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**Exam number:**

# Exam in Master / Financial Theory

## General instructions

- No technical aids are allowed.

- All calculations should be clearly motivated.

- Do not skip steps in the formal derivations.

- Answer the questions without providing additional / unrelated information. I deduct points for incorrect statements you make.

- If you cannot solve a question without making additional assumptions, state these assumptions clearly and explain in writing why they are necessary.

- The writing time is 5 hours. Write your examination number in the indicated space and on all papers you hand in.

- The total number of points is 50. For grade E is 25 points required, for grade D is 27.5 points required, for grade C is 32.5 points required, for grade B is 37.5 points required and for grade A is 45 points required.

Good luck!

## Problem 1

a) Write down the equation for the capital market line (CML), and define the different parts of it. (2 p)

b) —

c) How is the excess return $SMB$ in the Fama-French model defined? (2 p)

d) If a market satisfies the weak-form version of the efficient market hypothesis, must it also satisfy the strong-form version? Motivate your answer. (2 p)

e) In prospect theory, what is the form of the utility function to the left and to the right of the reference point, respectively? You may answer by drawing a graph. (2 p)
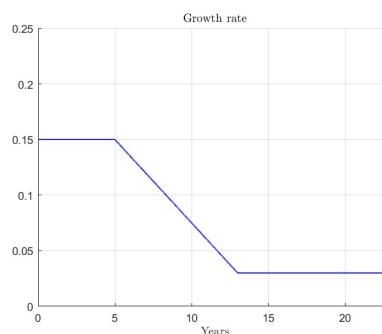
## Problem 2

a) Consider the following order book for the shares of a listed company.

| Number | Bid | Ask | Number |
|--------|-----|-----|--------|
| 100 | 106 | 107 | 230 |
| 125 | 101 | 112 | 120 |
| 50 | 98 | 118 | 150 |

An investor sells 150 shares in this market. How large is the bid-ask spread after these shares are sold? (2 p)

b) The following model has been suggested for a three period dividend growth model:



Describe the features that this model wants to capture. Be brief in your answer. (2 p)

c) Consider a model with $N \geq 2$ risky assets and a risk-free asset with return $r_f$ where short-selling is allowed. In equilibrium we can interpret the tangent portfolio as the market portfolio. Why do we need $r_f < \bar{\mu}_{min}$, where $\bar{\mu}_{min}$ is the expected return on the minimium variance portfolio, in order to get an economically reasonable model? Be brief in your answer.
(2 p)

d) In a dynamic consumption model the stochastic discount factor (SDF) is modelled as
$$m_{t+1} = 0.96e^{-0.5(C_{t+1}-C_t)}.$$

Give one example of a utility function that the representative consumer can have and that will result in this type of SDF. Motivate your answer.
(2 p)

e) A return is modelled using the following model with three factors:

$$r = 0.2 + 0.4F_1 - 0.5F_2 + 0.1F_3 + \varepsilon.$$

Here, the expected value of each of the three factors is zero, their variance is one and they are all uncorrelated with each other and with $\varepsilon$. How large is the idiosyncratic risk if $\text{Var}[r] = 0.46$? (2 p)

## Problem 3

The following government bondsare traded

| Bond no | Time to maturity | Coupon rate | Yield to maturity |
|---------|------------------|-------------|-------------------|
| 1       | 1                | 0           | 0.05              |
| 2       | 2                | 0.06        | 0.06              |
| 3       | 4                | 0           | 0.08              |

a) —

b) Is the forward rate $f_{1,4}$ larger or smaller than 0.08? Motivate your answer.
(2 p)

c) An investor wants to invest 1 million euros by investing in bond no 1 and bond no 3. How much of the invested amount should be put in each of the two bonds if the investor wants the bond portfolio to have duration $D = 3$?
(3 p)

d) Let $P_1$, $P_2$ and $P_3$ be the price of bond 1, 2 and 3 respectively when they have the same face value $F = 100$. Is it true that $P_3 < P_2 < P_1$? Motivate your answer. (3 p)

## Problem 4

Two assets have the following expected returns and standard deviations:

$$\mu_1 = 0.2, \ \mu_2 = 0.4, \ \sigma_1 = 0.1 \text{ and } \sigma_2 = 0.2.$$

a) How large is the expected return of the minimum variance portfolio if the correlation between the two asset returns is zero and short-selling is not allowed? (2 p)

b) For which values of the correlation between the two asset returns is the portfolio $(\pi_1, \pi_2) = (1, 0)$ the minimum variance portfolio if short-selling is not allowed? (3 p)

c) Determine the efficient frontier $\sigma(\overline{\mu})$ if the two assets are uncorrelated and short-selling is allowed. (5 p)

*Hint:* Use that $\pi_1 + \pi_2 = 1$ and express the portfolio weights in terms of the expected returns.

## Problem 5

A market consists of the following assets.

| Asset no | Price | No of shares | Expected return |
|----------|-------|--------------|-----------------|
| 1        | 20    | 100          | 0.05            |
| 2        | 25    | 200          | 0.1             |
| 3        | 20    | 150          | 0.2             |

The risk-free rate is $r_f = 0.02$.

a) Determine the market portfolio. (2 p)

b) Determine the security market line (SML) in this market. (2 p)

A firm's dividends has growth rate $g = 0.10$ and the firm's beta value is $\beta = 1.2$. In order to value the future dividend payments, a constant discount rate $r$ is used, and this is set according to the CAPM.

c) How large is the price earnings (P/E) ratio of the firm today if the plowback ratio is $b = 0.5$? (4 p)

The firm has invested in a project with cash flows $Y$ in one years time. The project's return has the same beta value as the firm.

d) What is the value today of this project if $E[Y] = 2\,280\,000$? (2 p)