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Statistical Machine Learning

Lecture 8
Convolutional neural networks
Numerical optimization for neural network training

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Course webpage

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Summary of Lecture 7 (I/IV)
Neural network

A neural network is a sequential construction of several generalized linear regression models.

Inputs	Hidden units	Output
1		
x_1		
$\vdots$		
x_p		

Summary of Lecture 7 (I/IV) Neural network

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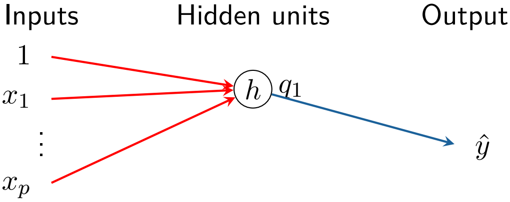

$$q_1 = h\left(b_1^{(1)} + \sum_{j=1}^{p} w_{1j}^{(1)} x_j\right) \qquad \hat{y} = w_1^{(2)} q_1$$

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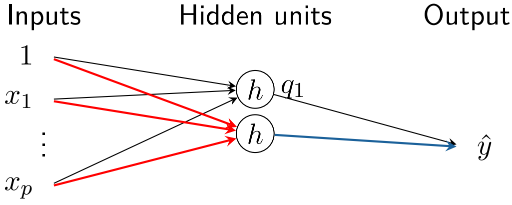

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$$\hat{y} = \sum_{i=1}^{2} w_i^{(2)} q_i$$

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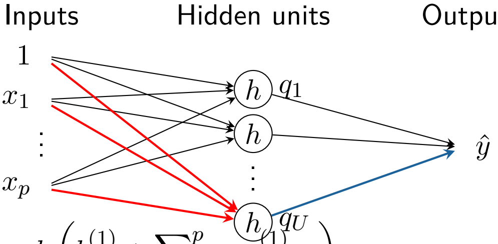

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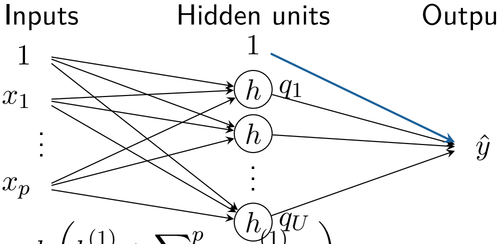

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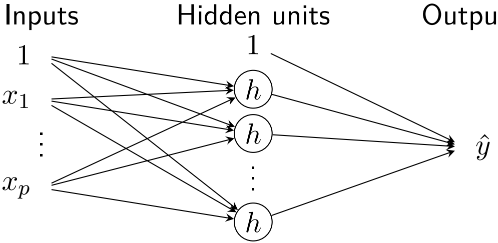

$$\mathbf{q} = h(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}) \qquad \hat{y} = \mathbf{W}^{(2)}\mathbf{q} + \mathbf{b}^{(2)}$$

$$\mathbf{W}^{(1)} = \begin{bmatrix} w_{11}^{(1)} & \dots & w_{1p}^{(1)} \\ \vdots & & \vdots \\ w_{U1}^{(1)} & \dots & w_{Up}^{(1)} \end{bmatrix}, \quad \mathbf{b}^{(1)} = \begin{bmatrix} b_1^{(1)} \\ \vdots \\ b_U^{(1)} \end{bmatrix}, \quad \mathbf{q} = \begin{bmatrix} q_1 \\ \vdots \\ q_U \end{bmatrix}$$

Weight matrix — Offset vector — Hidden units

$$\mathbf{b}^{(2)} = \begin{bmatrix} b^{(2)} \end{bmatrix}$$
$$\mathbf{W}^{(2)} = \begin{bmatrix} w_1^{(2)} & \dots & w_U^{(2)} \end{bmatrix}$$

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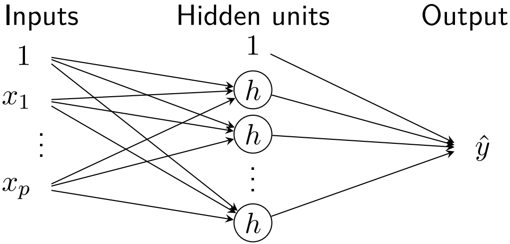

$$\mathbf{q} = h(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)})$$

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The non-linearity h acts element-wise.

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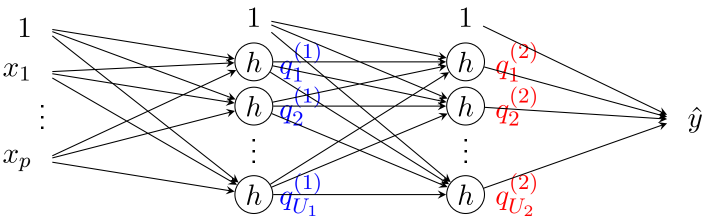

$$\mathbf{q}^{(1)} = h(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)})$$
$$\mathbf{q}^{(2)} = h(\mathbf{W}^{(2)}\mathbf{q}^{(1)} + \mathbf{b}^{(2)})$$
$$\hat{y} = \mathbf{W}^{(3)}\mathbf{q}^{(2)} + \mathbf{b}^{(3)}$$

In dense (or fully-connected) layers all input units are connected to all output units.

Summary of Lecture 7 (II/IV)

Parameters = weight matrices and offset vectors
All weight matrices and offset vectors in all layers combined are the parameters of the network

$$\boldsymbol{\theta} = \left[\mathsf{vec}(\mathbf{W}^{(1)})^\mathsf{T}, \quad \mathbf{b}^{(1)\mathsf{T}}, \quad \dots, \quad \mathsf{vec}(\mathbf{W}^{(L)})^\mathsf{T}, \quad \mathbf{b}^{(L)\mathsf{T}}\right]^\mathsf{T},$$

which constitutes the parametric model $\hat{y} = f(\mathbf{x}; \boldsymbol{\theta})$.

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Training
We train a network on training data $\{\mathbf{x}_i, y_i\}_{i=1}^n$ by considering the optimization problem

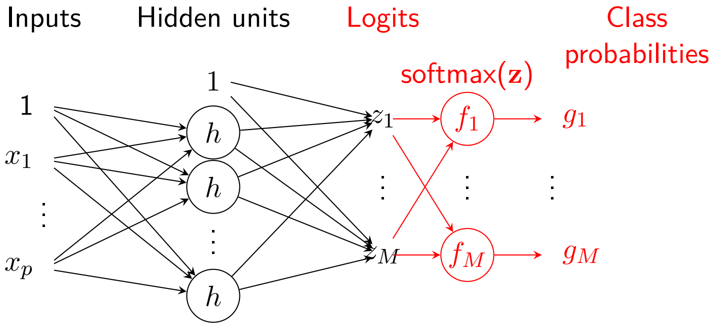
$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}), \qquad J(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n} L(y_i, \hat{y}_i)$$

where $\hat{y}_i = f(\mathbf{x}_i; \boldsymbol{\theta})$

Summary of Lecture 7 (III/IV) NN for classification ($M > 2$ classes)

For $M > 2$ classes we want to predict the class probability for all M classes $g_m = p(y = m|\mathbf{x})$. We extend the logistic function to the **softmax activation function**

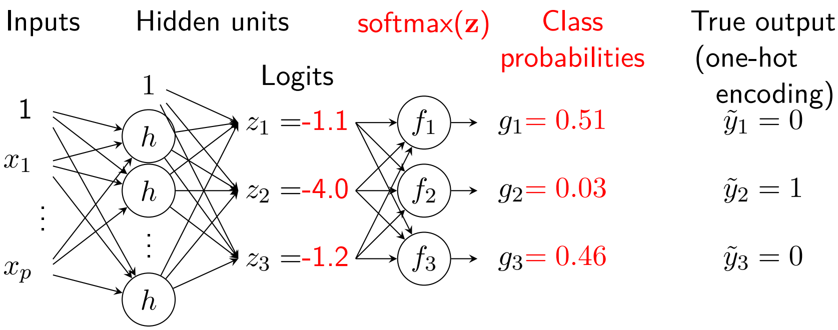
$$g_m = f_m(\mathbf{z}) = \frac{e^{z_m}}{\sum_{l=1}^{M} e^{z_l}}, \qquad m = 1, \ldots, M.$$



softmax($\mathbf{z}$) $= [f_1(\mathbf{z}), \ldots, f_M(\mathbf{z})]^\mathsf{T}$

Summary of Lecture 7 (IV/IV)
Example $M = 3$ classes

Consider an example with three classes $M = 3$ and where $y = 2$.



The network is trained by minimizing the **cross-entropy**

$$L(\tilde{\mathbf{y}}, \mathbf{g}) = -\sum_{m=1}^{M} \tilde{y}_m \ln(g_m) = -\ln 0.03 = 3.51$$

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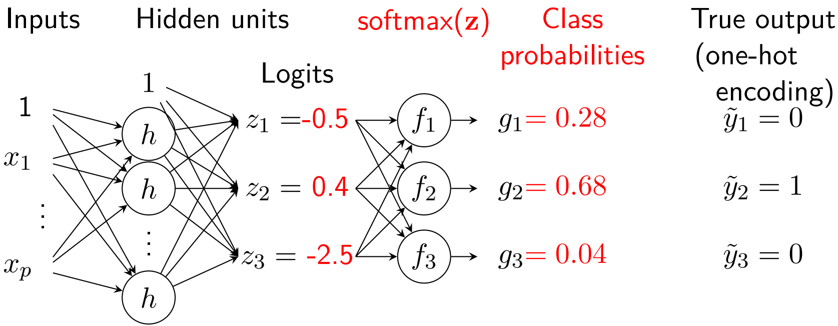

The network is trained by minimizing the **cross-entropy**

$$L(\tilde{\mathbf{y}}, \mathbf{g}) = -\sum_{m=1}^{M} \tilde{y}_m \ln(g_m) = -\ln 0.68 = 0.39$$

Summary of Lecture 7 (IV/IV) Example $M = 3$ classes

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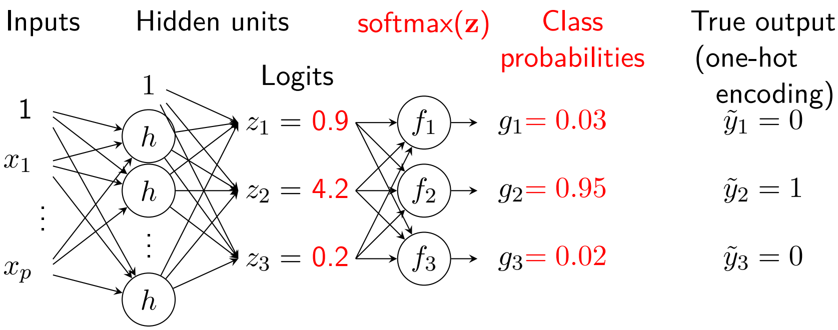

The network is trained by minimizing the **cross-entropy**

$$L(\tilde{\mathbf{y}}, \mathbf{g}) = -\sum_{m=1}^{M} \tilde{y}_m \ln(g_m) = -\ln 0.95 = 0.05$$

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Outline

1. **Previous lecture** The neural network model
   - Neural network for regression
   - Neural network for classification

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   - Neural network for regression
   - Neural network for classification
2. **This lecture**
   - Convolutional neural network
   - How to train a neural network

Convolutional neural networks

Convolutional neural networks (CNN) are a special kind neural networks tailored for problems where the input data has a grid-like structure.

Examples

- Digital images (2D grid of pixels)
- Audio waveform data (1D grid, times series)
- Volumetric data e.g. CT scans (3D grid)

The description here will focus on images.

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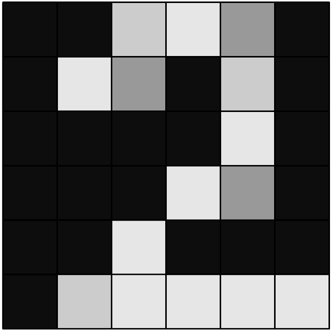
Data representation of images

Consider a grayscale image of 6×6 **pixels**.

- Each pixel value represents the color. The value ranges from 0 (total absence, black) to 1 (total presence, white)

Image



Data representation

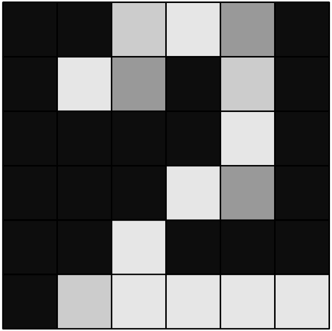
0.0	0.0	0.8	0.9	0.6	0.0
0.0	0.9	0.6	0.0	0.8	0.0
0.0	0.0	0.0	0.0	0.9	0.0
0.0	0.0	0.0	0.9	0.6	0.0
0.0	0.0	0.9	0.0	0.0	0.0
0.0	0.8	0.9	0.9	0.9	0.9

Data representation of images

Consider a grayscale image of 6×6 **pixels**.

- Each pixel value represents the color. The value ranges from 0 (total absence, black) to 1 (total presence, white)
- The pixels are the input variables $x_{1,1}, x_{1,2}, \ldots, x_{6,6}$.

Image



Input variables

$x_{1,1}$	$x_{1,2}$	$x_{1,3}$	$x_{1,4}$	$x_{1,5}$	$x_{1,6}$
$x_{2,1}$	$x_{2,2}$	$x_{2,3}$	$x_{2,4}$	$x_{2,5}$	$x_{2,6}$
$x_{3,1}$	$x_{3,2}$	$x_{3,3}$	$x_{3,4}$	$x_{3,5}$	$x_{3,6}$
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$x_{5,1}$	$x_{5,2}$	$x_{5,3}$	$x_{5,4}$	$x_{5,5}$	$x_{5,6}$
$x_{6,1}$	$x_{6,2}$	$x_{6,3}$	$x_{6,4}$	$x_{6,5}$	$x_{6,6}$

The convolutional layer

Consider a hidden layer with 6×6 hidden units.

Input variables $\quad 1 \quad$ Hidden units

$$
\begin{array}{cccccc}
x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} & x_{1,5} & x_{1,6} \\
x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} & x_{2,5} & x_{2,6} \\
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x_{6,1} & x_{6,2} & x_{6,3} & x_{6,4} & x_{6,5} & x_{6,6}
\end{array}
\qquad
\begin{array}{cccccc}
h & h & h & h & h & h \\
h & h & h & h & h & h \\
h & h & h & h & h & h \\
h & h & h & h & h & h \\
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h & h & h & h & h & h
\end{array}
$$

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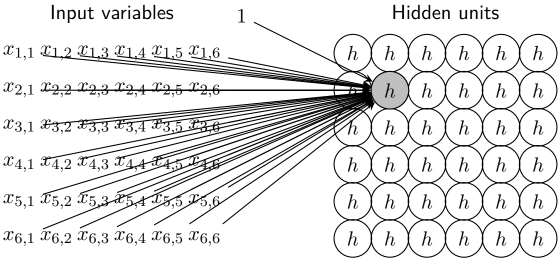
- **Dense layer** (previous lecture): Each hidden unit is connected with **all pixels**. Each pixel-hidden-unit-pair has its own **unique parameter**.



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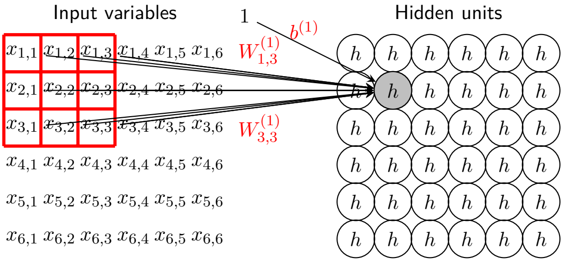

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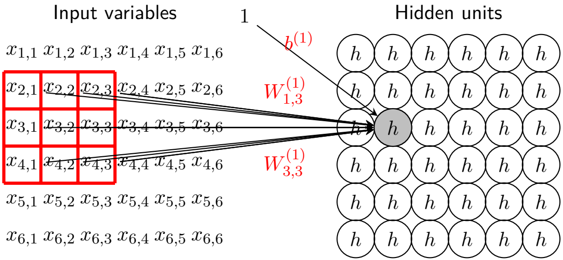

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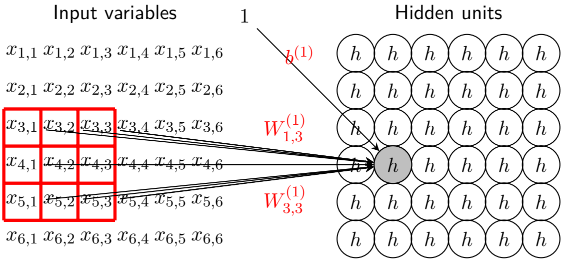

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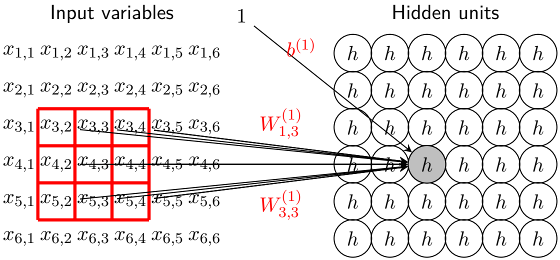

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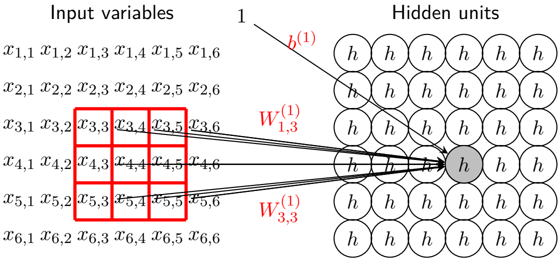

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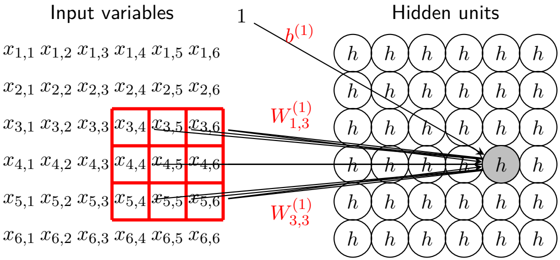

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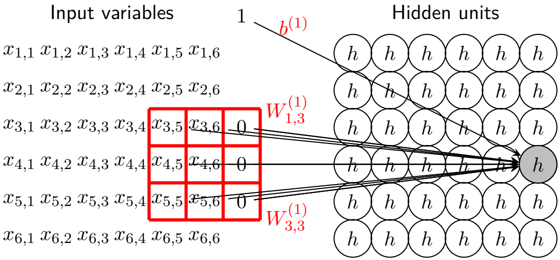

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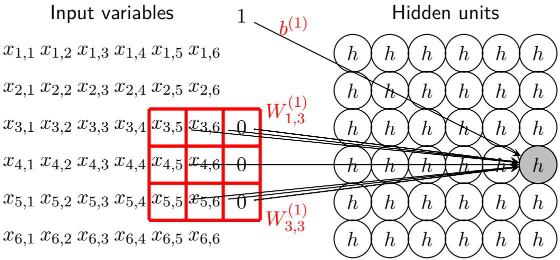
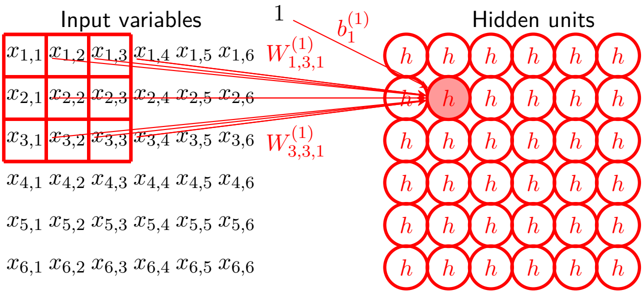


Conv. layer uses **sparse interactions** and **parameter sharing**

Multiple filters

- One filter per layer does not give enough flexibility. $\Rightarrow$
- We use **multiple filters** (visualized with different colors).
- Each filter produces its own set of hidden units – a **channel**.



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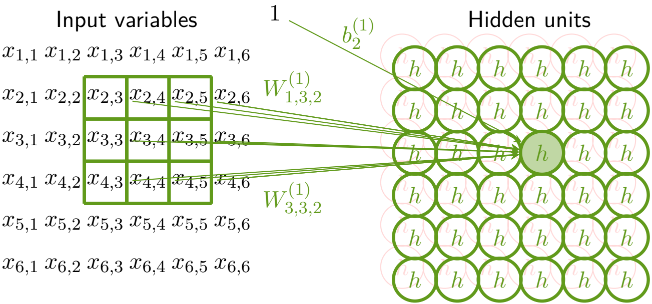

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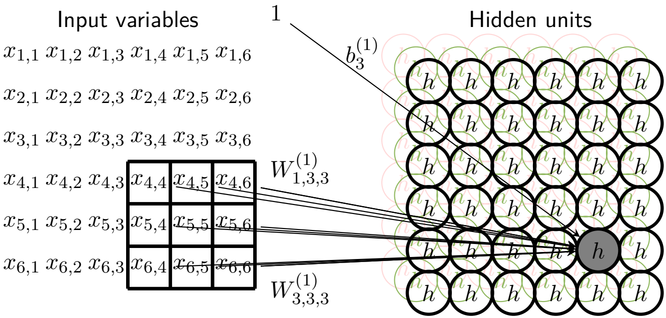

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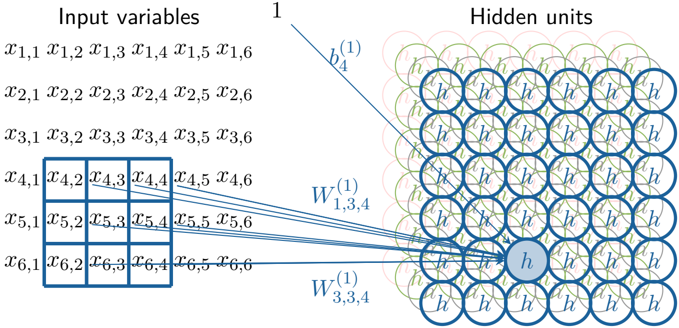

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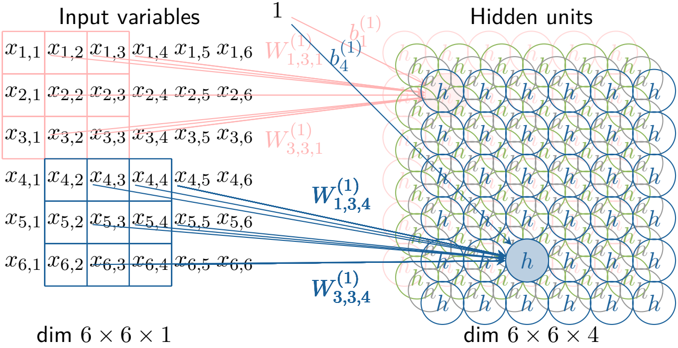
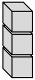
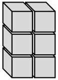
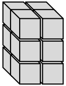


Hidden layers are organized in **tensors** of size (rows $\times$ columns $\times$ channels).

What is a tensor?

A **tensor** is a generalization of scalar, vector and matrix to arbitrary **order**.

Scalar order **0**	$a = 3$	
Vector order **1**	$\mathbf{b} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$	
Matrix order **2**	$W = \begin{bmatrix} 3 & 2 \\ -2 & 1 \\ -1 & 2 \end{bmatrix}$	
Tensor any order (here order **3**)	$\mathbf{T}_{:,:,1} = \begin{bmatrix} 3 & 2 \\ -2 & 1 \\ -1 & 2 \end{bmatrix}, \ \mathbf{T}_{:,:,2} = \begin{bmatrix} -1 & 4 \\ 1 & 2 \\ -5 & 3 \end{bmatrix}$	

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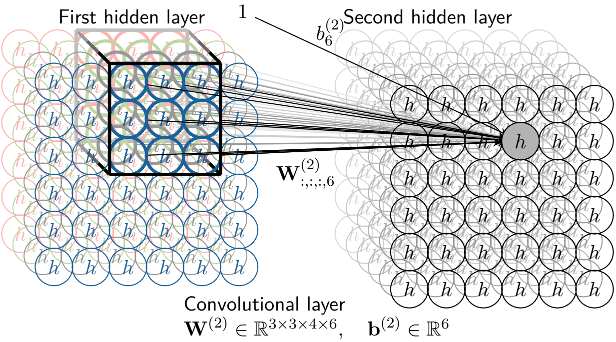
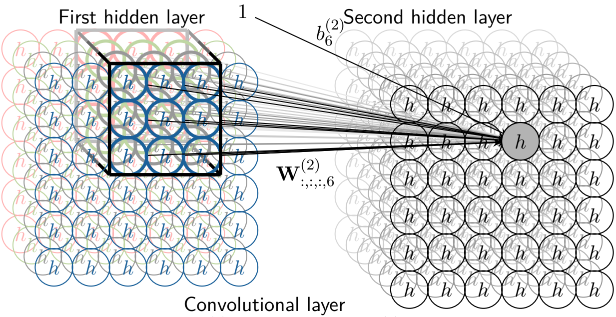
Multiple filters (cont.)

- A filter operates on **all channels** in a hidden layer.



Multiple filters (cont.)

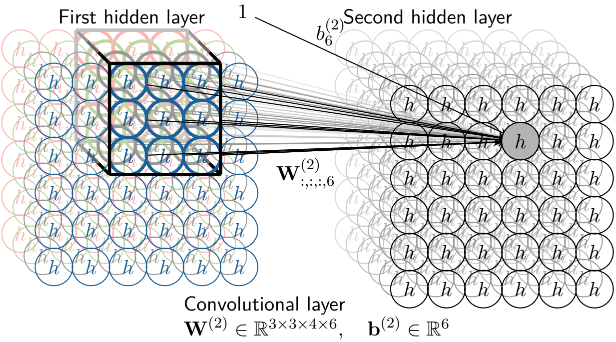
- A filter operates on **all channels** in a hidden layer.
- Each filter has the dimension (filter rows $\times$ filter colomns $\times$ input channels), here ($3 \times 3 \times 4$).



Convolutional layer
$\mathbf{W}^{(2)} \in \mathbb{R}^{3\times3\times4\times6}, \quad \mathbf{b}^{(2)} \in \mathbb{R}^6$

Multiple filters (cont.)

- A filter operates on **all channels** in a hidden layer.
- Each filter has the dimension (filter rows × filter colomns × input channels), here (3 × 3 × 4).
- We stack all filter parameters in a **weight tensor** with dimensions (filter rows × filter columns × input channels × output channels), here (3 × 3 × 4 × 6)



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Condensing information with strides

- **Problem**: As we proceed though the network we want to condense the information.

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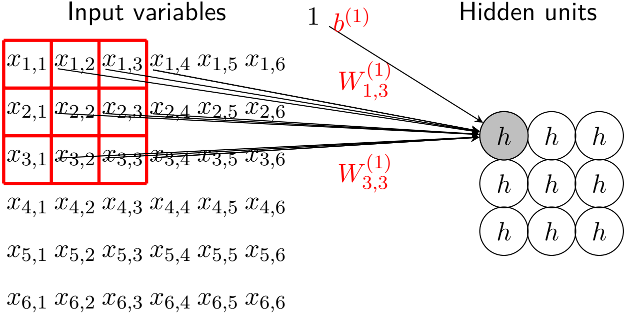

With stride 2 we get half the number of rows and columns in the hidden layer.

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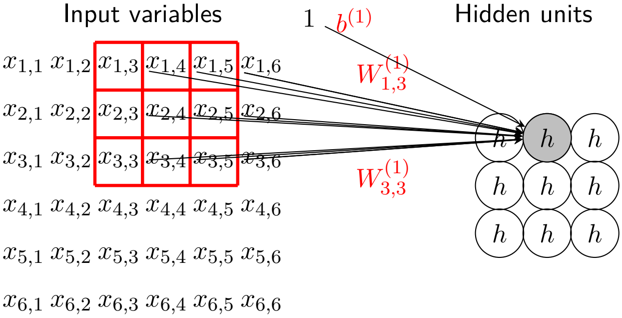

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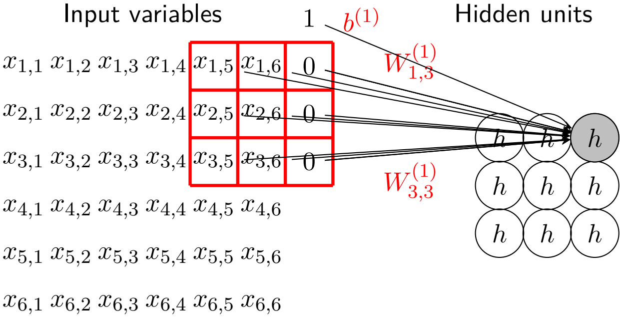

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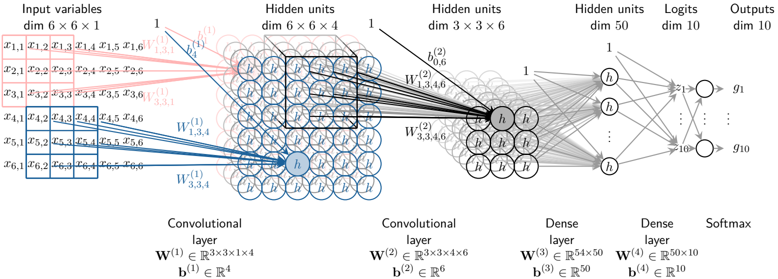
Full CNN architecture

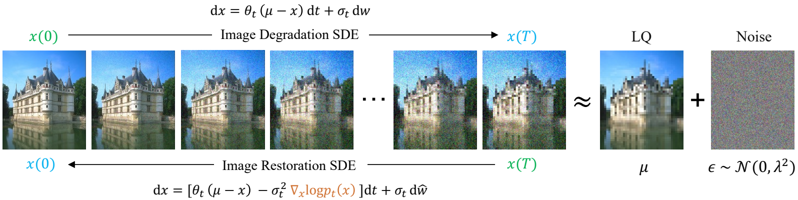
- A full CNN usually consist of multiple convolutional layers (here two) and a few final dense layers (here two).
- If we have a classification problem at hand, we end with a softmax activation function to produce class probabilities.



Here we use 50 hidden units in the last hidden layer and consider a classification problem with $M = 10$ classes.

CNN examples: image restoration

Using CNNs to remove degradations from images



Z. Luo, F.K. Gustafsson, Z. Zhao, J. Sjölund, T.B. Schön, *Image restoration with mean-reverting stochastic differential equations*, ICML, 2023.

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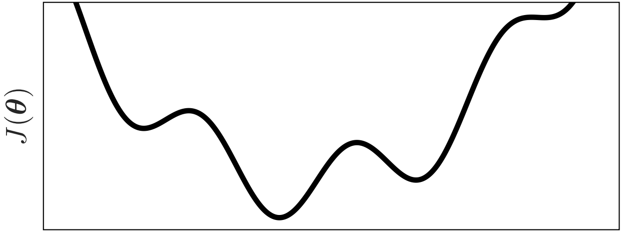
Numerical optimization

How to train a neural network

Unconstrained numerical optimization

We train a network by considering the optimization problem

$$\widehat{\boldsymbol{\theta}} = \arg\min_{\boldsymbol{\theta}} J(\boldsymbol{\theta}), \qquad J(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta})$$



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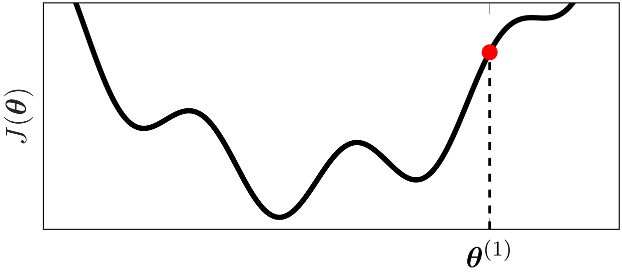

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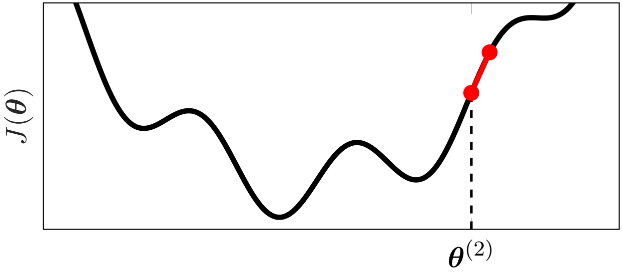

We solve the optimization problem by

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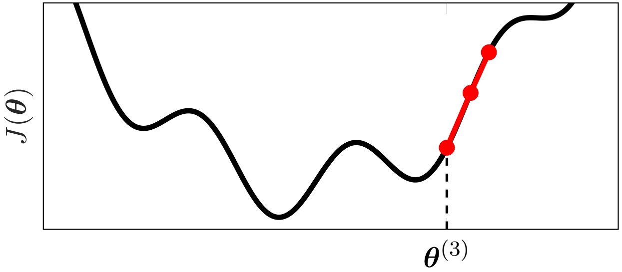

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- ... and updating $\boldsymbol{\theta}$ iteratively.

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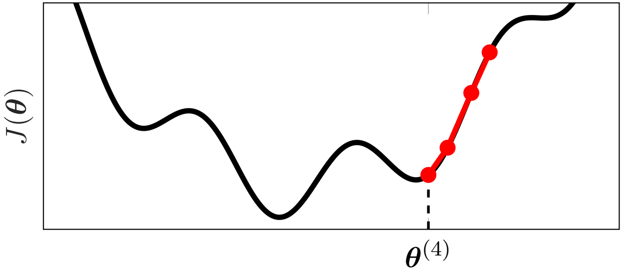

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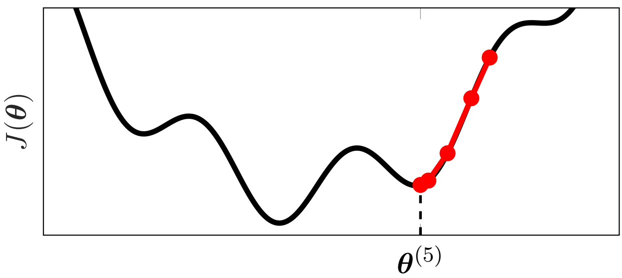

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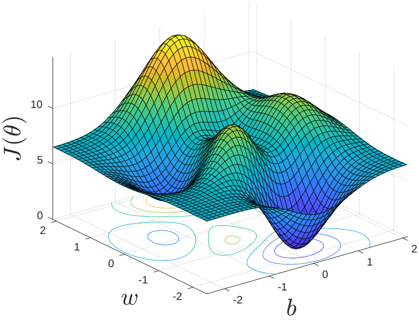
Iterative solution (gradient descent) - Example 2D



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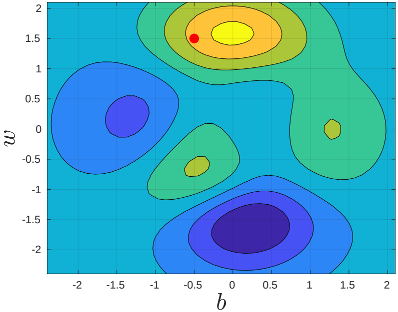

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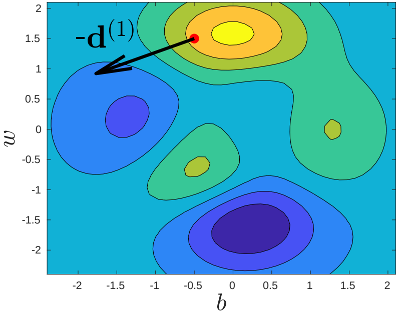

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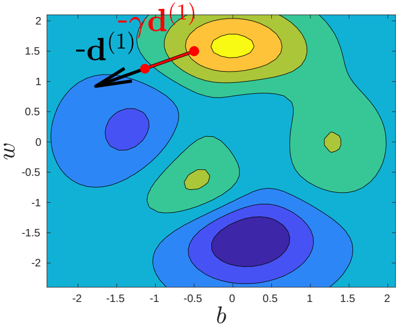

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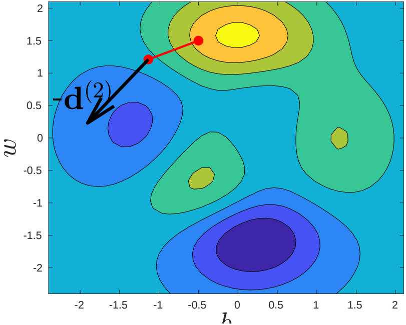

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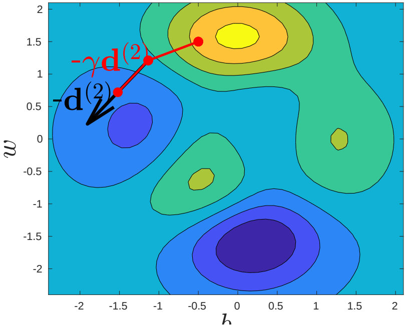

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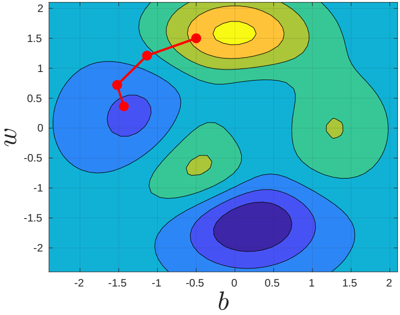

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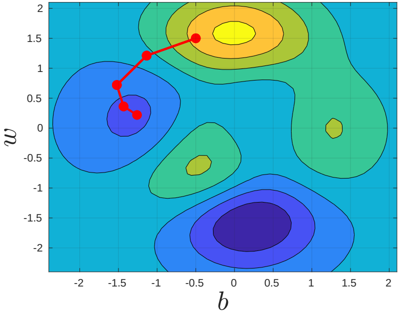

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We call $\gamma \in \mathbb{R}$ the **step length** or **learning rate**.

Computational challenge 1 - dim(θ) is big

At each optimization step we need to compute the gradient

$$\mathbf{d}^{(t)} = \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}^{(t)}) = \frac{1}{n}\sum_{i=1}^{n} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(t)}).$$

Computational challenge 1 - dim(θ) big: A neural network contains a lot of parameters. Computing the gradient is costly.

Solution: A NN is a composition of multiple layers. Hence, each term $\nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta})$ can be computed efficiently by repeatedly applying the chain rule. This is called the **back-propagation algorithm**. Not part of the course.

Computational challenge 2 - n is big

At each optimization step we need to compute the gradient

$$\mathbf{d}^{(t)} = \nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}^{(t)}) = \frac{1}{n}\sum_{i=1}^{n} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(t)}).$$

Computational challenge 2 - n big: We typically use a lot of training data n for training the neural netowork. Computing the gradient is costly.

Solution: For each iteration, we only use a small part of the data set to compute the gradient $\mathbf{d}^{(t)}$. This is called the **stochastic gradient descent**.

Stochastic gradient descent

A big data set is often redundant = many data points are similar.

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

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If the training data is big

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \approx \sum_{i=1}^{\frac{n}{2}} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}) \qquad \text{and}$$

$$\nabla_{\boldsymbol{\theta}} J(\boldsymbol{\theta}) \approx \sum_{i=\frac{n}{2}+1}^{n} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}).$$

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We can do the update with only half the computation cost!

$$\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \gamma \frac{1}{n/2} \sum_{i=1}^{\frac{n}{2}} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(t)}),$$

$$\boldsymbol{\theta}^{(t+2)} = \boldsymbol{\theta}^{(t+1)} - \gamma \frac{1}{n/2} \sum_{i=\frac{n}{2}+1}^{n} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(t+1)}).$$

Stochastic gradient descent

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$$\boldsymbol{\theta}^{(1)} = \boldsymbol{\theta}^{(0)} - \gamma \boldsymbol{\nabla}_{\boldsymbol{\theta}} L(\mathbf{x}_1, \mathbf{y}_1, \boldsymbol{\theta}^{(0)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
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$$\boldsymbol{\theta}^{(2)} = \boldsymbol{\theta}^{(1)} - \gamma \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_2, \mathbf{y}_2, \boldsymbol{\theta}^{(1)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

$$\boldsymbol{\theta}^{(3)} = \boldsymbol{\theta}^{(2)} - \gamma \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_3, \mathbf{y}_3, \boldsymbol{\theta}^{(2)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
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$$\boldsymbol{\theta}^{(4)} = \boldsymbol{\theta}^{(3)} - \gamma \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_4, \mathbf{y}_4, \boldsymbol{\theta}^{(3)})$$

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Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Mini-batch

$$\boldsymbol{\theta}^{(1)} = \boldsymbol{\theta}^{(0)} - \gamma \frac{1}{5} \sum_{i=1}^{5} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(0)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)
- We typically do something in between (not one data point, and not all data). We use a smaller set called **mini-batch**.

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Mini-batch

$$\boldsymbol{\theta}^{(2)} = \boldsymbol{\theta}^{(1)} - \gamma \frac{1}{5} \sum_{i=6}^{10} \boldsymbol{\nabla}_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(1)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)
- We typically do something in between (not one data point, and not all data). We use a smaller set called **mini-batch**.

Stochastic gradient descent

Training data

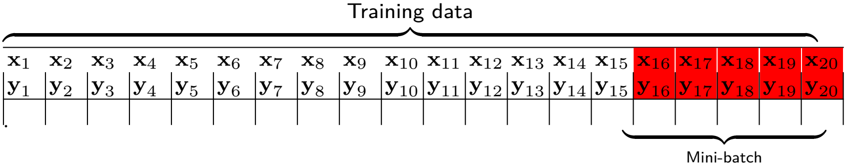
$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Mini-batch

$$\boldsymbol{\theta}^{(3)} = \boldsymbol{\theta}^{(2)} - \gamma \frac{1}{5} \sum_{i=11}^{15} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(2)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)
- We typically do something in between (not one data point, and not all data). We use a smaller set called **mini-batch**.

Stochastic gradient descent



$$\boldsymbol{\theta}^{(4)} = \boldsymbol{\theta}^{(3)} - \gamma \frac{1}{5} \sum_{i=16}^{20} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}^{(3)})$$

- The extreme version of this strategy is to use only one data point at each training step (called **online learning**)
- We typically do something in between (not one data point, and not all data). We use a smaller set called **mini-batch**.
- One pass through the training data is called an **epoch**.

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Iteration:

Epoch:

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.

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Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Iteration:

Epoch:

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.

Stochastic gradient descent

Training data

$\mathbf{x}_1$	$\mathbf{x}_2$	$\mathbf{x}_3$	$\mathbf{x}_4$	$\mathbf{x}_5$	$\mathbf{x}_6$	$\mathbf{x}_7$	$\mathbf{x}_8$	$\mathbf{x}_9$	$\mathbf{x}_{10}$	$\mathbf{x}_{11}$	$\mathbf{x}_{12}$	$\mathbf{x}_{13}$	$\mathbf{x}_{14}$	$\mathbf{x}_{15}$	$\mathbf{x}_{16}$	$\mathbf{x}_{17}$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{20}$
$\mathbf{y}_1$	$\mathbf{y}_2$	$\mathbf{y}_3$	$\mathbf{y}_4$	$\mathbf{y}_5$	$\mathbf{y}_6$	$\mathbf{y}_7$	$\mathbf{y}_8$	$\mathbf{y}_9$	$\mathbf{y}_{10}$	$\mathbf{y}_{11}$	$\mathbf{y}_{12}$	$\mathbf{y}_{13}$	$\mathbf{y}_{14}$	$\mathbf{y}_{15}$	$\mathbf{y}_{16}$	$\mathbf{y}_{17}$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{20}$

Iteration:

Epoch:

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration:

Epoch:

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration: 1

Epoch: 1

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration: 2

Epoch: 1

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration: 3

Epoch: 1

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration: 4

Epoch: 1

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

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Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_7$	$\mathbf{x}_{10}$	$\mathbf{x}_3$	$\mathbf{x}_{20}$	$\mathbf{x}_{16}$	$\mathbf{x}_2$	$\mathbf{x}_1$	$\mathbf{x}_{18}$	$\mathbf{x}_{19}$	$\mathbf{x}_{12}$	$\mathbf{x}_6$	$\mathbf{x}_{11}$	$\mathbf{x}_{17}$	$\mathbf{x}_{15}$	$\mathbf{x}_5$	$\mathbf{x}_{14}$	$\mathbf{x}_4$	$\mathbf{x}_9$	$\mathbf{x}_{13}$	$\mathbf{x}_8$
$\mathbf{y}_7$	$\mathbf{y}_{10}$	$\mathbf{y}_3$	$\mathbf{y}_{20}$	$\mathbf{y}_{16}$	$\mathbf{y}_2$	$\mathbf{y}_1$	$\mathbf{y}_{18}$	$\mathbf{y}_{19}$	$\mathbf{y}_{12}$	$\mathbf{y}_6$	$\mathbf{y}_{11}$	$\mathbf{y}_{17}$	$\mathbf{y}_{15}$	$\mathbf{y}_5$	$\mathbf{y}_{14}$	$\mathbf{y}_4$	$\mathbf{y}_9$	$\mathbf{y}_{13}$	$\mathbf{y}_8$

Iteration: 4

Epoch: 1

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_{19}$	$\mathbf{x}_{16}$	$\mathbf{x}_{18}$	$\mathbf{x}_{6}$	$\mathbf{x}_{9}$	$\mathbf{x}_{13}$	$\mathbf{x}_{1}$	$\mathbf{x}_{14}$	$\mathbf{x}_{20}$	$\mathbf{x}_{11}$	$\mathbf{x}_{3}$	$\mathbf{x}_{8}$	$\mathbf{x}_{7}$	$\mathbf{x}_{12}$	$\mathbf{x}_{4}$	$\mathbf{x}_{17}$	$\mathbf{x}_{5}$	$\mathbf{x}_{10}$	$\mathbf{x}_{2}$	$\mathbf{x}_{15}$
$\mathbf{y}_{19}$	$\mathbf{y}_{16}$	$\mathbf{y}_{18}$	$\mathbf{y}_{6}$	$\mathbf{y}_{9}$	$\mathbf{y}_{13}$	$\mathbf{y}_{1}$	$\mathbf{y}_{14}$	$\mathbf{y}_{20}$	$\mathbf{y}_{11}$	$\mathbf{y}_{3}$	$\mathbf{y}_{8}$	$\mathbf{y}_{7}$	$\mathbf{y}_{12}$	$\mathbf{y}_{4}$	$\mathbf{y}_{17}$	$\mathbf{y}_{5}$	$\mathbf{y}_{10}$	$\mathbf{y}_{2}$	$\mathbf{y}_{15}$

Iteration:

Epoch: 2

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.
- After each epoch we do another reshuffling and another pass through the data set.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_{19}$	$\mathbf{x}_{16}$	$\mathbf{x}_{18}$	$\mathbf{x}_{6}$	$\mathbf{x}_{9}$	$\mathbf{x}_{13}$	$\mathbf{x}_{1}$	$\mathbf{x}_{14}$	$\mathbf{x}_{20}$	$\mathbf{x}_{11}$	$\mathbf{x}_{3}$	$\mathbf{x}_{8}$	$\mathbf{x}_{7}$	$\mathbf{x}_{12}$	$\mathbf{x}_{4}$	$\mathbf{x}_{17}$	$\mathbf{x}_{5}$	$\mathbf{x}_{10}$	$\mathbf{x}_{2}$	$\mathbf{x}_{15}$
$\mathbf{y}_{19}$	$\mathbf{y}_{16}$	$\mathbf{y}_{18}$	$\mathbf{y}_{6}$	$\mathbf{y}_{9}$	$\mathbf{y}_{13}$	$\mathbf{y}_{1}$	$\mathbf{y}_{14}$	$\mathbf{y}_{20}$	$\mathbf{y}_{11}$	$\mathbf{y}_{3}$	$\mathbf{y}_{8}$	$\mathbf{y}_{7}$	$\mathbf{y}_{12}$	$\mathbf{y}_{4}$	$\mathbf{y}_{17}$	$\mathbf{y}_{5}$	$\mathbf{y}_{10}$	$\mathbf{y}_{2}$	$\mathbf{y}_{15}$

Iteration: 5

Epoch: 2

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.
- After each epoch we do another reshuffling and another pass through the data set.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_{19}$	$\mathbf{x}_{16}$	$\mathbf{x}_{18}$	$\mathbf{x}_{6}$	$\mathbf{x}_{9}$	$\mathbf{x}_{13}$	$\mathbf{x}_{1}$	$\mathbf{x}_{14}$	$\mathbf{x}_{20}$	$\mathbf{x}_{11}$	$\mathbf{x}_{3}$	$\mathbf{x}_{8}$	$\mathbf{x}_{7}$	$\mathbf{x}_{12}$	$\mathbf{x}_{4}$	$\mathbf{x}_{17}$	$\mathbf{x}_{5}$	$\mathbf{x}_{10}$	$\mathbf{x}_{2}$	$\mathbf{x}_{15}$
$\mathbf{y}_{19}$	$\mathbf{y}_{16}$	$\mathbf{y}_{18}$	$\mathbf{y}_{6}$	$\mathbf{y}_{9}$	$\mathbf{y}_{13}$	$\mathbf{y}_{1}$	$\mathbf{y}_{14}$	$\mathbf{y}_{20}$	$\mathbf{y}_{11}$	$\mathbf{y}_{3}$	$\mathbf{y}_{8}$	$\mathbf{y}_{7}$	$\mathbf{y}_{12}$	$\mathbf{y}_{4}$	$\mathbf{y}_{17}$	$\mathbf{y}_{5}$	$\mathbf{y}_{10}$	$\mathbf{y}_{2}$	$\mathbf{y}_{15}$

Iteration: 6

Epoch: 2

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.
- After each epoch we do another reshuffling and another pass through the data set.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_{19}$	$\mathbf{x}_{16}$	$\mathbf{x}_{18}$	$\mathbf{x}_{6}$	$\mathbf{x}_{9}$	$\mathbf{x}_{13}$	$\mathbf{x}_{1}$	$\mathbf{x}_{14}$	$\mathbf{x}_{20}$	$\mathbf{x}_{11}$	$\mathbf{x}_{3}$	$\mathbf{x}_{8}$	$\mathbf{x}_{7}$	$\mathbf{x}_{12}$	$\mathbf{x}_{4}$	$\mathbf{x}_{17}$	$\mathbf{x}_{5}$	$\mathbf{x}_{10}$	$\mathbf{x}_{2}$	$\mathbf{x}_{15}$
$\mathbf{y}_{19}$	$\mathbf{y}_{16}$	$\mathbf{y}_{18}$	$\mathbf{y}_{6}$	$\mathbf{y}_{9}$	$\mathbf{y}_{13}$	$\mathbf{y}_{1}$	$\mathbf{y}_{14}$	$\mathbf{y}_{20}$	$\mathbf{y}_{11}$	$\mathbf{y}_{3}$	$\mathbf{y}_{8}$	$\mathbf{y}_{7}$	$\mathbf{y}_{12}$	$\mathbf{y}_{4}$	$\mathbf{y}_{17}$	$\mathbf{y}_{5}$	$\mathbf{y}_{10}$	$\mathbf{y}_{2}$	$\mathbf{y}_{15}$

Iteration: 7

Epoch: 2

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.
- After each epoch we do another reshuffling and another pass through the data set.

Stochastic gradient descent

Training data (reshuffled)

$\mathbf{x}_{19}$	$\mathbf{x}_{16}$	$\mathbf{x}_{18}$	$\mathbf{x}_{6}$	$\mathbf{x}_{9}$	$\mathbf{x}_{13}$	$\mathbf{x}_{1}$	$\mathbf{x}_{14}$	$\mathbf{x}_{20}$	$\mathbf{x}_{11}$	$\mathbf{x}_{3}$	$\mathbf{x}_{8}$	$\mathbf{x}_{7}$	$\mathbf{x}_{12}$	$\mathbf{x}_{4}$	$\mathbf{x}_{17}$	$\mathbf{x}_{5}$	$\mathbf{x}_{10}$	$\mathbf{x}_{2}$	$\mathbf{x}_{15}$
$\mathbf{y}_{19}$	$\mathbf{y}_{16}$	$\mathbf{y}_{18}$	$\mathbf{y}_{6}$	$\mathbf{y}_{9}$	$\mathbf{y}_{13}$	$\mathbf{y}_{1}$	$\mathbf{y}_{14}$	$\mathbf{y}_{20}$	$\mathbf{y}_{11}$	$\mathbf{y}_{3}$	$\mathbf{y}_{8}$	$\mathbf{y}_{7}$	$\mathbf{y}_{12}$	$\mathbf{y}_{4}$	$\mathbf{y}_{17}$	$\mathbf{y}_{5}$	$\mathbf{y}_{10}$	$\mathbf{y}_{2}$	$\mathbf{y}_{15}$

Iteration: 8

Epoch: 2

- If we pick the mini-batches in order, they might be unbalanced and not representative for the whole data set.
- Therefore, we pick data points **at random** from the training data to form a mini-batch.
- One implementation is to randomly reshuffle the data before dividing it into mini-batches.
- After each epoch we do another reshuffling and another pass through the data set.

Mini-batch gradient descent

The full **stochastic gradient descent** algorithm (a.k.a **mini-batch gradient descent**) is as follows

1. Initialize $\boldsymbol{\theta}^{(0)}$, set $t \leftarrow 1$, choose batch size n_b and number of epochs E.
2. For $i = 1$ to E
   (a) Randomly shuffle the training data $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=1}^{n}$.
   (b) For $j = 1$ to $\frac{n}{n_b}$
      (i) Approximate the gradient of the loss function using the mini-batch $\{\mathbf{x}_i, \mathbf{y}_i\}_{i=(j-1)n_b+1}^{jn_b}$,
      $\hat{\mathbf{d}}^{(t)} = \frac{1}{n_b} \sum_{i=(j-1)n_b+1}^{jn_b} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta})\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}}$.
      (ii) Do a gradient step $\boldsymbol{\theta}^{(t+1)} = \boldsymbol{\theta}^{(t)} - \gamma \hat{\mathbf{d}}^{(t)}$.
      (iii) Update the iteration index $t \leftarrow t + 1$.

At each time we get a stochastic approximation of the true gradient $\hat{\mathbf{d}}^{(t)} \approx \frac{1}{n} \sum_{i=1}^{n} \nabla_{\boldsymbol{\theta}} L(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta})\Big|_{\boldsymbol{\theta}=\boldsymbol{\theta}^{(t)}}$, hence the name.

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Summary

1. **Previous lecture** The neural network model
    - Neural network for regression
    - Neural network for classification

Summary

1. **Previous lecture** The neural network model
    - Neural network for regression
    - Neural network for classification
2. **This lecture**
    - Convolutional neural network
    - How to train a neural network

A few concepts to summarize lecture 9

Convolutional neural network (CNN): A NN with a particular structure tailored for input data with a grid-like structure, like for example images.

Filter: (a.k.a kernel) A set of parameters that is convolved with a hidden layer. Each filter produces a new channel.

Channel: A set of hidden units produced by the same filter. Each hidden layer consists of one or more channels.

Stride: A positive integer deciding how many steps to move the filter during the convolution.

Tensor: A generalization of matrices to arbitrary order.

Gradient descent: An iterative optimization algorithm where we at iteration take a step proportional to the negative gradient.

Learning rate: (a.k.a step length). A scalar tuning parameter deciding the length of each gradient step in gradient descent.

Stochastic gradient descent (SGD): A version of gradient descent where we at each iteration only use a small part of the training data (a mini-batch).

Mini-batch: The group of training data that we use at each iteration in SG

Batch size: The number of data points in one mini-batch