^{-t^2/2}$.

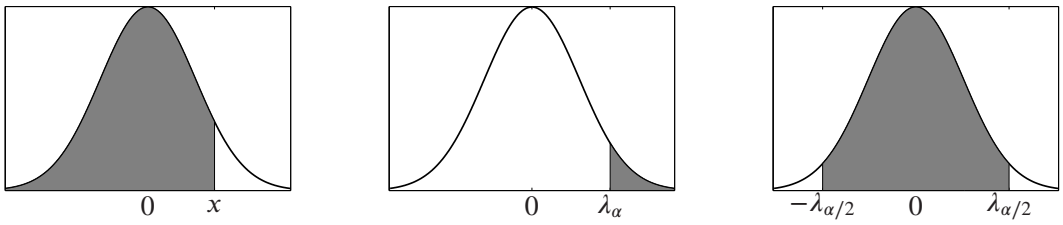
t	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	$.9^3$03	$.9^3$06	$.9^3$10	$.9^3$13	$.9^3$16	$.9^3$18	$.9^3$21	$.9^3$24	$.9^3$26	$.9^3$29
3.2	$.9^3$31	$.9^3$34	$.9^3$36	$.9^3$38	$.9^3$40	$.9^3$42	$.9^3$44	$.9^3$46	$.9^3$48	$.9^3$50
3.3	$.9^3$52	$.9^3$53	$.9^3$55	$.9^3$57	$.9^3$58	$.9^3$60	$.9^3$61	$.9^3$62	$.9^3$64	$.9^3$65
3.4	$.9^3$66	$.9^3$68	$.9^3$69	$.9^3$70	$.9^3$71	$.9^3$72	$.9^3$73	$.9^3$74	$.9^3$75	$.9^3$76
3.5	$.9^3$77	$.9^3$78	$.9^3$78	$.9^3$79	$.9^3$80	$.9^3$81	$.9^3$81	$.9^3$82	$.9^3$83	$.9^3$83
3.6	$.9^3$84	$.9^3$85	$.9^3$85	$.9^3$86	$.9^3$86	$.9^3$87	$.9^3$87	$.9^3$88	$.9^3$88	$.9^3$89
3.7	$.9^3$89	$.9^3$90	$.9^4$00	$.9^4$04	$.9^4$08	$.9^4$12	$.9^4$15	$.9^4$18	$.9^4$22	$.9^4$25
3.8	$.9^4$28	$.9^4$31	$.9^4$33	$.9^4$36	$.9^4$38	$.9^4$41	$.9^4$43	$.9^4$46	$.9^4$48	$.9^4$50
3.9	$.9^4$52	$.9^4$54	$.9^4$56	$.9^4$58	$.9^4$59	$.9^4$61	$.9^4$63	$.9^4$64	$.9^4$66	$.9^4$67
4.0	$.9^4$68	$.9^4$70	$.9^4$71	$.9^4$72	$.9^4$73	$.9^4$74	$.9^4$75	$.9^4$76	$.9^4$77	$.9^4$78

Ex. $.9^4$68=0.999968

Tabell 5. *Normalfördelningens kvantiler,* λ_α

Tabellen ger λ_α för $\alpha \leq 0.5$, där λ_α definieras av att $\Phi(\lambda_\alpha) = 1 - \alpha$, eller alternativt att $P(X > \lambda_\alpha) = \alpha$ då $X \sim \mathrm{N}(0, 1)$.
För $\alpha > 0.5$ utnyttjas att $\lambda_\alpha = -\lambda_{1-\alpha}$.

α	λ_α
0.5	0.0000
0.4	0.2533
0.3	0.5244
0.25	0.6745
0.2	0.8416
0.15	1.0364
0.1	1.2816
0.05	1.6449
0.025	1.9600
0.01	2.3263
0.005	2.5758
0.001	3.0902
0.0005	3.2905
0.0001	3.7190
0.00005	3.8906



Figur 1. *Arean till vänster om x är* $\Phi(x)$*, arean till höger om* λ_α *är* α *och arean mellan* $-\lambda_{\alpha/2}$ *och* $\lambda_{\alpha/2}$ *är* $1 - \alpha$.