Uppsala University
Department of Economics
Fredrik Armerin
070 – 251 75 55

Exam number:

Exam in Master / Financial Theory

General instructions

- No technical aids are allowed.
- All calculations should be clearly motivated.
- Do not skip steps in the formal derivations.
- Answer the questions without providing additional / unrelated information. I deduct points for incorrect statements you make.
- If you cannot solve a question without making additional assumptions, state these assumptions clearly and explain in writing why they are necessary.
- The writing time is 5 hours. Write your examination number in the indicated space and on all papers you hand in.
- The total number of points is 50. For grade E is 25 points required, for grade D is 27.5 points required, for grade C is 32.5 points required, for grade B is 37.5 points required and for grade A is 45 points required.

Good luck!

Problem 1

a) Write down Gordon's formula for the stock price P_t as a function of the dividends D_t, and define the different parts of it. (2 p)

b) A zero coupon bond has 4 years to maturity and is trading at a yield of $y = 0.043$. Determine the duration of the bond. (2 p)

c) The Fama-French model uses the return on the market portfolio as one factor. Name one of the other factors and how it is defined. (2 p)

d) An investor has utility function $u(x) = -5e^{-3x}$. Derive this investor's coefficient of absolute risk aversion. (2 p)

e) Three assets has the following mean and standard deviation of their rate of return:

Asset no i	$E[r_i]$	$\text{Std}[r_i]$
1	0.28	0.5
2	0.3	0.4
3	0.25	0.35

Can all three assets be on the mean-variance frontier? Motivate your answer. (2 p)

Problem 2

a) A market consists of the following assets.

Asset no i	β_i
1	0.9
2	1.2
3	1.8

How large is the beta value of a portfolio which has 300 000 euros invested in each of the assets? (2 p)

b) The rate of return of two assets have the following standard deviations:

$$\sigma_1 = 0.2 \text{ and } \sigma_2 = 0.4.$$

Determine the variance of the minimum variance portfolio if the rate of returns are uncorrelated and shortselling is allowed. (2 p)

c) Determine the $\alpha\%$ one-month Value-at-Risk if the monthly rate of return r satisfies $\ln(1+r) \sim N(\mu, \sigma^2)$. Here we interpret the Value-at-Risk as the number VaR such that $P(r \leq \text{VaR}) = \alpha/100$. (2 p)

d) Consider the factor model

$$r_i = E[r_i] + \beta_{i1}\big(F_1 - E[F_1]\big) + \beta_{i2}\big(F_2 - E[F_2]\big) + \varepsilon_i.$$

Write down the equation for the expected return $E[r_i]$ if there exists a risk-free asset and the conditions of the APT holds. (2 p)

e) Show that if the first n years of yields-to-maturity are all equal, $y_1 = y_2 = \ldots = y_n = y$, then every forward rate defined by this yield curve is equal to y. (2 p)

Problem 3

There are N risky assets, but no risk-free asset, and the rate of return vector $\boldsymbol{r} = (r_1, r_2, \ldots, r_N)^T$ has

$$E[\boldsymbol{r}] = \boldsymbol{\mu} \text{ and } \operatorname{Var}[\boldsymbol{r}] = \Sigma.$$

a) Derive the portfolio weights of the minimum variance portfolio, i.e. solve the problem

$$\begin{aligned} \min_{\boldsymbol{\pi}} \quad & \boldsymbol{\pi} \cdot \Sigma \boldsymbol{\pi} \\ \text{s.t.} \quad & \boldsymbol{\pi} \cdot \mathbf{1} = 1. \end{aligned}$$

(3 p)

b) The mean-variance frontier is given by the equation

$$\sigma(\bar{\mu}) = \sqrt{\frac{C\bar{\mu}^2 - 2B\bar{\mu} + A}{D}},$$

where A, B, C and D are constants. Use this equation to determine the variance of the minimum variance portfolio (expressed in the constants A, B, C and D). (3 p)

We now introduce a risk-free asset with rate of return r_f.

c) Explain how the mean-variance frontier will now look like, and describe the possible pairs of $(\sigma, \bar{\mu})$ that are available to any rational investor. You do not need to provide any formulas or equations, and can use a graph to complement your answer. (4 p)

Problem 4

A firm has beta value $\beta = 1.5$ with respect to the market portfolio, there is a risk-free asset with rate of return $r_f = 0.01$ and the expected return of the market portfolio is $E[r_m] = 0.11$.

a) How large is the discount rate at which the firm's dividends should be discounted according to the CAPM? (2 p)

The firm uses a plowback ratio of $b = 0.1$ and has return on equity $r_e = 0.40$. The earnings per share today, at time t, is $e_t = 8$ euros.

b) Calculate and give an economic interpretation of the present value of growth opportunities (PVGO). (4 p)

An investor buying shares in this firm wants to leverage up the investment by borrowing at the risk-free rate. The investor has the initial amount $E_0 = 50\,000$ euros and wants to invest $V_0 = 150\,000$ euros.

c) What is the investor's expected rate of return relative to the initial amount? (4 p)

Problem 5

A consumer wants to maximise

$$u(c_0) + \delta E[u(c_1)].$$

Given endowments and a possibility of investing in a portfolio of assets, the first-order condition of optimality can be written

$$P_{i0} = E\left[\left(P_{i1} + D_1\right)\delta\frac{u'(c_1)}{u'(c_0)}\right],\ i = 1, 2, \ldots, N,$$

where P_{it} is the price of asset i at time $t = 0, 1$, and D_i is the dividends paid by asset i at time 1.

a) Explain why we refer to

$$m = \delta\frac{u'(c_1)}{u'(c_0)}$$

as a stochastic discount factor (SDF). (2 p)

b) Under the assumption of the existence of a risk-free rate r_f, use the first-order condition above to derive the consumption-based CAPM equation

$$E\left[r_i\right] = r_f + \mathrm{Cov}\big(r_i, -(1 + r_f)m\big).$$

(5 p)

c) In order to improve this type of model in dynamic macro-finance modelling, one way is to introduce habit formation. Explain what is meant by internal and external habit formation respectively. (3 p)