Course information for 1MA170 – Graph Theory (Fall 2022)

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Generalities

The graph theory course will consist of 20 lectures, three of which will be exercise sessions. They are scheduled on timeedit.

The problems for the exercise sessions will be uploaded to canvas approximately one week prior to the session, and are generally supposed to be solved prior to the session, where solutions will be discussed. These exercises will **not** be part of the course assessment (see below), but material covered in the exercises may be relevant for later parts in the course.

Official course literature is Reinhard Diestel: *Graph Theory* (Springer). While the book is a good reference, there are considerable differences to the course content as mandated by the syllabus. Therefore, I would **not** recommend students buying it (lecture notes for this course will be provided anyways).

For questions, both concerning the organisation of the course and its mathematical content, please feel free to contact me via email:

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Assessment

The course assessment will consist of three homework assignments, and a written exam in January 2023. The succesful completion of the homework assignments is worth 3 credits, and the succesful completion of the written exam is worth 2 credits. **In order to pass the course, both parts need to be succesfully completed.**

The three written homework assignments will have a total of 20 points each, and will each cover roughly a third of the material of the course. In order to pass the homework assignments, one is required to score at least 27 points, out of a total of 60. A small bonus homework assignment will be provided in January, if needed. This part of the course is only evaluated by a pass-/fail grade (G/U).

The final exam will have a total of 40 points. In order to pass the final exam, one is required to score at least 18 points. Grading of the final exam follows the usual pattern (i.e. at least 18 points for grade 3, at least 25 points for grade 4, and at least 32 points for grade 5).

Over the entire course, the student can thus attain at most 100 points. The final grade is evaluated by how many points the student has attained in both homework assignments and the exam combined. If the total amount is at least $27 + 18 = 45$ points, the student is awarded a passing grade, 3. For a total of at least 63 points, the student is awarded a 4, and for a total of at least 81 points, the student is awarded a 5.

Preliminary course plan

The following provides a preliminary overview of the course structure and the contents of the lectures:

L1 Bridges of Königsberg, multigraphs, basic vocabulary, Eulerian graphs and Euler's theorem, handshaking lemma

L2 Simple graphs, morphisms, labelled graphs, special graphs, subgraphs, induced subgraphs

L3 Trees, leaves, characterising trees, existence of spanning trees, Cayley's theorem

L4 Adjacency and incidence matrix, basic algebraic graph theory, Kirchhoff's matrix-tree theorem

L5 Weighted graphs, minimal spanning trees, algorithms for finding MST's, distances, Dijkstra's algorithm

L6 Hamiltonian cycles, theorems of Dirac and Ore, closure, degree sequence. Examples: hypercubes and the Petersen graph

L7 Exercise session

L8 The max-flow-min-cut theorem, Ford-Fulkerson algorithm, applications

L9 Hall's theorem, perfect matchings, Tutte's 1-factor theorem

L10 Connectivity, Menger's theorem, characterisation of 2- and 3-connected graphs

L11 Planar graphs, Euler's formula, non-planarity of K_5 and $K_{3,3}$, minors, the Wagner-Kuratowski theorem(s)

L12 Vertex-colourings, the chromatic number and other graph invariants. Greedy colouring algorithm, Brooks' theorem, history of the 4-colour theorem

L13 5-colour theorem. Mycielski's construction, Erdős' theorem, some spectral bounds, perfect graphs and their characterisation

L14 Edge colourings, Vizing's theorem, line graphs. Basic Ramsey theory.

L15 Exercise session

L16 Szemerédi's regularity lemma and how it's applied

L17 The Rado graph, the extension property and its consequences, $G(p)$

L18 Erdös-Renyi random graphs $G(n, p)$, the probabilistic method, existence of small subgraphs

L19 Evolution of $G(n, p)$, birth of the giant component

L20 Exercise session