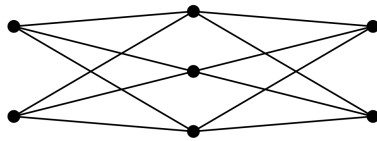


Sample exam questions

Johanna Strömberg
Uppsala Universitet

1. (Hamiltonicity)

- (a) Define *Hamilton cycle* and state Dirac's theorem. (2)
- (b) Prove that if G has a Hamilton cycle, then for each nonempty set $S \subseteq V(G)$, the graph $G \setminus S$ has at most $|S|$ components. (3)
- (c) Is the graph below Hamiltonian? (2)



- (d) Prove that the Cartesian product of two Hamiltonian graphs is Hamiltonian. Conclude that the hypercube, Q_k , is Hamiltonian for $k \geq 2$. (3)

2. (Graph Colourings)

- (a) Define the *chromatic number* of a graph G . (1)
- (b) Prove that the Petersen graph has chromatic number 3. (3)
- (c) The greedy colouring algorithm starts with an ordered list of the vertices and assigns to vertex v_i the lowest index colour not already used on one of its lower-indexed neighbours. Show that for any graph G , there is an ordering of the vertices so that the greedy algorithm uses exactly $\chi(G)$ colours. (3)
- (d) Prove that the chromatic number of a Cartesian product of two graphs G and H is the maximum of the chromatic numbers of G and H . (3)