i Informationsblad 1TD395

Exam Scientific Computing II

The exam is divided into two parts, part A and part B. Part A is related to grade 3, and part B is to grades 4 and 5.

Part A (grade 3)

The tasks in Part A are directly linked to one of the four course objectives: *Key Concepts, Algorithms, Analysis,* and *Argumentation*. In this part, there are 2 questions related to each objective (questions 1 to 8) and maximum 2 points per question.

Questions in part A are either multiple-choice questions or require entering a numerical value in a provided box. For these types of questions, you may need to solve the problem on paper and then select or enter the correct answer. You are able to answer this part in the Inspera only. As an option, you can write down your detailed solutions on paper and hand them in for review. We can then correct the information if you have entered the wrong answer due to something that we judge to be a simple careless error.

Part B (grades 4 and 5)

In part B there are 3 questions (questions 9, 10, 11). You can either type the detailed solution directly into Inspera or hand it in on paper to the invigilators. If you hand in paper-answers, make a note about it in Inspera in the corresponding question.

Questions 9 and 10 can give 0 or 10 points and question 11 can give 0, 10, or 20 points.

In both parts, if you hand in paper-answers please write in English and in neat and legible handwriting.

Grades

- **Grade 3:** At least 8 points. You must answer at least one question on each objective of part A. This corresponds to minimum 4 questions (i.e. 8 points) distributed among the four objectives. If you fail to meet a course objective entirely, it will result in a failing grade on the exam.
- **Grade 4:** At least 20 points. You must fulfill Grade 3 + solve one more question from part A (at least 5 correct answers) + fulfill one of the following items:
  -- solve either question 9 or question 10 completely
  -- solve question 11 partially (more than half)
- **Grade 5:** At least 32 points. You must fulfill Grade 3 + solve two more questions from part A (at least 6 correct answers) + fulfill one of the following items:
  -- solve both questions 9 and 10 completely
  -- solve question 11 completely
  -- solve question 9 completely and question 11 partially (more than half)
  -- solve question 10 completely and question 11 partially (more than half)

Allowed aid

- Pocket calculator.
- The formula sheet (is available as a link in Inspera. Sections 3 and 7 are related to this course)

Good Luck!

1 Algoritmer1_230529

Compute $y(0.1)$ using Heuns method for differential equation
$y'(t) - 10 \cdot y(t)^2 - t = 0,\ t \geq 0,\ y(0) = 0.7$
with steplength $h = 0.1$.
You can find the Heuns method in the formula sheet.
Perform the calculation on paper and choose your answer by selecting one of the options below (only one item is correct).

Select one alternative:

- 1.65805
- 0.90625
- 0.91125
- 2.70240
- 1.75000
- 2.72240
- 4.01240
- 1.24080

Maximum marks: 2

2 Algoritmer2_230529

Assume that you have an 8-sided die that is not completely symmetrical and now want to investigate its properties. You then perform a number of dice rolls and obtain the following outcomes:

Outcome	1	2	3	4	5	6	7	8
Frequency	21	33	12	25	8	15	26	10

Assume that the probability density function (pdf) and the cumulative density function (cdf) of this distribution are determined by the experiment above. To generate from this distribution we use a uniformly distributed random number between 0 and 1 (Inverse Transform Method). Suppose you randomly select the number 0.8012, which side of the dice corresponds to it?

Select one alternative:

- 2
- 3
- 4
- 7
- 8
- 6
- 5
- 1

Maximum marks: 2

3 Begrepp1_230529

When numerically solving differential equations, it is important to consider whether the equation is stiff. What are the characteristics of a stiff ODE?

One or more options may be correct, tick all correct items.

Select one or more alternatives:

- [ ] Explicit methods are beneficial
- [ ] The solution can have vastly different scales components
- [ ] The solution varies slowly
- [ ] The solution varies quickly
- [ ] Implicit methods are beneficial
- [ ] It requires a small steplength for stability in explicit methods
- [ ] The RK4 method can solve it with a relatively large steplength

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Maximum marks: 2

4 Begrepp2_230529

Classify the models and methods. To pass, all must be correct.

	Stochastic Model	Deterministic Method	Deterministic Model	Stochastic Method
Explicit Euler (Euler forward)	○	○	○	○
$\frac{dS}{dt} = \mu N - \mu S - \beta \frac{I}{N} S$ $\frac{dI}{dt} = \beta \frac{I}{N} S - \mu I - \gamma I$ $\frac{dR}{dt} = \gamma I - \mu R$	○	○	○	○
Mid-point rule for integration	○	○	○	○
$\emptyset \xrightarrow{\mu N} S$ $S \xrightarrow{\beta \frac{I}{N} S} I$ $I \xrightarrow{\gamma I} R$	○	○	○	○
SSA (Gillespies algorithm)	○	○	○	○
Monte Carlo integration	○	○	○	○

Maximum marks: 2

5 Analys1_230529

Suppose that we have estimated the integral $\int_a^b f(x)\,dx$ using Monte-Carlo method with $N = 10000$ random numbers. As an error estimate, we have also calculated the confidence interval length (or standard deviation) $\varepsilon = 0.2$. Now we want to improve the error and then analyze approximately how many random numbers would be required for the new confidence interval. How many random numbers are required to reduce the confidence interval length (or standard deviation) to $\varepsilon = 0.1$?

Calculate the required N and enter it here as an integer: $N =$ ______ .

Maximum marks: 2

6 Analys2_230529

We use the classical Runge-Kutta (RK4) method to solve an ODE. With the choice of steplength $h = 0.3$ we have estimated the global discretization error to 0.05 (what is estimated here is the absolute global error $|e_N|$). Now we want to reduce the steplength to improve the accuracy. What error will be approximately obtained if the steplength is reduced to $h = 0.1$?

Calculate the error and enter here to five decimal places*: ______ .

*For example 0.00123

Maximum marks: 2

7 Argumentation1_230529

Specify the suitable method (column) for each application (row).

	Explicit method	Implicit method
Systems with highly different scales	○	○
y'(t)=-(y-cos(t))+sin(t)	○	○
Non-stiff equation	○	○
Fast transients	○	○
Lower complexity per timestep	○	○
Stiff equation	○	○
y'(t)=-10000(y-cos(t))+sin(t)	○	○
Stability is crucial	○	○

Maximum marks: 2

8 Argumentation2_230529

Specify the suitable method (column) for each application (row).

Please match the values:

	Stokastisk metod	Deterministisk metod
Stochastic process	○	○
Solution is continuous (concentration, velocity, ...)	○	○
Integral 2D	○	○
Solution is discrete (individuals, number of molecules, ...)	○	○
Integral 10D	○	○
Scenarios in epidemic models with limited number of individuals	○	○
ODE	○	○

Maximum marks: 2

9 Betyg45_(1)_230529

Consider the integral

$$I = \int_{-\infty}^{\infty} (1+x^2) \frac{e^{-x}}{(1+e^{-x})^2} dx.$$

Let us consider this integral as an expectation value of a function of the "logistically distributed" random variable X: we write $X \sim Logistic$ if the probability density function of X is given by

$$f(x) = \frac{e^{-x}}{(1+e^{-x})^2}, \quad x \in (-\infty, \infty).$$

The cumulative density function can be obtained as

$$F(x) = \frac{1}{1+e^{-x}}, \quad x \in (-\infty, \infty).$$

First, explain with details how can we generate a random variable X from the above logistic density.

Then write I as an expectation value using such a random variable $X \sim Logistic$ and write a pseudocode that estimates the integral using the Monte Carlo method with a given number of samples from logistic distribution.

Fill in your answer here or write in the answer sheet and hand it in.

Format

Words: 0

Maximum marks: 10

10 Betyg45_(2)_230529

The absolute stability region of ODE solvers is obtained by applying the solver to test equation $y' = \lambda y$ for complex values λ with $Re(\lambda) \leq 0$.
In this task, you are asked to obtain the stability region of the following 2-stage method:

$$z_1 = y_k$$
$$z_2 = y_k + \frac{1}{2} h\, f(t_k, z_1)$$
$$y_{k+1} = y_k + h\, f(t_k + \frac{1}{2} h, z_2)$$

First, write down the algebraic equations for the stability region of the above method for complex values λ.
Then, derive the stability region (stability interval) by assuming that λ is a real number.

Fill in your answer here or write in the answer sheet and hand it in.

Format

Words: 0

Maximum marks: 10

11 Betyg45_(3)_230529

Differential equations in real applications often contain a number of parameters. The values of the parameters can be difficult to determine precisely and can be disturbed by, for example, measurement errors. As an example, we can consider the following application. Here we have two pendulums with the same mass and the same length that are connected by a spring (see Figure 1).

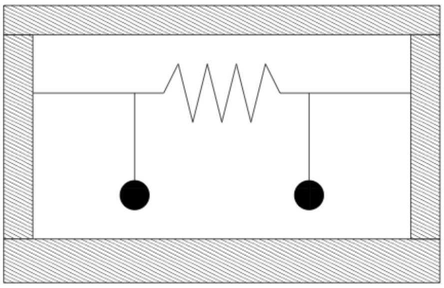

Figure 1: Two pendulums connected by a spring.

The equations of motion of two pendulums can be described by the following system of differential equations:

$$\theta_1'' + \sin(\theta_1) + \alpha(\theta_1 - \theta_2) = 0$$
$$\theta_2'' + \sin(\theta_2) - \alpha(\theta_1 - \theta_2) = 0$$

Here θ_1 and θ_2 correspond to the respective angles of the pendulums, and α is a constant that depends on the length and mass of the pendulum as well as the spring constant. The parameter α as well as the initial values of angles θ_1 and θ_2 are not known exactly.
Assume that the system starts from rest ($\theta_1'(0) = \theta_2'(0) = 0$) and use the following random values for parameters:

$\alpha \sim \mathbb{U}(10 - \delta, 10 + \delta)$ where $\delta = 0.2$ (uniform distribution)
$\theta_1(0) \sim \mathbb{N}(\pi/4, 0.02)$ (normal distribution)
$\theta_2(0) \sim \mathbb{N}(\pi/4, 0.02)$ (normal distribution)

The final time is $T = 10$.
Write a Matlab program that solves the above problem with repeated random values (1000 times) for parameters α, $\theta_1(0)$ and $\theta_2(0)$. The code must:
(1) plot various solutions for $\theta_1(t)$ over interval $t \in [0, T]$ in a figure,
(2) plot the histograms of variables $\theta_1(T)$ and $\theta_2(T)$,
(3) estimate the expectation and the standard deviation of $\theta_1(T)$.

You can use appropriate built-in solvers and functions in Matlab. It should be a detailed and executable program with justification for the choice of methods, e.g. why you choose a particular ODE solver.

Note: $\mathbb{U}(a, b)$ stands for uniform distribution in interval $[a, b]$, and $\mathbb{N}(\mu, \sigma^2)$ stands for normal distribution with mean μ and variance σ^2 (standard deviation σ).
Note: Both the code and the discussion should be written in the box below. Minor errors in the code are acceptable.

Fill in your answer here

Format

Words: 0

Maximum marks: 20