

## Markov Processes, Spring semester 2024

**Teacher:**

Örjan Stenflo, office 74108 Ångström.

**Literature:**

Gregory F. Lawler, Introduction to Stochastic Processes, Second edition, Chapman and Hall, 2006. (Ch. 1–3, 7.1–7.3, 8.1–8.3, 8.8)

Lecture notes with additional material and exercises will be available in Studium.

Other useful references:

P. Bremaud, Markov Chains. Gibbs fields, Monte Carlo simulation, and Queues, Second Edition, Springer 2020.

G.R. Grimmett, D.R. Stirzaker, Probability and Random Processes, Fourth edition, Oxford University Press, 2020.

O. Häggström, Finite Markov Chains and Algorithmic Applications, Cambridge University Press, 2002.

J. Norris, Markov Chains, Cambridge University Press, 1997.

D. Stirzaker, Stochastic Processes Models, Oxford University Press, 2005.

**Examination:** Written exam (max 35 points)+ project (max 5 points)+ assignments (max 3 bonuspoints).

**Written exam:** March 13, 2024

June 11 2024 (resit) (preliminary date)

August 2024 (resit)

**Project:** Short survey paper (4-6 pages) aimed at fellow students on a topic (free of choice) within the theory of Markov chains. The paper should be submitted electronically (in pdf form) no later than February 28 and should also be presented in class in a talk (on March 5 or March 6). The project gives up to a total of 5 points (paper 2p, presentation 2p, attending all presentations 1p) added to the result of the written exam on March 13. Group work (2-3 persons/group). Deadline for selecting topic: February 22.

**Assignments:** Three sets of assignment problems will be handed out during the course. Non-compulsory. Assignments will be marked and give up to a total of 3

bonuspoints added to the result of the written exam and project. (Bonus points are only valid at the first exam on March 13.)

| Assignments  | Out    | Due    |
|--------------|--------|--------|
| Assignment 1 | Jan 23 | Jan 31 |
| Assignment 2 | Feb 5  | Feb 14 |
| Assignment 3 | Feb 19 | Feb 27 |

For grade 5 the requirement is a total of at least 32 points, for grade 4 at least 25 points and the limit to pass (grade 3) is a total of 18 points.

**Course information:** Course plan, lecture notes, assignments, exercises etc. can be downloaded from Studium:

<https://login.studium.uu.se/>

### Exercises and Problems

“Basic exercises” and “Extra problems” can be downloaded from Studium.

See the lecture notes for suggested exercises and problems from Lawler’s book.

## Preliminary plan

L: Lecture. E: Exercises.

| Moment    | Date        | Contents (with suggested basic exercises and extra problems)   |
|-----------|-------------|--|
| L1        | 15/1        | Introduction, Stochastic processes   |
| L2        | 17/1        | Markov chains in discrete time, examples, Chapman-Kolmogorov equations. Basic exercises: 1–7.              |
| L3        | 18/1        | Classification and structure. Extra problems: A1, A2.  |
| L4        | 19/1        | First-step analysis and hitting times. Basic exercises: 11,14, Extra problems: a2, A2, A4, B1.             |
| L5        | 22/1        | Markov chains in the long run, Stationary distributions. Basic exercises: 8-10, 12-16, Extra problems: a1. |
| L6        | 23/1        | Convergence theorem, Branching processes. Extra problems: a3.  |
| L7        | 24/1        | Simulation of Markov chains, fractals. Basic exercises: 17.  |
| <b>E1</b> | <b>24/1</b> | <b>Exercises</b>   |
| L8        | 29/1        | Reversible Markov chains, Markov chain Monte Carlo. Extra problems: b2.                                    |
| L9        | 31/1        | More on MCMC methods, Propp-Wilson's perfect sampling method, Extra problems: b1.                          |
| L10       | 1/2         | Continuous time Markov chains, Basic exercises: 18, 20   |
| L11       | 5/2         | Forwards and backwards equations, examples, Basic exercises: 19, Extra problems: c2.                       |
| <b>E2</b> | <b>5/2</b>  | <b>Exercises</b>   |
| L12       | 7/2         | Classification. Basic exercises: 21-22, 27, Extra problems: b3, B3, B4,                                    |
| L13       | 12/2        | Birth and death processes, Basic exercises: 23-26, 28, Extra problems: c1, B2.                             |
| L14       | 15/2        | Applications   |
| <b>E3</b> | <b>15/2</b> | <b>Exercises</b>   |
| L15       | 19/2        | Introduction to Brownian motion, Basic exercises: 29-32, Extra problems: c3.                               |
| L16       | 21/2        | Forward and Backward equations, Diffusion processes  |
| L17       | 22/2        | Brownian bridge, geometric Brownian motion, Reflection principle   |
| <b>E4</b> | <b>22/2</b> | <b>Exercises</b>   |
| L18       | 4/3         | Repetition   |
| L19       | 5/3         | Project presentations  |
| L20       | 6/3         | Project presentations  |
| L21       | 8/3         | Repetition   |