L Viitasaari

8. Surface integrals and curve integrals of vector fields

We solve the problems together in the exercise sessions. Note that these problems are optional and for learning purposes: solving these does not provide extra points. Actual home assignments (giving you extra points) are given separately.

It is advised to take a look of the problems beforehand. Note that some of the problems might be very challenging, so do not feel bad if you are unable to solve them independently: we will go through the solutions together!

Problems for the session

8.1 Compute the area of $x^2+y^2+z^2=R^2$, $z \geq h$ with $0 \leq h \leq R$ (problem 8.30d from the book).

8.2 Compute the curve integral $\int_\gamma y \log \frac{x^2}{y} dx - \frac{x}{y} dy$, where γ is given by $y = x^2$ from $(1,1)$ to $(2,4)$ (problem 9.4 from the book).

Problems for individual practice

In addition to the problems below, one can get routine by solving similar exercises from the exercise-book "övningar i flerdimensionell analys".

8.1 Compute the area of the cylinder $x^2+y^2=4$, $0 \leq z \leq 3$ (problem 8.29c from the book).

8.2 Compute the area of $x^2+y^2+z^2=4, z \geq 1$ (problem 8.30c from the book).

8.3 Compute the area of the torus $(x,y,z) = ((2-\cos t)\cos s, (2-\cos t)\sin s, \sin t)$, $-\pi \leq s, t \leq \pi$ (problem 8.31 from the book).

8.4 Compute the curve integral $\int_\gamma y dx - dy$ where

(a) γ is a line from $(0,1)$ to $(1,-1)$.

(b) $\gamma = \gamma_1 + \gamma_2$, where γ_1 is a line from $(0,1)$ to $(1,1)$ and γ_2 is a line from $(1,1)$ to $(1,-1)$.

(problem 9.2 from the book).

8.5 Compute the curve integral $\int_\gamma (x^2 + xy)dx + (y^2 - xy)dy$, where

(a) γ is a line from $(0,0)$ to $(2,2)$.

(b) γ is a parabel $x^2 = 2y$ from $(0,0)$ to $(2,2)$.

(c) $\gamma = \gamma_1 + \gamma_2$, where γ_1 is a line from $(0,0)$ to $(2,0)$ and γ_2 is a line from $(2,0)$ to $(2,2)$. (problem 9.3 from the book).