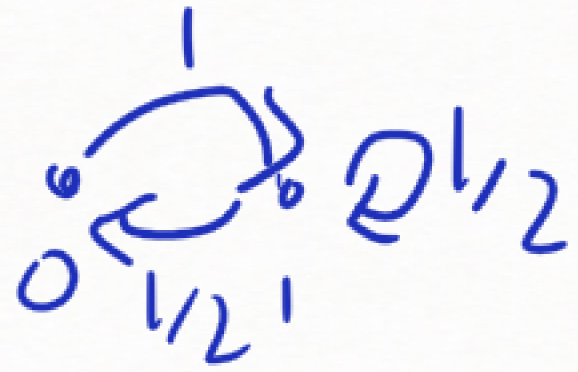
Extra pr. 61) We will construct an irr. aperiodic MC and repr. function such that w is not distributed acc. to the stat. distr. of the MC

Let (X_n) be an MC with 2 states (0 and 1) and tr. matrix $P = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$

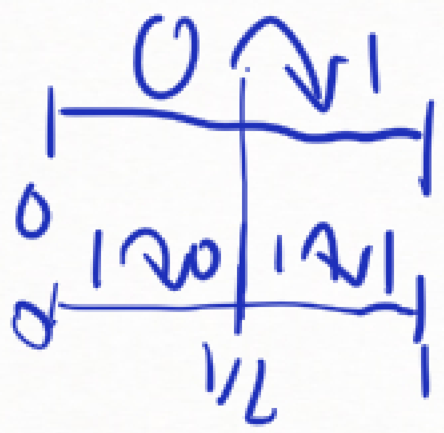


This MC has st. distr.

solving $\underset{\pi_1,\pi_2}{\pi} P = \pi \iff \begin{cases} \frac{\pi_2}{2} = \pi_1 \\ \pi_1 + \frac{\pi_2}{2} = \pi_2 \end{cases}$

i.e.

$$\pi = \left(\frac{1}{3}, \frac{2}{3}\right)$$



If $f_s(x) = \begin{cases} 1-x & s < 1/2 \\ 1 & s \geq 1/2 \end{cases}$

If $f_s(x) = \begin{cases} 1-x & \text{if } s < 0.5 \\ 1 & \text{if } s \geq 0.5 \end{cases}$

and $W = f_{I_N} \circ \dots \circ f_{I_1}(x)$ where $\{I_n\}$ i.i.d. $I_n \sim U[0,1]$

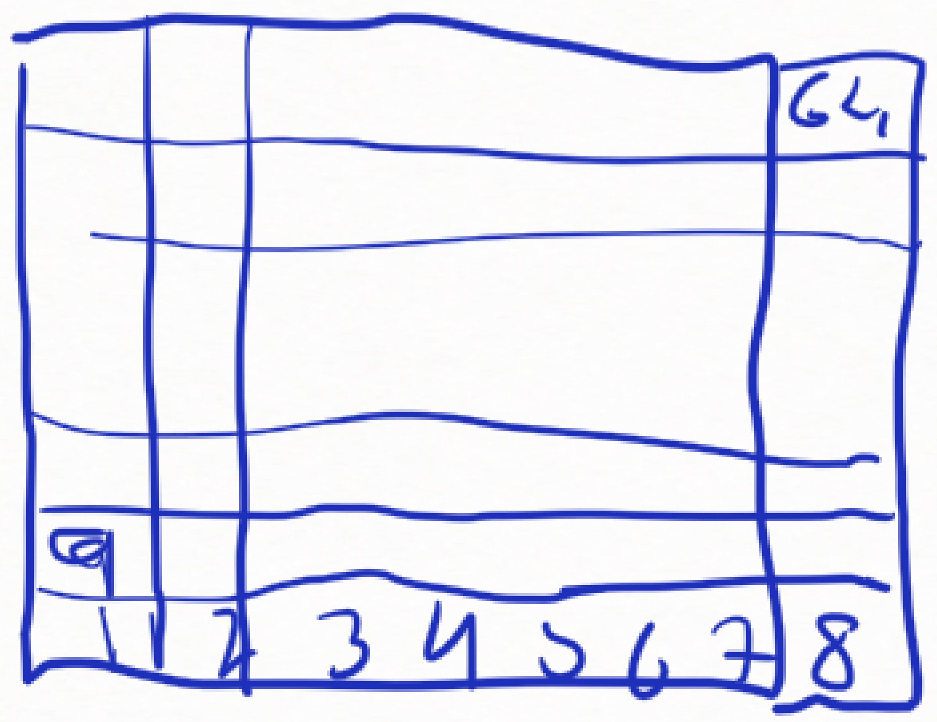
where N = smallest integer such that $f_{I_N} \circ \dots \circ f_{I_1}$ constant

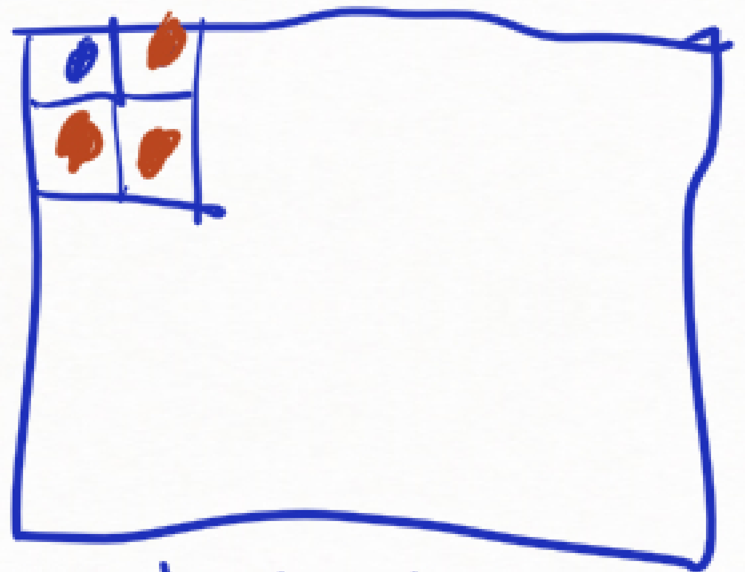
then $W = 1$ i.e.

$P(W=1) = 1$ so the dist. vector of W is

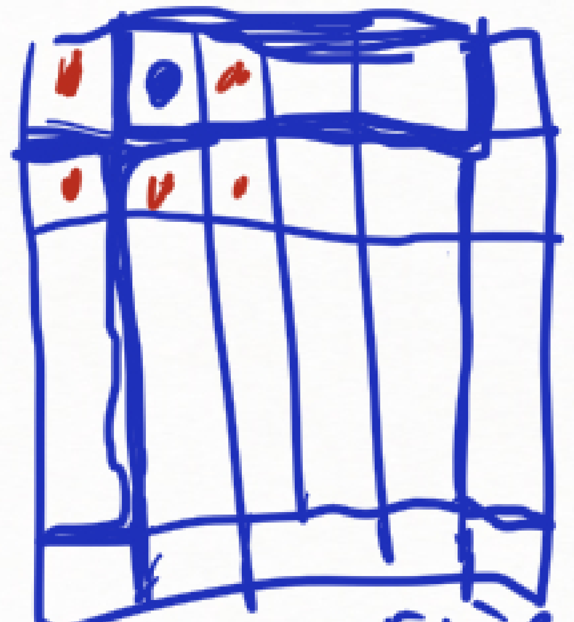
$$(0,1) \neq \underbrace{(1/3, 2/3)}_{\pi}$$

b2
extra pr.

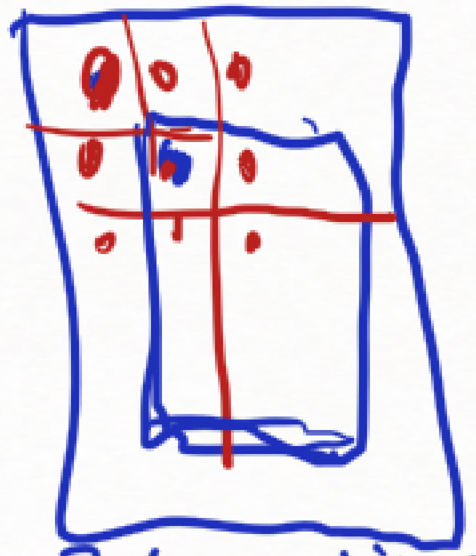




4 vertices have degree 3



24 vertices of deg. 5



36 vertices have deg. 8

Let X_n be the position of the King after n moves. Then (X_n) is a random walk on a connected graph with 64 vertices

The total degree of all vertices is $4\cdot 3+24\cdot 5+36\cdot 8=420$

so The the long run prob. of being in a particular vertex is the degree of that vertex divided by 420

so e.g if v is a given fixed corner then $\pi_v = \frac{3}{420}$.

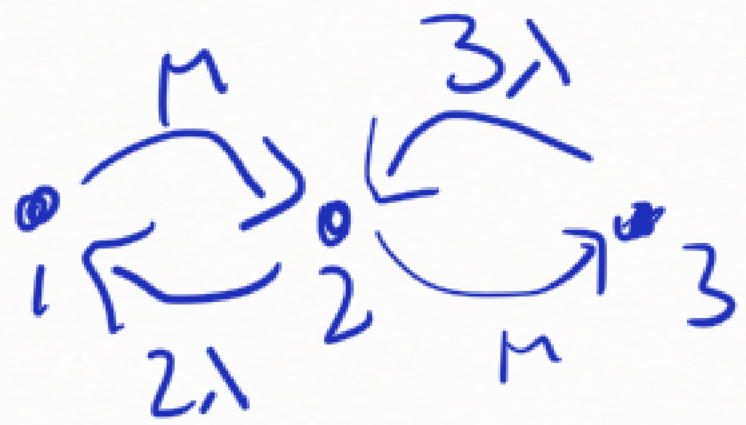
prob of being in v in the long run

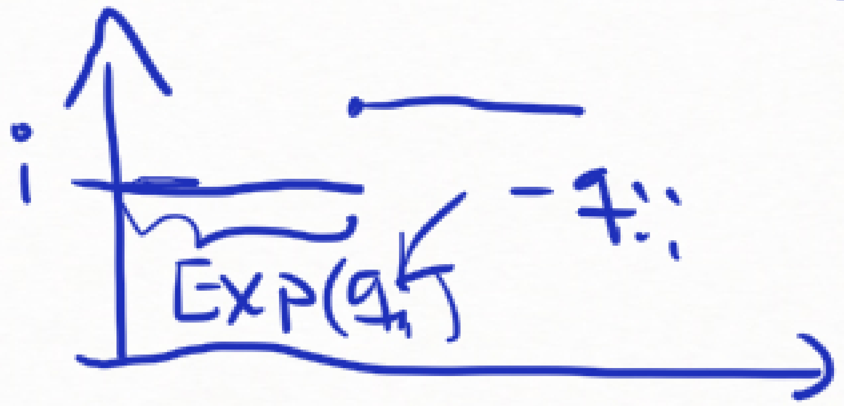
Thus the prob of being in some corner in long run is $4 \cdot \frac{3}{420} = \frac{1}{35}$

$\boxed{\text{Basic exercise 20}}$

Let $(X_t)_{t\geq 0}$ be an MP (on $\{1,2,3\}$) with generator

$$Q = \begin{pmatrix} -\mu & \mu & 0 \\ 2\lambda & -2\lambda-\mu & \mu \\ 0 & 3\lambda & -3\lambda \end{pmatrix}.$$





Let $\underbrace{v_{ij}}_{E(T_{ij})}$ = expected time to reach j from i

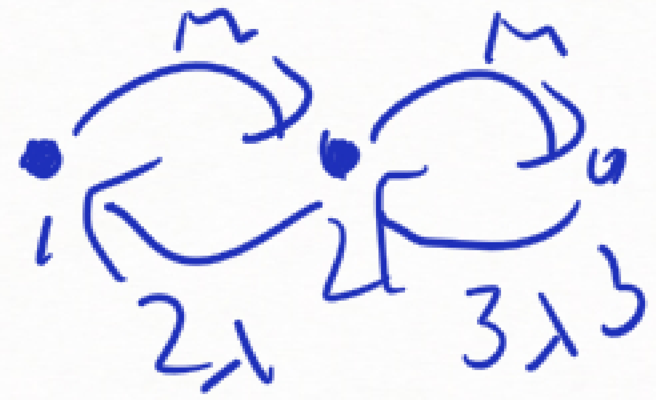
T_{ij}: time to reach j from i

$$T_{ij} = T_{L_i} + T$$

T_{L_i}: time to leave state i

T: time to reach j from new state reached after first jump

$$\Rightarrow v_{ij} = E(T_{L_i}) + \sum_{k\neq i} E(T \mid \text{new state is } k)\, P(\text{new state is } k)$$



$$\begin{cases} V_{13} = \dfrac{1}{\mu} + V_{23} \\[2ex] V_{23} = \dfrac{1}{2\lambda+\mu} + \dfrac{2\lambda}{2\lambda+\mu} \cdot V_{13} + \cancel{\dfrac{\mu}{2\lambda+\mu} \cdot 0} \end{cases}$$

$$\uparrow \quad T_{L_1} \sim Exp(\mu)$$

$$\uparrow \quad T_{L_2} \sim Exp(2\lambda+\mu)$$

$$\frac{q_{21}}{q_{21}+q_{23}} = \frac{q_{21}}{\sum_{j\neq 2} q_{2j}} = \frac{q_{21}}{q_2} = \frac{q_{21}}{-q_{22}}$$

E.g. if $\lambda = 1$ and $\mu = 10$ then

$$V_{13} = \underbrace{\dots}_{\text{linear system}} = 0.22$$

3.1 Lawler

Let X_t = # phone calls received at time t (measured in hours)

Suppose (X_t) is a Poisson process with with parameter $\lambda = 4$.

a) $P(X_1 < 2) = \underbrace{P(X_1 = 0)}_{\frac{e^{-4}4^0}{0!}} + \underbrace{P(X_1 = 1)}_{\frac{e^{-4}4^1}{1!}} = 5e^{-4}$

$X_t \sim Po(\lambda t)$

$\Rightarrow X_1 \sim Po(4)$

$P(X_1 = k) = \frac{e^{-4}4^k}{k!}$

b) $P(\underbrace{X_2 - X_1 \geq 2}_{} \mid \underbrace{X_1}_{} = 6) = P(\underbrace{X_2 - X_1}_{\sim Po(4)} \geq 2) = 1 - 5e^{-4}$

ind. (increments)

c) Let $T_k = \inf\{t \geq 0 : X_t = k\}$. Then

$E(T_{15}) = E(T_1) + E(T_2 - T_1) + \dots + E(T_{15} - T_{14}) = 15\underbrace{E(T_1)}_{\frac{1}{4}} = \frac{15}{4}$

since $T_1 \sim Exp(4)$