Institut for Matematik og Datalogi
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JBJ

MM851 – Fall 2019 – Exercises on Markov Chains

This is only for the danish math students who have already been informed that they should study Markov chains themselves!!

Exercises
Solve the following exercises and discuss them among yourselves.

- Consider the weather during a number of days as a stochastic process with the only possible states 0 : sun and 1 : rain. We assume for simplicity that the process is a Markov chain with transition matrix
  $P = \begin{pmatrix} 0.7 & 0.3 \\ 0.2 & 0.8 \end{pmatrix}$
  (a) Find the probability that a rainy day is followed by a sunny.
  (b) Formulate in words the event $X_{62} = 1$.
  (c) Find the conditional probability $Pr(X_{62} = 1|X_{61} = 0)$.
  (d) Find the probability that a rainy day is followed by two sunny days.
  (e) Find $Pr(X_{62} = 1|X_{60} = 0)$.
  (f) If Friday is sunny, what is the probability that the next following Sunday is also a sunny day?

- The weather changes at a tourist resort from one day to the next can somewhat simplified be described as a Markov chain with the three states:
  E_1 : sun, E_2 : clouds, E_3 : rain.

  Using weather statistics of the area the following transition probability matrix has been estimated: $P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.3 & 0.5 & 0.2 \\ 0.7 & 0.0 & 0.3 \end{pmatrix}$
  A vacationer intends to visit the resort during the period December 24-26. Under the assumption that there is still a lot of time before Christmas, derive the probability
  (a) that there will be three sunny days in a row;
  (b) of no rain at least during the first two days.

- Mitzenmacher and Upfal Excercises 7.6, 7.10, 7.21, 7.22, 7.24