A5

Consider the heat equation with constant coefficients in the interval $\mathcal{I} = (0, L)$:

$$\begin{aligned} u_t = a u_{xx}, &\quad x \in \mathcal{I}, \quad t > 0, \\ u = g, &\quad x = 0, \quad t > 0, \\ u_x = 0, &\quad x = L, \quad t > 0, \\ u = f, &\quad x \in \mathcal{I}, \quad t = 0, \end{aligned}$$

where g, $a > 0$, and $L > 0$ are real constants. We assume that the solution is real.

Let V be the space of functions $v(x,t)$ such that v and v_x are square-integrable over $\mathcal{I}$ for any t:

$$V = \{v(x,t) : ||v(\cdot,t)|| + ||v_x(\cdot,t)|| < \infty\}\,.$$

Further, define two spaces of functions that additionally satisfy Dirichlet boundary conditions at $x = 0$:

$$V_0 = \{v \in V : v(0,t) = 0\}$$

$$V_g = \{v \in V : v(0,t) = g\}\,.$$

Below is the weak form of the IBVP, but there are three gaps (X1, X2, and X3) that you need to fill in! Choose from the 7 alternatives listed below. To score 1 point, you need to get all gaps (X1, X2, and X3) right.

Find X1 such that

X2

for all X3.

Alt. 1: $u \in V_g$

Alt. 2: $u \in V_0$

Alt. 3: $v \in V_g$

Alt. 4: $v \in V_0$

Alt. 5: $u_t = a u_{xx}$

Alt. 6: $(v, u_t) = -a(v_x, u_x)$

Alt. 7: $(v, u_t) = -a(v_x, u_x) + agv(L,t)$

Solution

A correct statement of the weak form is:

Find $u \in V_g$ such that

$$(v, u_t) = -a(v_x, u_x)$$

for all $v \in V_0$.

That is, the correct answers are:

- X1 = Alt. 1
- X2 = Alt. 6
- X3 = Alt. 4