# Analysis of Time Series, L1

Rolf Larsson

Uppsala University

24 mars 2025

# Course overview

- Shumway and Stoffer:
  *Time Series Analysis and its Applications, with R examples, 5th ed.*
  Springer 2025.
- Chapters (from 4th ed.):
  1. Characteristics of Time Series (L1-2)
  2. Time Series Regression and Explanatory Data Analysis (L2)
  3. ARIMA Models (L3-9)
  4. Spectral Analysis and Filtering (L10-12)
  5. Additional Time Domain Topics (L13-16)
  6. State-Space Models (L17-18)

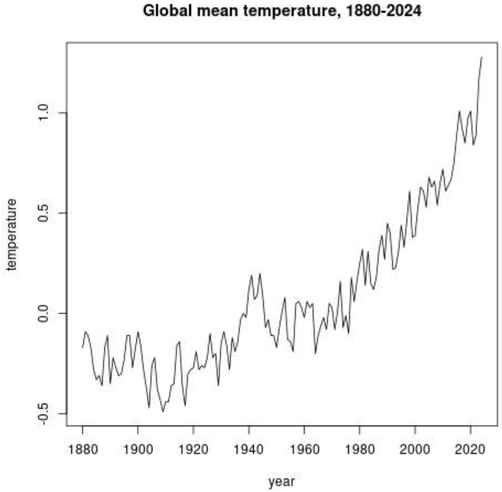
# Course overview

- 25 meetings:
  - 18 theory lectures
  - 5 problem sessions
  - 2 project presentation sessions
- Written exam (with book and/or homemade notes).
- Two hand in assignments, not compulsory but give bonus points.
- Project: Analyse your own time series. (Compulsory.)
- Please check Studium for further information!
  (The file “kursinfo”, including a list of recommended exercises.)

# Today

- 1.1-2: Introduction, examples
- 1.3: Statistical models
- 1.4: Measures of dependence
- Menti

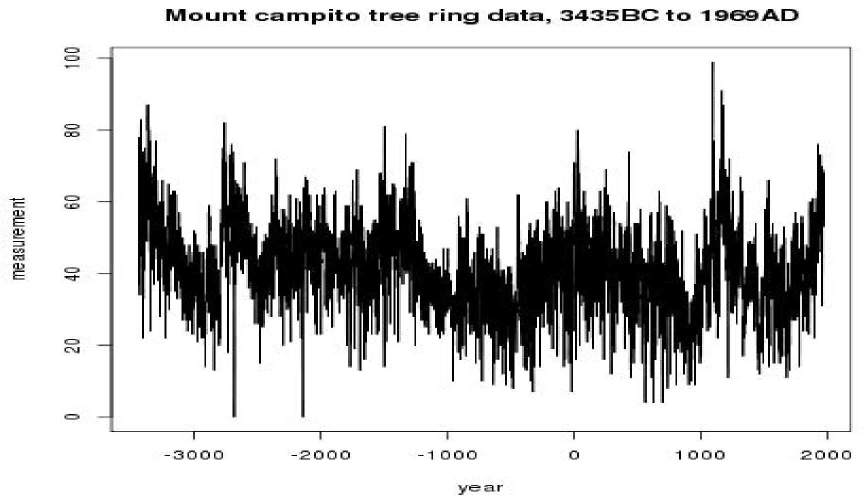
# Introduction, examples



In R:

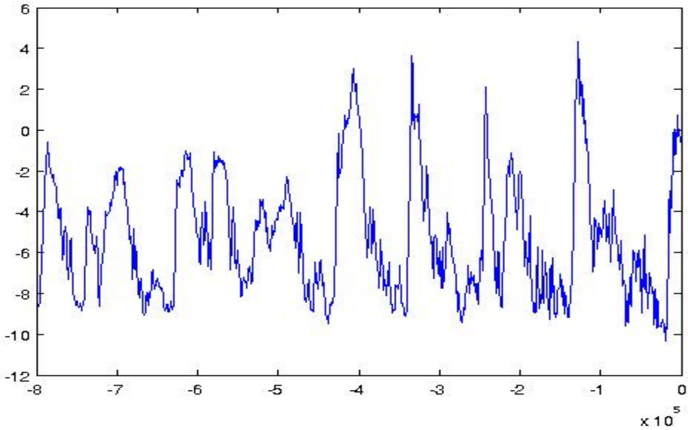
```
> year=read.table("temp.dat")$V1
> temp=read.table("temp.dat")$V2
> plot(year,temp,type='l',ylab='temperature',
main='Global mean temperature, 1880-2024')
```

# Introduction, examples

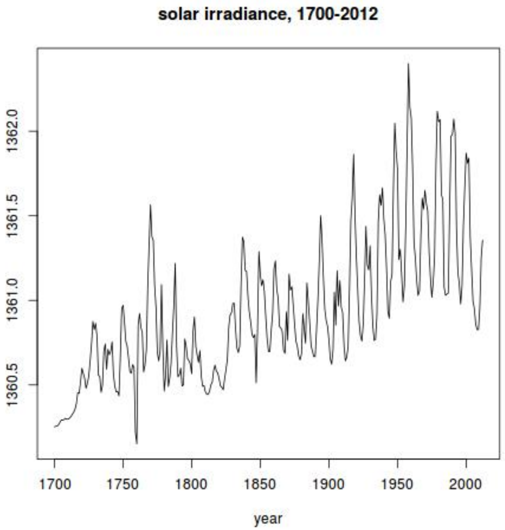


# Introduction, examples

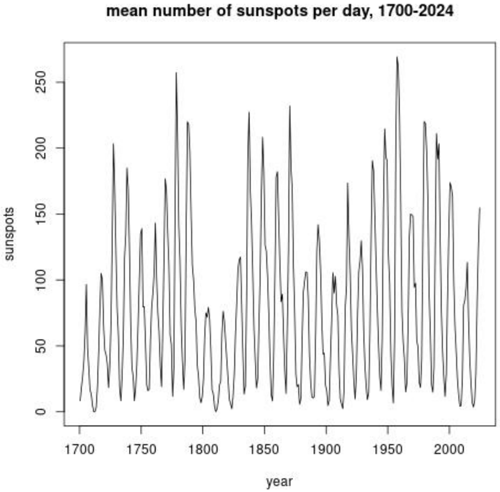
Antarctic ice core temperature proxies from about 800 000 BC to now.



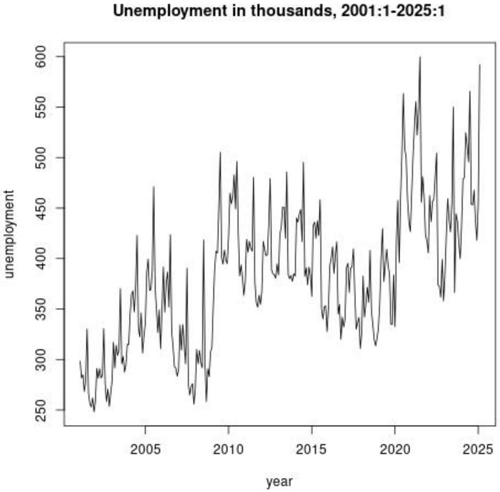
# Introduction, examples



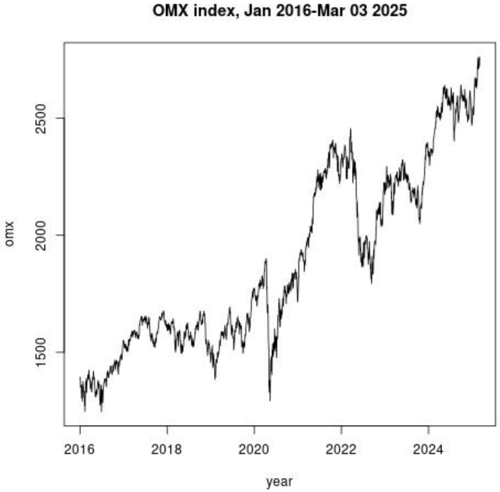
# Introduction, examples



# Introduction, examples

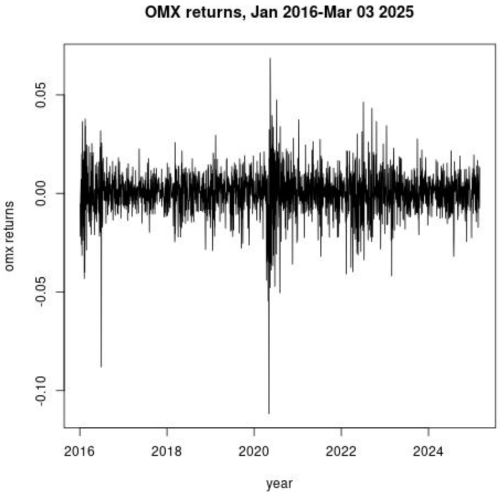


# Introduction, examples



# Introduction, examples

OMX index $x_t$, returns $r_t = \log x_t - \log x_{t-1}$.



# Statistical models

### Definition (Stochastic process)

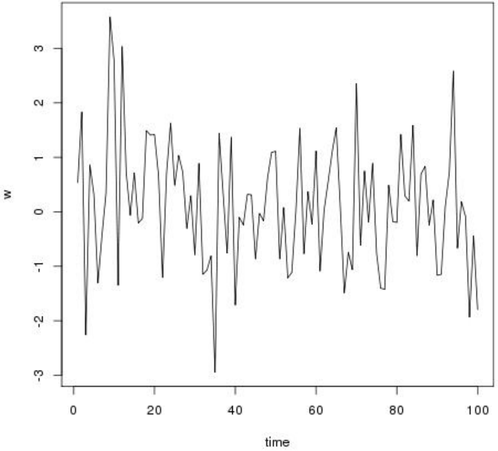
A collection of random variables $\{x_t\}$ where $t$ ranges over a set of integers, is called a *stochastic process in discrete time* (time series).

### Definition (Realization)

A collection of observed values of a stochastic process is called a *realization*.

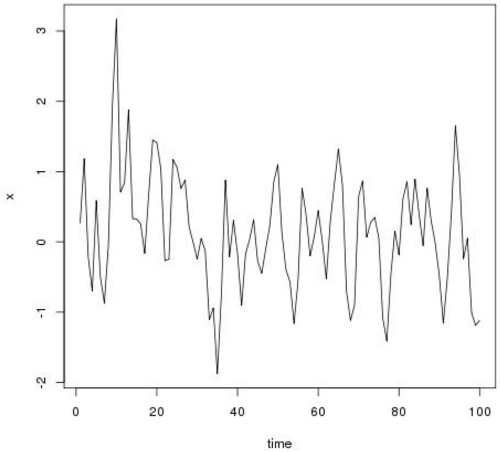
# Statistical models

Example 1: White noise, $w_t \sim N(0, \sigma_w^2)$, independent



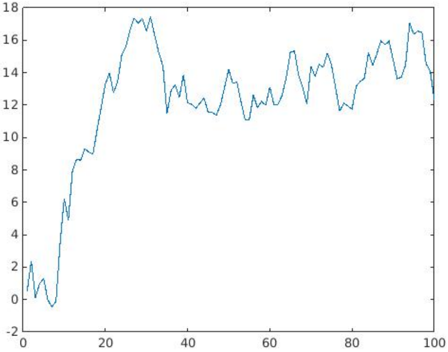
# Statistical models

Example 2: Moving average, $x_t = \frac{1}{2}(w_t + w_{t-1})$



# Statistical models

Example 3: Random walk, $x_t = x_{t-1} + w_t = x_0 + w_1 + \ldots + w_t$.



# Measures of dependence

### Definition (1.1)

The *mean function* of a stochastic process $\{x_t\}$ is defined as

$$\mu_t = E(x_t).$$

### Definition (1.2)

The *autocovariance function* of a stochastic process $\{x_t\}$ is defined as

$$\gamma(s,t) = \mathrm{cov}(x_s, x_t).$$

### Definition (1.3)

The *autocorrelation function* of a stochastic process $\{x_t\}$ is defined as

$$\rho(s,t) = \mathrm{corr}(x_s, x_t) = \frac{\gamma(s,t)}{\sqrt{\gamma(s,s)\gamma(t,t)}}.$$

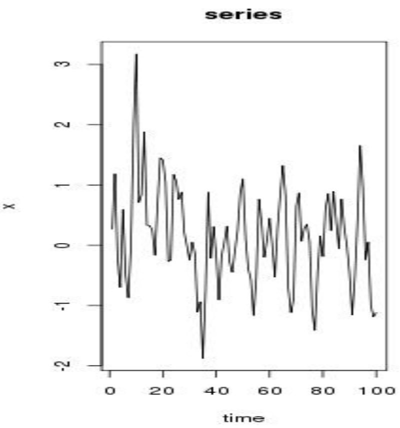
# Measures of dependence

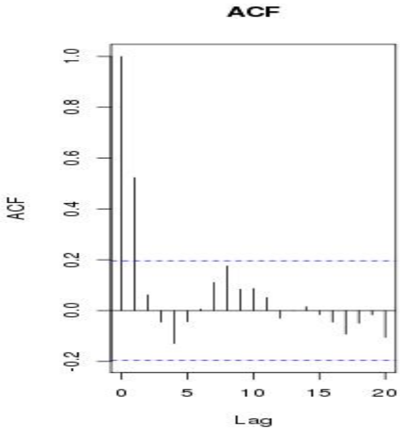
Calculate $\mu_t$, $\gamma(s,t)$ and $\rho(s,t)$ for

1. the white noise process $w_t$.
2. the moving average process $x_t = \frac{1}{2}(w_t + w_{t-1})$.
3. the random walk process $x_t = x_{t-1} + w_t$ where $x_0 = 0$.
4. In general, is it true that $\gamma(s,t) = \gamma(t,s)$ and $\rho(s,t) = \rho(t,s)$?

# Measures of dependence

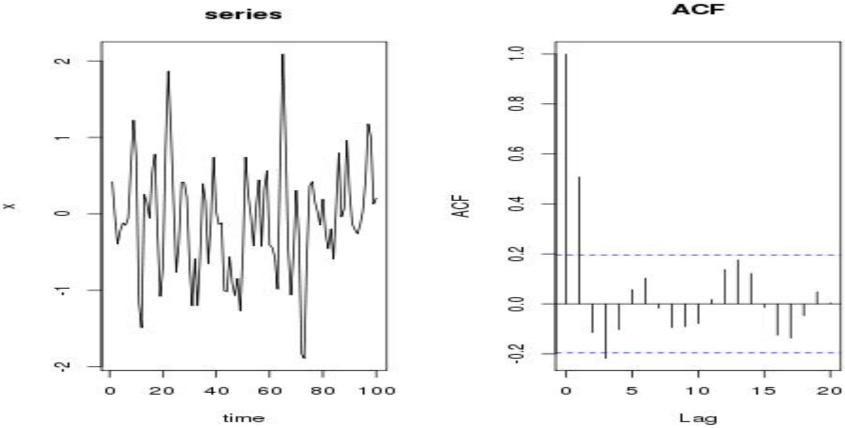
Simulation one of $x_t = \frac{1}{2}(w_t + w_{t-1})$



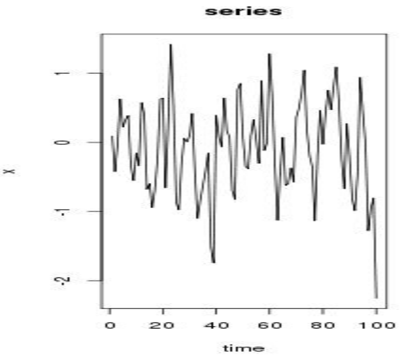
# Measures of dependence

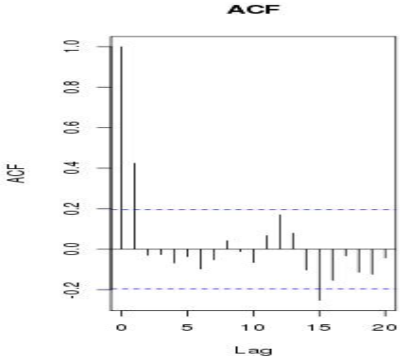
Simulation two of $x_t = \frac{1}{2}(w_t + w_{t-1})$



# Measures of dependence

Simulation three of $x_t = \frac{1}{2}(w_t + w_{t-1})$




In all three simulations, the series look quite different but the ACFs are similar.

# Measures of dependence

### Definition (1.4)

The *cross-covariance function* between two series $\{x_t\}$ and $\{y_t\}$ is defined as

$$\gamma_{xy}(s,t) = \mathrm{cov}(x_s, y_t).$$

### Definition (1.5)

The *cross-correlation function* between two series $\{x_t\}$ and $\{y_t\}$ is defined as

$$\rho_{xy}(s,t) = \mathrm{corr}(x_s, y_t) = \frac{\gamma_{xy}(s,t)}{\sqrt{\gamma_x(s,s)\gamma_y(t,t)}}.$$

# Measures of dependence

1. Let $x_t = \frac{1}{2}(w_t + w_{t-1})$ and $y_t = w_t$.
   1. Calculate $\gamma_{xy}(s,t)$ and $\rho_{xy}(s,t)$.
   2. Calculate $\gamma_{yx}(s,t)$ and $\rho_{yx}(s,t)$.
2. Let $x_t = x_{t-1} + w_t$ where $x_0 = 0$, and $y_t = w_t$.
   1. Calculate $\gamma_{xy}(s,t)$ and $\rho_{xy}(s,t)$.
   2. Calculate $\gamma_{yx}(s,t)$ and $\rho_{yx}(s,t)$.
3. In general, is it true that $\gamma_{xy}(s,t) = \gamma_{xy}(t,s)$ and $\rho_{xy}(s,t) = \rho_{xy}(t,s)$?
4. In general, is it true that $\gamma_{xy}(s,t) = \gamma_{yx}(t,s)$ and $\rho_{xy}(s,t) = \rho_{yx}(t,s)$?

# News of today

Definitions of

- a discrete time stochastic process (time series)
- the mean function
- the autocovariance function
- the autocorrelation function
- the cross-covariance function
- the cross-correlation function