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Financial Derivatives
Fall 2024

Assignment 2

Please hand in your solutions by Thursday December 12.

1. In the standard Black-Scholes model with interest rate r and volatility σ, consider the T_2-claim

$$\mathcal{X} = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} \ln S(t)\, dt,$$

where $0 < T_1 < T_2$.

(i) Determine the arbitrage-free price at time 0 of $\mathcal{X}$.

(ii) What is the price of $\mathcal{X}$ at time $t \in (T_1, T_2)$?

2. Let $c(K)$ denote the price of a call option with strike K and maturity T written on a particular dividend-free stock. Assume that $c(K_1) - c(K_2) > e^{-rT}(K_2 - K_1)$ for some strike prices $K_1 < K_2$, where r is a constant interest rate. Determine a static arbitrage opportunity.

3. A T-claim on a stock S pays the holder

$$\Phi(S(T)) = \begin{cases} 25 + S(T)/2 & \text{if } 0 < S(T) \leq 50 \\ 100 - S(T) & \text{if } 50 < S(T) \leq 100 \\ 0 & \text{if } S(T) > 100 \end{cases}$$

Express the value of this option at time $t < T$ as the value of a suitable portfolio consisiting of bonds, European call options and the underlying asset.

4. Consider a market with two stocks S_1 and S_2 satisfying

$$\begin{cases} dS_1(t) = \alpha_1 S_1(t)\, dt + \sigma_1 S_1(t)\, dW_1(t) \\ dS_2(t) = \alpha_2 S_2(t)\, dt + \sigma_2 S_2(t)\, (\rho dW_1(t) + \sqrt{1-\rho^2} dW_2(t)), \end{cases}$$

where $\rho \in (-1, 1)$, α_i and σ_i are constants and W_1, W_2 are independent Brownian motions. Determine the price of a contract that at time T pays the amount $\mathcal{X} = \min\{S_1(T), S_2(T)\}$.