Fourier Analysis, 1MA211
Fall 2022, period 2
Course Information

Uppsala University
Department of Mathematics

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1 Introduction

Welcome to Fourier Analysis at Uppsala University! Topics discussed in this course will include Laplace transform, Fourier series, Fourier transform, distributions, and applications to differential equations. Compared to the course Transform Methods, we will do things in a more mathematical way and dive deeper into the theoretical aspects of the subject.

The entire course, including homework, group assignments and exam, with be in **English**.

2 Practical Information

The instructors are:

Luís Diogo, luis.diogo@math.uu.se
Alireza Tavakoli, alireza.tavakoli@math.uu.se

You can find the course schedule on TimeEdit.

In Studium you find a collection of exercises. Exercises with bold enumeration are recommended for the lessons. The other exercises are for the purpose of more practice. A star (*) next to the exercise number indicates that the exercise is on a more advanced level.

3 Course Literature

We will mostly follow:

- [V] *Fourier Analysis and Its Applications*, Anders Vretblad (in English).

Sometimes we may also use:

- [L] *Fourieranalys*, Lars-Åke Lindahl (notes in Swedish, available on Studium),
- [SS] *Fourier Analysis: an Introduction*, Elias Stein and Rami Shakarchi (in English).

4 Schedule

There are 20 lectures and 10 lessons. The following is a tentative schedule for the course (it may change a little during the period).

Subject	Lectures	Lessons	Course Literature
Introduction	1	1	
Background: complex numbers, Riemann integrals	1-2	1	[L] 2.1
Laplace transform	3-4	1-2	[V] 3.1-4; [L] 9.1-4
Sequences and series of functions	4-5	3	[L] 2.2
Fourier series. Uniform and pointwise convergence	6-8	3-4	[V] 4.1, 4.3-6; [L] 3.1-7, 4.3-8
Separation of variables for partial differential equations	9	5	[V] 6.1-6.3; [L] 5
Mean square convergence (also called L^2-convergence)	10-12	5	[V] 5.1-6
Fourier transform	13-16	6-7	[V] 7.1-8; [L] 6.1-7, 7.1-2, 7.4, 8.1, 8.4
Distribution theory	17-18	8	[V] 8.1-8.9
Group oral presentations		9	
Review	19-20	10	

5 Course Material

The course material will be available on Studium. There you will find:

- Course information (this document),
- Table of formulas (provided on the exam),
- Collection of exercises (possibly updated during the period),
- *Fourieranalys*, Lars-Åke Lindahl (notes in Swedish),
- Home assignments,
- Old exams for practicing.

Note that you will have to register to the course before you get access to the material in Studium. Also, some of the content might be updated during the course.

6 Home Assignments

There are two **optional** home assignments. Do not confuse them with the collection of exercises (which is for your own practice only).

- The due dates are as follows:

  Home assignment 1: 2022-11-26

  Home assignment 2: 2022-12-22

- The assignments can increase the exam grade by a maximum of 3 bonus points. Let P be the sum of the points you get in the two assignments and let T be the sum of the total number of points available in the two assignments. You get:

  1 bonus point: if $P \geq 40\%$ of T;

  2 bonus points: if $P \geq 60\%$ of T;

  3 bonus points: if $P \geq 75\%$ of T.

- The bonus points can only be used on the ordinary exam, and can only increase your exam grade to 31 points (so they cannot raise your course grade to 5).

- You are allowed to cooperate but not to copy from each other.

- The solutions to the assignments are submitted in Studium in pdf-format, either scanned or written in LaTeX. Please make sure that the hand-ins are clearly readable!

- You need to write your answers in **English**.

7 Group oral assignments

There is a **mandatory** group assignment, scheduled for **Lesson 9**. Each group shall prepare a 12 minute presentation about one of the topics below (no two groups should choose the same topic). For each topic, you should explain the main result, indicate why it is important and give an idea of the proof. You should also be able to answer related questions. I expect you to email me the notes of your presentation. This part of the course is graded as Pass or Fail. The presentations should be done in **English**.

The registration for the groups and the choice of topics will be done through Studium, later in the course.

Here is a list of possible topics, with an indication of relevant course literature:

1. Shannon's sampling theorem ([V] and [L])
2. Heisenberg's uncertainty principle ([L] and [SS])
3. Gibbs' phenomenon ([V] and [L])
4. Weierstrass' approximation theorem and applications, also known as the Stone–Weierstrass theorem ([V], [L] and [SS])
5. discrete Fourier transform ([V], [L] and [SS])
6. derivation of the wave equation for the vibrating string ([SS])
7. Poisson kernel and Dirichlet's problem in the disk ([V], [L] and [SS])
8. linear time-invariant systems ([L])
9. central limit theorem ([L])
10. the Z-transform ([V] and [L])
11. multi-dimensional Fourier analysis ([V] and [SS])
12. Fejér's theorem and consequences ([V] and [SS])

8 Exam

- The exam will take place on 2023-01-09. Look up the time and place on Ladok.
- **Do not forget to register for the exam.** You can do so on Ladok, until 2 weeks before the exam.
- The exam will be graded for a total of 40 points. The grade limits for grades 3, 4 and 5 are 18, 25 and 32 respectively.

9 Syllabus

The course syllabus can be found here.

9.1 Course Content

Fourier series in complex and trigonometric form. Pointwise and uniform convergence. The Dirichlet kernel. Summability methods. L^2-theory: Orthogonality, completeness, ON systems. Applications to partial differential equations. Separation of variables. Distributions.

The Fourier transform and its properties. Convolution. The inversion formula. The Plancherel theorem.

The Laplace transform and its properties. Convolution. Applications to initial value problems and integral equations.

9.2 Learning Outcomes

On completion of the course, the student should be able to

- account for basic concepts and theorems within the Fourier analysis,
- demonstrate basic numeracy skill concerning the concepts in the previous point,
- use the numeracy skill at the solution of mathematical and physical problems formulated as ordinary or partial differential equations.

10 For those Interested

Other interesting courses that are related to Fourier analysis include:

- Integration theory
- Partial differential equations (10 credit version)
- Functional analysis
- Mathematical methods of physics (for Physics students)

Other interesting books on the topic:

- Fourier Analysis and Applications, Folland,
- Introduction to Fourier Analysis and Wavelets, Pinsky.