Remember to justify your answers clearly. You may answer in Swedish or in English. There are 8 problems, each worth of 0–5 points. Grade limits are 16-24 (grade 3), 25-31 (Grade 4), and 32-40 (grade 5).

1. Is the function $f(x,y) = \frac{x^2y^2}{x^2y^2+(x-y)^2}$, $\quad (x,y) \neq (0,0)$, $f(0,0) = 1$ continuous? (5p)

2. Find the largest and smallest value of $f(x,y,z) = x + y$ on $x^2 + y^2 + z^2 \leq 1$. (5p)

3. Find all $u(x,y) = f(xy)$, where f is a function of one-variable, that satisfy $xu'_x + yu'_y = x^2y^2$. (5p)

4. Let $f(x,y,z) = \cos(xyz) + xz^2 - y^2x$. (5p)
   (a) Find the direction of most decrease at point $(x_0,y_0,z_0) = (0,0,1)$.
   (b) Compute the directional derivative into the direction $v = (1,1,1)$ at the point $(x_0,y_0,z_0) = (0,0,1)$.

5. Show that the equation $yx - \sin x = \cos x + 1$ has a solution $y = y(x)$ that is close to 0 whenever x is close to π. Find the coefficients a_0, a_1, and a_2 in the Taylor approximation

$$y(x) \approx a_0 + a_1x + a_2x^2$$

   around $x = \pi$. (5p)

6. Compute $\int\int_D xy(x^2+y^2)^3 dxdy$, where $D = \{(x,y) : 0 \leq x, y \leq 1\}$. (5p)

7. Compute the curve integral $\int_\gamma \left(y + y\sin(e^{xy}) + x^2 + y^2\right) dx + (x + x\sin(e^{xy}))\, dy$, where γ is given by the parametrisation $r(t) = (x(t), y(t)) = (2\cos t, 2\sin t)$, $\quad 0 \leq t \leq \pi$. (5p)

8. Find the values of α such that the integral $\int\int\int_{\mathbb{R}^3} \frac{\min(1,x^2+y^2+z^2)}{(x^2+y^2+z^2)^\alpha} dxdydz$ converges. (5p)