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Exam number:

Exam in Master / Financial Theory

General instructions

- No technical aids are allowed.
- All calculations should be clearly motivated.
- Do not skip steps in the formal derivations.
- Answer the questions without providing additional / unrelated information. I deduct points for incorrect statements you make.
- If you cannot solve a question without making additional assumptions, state these assumptions clearly and explain in writing why they are necessary.
- The writing time is 5 hours. Write your examination number in the indicated space and on all papers you hand in.
- The total number of points is 50. For grade E is 25 points required, for grade D is 27.5 points required, for grade C is 32.5 points required, for grade B is 37.5 points required and for grade A is 45 points required.

Good luck!

Problem 1

a) —

b) Write down the pricing formula for a zero-coupon bond, and define the different parts of it. (2 p)

c) How is the excess return SMB in the Fama-French model defined?(2 p)

d) What does the semistrong-form version of the efficient market hypothesis say? (2 p)

e) —

Problem 2

a) What is meant by the bid and ask price respectively? (2 p)

b) Derive an expression for the forward rate $f_{2,3}$ in terms of spot rates at time $t = 0$. (2 p)

c) Consider the relationship

$$r_{t,t+1} = \frac{P_{t+1} + D_{t+1} - P_t}{P_t},$$

where it is assumed that $E_t[r_{t,t+1}] = r$. Show that in this case

$$P_t = E_t\left[\frac{P_{t+1} + D_{t+1}}{1 + r}\right].$$

(2 p)

d) A market consists of the following assets.

Asset no	**Price**	**No of shares**	**Expected return**
1	10	300	0.12
2	12.5	200	0.20
3	45	100	0.10

Determine the expected return of the market portfolio. (2 p)

e) In what way is the "liquidity premium theory" trying to explain the form of yield curves? (2 p)

Problem 3

Two assets have the following expected returns and standard deviations:

$$\bar{\mu}_1 = 0.3, \bar{\mu}_2 = 0.2, \sigma_1 = 0.2 \text{ and } \sigma_2 = 0.1.$$

The correlation between them is

$$\rho = 0.75.$$

a) How large is the expected return on the minimum variance portfolio if short-selling is not allowed? (3 p)

b) How large is the expected return on the minimum variance portfolio if short-selling is allowed? (3 p)

c) Still allowing for short-selling we now add a risk-free rate $r_f = 0.02$ to the model. Is the portfolio with weights $(0.5, 0.5)$ the tangent portfolio? Motivate your answer. (4 p)

Problem 4

In order to model a financial market two uncorrelated factors F_1 and F_2 are used. They are normalised to have expected value equal to zero and variance equal to one. In this model the return of two financial assets are given by

$$\begin{aligned} r_1 &= 0.26 + 0.2F_1 + 0.4F_2 + \varepsilon_1 \\ r_2 &= 0.12 + 0.1F_1 - 0.3F_2 + \varepsilon_2, \end{aligned}$$

where ε_1 and ε_2 are uncorrelated with the indexes, have zero expected value and standard deviation σ_1 and σ_2 respectively.

a) How large is the total risk in asset 1 if $\sigma_1 = 0.4$? (2 p)

b) How large is the idiosyncratic risk in asset 2 if $\mathrm{Std}[r_2] = 0.26$? (2 p)

In order to determine the expected return $\bar{\mu}_3$ of a third asset, the exact factor model

$$\begin{aligned} r_1 &= 0.26 + 0.2F_1 + 0.4F_2 \\ r_2 &= 0.12 + 0.1F_1 - 0.3F_2 \end{aligned}$$

is used. This third asset has factor loadings $b_{31} = 0.2$ and $b_{32} = 0.1$.

c) How large is the total risk in asset 3? (2 p)

d) How large is the expected return of asset 3 if the risk-free rate is $r_f = 0.02$ and we assume that the conditions of APT holds? (4 p)

Problem 5

A representative consumer has preferences over consumption over two time periods given by

$$U(c_0, c_1) = u(c_0) + \delta u(c_1),$$

where $0 < \delta < 1$ and u is an increasing and concave function.

a) Write down the stochastic discount factor (SDF) for this case. (2 p)

Now assume that there also exists a risk-free asset with rate of return r_f.

b) Derive an expression for r_f that shows how it depends on δ and u. (3 p)

c) How large is the risk-free rate r_f if $u(x) = \ln x$ and the consumption growth rate is normally distributed with mean μ and standard deviation σ? (5 p)

Hint: If X is normally distributed with mean μ and standard deviation σ, then $E\left(e^X\right) = e^{\mu+\sigma^2/2}$.