UPPSALA UNIVERSITY
Dept of Mathematics
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Matematical statistics
Inference II 1MS037
2023–11–15

Hand-in assignment 1

There are three compulsory home assignments. You may hand in the assignments in groups of two students. To pass the assignment, 10p are needed. Solutions should be well motivated. Hand in your solutions via Studium *as a single pdf file*. If you have worked together with somebody else, it is enough to hand in for one of you as long as you write both names.

The deadline for this assignment is November 22.

1. Consider the discrete random variable X with probability function

$$P(X=k)=\begin{cases} 9\theta_1^2, & \text{if } k=0,\\ 6\theta_1\theta_2, & \text{if } k=1,\\ \theta_2^2, & \text{if } k=2,\\ 0, & \text{otherwise,}\end{cases}$$

where $3\theta_1+\theta_2=1$.

Consider an independent sample $\mathbf{X}=(X_1,...,X_n)$ where all X_i, $i=1,2,...,n$, are distributed as X.

   (a) Does the distribution belong to a strictly k-parametric exponential family? (3p)

   (b) Derive k and the corresponding sufficient statistic(s). (2p)

2. Consider a random sample $\mathbf{X}=(X_1,...,X_n)$ where the X_i are independent continuous random variables with density function

$$f(x)=\frac{4}{\theta}x^3\exp\left(-\frac{x^4}{\theta}\right),$$

for $x\geq 0$ and 0 otherwise.

   (a) Calculate the score function. (2p)

   (b) Calculate the Fisher information. (3p)

   Hint: Without proof, you may use that $E(X^k)=\theta^{k/4}\Gamma\left(1+\frac{k}{4}\right)$, for $k=1,2,...$, where $\Gamma(\cdot)$ is the Gamma function.

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3. Consider a random sample $\mathbf{X} = (X_1, ..., X_n)$ where the X_i are i.i.d. and discrete with probability function

$$p(x;\theta) = \theta(1-\theta)^{x-1},\ x = 1, 2, ...,$$

where $0 \leq \theta \leq 1$.

(a) Show that the statistic $T = (X_1, ..., X_{n-1})$ is *not* sufficient for θ. (3p)

(b) Find a sufficient statistic for θ. (2p)

4. Consider a random sample $\mathbf{X} = (X_1, ..., X_n)$ where the X_i are i.i.d. and continuous random variables with density function

$$f(x) = \theta x^{-2} \exp(-\theta x^{-1}),$$

for $x \geq 0$, $\theta > 0$, and $f(x) = 0$ for $x < 0$.

Find a minimal sufficient statistic for θ. (5p)