Lecture 1 (p 1-16 in Björk's book)

Motivating discussion (What is an option?)

A Swedish company has signed a contract to buy a machine from a US company. The price is 100.000 USD to be paid <u>at delivery</u> (6 months from now, i.e. $T = \frac{1}{2}$). Current exchange rate is 11 SEK/USD.

Big currency risk! Three possible strategies below.

1. Buy 100.000 USD today and put in a eurodollar account.
   - \+ Risk completely eliminated.
   - \- Money is tied up for a long time.
   - \- The company may not even have 1,100.000 SEK today

2. Buy a forward contract from a bank:
   - The bank delivers 100.000 USD at $t=T$
   - The company pays $K \cdot 100.000$ SEK at $t=T$.
   - K is chosen (at $t=0$) so that no transfer of money is needed at $t=0$.
   - \+ Risk completely eliminated.
   - \- If the exchange rate drops (below K) then the cost is too large.

(3) Buy a European call option on 100.000 USD with strike price K and exercice date T.
It gives the right, but not the obligation, to buy 100.000 USD at $t=T$ at price $K \cdot 100.000$ SEK.

- If the exchange rate at $t=T$ is above K, use the option.
- If the exchange rate at $t=T$ is below K, do not use the option.

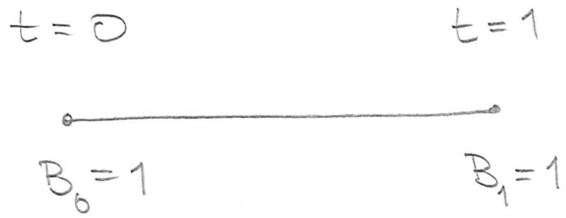
Two main problems in this course:

- What is a fair price of an option?
- If you are the seller of an option, how to protect yourself (hedge) against risk?

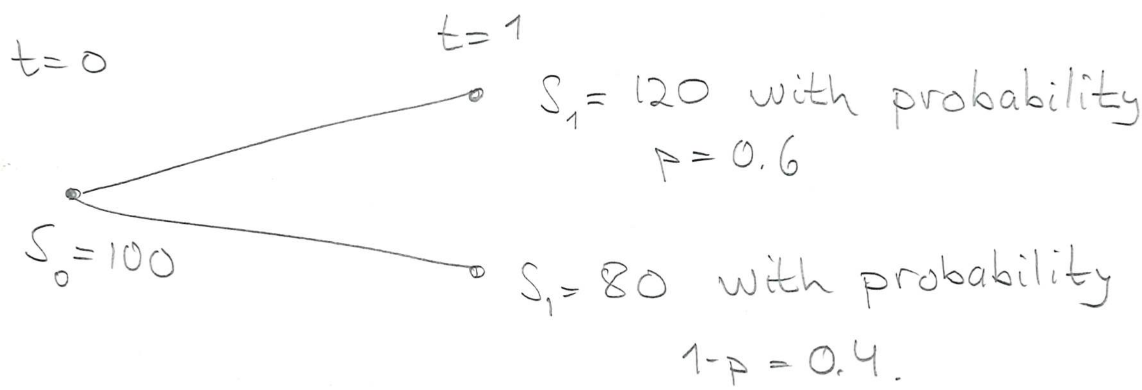
A motivating example in discrete time

At $t=0$ we can trade in a market with two assets.

- A bank account (non-risky asset):



- A stock (risky asset):



A <u>call option</u> is a contract that gives its holder the right (but not the obligation) to buy the stock at $t=1$ at a certain (pre-determined) price K. Assume $K=110$.

Thus, at time $t=1$, the option is worth

$120-110$ if the stock price is 120 $(p=0.6)$

0 _____ " _____ 80 $(1-p=0.4)$

Question: What is the option value at time $t=0$?

Idea: Try to replicate the option, i.e. find a trading strategy (in B and S) so that the value at $t=1$ is the same as the option value.

Let

x = amount invested in B

y = number of shares of S

$$\begin{cases} x + 120y = 10 \\ x + 80y = 0 \end{cases} \quad \text{gives} \quad \begin{cases} x = -20 \\ y = \frac{1}{4} \end{cases}$$

The strategy of borrowing 20 from the bank and buying $\frac{1}{4}$ of a share of S at $t=0$ has value

$$\frac{1}{4}S_1 - 20 = \begin{cases} 10 & \text{if } S_1 = 120 \\ 0 & \text{if } S_1 = 80 \end{cases}$$

at $t=1$. Moreover the initial cost is

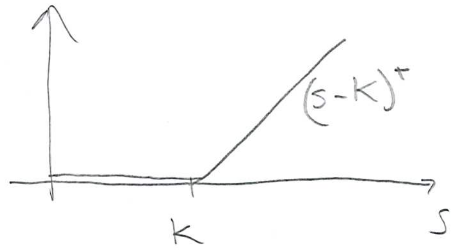
$$\frac{1}{4}S_0 - 20 = 5.$$

Conclusion: In order to not to introduce arbitrage (risk-free profit) possibilities, the option price at $t=0$ has to be 5!

Remark: The value of p does not influence the option value!

Notation: We write $a^+ = \max\{a, 0\}$. In particular

$$(s-K)^+ = \begin{cases} s-K & \text{if } s \geq K \\ 0 & \text{if } s < K \end{cases}$$



Remark: Let us change the probability p into a value q such that $E^Q[S_1] = S_0$ (i.e. $q=0.5$).

Then $E^Q\left[(S_1-K)^+\right] = \frac{1}{2}\cdot 10 + \frac{1}{2}\cdot 0 = 5$

The option value!

In general, the option price is $E^Q\left[\frac{B_0}{B_1}(S_1-K)^+\right]$ where Q is such that $E^Q\left[\frac{B_0}{B_1}S_1\right] = S_0$. (We do not prove this here.)

Exercise: 1) In the above example, find a replicating strategy for a put option (the right, but not the obligation, to sell the stock at $K=110$).

2. Find the value of the put option at $t=0$.

Answer: $\begin{cases} x = 90 \\ y = -3/4 \end{cases}$, option value is 15.