Financial Theory – Lecture 15

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Agenda

- Macro-finance.
- ESG investing

The ESG investing part is based on

- Section 13.4 in the course book.

Introduction

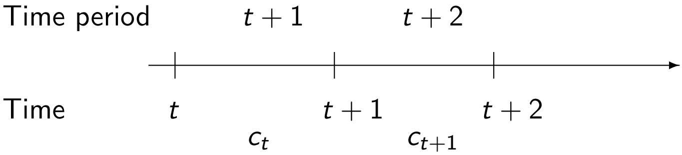
What is macro-finance?

> *Macro-finance studies the relationship between asset prices and economic fluctuations.*

John H. Cochrane (2017), "Macro-Finance", *Review of Finance*, p. 945-985.

The utility of a consumer over time

A consumer has at time t to choose his/her consumption c_t over the next period.



But a consumer also has to choose consumption over all future time periods.

The utility of a consumer over time

What is the utility of a consumption stream

$$(c_t, c_{t+1}, c_{t+2}, \ldots)?$$

We use expected utility, and let the utility of this stream at time t be given by

$$E_t\left[U(t, c_t, c_{t+1}, c_{t+2}, \ldots)\right].$$

In order to get more explicit results, we assume some more structure on the utility function.

The utility of a consumer

- The utility function is additive:

$$
\begin{aligned}
U(t, c_t, c_{t+1}, c_{t+2}, \ldots) &= U_t(c_t) + U_t(c_{t+1}) \\
&\quad U_t(c_{t+2}) + \cdots \\
&= \sum_{s=0}^{\infty} U_t(c_{t+s}).
\end{aligned}
$$

- We have time-separability:

$$U_t(c_{t+s}) = \delta^s u(c_{t+s}).$$

This leads to the following utility at time t

$$E_t\left[\sum_{s=0}^{\infty} \delta^s u(c_{t+s})\right].$$

The utility of a consumer

This is a generalisation of the utility function

$$u(c_0) + \delta E\left[u(c_1)\right]$$

we used when deriving the consumption-based CAPM equation.

At every t there is a non-negative endowment e_t given to the investor.

At time t, the investor chooses the portfolio

$$\mathbf{x}_t = \left(x_{1t}, x_{2t}, \ldots, x_{Nt}\right)^T$$

and holds it during $(t, t+1]$.

The utility of a consumer

The budget constraint when portfolio $\mathbf{x}_t$ is bought at time t is

$$\begin{array}{rcl} c_t + \displaystyle\sum_{i=1}^{N} x_{it} P_{it} & = & e_t + \displaystyle\sum_{i=1}^{N} x_{i,t-1} \big(P_{it} + D_{it} \big) \\ \text{Consumption} + \text{Cost of new portfolio} & = & \text{Endowment} + \text{Value of old portfolio} \end{array}$$

Using vector notation:

$$c_t + \mathbf{x}_t \cdot \mathbf{P}_t = e_t + \mathbf{x}_{t-1} \cdot (\mathbf{P}_t + \mathbf{D}_t).$$

In the same way, we get the following budget constraint when portfolio $\mathbf{x}_t$ is sold:

$$c_{t+1} + \mathbf{x}_{t+1} \cdot \mathbf{P}_{t+1} = e_{t+1} + \mathbf{x}_t \cdot (\mathbf{D}_{t+1} + \mathbf{P}_{t+1}).$$

The utility of a consumer

At time t we want to maximise

$$E_t\left[\sum_{s=0}^{\infty}\delta^s u(c_{t+s})\right] = u(c_t) + E_t\left[\delta u(c_{t+1}) + \delta^2 u(c_{t+1})\dots\right]$$

over $\mathbf{x}_t$ subject to the two constraints from the previous slide.

Only the first two terms contain $\mathbf{x}_t$ and replacing c_t and c_{t+1} from the previous slide results in

$$\max_{\mathbf{x}_t} u\big(c_t(\mathbf{x}_t)\big) + E_t\left[\delta u\big(c_{t+1}(\mathbf{x}_t)\big)\right].$$

The FOC with respect to x_{it} is

$$u'(c_t)\frac{\partial c_t}{\partial x_{it}} + \delta E_t\left[u'(c_{t+1})\frac{\partial c_{t+1}}{\partial x_{it}}\right] = 0.$$

The utility of a consumer

Using the constraints we get

$$u'(c_t)\cdot(-P_{it})+\delta E_t\left[u'(c_{t+1})\cdot(D_{i,t+1}+P_{i,t+1})\right]=0$$

$$\Leftrightarrow$$

$$P_{it}=E_t\left[\delta\frac{u'(c_{t+1})}{u'(c_t)}\cdot(D_{i,t+1}+P_{i,t+1})\right].$$

$$\Leftrightarrow$$

$$1=E_t\left[\delta\frac{u'(c_{t+1})}{u'(c_t)}\cdot\frac{D_{i,t+1}+P_{i,t+1}}{P_{it}}\right]=E_t\left[\delta\frac{u'(c_{t+1})}{u'(c_t)}\cdot R_{i,t+1}\right].$$

The stochastic discount factor

The stochastic discount factor (SDF) for discounting from $t+1$ to t is

$$m_{t+1} = \delta \frac{u'(c_{t+1})}{u'(c_t)}.$$

Looking at m_{t+1}, we see again that the value of the SDF is high in the future states where $u'(c_{t+1})$ is high, and $u'(c_{t+1})$ is high when c_{t+1} is low.

The SDF from t to 0 is given by

$$\begin{aligned} m_t \cdot m_{t-1} \cdot \ldots \cdot m_1 &= \delta \frac{u'(c_t)}{u'(c_{t-1})} \cdot \delta \frac{u'(c_{t-1})}{u'(c_{t-2})} \cdot \ldots \cdot \delta \frac{u'(c_1)}{u'(c_0)} \\ &= \delta^t \frac{u'(c_t)}{u'(c_0)}. \end{aligned}$$

Taking a positive view

We will now continue by taking a positive rather than normative view.

Instead of taking the returns R_{it} as given and finding the optimal consumption, we instead take the consumption as given and wants to know what this says about the returns.

In order to do this, we need to use aggregate consumption. The same FOC will hold if we e.g. assume the existence of a representative consumer.

The risk premium

If there exists a risk-free asset with gross return $R_{f,t}$, then

$$1 = E_t\left[m_{t+1}R_{f,t}\right] \quad \Leftrightarrow \quad 1 = R_{f,t}E_t\left[m_{t+1}\right] \quad \Leftrightarrow \quad R_{f,t} = \frac{1}{E_t\left[m_{t+1}\right]}.$$

Now write the pricing equation

$$1 = E_t\left[m_{t+1}R_{i,t+1}\right]$$

as

$$1 = \underbrace{E_t\left[m_{t+1}\right]}_{=1/R_{f,t}} E_t\left[R_{i,t+1}\right] + \mathrm{Cov}_t\left[m_{t+1}, R_{i,t+1}\right],$$

or

$$R_{f,t} = E_t\left[R_{i,t+1}\right] + R_{f,t}\mathrm{Cov}_t\left[m_{t+1}, R_{i,t+1}\right].$$

The risk premium

This can be written

$$E_t\left[R_{i,t+1}\right] - R_{f,t} = R_{f,t}\mathrm{Cov}_t\Big[-m_{t+1}, R_{i,t+1}\Big],$$

or

$$E_t\left[r_{i,t+1}\right] - r_{f,t} = R_{f,t}\mathrm{Cov}_t\Big[-m_{t+1}, r_{i,t+1}\Big].$$

We recognise the left-hand side as the risk premium.

> *Explaining why different assets have different risk premia is the main goal of asset pricing theory.*

Kerry Back (2017), "Asset Pricing and Portfolio Choice Theory", 2nd Ed., *Oxford University Press*.

Bounding the Sharpe ratio

Now,

$$
\begin{aligned}
\left|E_t\left[r_{i,t+1}\right]-r_{f,t}\right| &= \Big|R_{f,t}\mathrm{Cov}_t\big[-m_{t+1}, r_{i,t+1}\big]\Big| \\
&= R_{f,t}\Big|\mathrm{Std}_t[m_{t+1}]\mathrm{Std}[r_{i,t+1}]\mathrm{Corr}_t[m_{t+1}, r_{i,t+1}]\Big| \\
&\leq R_{f,t}\mathrm{Std}_t[m_{t+1}]\mathrm{Std}[r_{i,t+1}].
\end{aligned}
$$

This implies that

$$|\mathrm{SR}_{i,t}| \leq R_{f,t}\mathrm{Std}_t[m_{t+1}].$$

Hence, the standard deviation of the SDF bounds the Sharpe ratio.

CRRA utility functions

When

$$u'(x) = x^{-\gamma}, \ \gamma \geq 0,$$

we have the CRRA (Constant Relative Risk Aversion) class of utility functions.

For such a utility function

$$\begin{aligned} m_{t+1} &= \delta \frac{u'(c_{t+1})}{u'(c_t)} \\ &= \delta \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \\ &= \delta \left(\frac{c_{t+1}}{c_t} \right)^{-\gamma}. \end{aligned}$$

CRRA utility functions

With

$$c_t^{\log} = \ln c_t \quad \Leftrightarrow \quad c_t = e^{c_t^{\log}}$$

we can write

$$\begin{aligned} m_{t+1} &= \delta \left(\frac{e^{c_{t+1}^{\log}}}{e^{c_t^{\log}}} \right)^{-\gamma} = \delta \left(e^{c_{t+1}^{\log} - c_t^{\log}} \right)^{-\gamma} \\ &= \delta e^{-\gamma (c_{t+1}^{\log} - c_t^{\log})} \\ &= \delta e^{-\gamma \Delta c_{t+1}^{\log}}. \end{aligned}$$

Here, $\Delta c_{t+1}^{\log}$ is the consumption growth.

CRRA utility functions

When

$$m_{t+1} = \delta e^{-\gamma \Delta c_{t+1}^{\log}}$$

we get

$$E_t \left[r_{i,t+1} \right] - r_f = -R_{f,t} \delta \mathrm{Cov}_t \left[e^{-\gamma \Delta c_{t+1}^{\log}}, r_{i,t+1} \right].$$

Hence, the risk premium is dependent on the growth in consumption.

We now use the Taylor approximation

$$e^{-\gamma \Delta c_{t+1}^{\log}} \approx 1 - \gamma \Delta c_{t+1}^{\log}.$$

This implies

$$\begin{aligned} E_t \left[r_{i,t+1} \right] - r_f &\approx -R_{f,t} \delta \mathrm{Cov}_t \left[1 - \gamma \Delta c_{t+1}^{\log}, r_{i,t+1} \right] \\ &= R_{f,t} \delta \gamma \mathrm{Cov}_t \left[\Delta c_{t+1}^{\log}, r_{i,t+1} \right]. \end{aligned}$$

The CRRA-lognormal model

Assumptions

- The utility function is CRRA:

$$u'(x) = x^{-\gamma}$$

  for some $\gamma \geq 0$.

- The growth rate in consumption is IID and normally distributed:

$$\Delta c_{t+1}^{\log} \sim N(g, \sigma_g).$$

The CRRA-lognormal model

Recall the following result: If X is normally distributed with mean μ and variance σ^2, i.e. $X \sim N(\mu, \sigma^2)$, then

$$E\left[e^X\right] = e^{\mu + \frac{\sigma^2}{2}}.$$

It follows from this that

$$E\left[e^{aX}\right] = e^{a\mu + \frac{a^2\sigma^2}{2}}.$$

The CRRA-lognormal model

The pricing equation for the risk-free return:

$$E_t\left[\delta e^{-\gamma \Delta c_{t+1}^{\text{log}}} R_{f,t}\right] = 1.$$

$$\Leftrightarrow$$

$$\delta E_t\left[e^{-\gamma \Delta c_{t+1}^{\text{log}}}\right] = \frac{1}{R_{f,t}}.$$

Under our assumptions (use the formula above with $a = -\gamma$):

$$\delta e^{-\gamma g + \frac{\gamma^2 \sigma_g^2}{2}} = \frac{1}{R_{f,t}},$$

or

$$1 + r_{f,t} = R_{f,t} = e^{-\ln \delta + \gamma g - \frac{\gamma^2 \sigma_g^2}{2}}.$$

Given the parameters of the model, this should be the risk-free rate.

The CRRA-lognormal model

Now,

$$
\begin{aligned}
\mathrm{Cov}_t\left[\Delta c_{t+1}^{\log}, r_{i,t+1}\right] &= \mathrm{Corr}_t\left[\Delta c_{t+1}^{\log}, r_{i,t+1}\right] \cdot \mathrm{Std}_t\left[\Delta c_{t+1}^{\log}\right] \cdot \mathrm{Std}_t\left[r_{i,t+1}\right] \\
&= \rho_t \cdot \sigma_g \cdot \sigma_{it}.
\end{aligned}
$$

It follows that the risk premium is

$$
\begin{aligned}
E_t\left[r_{i,t+1}\right] - r_{f,t} &\approx R_{f,t}\delta\gamma \mathrm{Cov}_t\left[\Delta c_{t+1}^{\log}, r_{i,t+1}\right] \\
&= R_{f,t}\delta\gamma\rho_t\sigma_g\sigma_{it} \\
&\approx \left\{R_{f,t}\delta = e^{\gamma g - \frac{\gamma^2\sigma_g^2}{2}} \approx 1\right\} \\
&\approx \gamma\rho_t\sigma_g\sigma_{it},
\end{aligned}
$$

and the Sharpe ratio

$$
\frac{E_t\left[r_{i,t+1}\right] - r_{f,t}}{\sigma_{it}} \approx \gamma\rho_t\sigma_g.
$$

The equity premium puzzle

Now let the theory meet data. Using

$$E_t\left[r_{i,t+1}\right] - r_f \approx \gamma \rho_t \sigma_g \sigma_{it},$$

we can estimate the market's risk premium γ.

This analysis was done by Mehra & Prescott in their paper "The Equity premium: A puzzle" from 1985.

This resulted in a very high value of γ.

Using reasonable values on the parameters, it can be shown that γ is around 65, while a value of γ between 2 and 5 seems more realistic (p. 295 in "Financial Asset Pricing Theory" by Claus Munk).

This is know as the equity premium puzzle.

How can this puzzle be resolved?

Extended models

To explain among other things the equity premium puzzle, the following are examples of what have been suggested.

- Habit formation.
- Recursive utility.
- Heterogeneous preferences.
- Ambiguity aversion.

For a fuller list, see Cochrane's paper on macro-finance.

Habit formation

There are two types of habit formation.

Internal habit formation
The level of habit h_t is formed from previous consumption:

$$h_t = f(c_{t-1}, c_{t-2}, \dots)$$

External habit formation
"Keeping up with the Joneses." Consumption is compared to an external habit benchmark X_t:

$$\text{Utility} = u(c_t - X_t).$$

Recursive utility

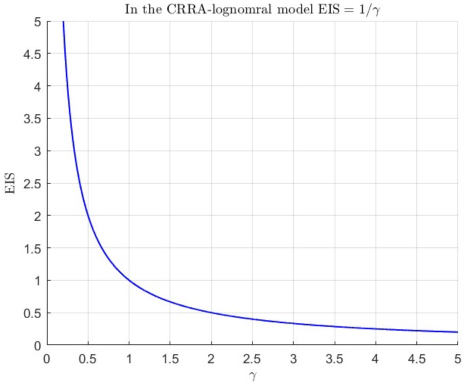
One problem with the standard model is that γ both measures risk-aversion and intertemporal effects.

The elasticity of intertemporal substitution in consumption (EIS) is defined as

$$\text{EIS} = -\frac{d\ln(c_{t+1}/c_t)}{d\ln\left(u'(c_{t+1})/u'(c_t)\right)}.$$

With $u'(x) = x^{-\gamma}$:

$$\text{EIS} = \frac{1}{\gamma}.$$



Recursive utility

One way to disentangle these two effects is to introduce recursive utility.

The general formula is

$$U_t = f(c_t, \mathrm{CE}_t(U_{t+1})),$$

where f is a function and $\mathrm{CE}_t(U_{t+1})$ is the certainty equivalent at time t of the utility at time $t+1$:

$$u\big(\mathrm{CE}_t(U_{t+1})\big) = E_t\left[u\big(U_{t+1}\big)\right]$$

for some utility function u.

One example is the Epstein-Zin recursive utility, which uses $u'(x) = x^{-\gamma}$.

Heterogeneous preferences

Different investors have different risk aversion.

Groups of investors with low risk aversion will buy more stocks than groups with higher risk aversion.

This means that when the stock market goes down, the wealth of the less risk averse will decrease and the average (wealth weighted) risk aversion decreases.

Ambiguity aversion

If the probabilities are not known, then we need to understand how the model behaves.

The case when probabilites are not known is referred to as Knightian uncertainty, or ambiguity.

There are several ways of handle this type of uncertainty.

One way is to use some type of max-min preferences.

A general approach

It turns out that in the cases we have considered above, the SDF can be written

$$m_{t+1} = \delta \frac{u'(c_{t+1})}{u'(c_t)} \cdot Y_{t+1}$$

for a stochastic process Y_t.

ESG investing

The recent years, there has been a hightend interest in ESG investing.

ESG investing

Regarding environmental aspects, it is common to look at the differences between "Green" and "Brown" investments.

Firms and countries issue green bonds, which are bonds that is only allowed to finance green investments.

One potential problem is greenwashing.

ESG investing

Not all investments are purely Green or Brown. Instead investors take ESG profiles into account when they choose their investments.

There are several ways of doings this:

- Active ownership in the firms invested in.
- Systematic and explicit inclusion of ESG in the investment analysis or in the portfolio construction.
- In the security selection, choose the firms with the best ESG profile.
- Include/exclude firms with good/bad ESG profile.

ESG investing

Introduction of ESG rating in portfolio optimisation is an ongoing area of research.

This leads to "ESG portfolio frontiers".

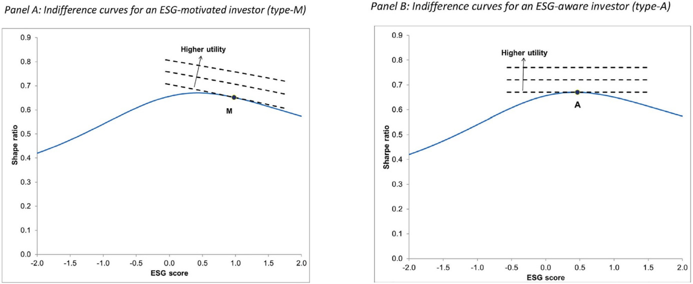


Figure 13.8: The ESG-efficient frontier and investor indifference curves. This is Figure 3 from Pedersen, Fitzgibbons, and Pomorski (2021).

ESG investing

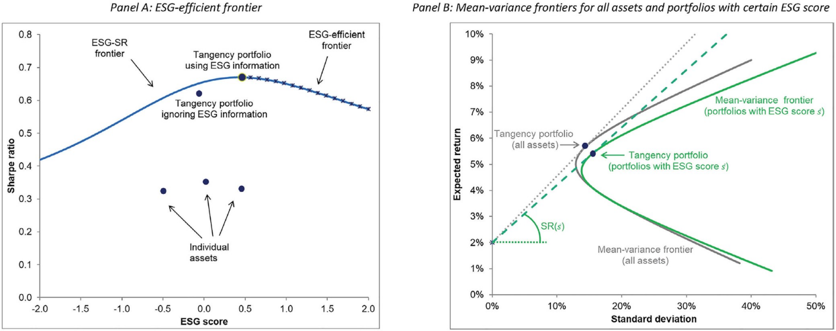


Figure 13.9: The ESG-efficient frontier and mean-variance frontiers. This is Figure 1 from Pedersen, Fitzgibbons, and Pomorski (2021).