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Exam number:

Exam in Master / Financial Theory

General instructions

- No technical aids are allowed.

- All calculations should be clearly motivated.

- Do not skip steps in the formal derivations.

- Answer the questions without providing additional / unrelated information. I deduct points for incorrect statements you make.

- If you cannot solve a question without making additional assumptions, state these assumptions clearly and explain in writing why they are necessary.

- The writing time is 5 hours. Write your examination number in the indicated space and on all papers you hand in.

- The total number of points is 50. For grade E is 25 points required, for grade D is 27.5 points required, for grade C is 32.5 points required, for grade B is 37.5 points required and for grade A is 45 points required.

Good luck!

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Problem 1

a) What is meant by the plowback ratio? (2 p)

b) The price of stocks in a company is 100 euros per share today. How large is the leverage ratio of an investor who wants to buy 200 stocks by investing 5 000 euros of his or her own money? (2 p)

c) Determine the coefficient of relative risk aversion for an investor with utility function $u(x) = 2 \ln x$. (2 p)

d) What is, according to CAPM, the expected return on an asset whose rate of return is uncorrelated with the rate of return of the market portfolio? (2 p)

e) Give two examples of behavioural biases. (2 p)

Problem 2

a) A market consists of the following assets.

Asset no	Price	No of shares
1	22.5	400
2	30	200
3	15	1 000

Determine the market portfolio. (2 p)

b) The rate of return of two assets have the following standard deviations:

$$\sigma_1 = 0.1 \text{ and } \sigma_2 = 0.215.$$

Determine the variance of the minimum-variance portfolio if the correlation between the rates of return is 1 and short-selling is not allowed. (2 p)

c) Describe using a graph together with a text explaining the graph, the difference between Value-at-Risk and Expected shortfall. (2 p)

d) In the one period consumption-based CAPM the stochastic discount factor (SDF) is given by

$$m = \delta \frac{u'(c_1)}{u'(c_0)}.$$

Explain why the high values of m occurs at bad future states of the world. (2 p)

e) What is meant by the expression the "factor zoo"? (2 p)

Problem 3

There are N risky assets and the rate of return vector $\boldsymbol{r} = (r_1, r_2, \ldots, r_N)^T$ has

$$E[\boldsymbol{r}] = \boldsymbol{\mu} \quad \text{and} \quad \text{Var}[\boldsymbol{r}] = \Sigma.$$

There is also a risk-free asset with rate of return r_f, and the portfolio weights in the risky assets is denoted $\boldsymbol{\pi}$.

a) Show that the problem of finding the portfolio weights in the risky assets that gives the portfolio with minimum variance having expected rate of return $\bar{\mu}$ is given by

$$
\begin{aligned}
\min_{\boldsymbol{\pi}} \quad & \boldsymbol{\pi} \cdot \Sigma \boldsymbol{\pi} \\
\text{s.t.} \quad & r_f + \boldsymbol{\pi} \cdot (\boldsymbol{\mu} - r_f \mathbf{1}) = \bar{\mu}.
\end{aligned}
$$

(3 p)

b) Derive the portfolio weights for the tangent portfolio. (5 p)

Hint: The portfolio weights $\boldsymbol{\pi}_{\text{tan}}$ of the tangent portfolio solves the problem in a) and satisfies the relation $\boldsymbol{\pi}_{\text{tan}} \cdot \mathbf{1} = 1$.

c) Why is it reasonable from an economic point of view that the risk-free rate of return should be lower than the mean rate of return of the minimum-variance portfolio?

(2 p)

Problem 4

In order to model a financial market three uncorrelated factors F_1, F_2 and F_3 are used. They are normalised to have expected value equal to zero and variance equal to one. In this model the return of two financial assets are given by

$$
\begin{aligned}
r_1 &= E[r_1] + 0.2F_1 - 0.2F_2 + 0.3F_3 \\
r_2 &= E[r_2] + 0.1F_1 + 0.1F_2 + 0.5F_3.
\end{aligned}
$$

a) Which of the two assets has the largest total risk? (2 p)

b) Explain why there is no idiosyncratic risk in any of the two assets. (2 p)

The risk premia of the three factors are

$$RP_1 = 0.1, \ RP_2 = 0.05 \text{ and } RP_3 = 0.2$$

respectively.

c) How large is the expected return of return on asset 2 if the expected rate of return of asset 1 is 0.12 and we assume that the conditions of APT holds?

(4 p)

d) Now assume that there exists a risk-free asset with rate of return $r_f = 0.04$. How large is now the expected rate of return on asset 1 if we assume that the conditions of APT holds? (2 p)

Problem 5

The following government bonds, all with face value 100, are traded on a market.

Bond no	Time to maturity	Coupon rate	Price
1	1	0	98
2	2	0.05	101.9
3	3	0	95

a) What would the price be of a zero-coupon bond with face value 100 and time to maturity 2 years? (4 p)

b) Is the forward rate $f_{1,3}$ larger or smaller than 0.01? Motivate your answer. (4 p)

c) Which of the bonds have the largest duration? Motivate your answer. (2 p)