Some basic microeconomics

Preferences and utility functions

Every consumer have preferences $\succeq$ over consumption bundles.

If x is preferred to y, then we write $x \succeq y$.

We use a utility function U to represent preferences:

$$x \succeq y \quad \Leftrightarrow \quad U(x) \geq U(y).$$

The goal of a consumer is to maximize his or her utility.

Deriving the optimal quantities (1)

In order to choose the optimal consumption bundle, the consumer needs to take the prices of the goods into account.

If there are n number of goods, then there are n number of prices:

$$p_1, p_2, \ldots, p_n.$$

Deriving the optimal quantities (2)

Let

$$x = (x_1, x_2, \ldots, x_n)$$

denote the vector of quantities of each good and m the income, or budget, available to the consumer.

Then the budget constraint, or budget line, is

$$p_1x_1 + p_2x_2 + \ldots + p_nx_n = m.$$

Deriving the optimal quantities (3)

The consumer wants to maximize his or her utility $U(x)$ given the budget constraint.

Mathematically, we get

$$\max_x U(x) \quad \text{subject to} \quad p_1 x_1 + p_2 x_2 + \ldots + p_n x_n = m.$$

Deriving the optimal quantities (4)

From now on we assume that $n = 2$:

$$\max_{x_1, x_2} U(x_1, x_2) \text{ subject to } p_1 x_1 + p_2 x_2 = m.$$

To solve this problem we start by using the constraint to get

$$p_1 x_1 + p_2 x_2 = m \quad \Leftrightarrow \quad x_2 = \frac{m - p_1 x_1}{p_2}.$$

We then replace x_2 with this in U:

$$\max_{x_1} U\left(x_1, \frac{m - p_1 x_1}{p_2}\right).$$

Deriving the optimal quantities (5)

The first-order condition (FOC) with respect to x_1 is

$$\frac{\partial U}{\partial x_1} + \frac{\partial U}{\partial x_2} \cdot \left(-\frac{p_1}{p_2}\right) = 0$$

$$\Leftrightarrow$$

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{p_1}{p_2}.$$

Deriving the optimal quantities (6)

Using

$$\frac{\frac{\partial U}{\partial x_1}}{\frac{\partial U}{\partial x_2}} = \frac{p_1}{p_2}.$$

together with the budget constraint

$$p_1 x_1 + p_2 x_2 = m$$

makes it possible to solve for the optimal quantities:

$$x_1 = D_1(p_1, p_2, m) \text{ and } x_2 = D_2(p_1, p_2, m).$$

These are the demand functions for the individual.

The Cobb-Douglas utility function (1)

A commonly used utility function is the Cobb-Douglas utility function:

$$U(x_1, x_2) = x_1^a x_2^{1-a}$$

for $0 < a < 1$.

In this case

$$\frac{\partial U}{\partial x_1} = a x_1^{a-1} x_2^{1-a}$$

and

$$\frac{\partial U}{\partial x_2} = (1-a) x_1^a x_2^{-a}.$$

It follows that

$$\frac{\partial U / \partial x_1}{\partial U / \partial x_2} = \frac{a x_2}{(1-a) x_1}.$$

The Cobb-Douglas utility function (2)

In the Cobb-Douglas case, the optimal quantities are found by solving

$$\begin{cases} \frac{ax_2}{(1-a)x_1} & = & \frac{p_1}{p_2} \\ p_1 x_1 + p_2 x_2 & = & m. \end{cases}$$

The solution is given by

$$x_1 = \frac{am}{p_1} \quad \text{and} \quad x_2 = \frac{(1-a)m}{p_2}.$$

Hence, the demand functions are

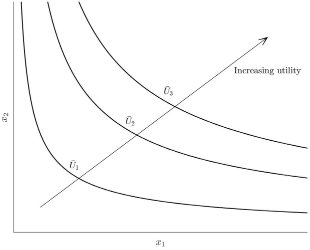
$$D_1(p_1, p_2, m) = \frac{am}{p_1} \quad \text{and} \quad D_2(p_1, p_2, m) = \frac{(1-a)m}{p_2}.$$

Indifference curves (1)

Now fix a utility level $\bar{U}$. We define the indifference curve as:

$$U(x_1, x_2) = \bar{U}.$$

Two indifference curves representing two different utility levels do not cross.



The utility increases in the north-east direction (so $\bar{U}_1 < \bar{U}_2 < \bar{U}_3$).

Indifference curves (2)

We can differentiate the equation

$$U(x_1, x_2) = \bar{U}$$

to get

$$dU(x_1, x_2) = d\bar{U} \quad \Leftrightarrow \quad \frac{\partial U}{\partial x_1} dx_1 + \frac{\partial U}{\partial x_2} dx_2 = 0.$$

Now rearrange this equation into:

$$\frac{dx_2}{dx_1} = -\frac{\partial U / \partial x_1}{\partial U / \partial x_2}.$$

Slopes

We saw above that along an indifference curve

$$\frac{dx_2}{dx_1} = -\frac{\partial U/\partial x_1}{\partial U/\partial x_2}.$$

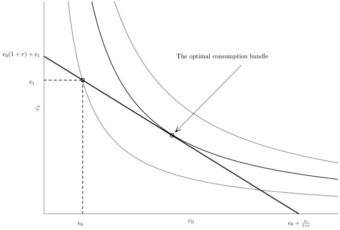
The budget constraint in this ($n = 2$) case is

$$p_1x_1 + p_2x_2 = m \quad \Leftrightarrow \quad x_2 = -\frac{p_1}{p_2}x_1 + \frac{m}{p_2}.$$

Hence, the slope of the budget constraint is $-p_1/p_2$.

Equal slopes

$$\begin{aligned}\text{Slope of the indifference curve} &= -\frac{\partial U/\partial x_1}{\partial U/\partial x_2} \\ &= \{\text{FOC}\} \\ &= -p_1/p_2 = \text{Slope of the budget line.}\end{aligned}$$



At the optimal point, the slope of the indifference curve is equal to the slope of the budge constraint.

The market's demand function

The demand function D_i gives the consumer's demand for good i given the prices $p_1, \ldots, p_n$ and the income m:

$$x_i = D_i(p_1, \ldots, p_n, m).$$

To get the market's demand function we add the demand functions from all the consumers in the market.

We let Q_i denote the total amount demanded of good i in the market.

More on demand functions

Let us assume that there exists one good (with $Q = x_1$).

In many cases we model the market's inverse demand function:

$$p = p(Q).$$

It is not uncommon to assume that the inverse demand function is linear:

$$p = a - bQ$$

for some constants $a, b > 0$. Since both price p and quantity Q must be positive, the function is only valid when

$$p \geq 0 \quad \text{and} \quad Q \geq 0.$$

The supply function

The supply of good i on the market is determined by the supply function:

$$Q_i = S_i(p_1, \ldots, p_n).$$

This is for given prices the quantity the firms in the market want to produce.

How the supply function looks like depends on how the firms produce the goods, and on the market form.

Equilibrium

A market is in equilibrium if the traders on a market are able to buy or sell the quantity they want.

The price at which trading occurs is the equilibrium price, and the quantity sold and bought is called the equilibrium quantity.

In equilibrium supply equals demand.