L Viitasaari

7. Volume,mass, and curve integrals

We solve the problems together in the exercise sessions. Note that these problems are optional and for learning purposes: solving these does not provide extra points. Actual home assignments (giving you extra points) are given separately.

It is advised to take a look of the problems beforehand. Note that some of the problems might be very challenging, so do not feel bad if you are unable to solve them independently: we will go through the solutions together!

Problems for the session

7.1 The ball $x^2 + y^2 + z^2 \leq 49$ cuts the hyperboloid $x^2 + y^2 - z^2 + 1 = 0$ into three pieces. Two of them has the same volume. What is this volume?

7.2 Compute the volume of the body determined by $0 \leq z \leq 10 - x^2 - y^2$, $x + 1 - y^2 \geq 0$, and $x + y^2 - 1 \leq 0$.

7.3 Compute the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1$. Conclude that the volume of a ball of radius r is given by $V_r = \frac{4}{3}\pi r^3$.

7.4 Find the center of mass for $x^2 + y^2 + z^2 \leq R^2$, $x, z \geq 0$ with a constant density $\rho = 1$.

7.5 Compute the length of the curve defined by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

7.6 Compute the curve integral $\int_\gamma x^5 ds$ along the curve defined by $y = x^3$, $-1 \leq x \leq 1$.

Problems for individual practice

In addition to the problems below, one can get routine by solving similar exercises from the exercise-book ”övningar i flerdimensionell analys”.

7.1 Compute the volume of the body that lies between paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.

7.2 Compute the volume of the body defined by $x^2 + y^2 \leq 4x$ and $|z| \leq x^2 + y^2$.

7.3 A body is defined by $0 \leq y \leq x$ and $0 \leq z \leq \sqrt{4 - x^2 - y^2}$ and has a density $\rho(x, y, z) = x^2 + y^2$. Compute the mass.

7.4 Compute the volume of a body that lies between $z = 2 - x^2 - y^2$ and $x = y^2$, and find the center of mass with a constant density $\rho = 1$.