Partial Differential Equations with Applications in Finance, VT25

Topias Tolonen-Weckström

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Lecturer:	Topias Tolonen-Weckström	**Course Page:**	On Studium here
Email:	topias.tolonen@math.uu.se	**Schedule:**	On TimeEdit here

1 General information

- **Lectures:** There will be 13 lectures. Regular attendance is essential but not required. Locations of the lectures can be found in TimeEdit.
- **Exercise sessions:** There will be 2 exercise sessions, during which we will solve some of problems together.
- **Syllabus and Reading list:** On UU's Website.

2 Course Assessment

- **Assignments:** There will be two non-mandatory assignments, each worths 2 bonus points. The goal of the assignments is to introduce you to exam-like questions and to incentive you in solving problems. I will grade them as if they were exam questions, so that as feedback you can see how I will be grading exam questions. Collaboration is encouraged. However, each student should submit their solutions independently.
- **Final exam:** There will be one mandatory closed-book exam on *June 2nd*. You will get the exam paper with 5 problems each worth 8 points. Some old exams are to be shared for practice.
- **Retakes:** The retakes are scheduled in August and Easter.
  Note: any possible bonus points cannot be used for the retakes.

The final grade is calculated as follows (these may be adjusted downwards):

- $18 - 24$ points: grade 3
- $25 - 31$ points: grade 4
- $32 - 44$ points: grade 5

3 Course content

- **Objectives:** This course is primarily designed for graduate students in mathematics and others interested in the themes of the course. We will discuss the basics of stochastic processes, stochastic calculus, SDEs and their connection to parabolic PDEs, and further discuss topics in optimal stopping and stochastic control theory.
  Like most courses in mathematics, we will define mathematical objects, make statements and show theorems. However, instead of formulating the theories step by step, we will look at the general mathematical mechanisms and intuition of those theories: how different aspects are correlated (perhaps surprisingly), and formulate concrete examples and solve them.
  This course is, generally speaking, a probability course. We will not go through general and rigorous PDE theory and we will not do numerics, data, or coding. If the course is viewed from the lenses of the democratic aspect of higher education, the main aim would be for the students to deepen their enjoyment in probability and be inspired by the applications of it.

- **Prerequisites:** The syllabus defines prerequisites as "90 credits in mathematics. Financial Derivatives. Participation in Probability Theory II or Integration Theory". This is somewhat accurate, and most important here is a working knowledge of (measure theory based – if possible) probability theory. Previous knowledge on stochastic calculus and financial derivatives would be a plus and makes the exercises less tedious.

- **Tentative Course Plan:** The course consists of five modules as shown below, and each module will be associated with one or two basic examples in financial mathematics. Please check the last page of this document for the detailed preliminary lecture plan.

  1. Review of Probability Theory and Itô calculus
  2. Feynman-Kac theorem and its applications
  3. The Heat Equation and the Kolmogorov Equations
  4. Optimal Stochastic Control
  5. Optimal stopping and free boundary problems

4 Course Literature

The course has extensive lecture notes which will updated as the course goes:

- Tolonen-Weckström, Topias: Partial Differential Equations with Applications in Finance, Lecture notes, Uppsala University, 2024.

The lecture notes aim to be a somewhat exhaustive source for the students to study, but they are ultimately meant to be complimented by the lectures. Both the lectures and the notes are largely based on the following material:

- Björk, Tomas: Arbitrage theory in continuous time, Fourth edition., Oxford, Oxford University Press, 2020
- Kohn, Robert: PDE for Finance, Courant Institute, New York University, 2015
- Karatzas, Ioannis; Shreve, Steven E.: Brownian motion and stochastic calculus, 2. ed., Berlin, Springer, cop. 1991.

This book is not listed in the course syllabus, but it is a wonderful source for SDE-related knowledge. Moreover, it touches upon almost all concepts covered during this course:

- Øksendal, Bernt. Stochastic Differential Equations: An Introduction with Applications, Springer. 2014

5 Course Evaluation

There will be two anonymous surveys, the **mid-course evaluation** and the **course evaluation** on Studium. All course evaluations are non-mandatory, but they help all of us identify what is working and what could use improvement. It is therefore recommended that you take some time and answer the surveys.

6 Preliminary Lecture Plan

F1. Course introduction, probabilities. 28/3
F2. The Ito integrals and their properties 1/4
F3. Stochastic differential equations and the Ito's formula 4/4
F4. The infinitesimal generator and the Feynman-Kac Theorem 7/4
F5. Brownian motion and applications of the FK theorem 10/4
F6. Brownian Motion Continued 22/4
F7. The heat equation and its properties 25/4
Assignment I Deadline 27/4
F8. The heat equation II, Kolmogorov equations for BM 2/5
L1. Exercise session I 5/5
F9. Markov processes and the Kolmogorov equations and Dupire's local volatility 7/5
F10. Optimal control problems, the Hamilton-Jacobi-Bellman Equation . 12/5
F11. Merton's portfolio allocation problems 13/5
F12. Optimal stopping theory I 15/5
Assignment II Deadline 18/5
F13. Optimal stopping theory II, the American options 19/5
L2. Exercise session II 22/5
Exam 2/6