Financial Theory – Lecture 9

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Agenda

- Data from financial markets.
- Some financial econometrics.
- Efficient markets.
- Behavioural finance.

The lecture is based on

- Chapter 12 in the course book.
- Some additional literature.

Prices on financial markets

We use price data to create a return time series.

I would recommend that you use prices as your raw data and then calculate the rate of return or the log-return.

The data source I use is Refinitiv Eikon. This is available through the library at Ekonomikum.

There are introductory courses via Zoom and you can also ask the library for a one-on-one introduction to Refinitiv Eikon at the computer in the library.

There is also data available from Swedish House of Finance.

Prices on financial markets

An exchange is usually using a limit order book.

There is a bid and an ask price.

The bid price is the highest price a buyer want to buy for, and the ask price is the lowest price a seller wants to sell for.

Orderdjup

Antal	Köp	Sälj	Antal
34	**76,52**	**76,53**	1 125
1 635	76,51	76,54	3 596
1 510	76,50	76,55	1 849
4 489	76,49	76,56	7 469
4 726	76,48	76,57	8 170

$$\text{Bid price} = 76.52. \ \text{Ask price} = 76.53.$$
$$\text{Bid-ask spread} = 76.53 - 76.52 = 0.01.$$

(Market depth for Ericsson B, 29 August 2022 from Avanza.)

Prices on financial markets

On and after a trading day the following prices are collected:

- Closing: The last price during the day at which a trade was made.
- Opening: The first price during the day at which a trade was made.
- High: The highest price during the day at which a trade was made.
- Low: The lowest price during the day at which a trade was made.

Both academics and practitioners are also interested in how large the volume, i.e. the number of shares that was traded, is in a transaction.

The VWAP is the volume-weighted average price.

Estimating the risk premia

Start with a K-factor model:

$$r_i = E\left[r_i\right] + \beta_{i1}\big(F_1 - E\left[F_1\right]\big) + \ldots + \beta_{iK}\big(F_K - E\left[F_K\right]\big) + \varepsilon_i$$

and $\mathrm{Cov}[F_k, \varepsilon_i] = 0$ for every k and i, and $\mathrm{Cov}[\varepsilon_i, \varepsilon_j] = 0$ when $i \neq j$.

We then add the condition of no arbitrage to the model $\longrightarrow$ the APT equation

$$E\left[r_i\right] = \mathrm{RP}_0 + \beta_{i1}\mathrm{RP}_1 + \ldots + \beta_{iK}\mathrm{RP}_K,$$

with $\mathrm{RP}_0 = r_f$ is there is a risk-free asset.

There are two standard ways of estimating the risk premia:

1) The two-stage or cross-sectional approach.
2) The Fama-MacBeth approach.

Estimating the risk premia

Assume that data r_{it} are given for $i = 1, 2, \ldots, N$ assets and for $t = 1, 2, \ldots, T$ times.

For ease of exposition I assume that the only factor is the rate of return on the market portfolio, i.e. we want to estimate the risk premium in the Single-Index Model.

In both approaches we start by using the data and for each asset i estimate the β_{im}'s by running the regressions

$$r_{it} = a_i + \beta_{im} r_{mt} + \varepsilon_{it}.$$

This leads to estimated values on the beta's: $\hat{\beta}_{im}$, $i = 1, \ldots, N$.

Estimating the risk premia

In the two-step approach we treat the estimated betas are the true betas and run the regression

$$\bar{r}_i = \mathrm{RP}_0 + \mathrm{RP}_m \hat{\beta}_{im} + \eta_i,$$

where

$$\bar{r}_i = \frac{1}{T} \sum_{t=1}^{T} r_{it}$$

and η_i is an error term.

This gives the estimates $\widehat{\mathrm{RP}}_0$ and $\widehat{\mathrm{RP}}_m$.

Estimating the risk premia

In the Fama-MacBeth approach we again treat the estimated betas as the true betas, but now we run the regressions

$$r_{it} = \mathrm{RP}_{0t} + \mathrm{RP}_{mt}\hat{\beta}_{im} + \eta_{it}$$

for $t = 1, 2, \ldots, T$.

This gives the estimates $\widehat{\mathrm{RP}}_{0t}$ and $\widehat{\mathrm{RP}}_{mt}$.

We then get the estimates of the risk premia from

$$\widehat{\mathrm{RP}}_0 = \frac{1}{T}\sum_{t=0}^{T}\widehat{\mathrm{RP}}_{0t} \quad \text{and} \quad \widehat{\mathrm{RP}}_m = \frac{1}{T}\sum_{t=0}^{T}\widehat{\mathrm{RP}}_{mt}.$$

Estimating the risk premia

If there is a risk-free rate, then we estimate the betas using the time series, but regressions in the second step are

$$\bar{r}_i - r_f = \mathrm{RP}_m \hat{\beta}_{im} + \eta_i$$

and

$$r_{it} - r_f = \mathrm{RP}_{mt} \hat{\beta}_{im} + \eta_{it}$$

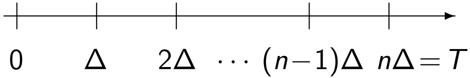
for $t = 1, 2, \ldots, T$ respectively.

Note that there is no constant in these regressions.

Estimating the mean and the variance of returns

The difference between estimation of the mean and the variance of a time series was shown in the following example in Merton (1980).

The total time length T is divided into n parts each length Δ:



One example is $T = 10$ years and $\Delta = 1$ month.

We model the return r_t over period t for $t = 1, 2, \ldots, n$ as

$$r_t = \mu\Delta + \sigma\sqrt{\Delta}\,\varepsilon_t,$$

where each $\varepsilon_t \sim N(0, 1)$ and independent of each other.

Estimating the mean and the variance of returns

In this model each r_t is independent of the others.

Furthermore

$$r_t \sim N(\mu\Delta, \sigma^2\Delta).$$

This is a discrete time version of a Brownian motion (or Wiener process) with drift.

Estimating the mean and the variance of returns

To estimate μ we use the estimator

$$\hat{\mu} = \frac{1}{T}\sum_{t=1}^{n} r_t = \frac{1}{\Delta}\cdot\frac{1}{n}\sum_{t=1}^{n} r_t.$$

This is an unbiased estimator,

$$E\left[\hat{\mu}\right] = \frac{1}{T}\sum_{t=1}^{n} E\left[r_t\right] = \frac{1}{n\Delta}\cdot n\cdot\mu\Delta = \mu,$$

with variance

$$\mathrm{Var}[\hat{\mu}] = \mathrm{Var}\left[\frac{1}{T}\sum_{t=1}^{n} r_t\right] = \frac{1}{T^2}\sum_{t=1}^{n}\mathrm{Var}[r_t] = \frac{1}{T^2}\cdot n\cdot\sigma^2\Delta = \frac{\sigma^2}{T}.$$

Estimating the mean and the variance of returns

For σ^2 we use the estimator

$$\widehat{\sigma^2} = \frac{1}{T}\sum_{t=1}^{n} r_t^2.$$

This estimator is not unbiased:

$$\begin{aligned}
E\left[\widehat{\sigma^2}\right] &= \frac{1}{T}\sum_{t=1}^{n} E\left[r_t^2\right] \\
&= \frac{1}{T}\sum_{t=1}^{n}\left(\mathrm{Var}[r_t] + E\left[r_t\right]^2\right) \\
&= \frac{1}{T}\cdot n\left(\sigma^2\Delta + \mu^2\Delta^2\right) \\
&= \sigma^2 + \mu^2\Delta = \sigma^2 + \mu^2\frac{T}{n}.
\end{aligned}$$

Estimating the mean and the variance of returns

Merton prefers this estimator to

$$\frac{1}{T}\sum_{t=1}^{n}\left(r_t - \bar{r}\right)^2$$

because you do not need to estimate the mean $\bar{r}$ in order to find its value..

Also note that

$$E\left[\widehat{\sigma^2}\right] = \sigma^2 + \mu^2\frac{T}{n} \to \sigma^2$$

as $n \to \infty$.

Estimating the mean and the variance of returns

One can show that

$$\mathrm{Var}\left[\widehat{\sigma^2}\right] = \frac{2\sigma^2}{n} + \frac{4\mu^2 T}{n^2}.$$

Recall that

$$\mathrm{Var}\left[\hat{\mu}\right] = \frac{\sigma^2}{T}.$$

If we want a better precision (=lower variance) in the estimtion of σ we can inrease the sampling frequence (increase n).

If we want a better precision (=lower variance) in the estimtion of μ we must inrease the sampling period (increase T), i.e. collect more data.

Efficiency

We now turn to the question of how informationally efficient financial markets are.

The efficient market hypothesis (EMH) states that

> *The prices in a financial market fully reflect all available information.*

In this definition, what is meant by all avilable information?

To be more precise, there are three different versions of efficiency.

Efficiency

Some inital comments.

- Getting information is costly. It takes time to collect information and it can be costly to obtain it.
- We expect that information that is cheap to acquire is probably known to more investors than information that is costly to acquire.
- In an efficient market prices move only when new information arrives.
- When testing for efficiency you have the "joint hypothesis problem": You have to also have a model for expected returns.

Weak-form efficiency

Definition:

- All information in historical trading such as prices and volumes is fully reflected in today's prices.

One concequence of this is that technical analysis of asset prices is useless.

We test this type of EMH by looking at if it is possible to predict returns by using historical trading data.

This type of data can typically be obtained at a low cost.

Semistrong-form efficiency

Semistrong-form efficiency

Definition:

- All historical publically availabe information is fully reflected in today's prices.

This means that we should also include quarterly and yearly reports from companies, announcements (such as a profit warning and the sacking of a CEO) and other relevant public news.

To test this type of efficiency event studies can be used. There are also alternative ways of testing for the semistrong form.

Strong-form efficiency

Strong-form efficiency

Definition:

- All historical informaton, public and private, is fully reflected in today's prices.

One concequence of this is that there is no extra gain for traders with inside information.

This is indeed a strong assumption.

Efficiency today

Nowadays most economicst (but not all!) agree that markets are efficient to some degree, but that it is not prefectly true.

Since there are many asset managers making money by collecting and processing information it is reasonable to take the stand that markets are enough inefficient for them to be compensated for the cost they have colleting and processing information.

This has led Pedersen to use the phrase "efficiently inefficient".

Empirical results

There is a difference in what the researchers knew in the 1970's, and what they knew in the 2000's.

The following two slides are based on p. 389-391 in Cochrane (2005).

Empirical results

In the 1970's

- CAPM explains why some assets have higher average returns than others.
- Stock returns are close to unpredictable.
- Professional asset managers do not outperform passive portfolios when corrected for risk.

Empirical results

In the 2000's

- There are assets whose expected return cannot be explained by their beta values with respect to the market return.
- Stock returns are predictable over longer time horizons, but close to unpredictable measured daily, weekly or monthly.
- Some fund managers seem to outperform simple indexes, even after corrected for risk.

Normative and positive results

So far, we have mainly taken a normative view.

One example is the mean-variance approach to investments: Given the vector of mean returns and the covariance matrix, we calculate the optimal weights.

But is this how investors behave?

That is, what are the positive results of financial economics?

Behavioural finance and economics

There are a number of empirical facts about how individuals and investors behave that seems to be at odds with assumptions and theory.

They are known as biases, and are studied in behavioural finance and behavioural economics.

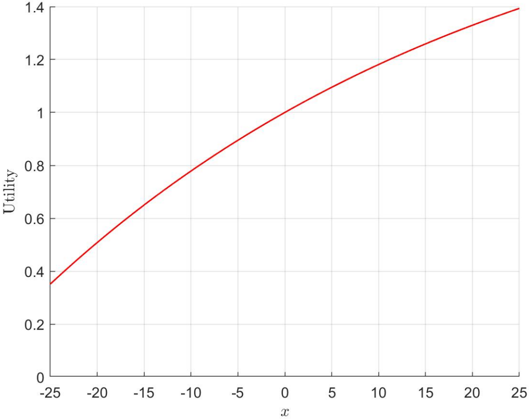
The Allais paradox

Already in 1953, Maurice Allais in "Le comportement de l'homme rationnel devant le risque, critique des postulats et axiomes de l'école Américaine" raised a critique against the axioms of expected utility theory.

Allais argued that individuals do not behave as predicted by expected utility theory.

Prospect theory

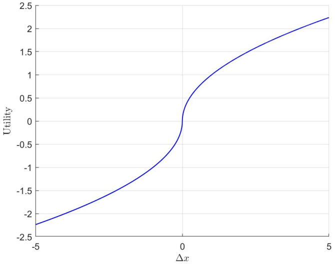
Recall the notion of utility over monetary outcomes.



Prospect theory

A refinement of expected utility theory is prospect theory.

- Utility is measured relative to a reference point.



- The utility function over changes in monetary values is concave in gains and convex in losses.
- Probabilities are weighted using a weighting function.

The extension cumulative prospect theory has better theoretical properties.

Behavioural biases

Anchoring
Individuals and investors anchor their views.

Example
When estimating the expected return of a new asset, expected returns of existing assets might influence the estimation fo this new asset.

Loss aversion or Disposition effect
There is an aversion in realising losses.

Example
Investments that has decreased in value are hard to sell for an investor.

Behavioural biases

Overconfidence

Individuals tend to too confident in their views.

Example

Investors are too sure about their subjective views on estimates, and their own ability in general.

Mental accounting

Different attitudes to risk for different parts of an investor's portfolio. The portfolio is not considered as a whole

Example

Gains made can be considered as less problematic to lose than the money that made up the original investment.

Behavioural biases

Framing

The answer to a question is dependent of how it is framed.

Example

In finance, an example is if an investment opportunity is described using possible gains or possible losses.

Behavioral game theory

In some cases the experimental outcomes of games seems to contradict rational behavior.

A standard example is the ultimatum game.

Another interesting example is the dictator game.

An example from auctions is the winner's curse: In a common value auction (where the item being auctioned has the same value for all bidders) the winning bid exceeds the value of the item.

Answers to the critique from behavioural finance

Financial economists such as Stephen A. Ross and John H. Cochrane have contested the claims of behavioural finance.

> *Note that to obtain a significant effect on equilibrium prices, such behavioural biases have to be systematic across individuals.*

From Munk (2013).

References

[1] Cochrane, J. H. (2005), "Asset Pricing", revised edition, *Princeton University Press*.

[2] Merton, R. C. (1980), "On Estimating the Expected Return on the Market: An Explanatory investigation", *Journal of Financial Economics*, pp. 323-361.

[3] Munk, C. (2013), "Financial Asset Pricing Theory", *Oxford University Press*.