

FINAL EXAMINATION**Analysis of survival data (7.5hp) 2025-01-17****INFORMATION:****A.** Allowed means of assistance:

Handouts from lectures L1-L7 (handwritten notes allowed), ethical guidelines by ISI, RSS, ASA, and Svenska statistikfrämjandet. Calculator, ruler, dictionary.

B. Writing time: 5 hours.**C.** The examination consists of 5 tasks, for a total of 80 points.

Including any bonus points, at least 48 points are needed to pass (G), and 72 points to pass with distinction (VG).

D. For every task the maximum score is shown (for every part of the task). Sometimes the parts cannot be judged independent of each other, which means that points might not be able to be set for a later part if the previous part has not been solved in a correct way (in principle). Negative points will never be set.**E.** You can write your answers in English or Swedish.**F.** If you desire clarification regarding the test, especially the wording of a problem, please contact an examination proctor. The examination proctors can contact the responsible teacher.**G.** After turning in your test, you will keep the test pages with the question statements (not to be handed in!). Preliminary solutions will be posted at Studium.**INSTRUCTIONS:****A.** Follow the instructions on the front page to be stapled to your solutions. E.g., the solutions for each task should be started on a new sheet.**B.** Present all your solutions in a way that makes it easy to follow your way of thinking! What is unclearly presented is assumed to be unclearly thought. Motivate all important steps of your solution, including any assumptions that need to be fulfilled (and check if they are).**C.** When constructing confidence intervals you must (besides what is presented in B above) state what the interval is intended to cover, and present the formula for the interval before you present the calculation (if needed), and interpret the calculated interval.**D.** When performing hypothesis testing you must (besides what is presented in B above) present null and alternative hypotheses, choice of significance level, choice of test, P -value, result, and conclusion.

Good luck!

(6) **Task 1**

- (3) **A** You are about to analyze time-to-event data from a certain study. Why is it important to know which types of censoring and/or truncation that is/are present in the study?
- (3) **B** Why do you think that nonparametric/semiparametric methods of analyzing time-to-event data have become more popular than parametric methods?

(14) **Task 2**

A mobile app developer studied how long users actively engage with their app after installation. The company began tracking new installations starting on January 1, 2019. The dataset includes the installation date and the date of the user's last activity on the app. For some users, the app is still actively used as of December 2024. Additionally, the company has records of users who installed the app before January 1, 2019, but were still active on or after January 1, 2019. There is no information about users who installed and stopped using the app before January 1, 2019. Time is measured in months.

- (2) **A** Describe the type(s) of truncation represented in this study. Motivate your answer. If you find that no truncation is present, make up an example of truncation that could have been present in a study like this one.
- (8) **B** For the four following users, identify the type(s) of censoring present. Motivate your answer.
- i) A user who installed the app in March 2023 and is still using it as of December 2024.
 - ii) A user who installed the app in January 2020 and last used it in May 2021.
 - iii) A user who installed the app in October 2018 and is still using it as of December 2024.
 - iv) A user who installed the app in July 2019, actively used it in August 2020, and, after a glitch in the software recording the usage (which was debugged in October 2020), it was discovered that the user had stopped using the app.
- (4) **C** Confining your attention to the four users described above, write down the likelihood for this portion of the study.

(26) Task 3

In a study examining career longevity in professional basketball, researchers investigated the relationship between team performance and the time until retirement. Data was collected for 2,500 randomly selected players who began their professional careers between 2005 and 2015. Players who were still active at the study's conclusion in December 2020 were considered censored.

Variable specification:

- *time*: Time in years from the start of the professional career to retirement or the study's conclusion
- *censored*: Indicator for whether the player was still active at the end of the study (1 = yes, 0 = no)
- *team_performance*: Team's average performance level (High, Medium, Low)

Source: Fictive statistics

Use the SAS output on the following pages to answer the questions below. Note that survival estimates are presented only for selected time points (all relevant information for answering the questions is included).

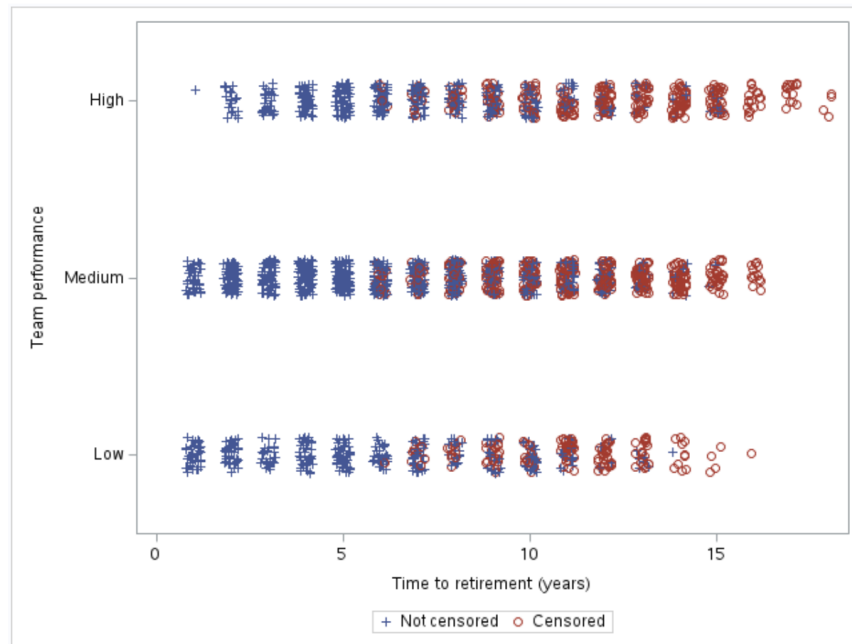
NOTE: In each of the questions, interpret “survival” in the context of this study.

- (8) A** Present, interpret, and compare the following estimates for the team performance groups with the highest and lowest survival probabilities (i.e., only two of the three team performance groups):

- The 25th percentile, 75th percentile and median (50th percentile) of “survival” times.

If any measures cannot be estimated, explain why and present the minimum time for those measures.

- (3) B** Present and compare the approximative probability of “surviving” at least 10 years for the two team performance groups in A.
- (2) C** Present and interpret the approximative cumulative hazard at 10 years for the team performance group with the lowest cumulative hazard.
- (13) D** Is there a significant difference in retirement probabilities between the three team performance groups? Perform an appropriate hypothesis test to determine this. Remember to follow the instructions on the front page.

SAS OUTPUT Task 3

Summary of the Number of Censored and Uncensored Values					
Stratum	performance	Total	Failed	Censored	Percent Censored
1	High	748	450	298	39.84
2	Low	509	356	153	30.06
3	Medium	1243	853	390	31.38
Total		2500	1659	841	33.64

Stratum 1: Team performance = High					
Product-Limit Survival Estimates					
time	Survival	Failure	Survival Standard Error	Number Failed	Number Left
0.0000	1.0000	0	0	0	748
1.0000	0.9987	0.00134	0.00134	1	747
2.0000	.	.	.	2	746
5.0000	.	.	.	162	586
5.0000	0.7821	0.2179	0.0151	163	585
6.0000	.	.	.	164	584
6.0000	.	.	.	216	532
6.0000	0.7099	0.2901	0.0166	217	531
6.0000	*	.	.	217	530
9.0000	.	.	.	339	380
9.0000	0.5406	0.4594	0.0184	340	379
9.0000	*	.	.	340	378
10.0000	.	.	.	380	313
10.0000	0.4778	0.5222	0.0187	381	312
10.0000	*	.	.	381	311
15.0000	.	.	.	449	73
15.0000	0.3207	0.6793	0.0214	450	72
15.0000	*	.	.	450	71
18.0000	*	.	.	450	1
18.0000	*	.	.	450	0

Note: The marked survival times are censored observations.

Stratum 2: Team performance = Low

Product-Limit Survival Estimates					
time	Survival	Failure	Survival Standard Error	Number Failed	Number Left
0.0000	1.0000	0	0	0	509
1.0000	.	.	.	1	508
3.0000	.	.	.	108	401
3.0000	0.7859	0.2141	0.0182	109	400
4.0000	.	.	.	110	399
4.0000	.	.	.	150	359
4.0000	0.7033	0.2967	0.0202	151	358
5.0000	.	.	.	152	357
6.0000	.	.	.	226	283
6.0000	0.5540	0.4460	0.0220	227	282
6.0000 *	.	.	.	227	281
7.0000	.	.	.	259	249
7.0000	0.4890	0.5110	0.0222	260	248
7.0000 *	.	.	.	260	247
12.0000	.	.	.	348	70
12.0000	0.2715	0.7285	0.0219	349	69
12.0000 *	.	.	.	349	68
13.0000	.	.	.	354	37
13.0000	0.2327	0.7673	0.0238	355	36
13.0000 *	.	.	.	355	35
15.0000 *	.	.	.	356	1
16.0000 *	.	.	.	356	0

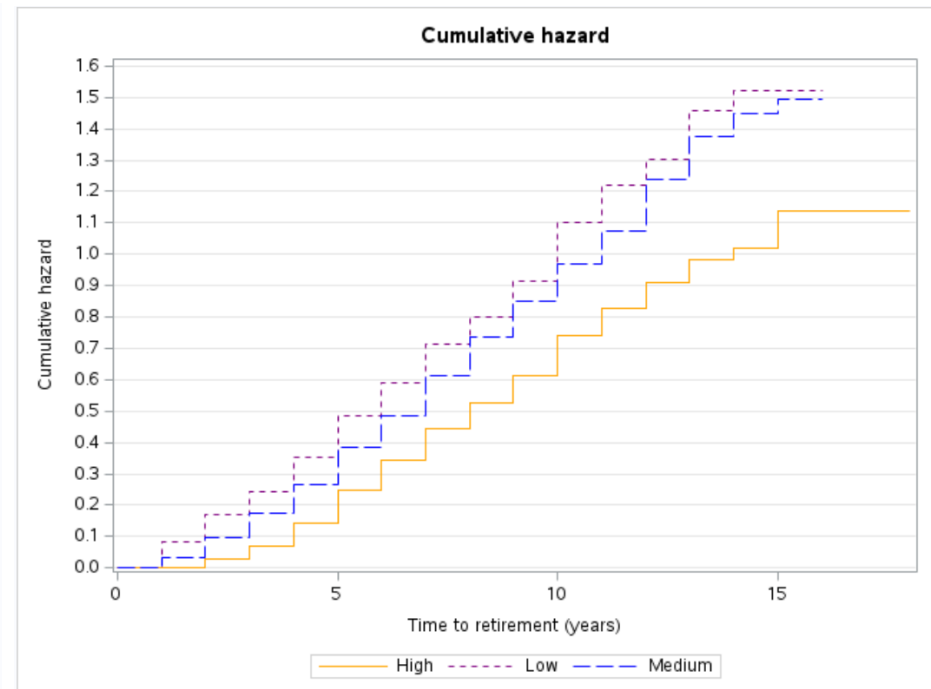
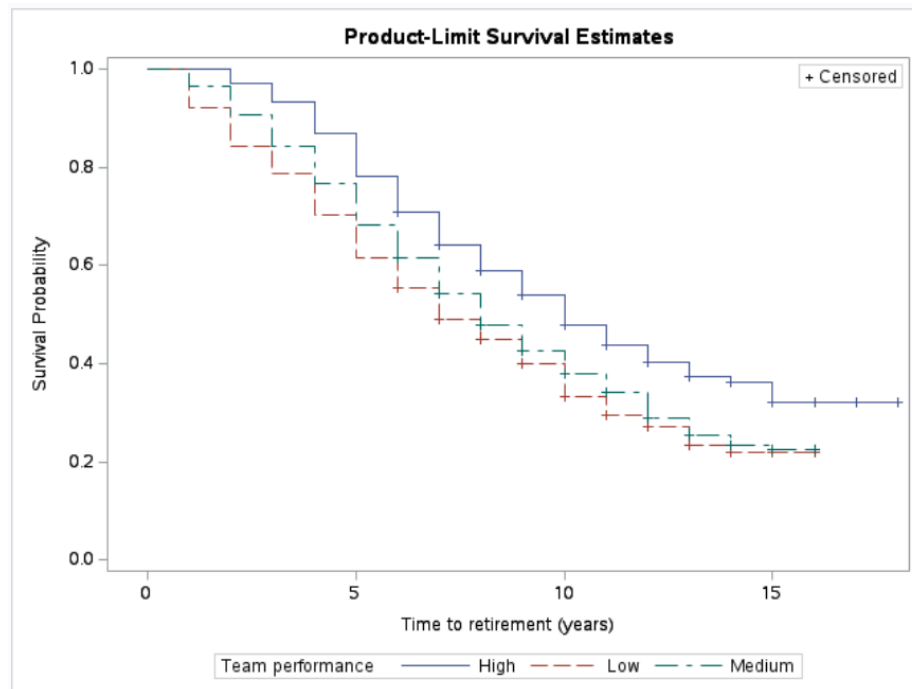
Note: The marked survival times are censored observations.

Stratum 3: Team performance = Medium

Product-Limit Survival Estimates					
time	Survival	Failure	Survival Standard Error	Number Failed	Number Left
0.0000	1.0000	0	0	0	1243
1.0000	.	.	.	1	1242
4.0000	.	.	.	200	999
4.0000	0.7675	0.2325	0.0120	289	954
5.0000	.	.	.	290	953
5.0000	.	.	.	394	849
5.0000	0.6822	0.3178	0.0132	395	848
6.0000	.	.	.	396	847
7.0000	.	.	.	560	663
7.0000	0.5411	0.4589	0.0142	569	662
7.0000 *	.	.	.	569	661
8.0000	.	.	.	644	568
8.0000 *	.	.	.	644	567
13.0000	.	.	.	843	146
13.0000	0.2526	0.7474	0.0145	844	145
13.0000 *	.	.	.	844	144
14.0000	.	.	.	850	94
14.0000	0.2349	0.7651	0.0150	851	93
14.0000 *	.	.	.	851	92
16.0000 *	.	.	.	853	1
16.0000 *	.	.	.	853	0

Note: The marked survival times are censored observations.

Test of Equality over Strata			
Test	Chi-Square	DF	Pr > Chi-Square
Log-Rank	48.8282	2	<.0001
Wilcoxon	58.3474	2	<.0001
-2Log(LR)	39.7642	2	<.0001



END OF TASK 3

(30) Task 4

In the study of the career longevity of basketball players described in Task 3, the researchers also investigated the relationship between a number of possible covariates and time to retirement.

The following variables are included in the data:

- *time*: Time in years from the start of the professional career to retirement or the study's conclusion
- *censored*: Indicator for whether the player was still active at the end of the study (1 = yes, 0 = no)
- *team_performance*: Team's average performance level (High, Medium, Low)
- *nationality*: Player's nationality (1 = domestic, 2 = international)
- *age*: Age at debut (years)
- *age_group*: Age divided into groups (18-23, 24-30, 31-35)
- *position*: Playing position (Guard, Forward, Center)

Use the output below, generated from SAS PROC PHREG, and answer the following questions:

- (3) **A** Do any of the covariates above need to be handled in any special way (e.g. by recoding or transforming them), or can they be used as they are in the regression analysis? Motivate your answer.

(You don't need to take the PH assumption into account here.)

- (17) **B** Which of the presented six models below would you choose to analyze the data? Motivate your choice carefully.

- (8) **C** Explain/interpret the relationships between the covariates in the model you chose above and time to retirement.

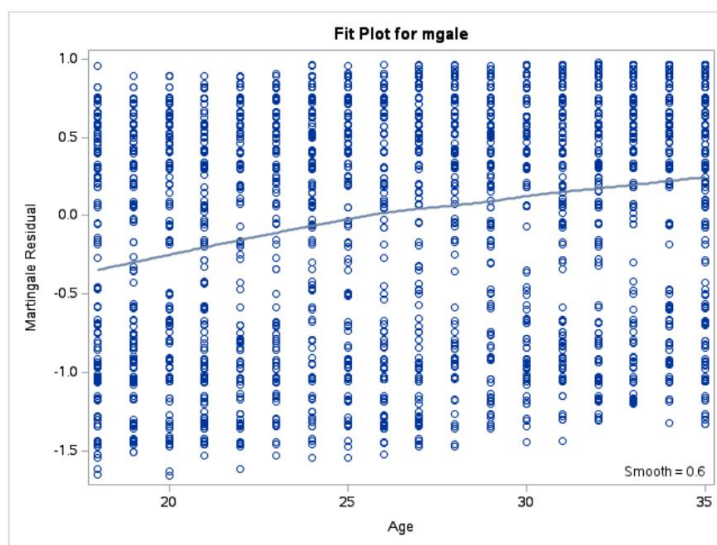
If you've chosen a stratified model, remember to also interpret the stratifying variable(s).

If you've chosen a model with a time-dependent coefficient, calculate the hazard ratio at 5, 10, and 15 years, respectively (no confidence intervals needed).

- (2) **D** Calculate and interpret the generalized R^2 for the selected model.

SAS OUTPUT Task 4

Summary of the Number of Event and Censored Values			
Total	Event	Censored	Percent Censored
2500	1659	841	33.64

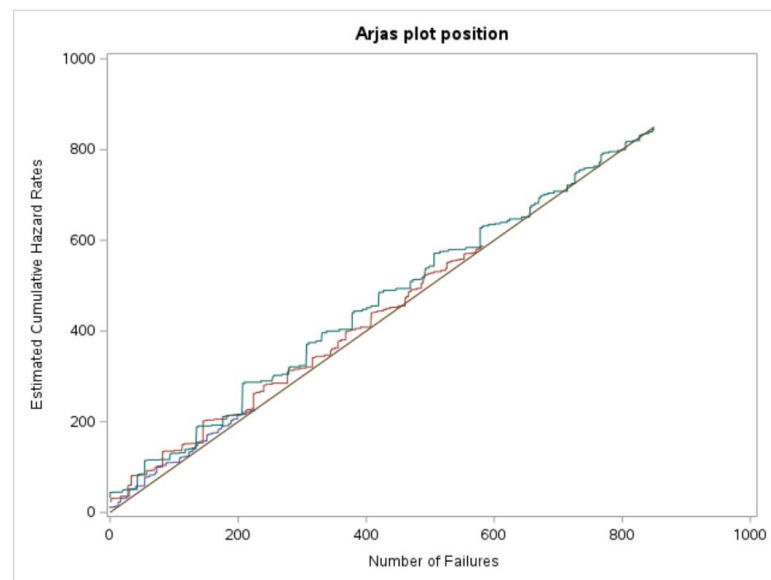
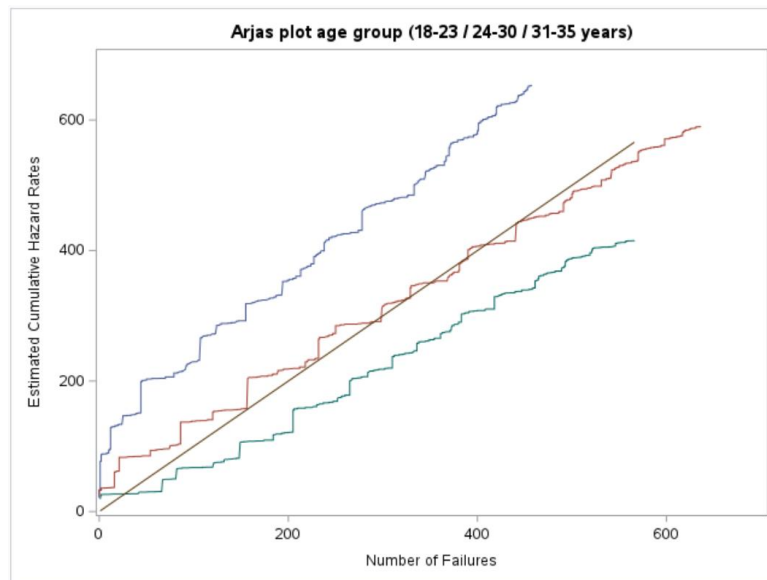
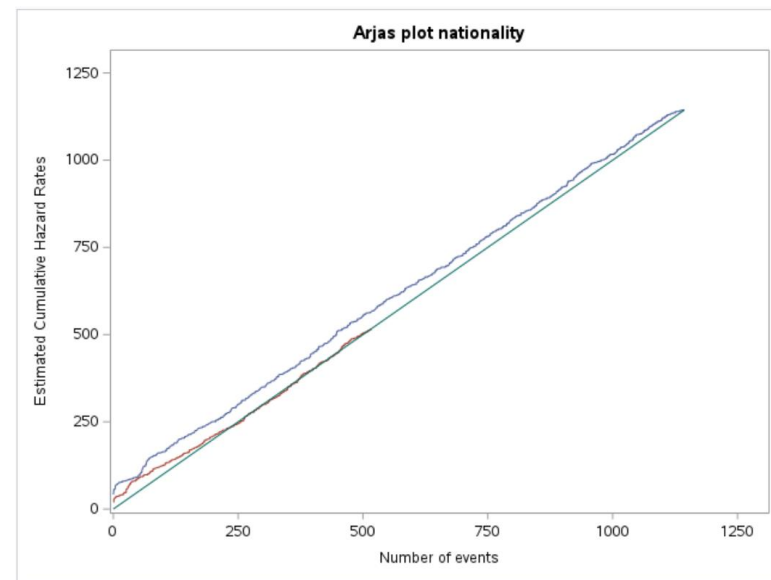
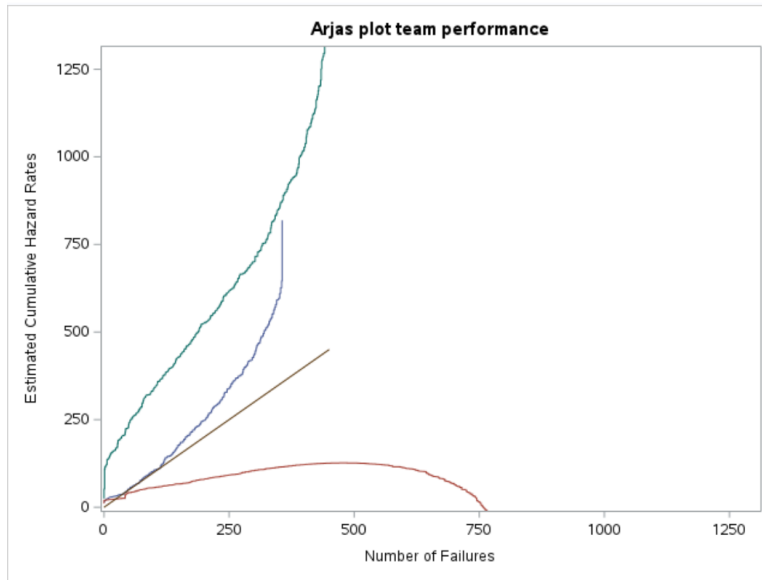


Analysis of Maximum Likelihood Estimates						
Parameter	DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio
nationality (domestic)	1	0.10134	0.14732	0.4732	0.4915	1.107
lnt_nationality (domestic)	1	-0.07178	0.08242	0.7585	0.3838	0.931
age	1	0.14771	0.01443	104.7992	<.0001	1.159
lnt_age	1	-0.05476	0.00797	47.2092	<.0001	0.947

NOTE: "lnt_x" denotes a time-dependent covariate, calculated as $x \cdot \log(\text{time})$

Linear Hypotheses Testing Results			
Test	DF	Wald Chi-Square	Pr > ChiSq
lnt_performance_high= lnt_performance_low = 0	2	56.2997	<.0001
lnt_position_guard= lnt_position_forward = 0	2	0.3140	0.8547
lnt_age_group21_30 = lnt_age_group31_35 = 0	2	49.5938	<.0001

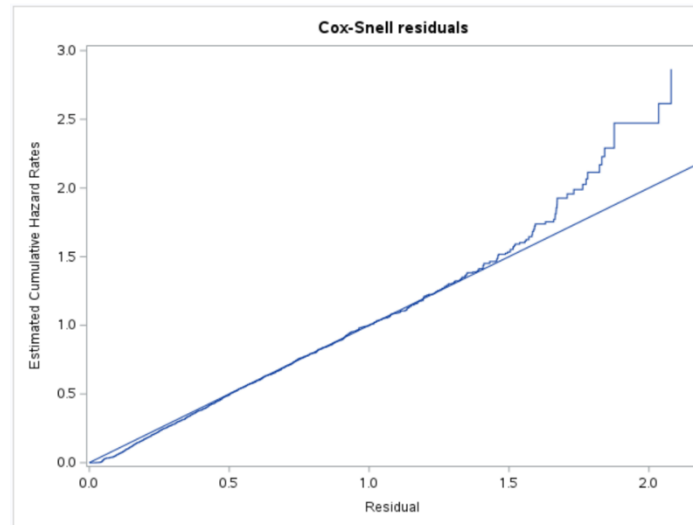
NOTE: Dummy variables have been created above for two of the three categories of the variables *team_performance*, *position*, and *agegroup*.



Model 1: team_performance, nationality, age, position

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	11149.868	10961.096
AIC	11149.868	10973.096
SBC	11149.868	11005.580

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
team_performance	2	46.1762	<.0001
nationality	1	0.0015	0.9695
age	1	138.0854	<.0001
position	2	0.1506	0.9275

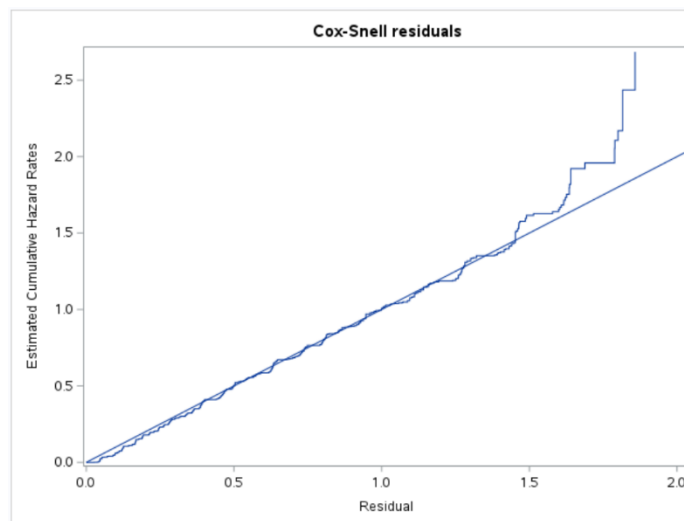


Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
team_performance	High	1	-0.31664	0.05865	29.1474	<.0001	0.729	0.649	0.817
team_performance	Low	1	0.13938	0.06327	4.8535	0.0276	1.150	1.014	1.300
nationality	International	1	0.00203	0.05316	0.0015	0.9695	1.002	0.902	1.111
age		1	0.05498	0.00468	138.0854	<.0001	1.057	1.047	1.066
position	Forward	1	0.00315	0.07839	0.0016	0.9680	1.003	0.862	1.172
position	Guard	1	0.02128	0.07495	0.0806	0.7764	1.022	0.884	1.186

Model 2: team performance, nationality, age group, position

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	11149.868	10969.945
AIC	11149.868	10983.945
SBC	11149.868	11021.843

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
team_performance	2	45.6445	<.0001
nationality	1	0.0263	0.8713
age_group	2	125.6227	<.0001
position	2	0.0882	0.9569

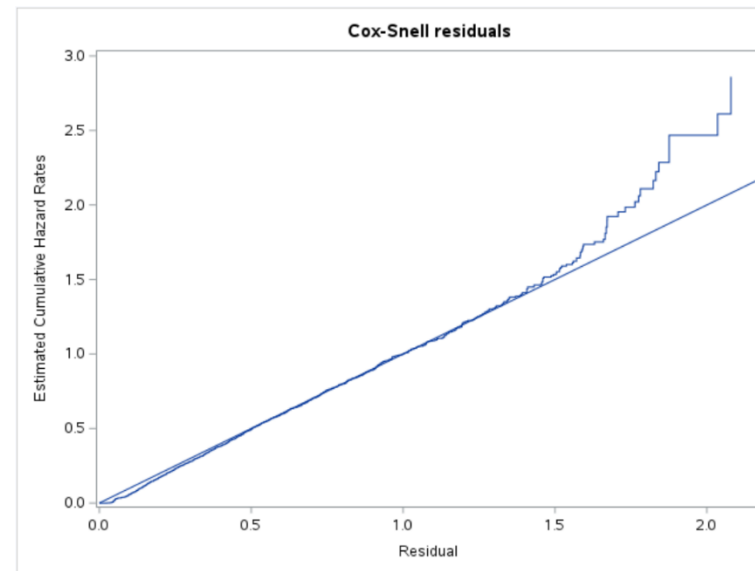


Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
team_performance	High	1	-0.31820	0.05867	29.4174	<.0001	0.727	0.648	0.816
team_performance	Low	1	0.13320	0.06327	4.4319	0.0353	1.142	1.008	1.292
nationality	International	1	-0.00863	0.05326	0.0263	0.8713	0.991	0.893	1.100
age_group	24-30	1	0.45658	0.06163	54.8854	<.0001	1.579	1.400	1.782
age_group	31-35	1	0.70598	0.06343	123.8591	<.0001	2.026	1.789	2.295
position	Forward	1	0.0004507	0.07844	0.0000	0.9954	1.000	0.859	1.169
position	Guard	1	0.01496	0.07506	0.0397	0.8420	1.015	0.878	1.178

Model 3: team performance, nationality, age, lnt age, position

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	11149.868	10912.672
AIC	11149.868	10926.672
SBC	11149.868	10964.570

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
team_performance	2	44.6620	<.0001
nationality	1	0.0057	0.9397
age	1	102.7476	<.0001
lnt_age	1	45.8005	<.0001
position	2	0.2231	0.8945



Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
team_performance	High	1	-0.30962	0.05862	27.9024	<.0001	0.734	0.654	0.823
team_performance	Low	1	0.13945	0.06325	4.8610	0.0275	1.150	1.015	1.300
nationality	International	1	0.00402	0.05316	0.0057	0.9397	1.004	0.904	1.114
age		1	0.14628	0.01443	102.7476	<.0001	1.158	1.126	1.191
lnt_age		1	-0.05401	0.00798	45.8005	<.0001	0.947	0.933	0.962
position	Forward	1	0.00152	0.07839	0.0004	0.9846	1.002	0.860	1.170
position	Guard	1	0.02434	0.07495	0.1055	0.7454	1.025	0.887	1.189

Model 4: team performance, nationality, position, stratified by age group

Summary of the Number of Event and Censored Values					
Stratum	age_group	Total	Event	Censored	Percent Censored
1	18-23	830	457	373	44.94
2	24-30	909	636	273	30.03
3	31-35	761	566	195	25.62
Total		2500	1659	841	33.64

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	10756.945	10707.776
AIC	10756.945	10717.776
SBC	10756.945	10744.846

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
team_performance	2	47.1203	<.0001
nationality	1	0.0297	0.8633
position	2	0.0676	0.9668

age_group=18-23

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	3198.860	3183.646
AIC	3198.860	3193.646
SBC	3198.860	3214.269

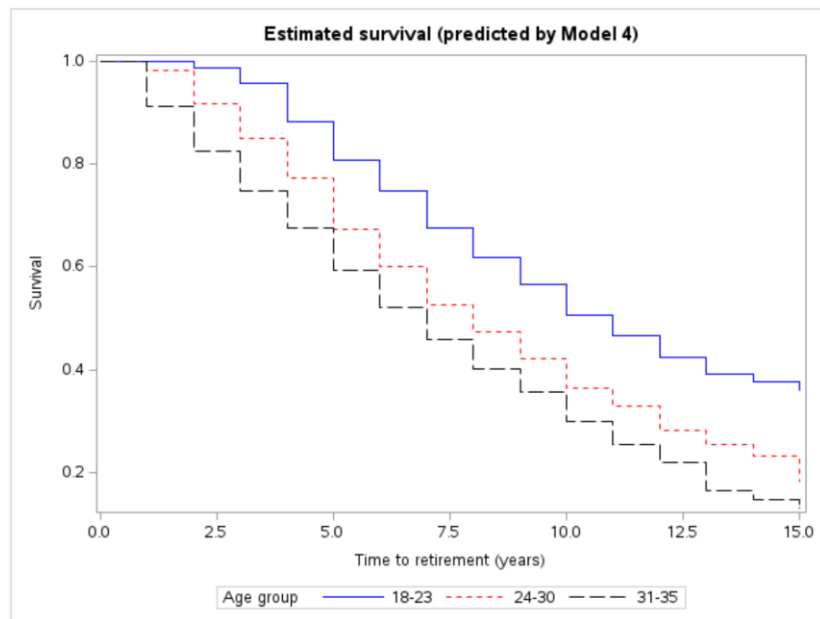
age_group=24-30

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	4080.262	4064.122
AIC	4080.262	4074.122
SBC	4080.262	4096.398

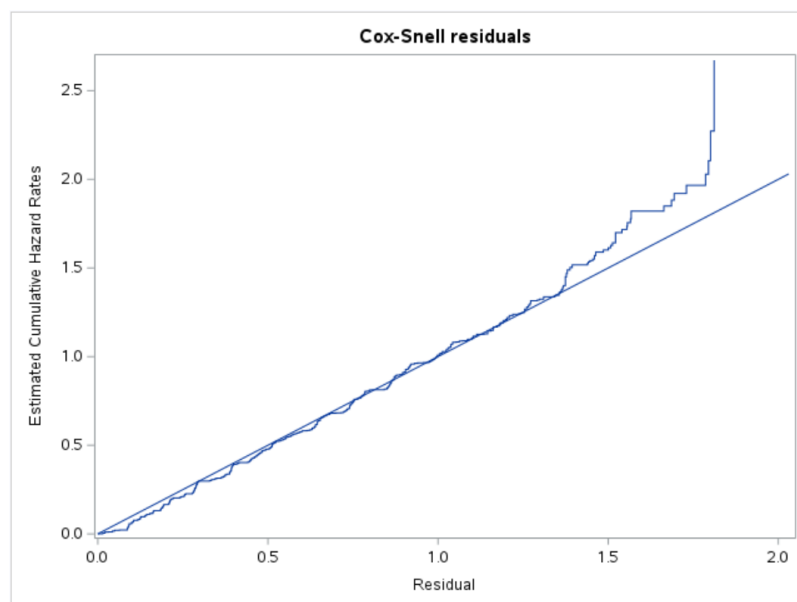
age_group=31-35

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	3477.823	3448.117
AIC	3477.823	3458.117
SBC	3477.823	3479.810

Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
team_performance	High	1	-0.32374	0.05888	30.2335	<.0001	0.723	0.644	0.811
team_performance	Low	1	0.13746	0.06328	4.7180	0.0298	1.147	1.012	1.298
nationality	International	1	-0.00918	0.05329	0.0297	0.8633	0.991	0.892	1.099
position	Forward	1	-0.00819	0.07847	0.0109	0.9168	0.992	0.852	1.159
position	Guard	1	0.00583	0.07509	0.0060	0.9381	1.006	0.870	1.168



The estimated survival above shows the predicted survival for reference value(s) of any categorical variable(s), and mean value(s) of any continuous variable(s). Any time-dependent variable(s) are excluded.



Model 5: nationality, position, age, Int_age, stratified by team performance

Summary of the Number of Event and Censored Values					
Stratum	team_performance	Total	Event	Censored	Percent Censored
1	High	748	450	298	39.84
2	Low	509	356	153	30.06
3	Medium	1243	853	390	31.38
Total		2500	1659	841	33.64

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	10891.471	10702.293
AIC	10891.471	10712.293
SBC	10891.471	10739.363

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
nationality	1	0.0048	0.9448
age	1	103.6251	<.0001
Int_age	1	46.3367	<.0001
position	2	0.1896	0.9095

Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
nationality	International	1	0.00368	0.05318	0.0048	0.9448	1.004	0.904	1.113
age		1	0.14700	0.01444	103.6251	<.0001	1.158	1.126	1.192
Int_age		1	-0.05439	0.00799	46.3367	<.0001	0.947	0.932	0.962
position	Forward	1	0.0003368	0.07842	0.0000	0.9966	1.000	0.859	1.169
position	Guard	1	0.02170	0.07498	0.0837	0.7723	1.022	0.884	1.186

team_performance=High

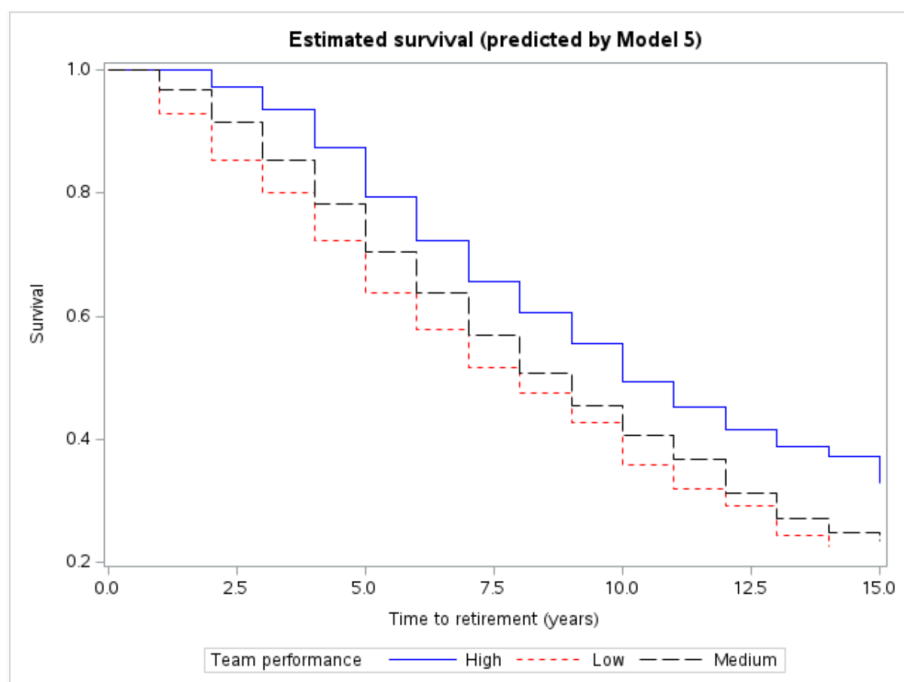
Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	3076.299	3017.477
AIC	3076.299	3027.477
SBC	3076.299	3048.023

team_performance=Low

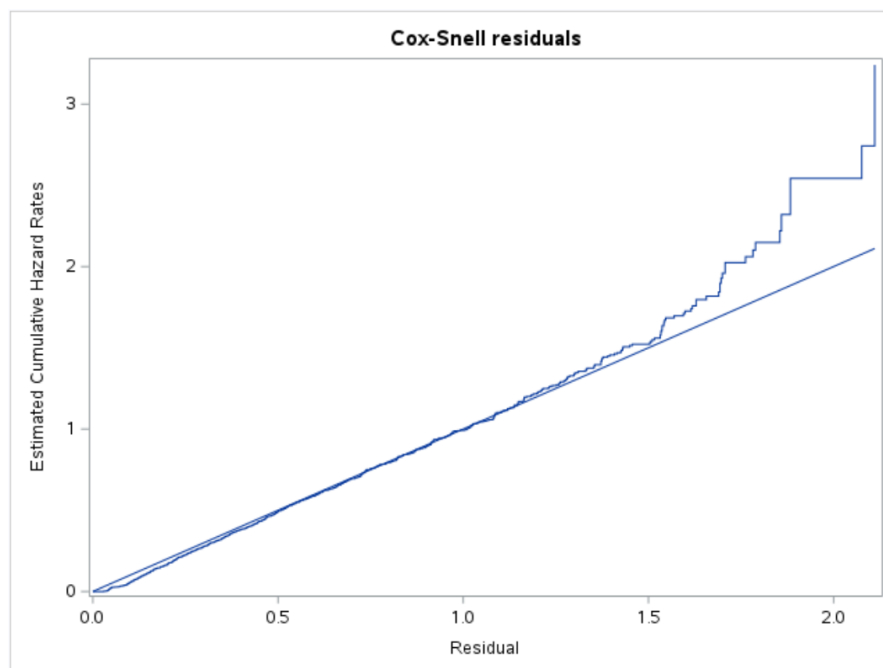
Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	2238.178	2212.392
AIC	2238.178	2222.392
SBC	2238.178	2241.767

team_performance=Medium

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	5576.994	5456.050
AIC	5576.994	5466.050
SBC	5576.994	5489.794



The estimated survival above shows the predicted survival for reference value(s) of any categorical variable(s), and mean value(s) of any continuous variable(s). Any time-dependent variable(s) are excluded.



Model 6: nationality, position, stratified by age group and team performance

Summary of the Number of Event and Censored Values						
Stratum	age_group	team_performance	Total	Event	Censored	Percent Censored
1	18-23	High	273	137	136	49.82
2	18-23	Low	165	101	64	38.79
3	18-23	Medium	392	219	173	44.13
4	24-30	High	258	170	88	34.11
5	24-30	Low	193	138	55	28.50
6	24-30	Medium	458	328	130	28.38
7	31-35	High	217	143	74	34.10
8	31-35	Low	151	117	34	22.52
9	31-35	Medium	393	306	87	22.14
Total			2500	1659	841	33.64

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	10255.114	10255.056
AIC	10255.114	10261.056
SBC	10255.114	10277.298

Type 3 Tests			
Effect	DF	Wald Chi-Square	Pr > ChiSq
nationality	1	0.0267	0.8702
position	2	0.0304	0.9849

Analysis of Maximum Likelihood Estimates									
Parameter		DF	Parameter Estimate	Standard Error	Chi-Square	Pr > ChiSq	Hazard Ratio	95% Hazard Ratio Profile Likelihood Confidence Limits	
nationality	International	1	-0.00872	0.05338	0.0267	0.8702	0.991	0.892	1.100
position	Forward	1	-0.00453	0.07874	0.0033	0.9541	0.995	0.855	1.164
position	Guard	1	0.00490	0.07527	0.0042	0.9480	1.005	0.869	1.167

age_group=18-23 team_performance=High

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	941.650	939.276
AIC	941.650	945.276
SBC	941.650	954.036

age_group=18-23 team_performance=Low

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	635.488	634.057
AIC	635.488	640.057
SBC	635.488	647.902

age_group=18-23 team_performance=Medium

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	1461.522	1459.442
AIC	1461.522	1465.442
SBC	1461.522	1475.609

age_group=24-30 team_performance=High

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	1068.236	1064.305
AIC	1068.236	1070.305
SBC	1068.236	1079.712

age_group=24-30 team_performance=Low

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	825.683	824.604
AIC	825.683	830.604
SBC	825.683	839.385

age_group=24-30 team_performance=Medium

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	2012.467	2006.674
AIC	2012.467	2012.674
SBC	2012.467	2024.053

age_group=31-35 team_performance=High

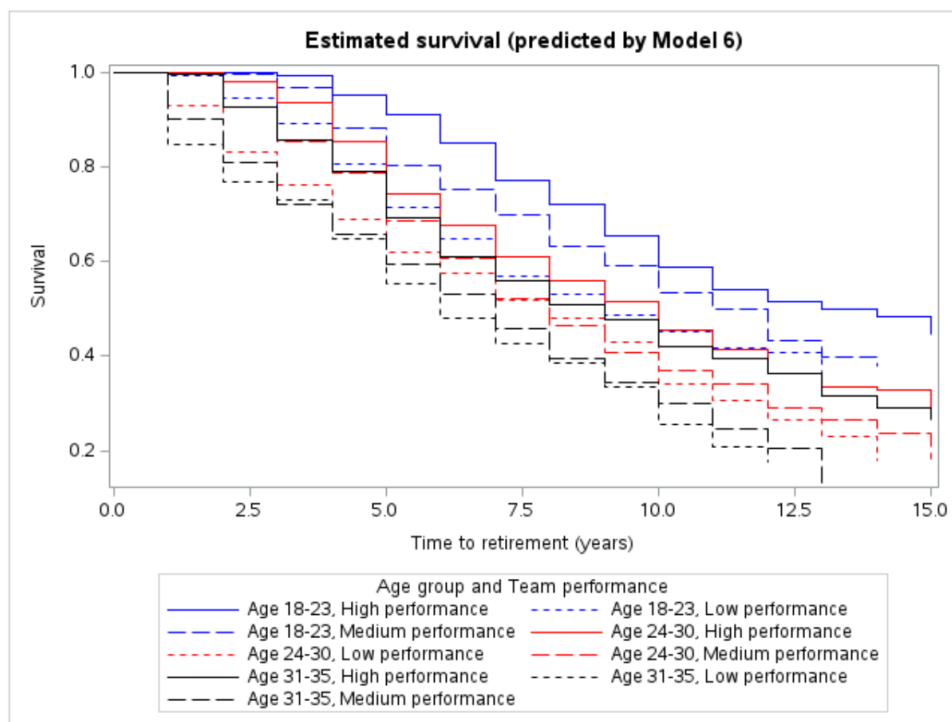
Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	891.908	889.155
AIC	891.908	895.155
SBC	891.908	904.044

age_group=31-35 team_performance=Low

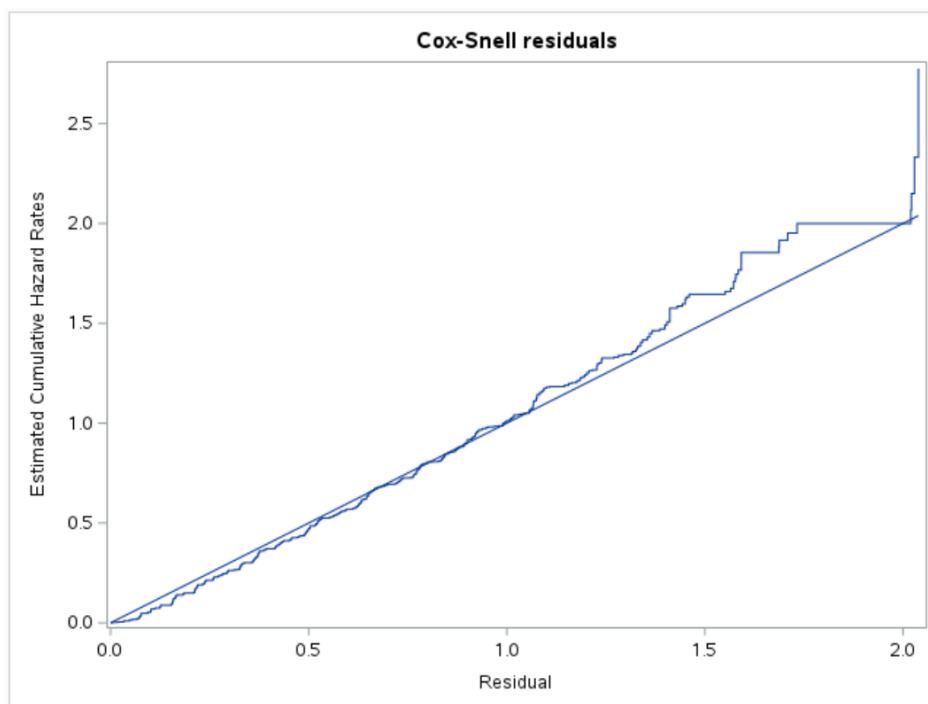
Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	635.569	626.143
AIC	635.569	632.143
SBC	635.569	640.430

age_group=31-35 team_performance=Medium

Model Fit Statistics		
Criterion	Without Covariates	With Covariates
-2 LOG L	1782.591	1779.885
AIC	1782.591	1785.885
SBC	1782.591	1797.055



The estimated survival above shows the predicted survival for reference value(s) of any categorical variable(s), and mean value(s) of any continuous variable(s). Any time-dependent variable(s) are excluded.



END OF TASK 4

(4) Task 5

You are working your first job as a statistician at a streaming company.

You are analyzing data using Cox regression, where the time-to-event variable is subscription time.

During your work on the data analysis, you discover that one covariate violates the proportional hazards assumption, and you ask your employer for more time to investigate the best way to handle this issue. However, your employer is under pressure to present the results of your analysis at the next board meeting and instructs you to ignore the violation.

What do you do?

What would be the ethically correct thing to do? Refer to relevant part(s) of a code of ethics document that would be applicable in this situation.

TABLE C.2*Upper Percentiles of a Chi-Square Distribution*

<i>Degrees of Freedom</i>	<i>Upper Percentile</i>				
	<i>0.1</i>	<i>0.05</i>	<i>0.01</i>	<i>0.005</i>	<i>0.001</i>
1	2.70554	3.84146	6.63489	7.87940	10.82736
2	4.60518	5.99148	9.21035	10.59653	13.81500
3	6.25139	7.81472	11.34488	12.83807	16.26596
4	7.77943	9.48773	13.27670	14.86017	18.46623
5	9.23635	11.07048	15.08632	16.74965	20.51465
6	10.64464	12.59158	16.81187	18.54751	22.45748
7	12.01703	14.06713	18.47532	20.27774	24.32130
8	13.36156	15.50731	20.09016	21.95486	26.12393
9	14.68366	16.91896	21.66605	23.58927	27.87673
10	15.98717	18.30703	23.20929	25.18805	29.58789
11	17.27501	19.67515	24.72502	26.75686	31.26351
12	18.54934	21.02606	26.21696	28.29966	32.90923
13	19.81193	22.36203	27.68818	29.81932	34.52737
14	21.06414	23.68478	29.14116	31.31943	36.12387
15	22.30712	24.99580	30.57795	32.80149	37.69777
16	23.54182	26.29622	31.99986	34.26705	39.25178
17	24.76903	27.58710	33.40872	35.71838	40.79111
18	25.98942	28.86932	34.80524	37.15639	42.31195
19	27.20356	30.14351	36.19077	38.58212	43.81936
20	28.41197	31.41042	37.56627	39.99686	45.31422
21	29.61509	32.67056	38.93223	41.40094	46.79627
22	30.81329	33.92446	40.28945	42.79566	48.26762
23	32.00689	35.17246	41.63833	44.18139	49.72764
24	33.19624	36.41503	42.97978	45.55836	51.17897
25	34.38158	37.65249	44.31401	46.92797	52.61874
26	35.56316	38.88513	45.64164	48.28978	54.05114
27	36.74123	40.11327	46.96284	49.64504	55.47508
28	37.91591	41.33715	48.27817	50.99356	56.89176
29	39.08748	42.55695	49.58783	52.33550	58.30064
30	40.25602	43.77295	50.89218	53.67187	59.70221
31	41.42175	44.98534	52.19135	55.00248	61.09799
32	42.58473	46.19424	53.48566	56.32799	62.48728
33	43.74518	47.39990	54.77545	57.64831	63.86936
34	44.90316	48.60236	56.06085	58.96371	65.24710
35	46.05877	49.80183	57.34199	60.27459	66.61917
36	47.21217	50.99848	58.61915	61.58107	67.98495
37	48.36339	52.19229	59.89256	62.88317	69.34759
38	49.51258	53.38351	61.16202	64.18123	70.70393
39	50.65978	54.57224	62.42809	65.47532	72.05504