Multivariate calculus M
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Exercise 6.

6. Generalised multidimensional integrals

We solve the problems together in the exercise sessions. Note that these problems are optional and for learning purposes: solving these does not provide extra points. Actual home assignments (giving you extra points) are given separately.

It is advised to take a look of the problems beforehand. Note that some of the problems might be very challenging, so do not feel bad if you are unable to solve them independently: we will go through the solutions together!

Problems for the session

6.1 Compute $\int\int_{\mathbb{R}^2} x^2 e^{-\sqrt{x^2+y^2}} dxdy$.

6.2 Let $D = \{(x,y) : 0 \leq y \leq 1-x, 0 \leq x \leq 1\}$. For which values $\alpha \in \mathbb{R}$ the integral $\int\int_D \frac{1}{(x+y)^\alpha} dxdy$ converges?

6.3 Is the generalised integral $\int\int_{\mathbb{R}^2} \frac{x^2}{(1+x^2)(x^2+y^2)^{\frac{3}{2}}} dxdy$ convergent or divergent?

6.4 Compute $\int\int\int_{\mathbb{R}^3} \frac{e^{-(x+2y+2z)^2}}{[1+(2x-2y+z)^2)][1+(2x+y-2z)^2]} dxdydz$.

Problems for individual practice

In addition to the problems below, one can get routine by solving similar exercises from the exercise-book "övningar i flerdimensionell analys".

6.1 Compute $\int\int_D \frac{1}{1+x^2y^2} dxdy$ for $D = \{(x,y) : 1 \leq x \leq 2, y \geq 0\}$.

6.2 Let D be a triangle determined by $(0,0)$, $(0,1)$, and $(1,0)$. Show that the integral $\int\int_D (1-x-y)^\alpha dxdy$ converges for all values $\alpha > -1$, and determine the value of the integral.

6.3 For which values $\alpha \in \mathbb{R}$ the integral $\int\int_{\mathbb{R}^2} (x^2+y^2)^{-\frac{\alpha}{2}} dxdy$ is convergent?

6.4 Is $\int\int_D \frac{x^2+1}{x^2(x^2+y^2)^{\frac{3}{2}}} dxdy$ for $D = \{(x,y) : x^2+y^2 \leq 1\}$ convergent or divergent?

6.5 Compute $\int\int\int_K e^{x+y+z} dxdydz$, where $K = \{(x,y,z) : 0 \leq x,y,z \leq 1\}$.

6.6 Compute $\int\int\int_K \sqrt{x^2+y^2+z^2} dxdydz$, where $K = \{(x,y,z) : x^2+y^2+z^2 \leq 1, z \geq \sqrt{x^2+y^2}\}$.

6.7 Compute $\int\int\int_K \frac{1}{x^2+y^2+z^2} dxdydz$, where $K = \{(x,y,z) : 1 \leq x^2+y^2+z^2 \leq 4\}$.

6.8 Compute $\int\int\int_K \frac{z}{1+x^2+y^2} dxdydz$, where $K = \{(x,y,z) : x^2+y^2+z^2 \leq 1, z \geq \sqrt{x^2+y^2}\}$.