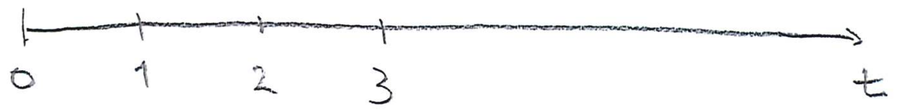
Lecture 8

6 Portfolio dynamics

Let the time axis be discrete



Def 6.1

N = # of different assets

S_n^i = the price of one unit of asset i at time n

h_n^i = # of units of asset i bought at time n.

$h_n = (h_n^1, h_n^2, \dots, h_n^N)$ is a portfolio

V_n = the value of a portfolio h_n at time n.

$$= \sum_{i=1}^{N} h_n^i S_n^i = h_n \circ S_n$$

vector mult

Interpretation:

1. At time $n-$ we have an old portfolio h_{n-1} from the previous period.
2. At time n, S_n becomes observable
3. At time n, after observing S_n, we choose h_n.

Budget equation: $h_n \circ S_{n+1} = h_{n+1} \circ S_{n+1}$

Notation: If $\{x_n\}_{n=0}^{\infty}$ is a sequence of real numbers, let $\Delta x_n := x_{n+1} - x_n$.

The budget equation becomes

$$S_{n+1} \cdot \Delta h_n = 0$$

← Backward increments!

Do not take limits $\Delta t \to 0$ yet!

Recall $V_n = h_n \cdot S_n$.

Since $\Delta V_n = h_{n+1} \cdot S_{n+1} - h_n \cdot S_n = h_{n+1} \cdot S_{n+1} - h_n \cdot S_{n+1} + h_n \cdot S_{n+1} - h_n \cdot S_n$

we have $\quad = S_{n+1} \cdot \Delta h_n + h_n \cdot \Delta S_n$

$\Delta V_n = h_n \cdot \Delta S_n$ if the budget equation is fulfilled.

Below we use this relation to <u>define</u> what is meant by a self-financing portfolio in continuous time.

Def 6.10 Let $\{S_t, t \geq 0\}$ be an N-dim price process.

1. A portfolio h is an $\mathcal{F}^S$-adapted N-dim. process.
2. h is Markovian if $h_t = h(t, S_t)$ for some function h.
3. The value process V^h of h is

$$V_t^h = \sum_{i=1}^{N} h_t^i S_t^i = h_t \cdot S_t$$

4. A portfolio h is self-financing if

$$dV_t^h = h_t \cdot dS_t$$

5. For a given portfolio h, the corresponding relative portfolio w is

$$w_t^i = \frac{h_t^i S_t^i}{V_t^h} \qquad i = 1, \ldots, N$$

Note that $\sum_{i=1}^{N} w_t^i = 1$.

Also, h is self-financing if and only if

$$dV_t^h = V_t^h \sum_{i=1}^{N} \frac{w_t^i}{S_t^i} dS_t^i .$$