1 teori, se boken

2

A	B	C	$(A \to B)$	$\to$	$(C \land A)$	
1	1	1	1	1	1	√
1	1	0	1	0	0	
1	0	1	0	1	1	√
1	0	0	0	1	0	√
0	1	1	1	0	0	
0	1	0	1	0	0	
0	0	1	1	0	0	
0	0	0	1	0	0	
			1		2	

DNF: $(A \land B \land C) \lor (A \land \neg B \land C) \lor (A \land \neg B \land \neg C)$

eq $(A \land B \land C) \lor (A \land \neg B)$ DNF

eq $(A \lor A) \land (A \lor \neg B) \land (B \lor A) \land (B \lor \neg B) \land (C \lor A) \land (C \lor \neg B)$

eq $A \land (A \lor \neg B) \land (A \lor B) \land (A \lor C) \land (\neg B \lor C)$

KNF

3a) $\neg A \vee B \vdash \neg(A \wedge \neg B)$

$$\dfrac{\dfrac{\neg A \vee B \qquad \dfrac{\overset{(1)}{\cancel{\neg A}} \qquad \dfrac{\overset{(3)}{\cancel{A \wedge \neg B}}}{A}}{\bot} \qquad \dfrac{\overset{(2)}{\cancel{B}} \qquad \dfrac{\overset{(4)}{\cancel{A \wedge \neg B}}}{\neg B}}{\bot}}{\bot}\ \vee E\ (1)(2)}{\neg(A \wedge \neg B)}\ \neg I\ (3)(4)$$

3b) $A \to (B \vee C),\ A,\ \neg C \vdash B \wedge A$

$$\dfrac{\dfrac{A \qquad A \to B \vee C}{B \vee C} \qquad \dfrac{\overset{(1)}{\cancel{B}} \qquad A}{B \wedge A} \qquad \dfrac{\dfrac{\overset{(2)}{\cancel{C}} \qquad \neg C}{\bot}}{B \wedge A}}{B \wedge A}\ \vee E\ (1)(2)$$

4a) $A \to (B \wedge C) \models (A \to B) \wedge C$

Motexempel σ-struktur A, B, C falska.
Då är $A \to (B \wedge C)$ sann

men $(A \to B) \wedge C$ blir falsk.

$\therefore \not\models$

4b) $(A \to B) \wedge C \models A \to (B \wedge C)$

A	$\to$	B	$\wedge$	C	A	$\to$	B	$\wedge$	C
1	1	1	1	1	1	1	1	1	1
1	1	1	0	0	1	0	1	0	0
1	0	0	0	1	1	0	0	0	1
1	0	0	0	0	1	0	0	0	0
0	1	1	1	1	0	1	1	1	1
0	1	1	0	0	0	1	1	0	0
0	1	0	1	1	0	1	0	0	1
0	1	0	0	0	0	1	0	0	0

Ja $\models$.

Nat. deduktion:

$$\dfrac{\dfrac{\dfrac{\cancel{A}^{(1)} \qquad \dfrac{(A \to B) \wedge C}{A \to B}}{B} \qquad \dfrac{(A \to B) \wedge C}{C}\,\wedge E}{B \wedge C}}{A \to (B \wedge C)}\,\to I\ (1)$$

5.a) $A \vee B,\ \neg A \vee \neg C,\ C \to \neg B \overset{?}{\vdash} C$

A	B	C	$A \vee B$	$\neg A$	$\vee$	$\neg C$	C	$\to$	$\neg B$	C
1	1	1	1	0	0	0	1	0	0	1
1	1	0	1	0	1	1	0	1	0	0
1	0	1	1	0	0	0	1	1	1	1
1	0	0	1	0	1	1	0	1	1	0
0	1	1	1	1	1	0	1	0	0	1
0	1	0	1	1	1	1	0	1	0	0
0	0	1	0	1	1	0	1	1	1	1
0	0	0	0	1	1	1	0	1	1	0

Eftersom $\Gamma \not\models C$ (t.ex rad 2: $\left.\begin{array}{l} A=B=1 \\ C=0 \end{array}\right\}$ ger Γ sanna men C falsk.

Sundhetssatsen $\Rightarrow \Gamma \not\vdash C$.

5b) $A, B \vdash C \vee \neg C$

$C \vee \neg C$ är sann i varje σ-struktur, därför gäller $\models C \vee \neg C$, fullständighetssatsen ger att $\vdash C \vee \neg C$, och också $A, B \vdash C \vee \neg C$ (onödiga premisser)

6a) $\langle \bar{c}; \bar{F}; \bar{P} \rangle$ $\quad \langle 0; 1; 2 \rangle$

a) Slutna termer: $\bar{c}$, $\bar{F}(\bar{c})$, $\bar{F}(\bar{F}(\bar{c}))$, ...

b) slutna atomära formler

$t \doteq t$ där t en sluten term,

t.ex $\bar{F}(\bar{c}) \doteq \bar{F}(\bar{F}(\bar{c}))$

$\bar{P}(t_1, t_2)$ där t_1, t_2 är slutna termer

t.ex

$\bar{P}(\bar{F}(\bar{c}), \bar{c})$

c) τ : $\bar{P}(\bar{c}, \bar{c}) \wedge \forall x \exists y \bar{P}(x, y)$

$A = \{0\}$ $\quad P = \{(0,0)\}$ $\quad F(0) = 0$

$\mathfrak{A} = \langle A, 0, F, P \rangle$

$\mathfrak{A} \models \tau$

$B = \{0, 1\}$ $\quad P = \{(0,0)\}$ $\quad F(0) = 0$, $F(1) = 1$

$\mathfrak{B} = \langle B, 0, F, P \rangle$

$\mathfrak{B} \not\models \forall x \exists y \bar{P}(x, y)$ eftersom $(1,0) \notin P$ och $(1,1) \notin P$.

$\therefore \mathfrak{B} \not\models \tau$.

7) $\langle ; \bar{F} ; \bar{P}, \bar{Q} \rangle$ sign $\langle ; 2 ; 1, 2 \rangle$

a) $\forall x \exists y \exists z (\bar{P}(y) \wedge \bar{P}(z) \wedge \bar{F}(y, z) \doteq x)$

b) $\forall x (\bar{P}(x) \to \exists y (\neg x \doteq y \wedge \bar{P}(x, y)))$

8. $\langle\ ;\ ;\ \bar{P}, \bar{Q}, \bar{R} \rangle \qquad \langle\ ;\ ;\ 1, 1, 1 \rangle$

a) $\forall x(\bar{P}(x) \to \neg \bar{Q}(x)),\ \forall x(\bar{P}(x) \to \bar{R}(x)),\ \exists x \bar{P}(x) \vdash \exists x(\bar{R}(x) \wedge \neg \bar{Q}(x))$

$$
\dfrac{\exists x \bar{P}(x) \qquad \dfrac{\dfrac{\dfrac{\overset{①}{\cancel{\bar{P}(x)}} \qquad \dfrac{\forall x(\bar{P}(x) \to \neg \bar{Q}(x))}{\bar{P}(x) \to \neg \bar{Q}(x)}\ \forall E}{\neg Q(x)}\ \to E \qquad \dfrac{\dfrac{\forall x(\bar{P}(x) \to \bar{R}(x))}{\bar{P}(x) \to R(x)}\ \forall E \qquad \overset{②}{\cancel{\bar{P}(x)}}}{\bar{R}(x)}\ \to E}{\bar{R}(x) \wedge \neg \bar{Q}(x)}\ \wedge I}{\exists x(\bar{R}(x) \wedge \neg \bar{Q}(x))}}{\exists x(\bar{R}(x) \wedge \neg \bar{Q}(x))}\ \exists E\ ①②
$$

$\vdash \exists x \neg \bar{P}(x) \to \neg \forall x \bar{P}(x)$

b)

$$
\dfrac{\dfrac{\exists x \neg \bar{P}(x) \qquad \dfrac{\dfrac{\overset{②}{\cancel{\neg \bar{P}(x)}} \qquad \dfrac{\cancel{\forall x \bar{P}(x)}\ ①}{\bar{P}(x)}}{\bot}}{\neg \forall x \bar{P}(x)}\ ①}{\neg \forall x \bar{P}(x)}\ \exists E\ ②}{\exists x \neg \bar{P}(x) \to \neg \forall x \bar{P}(x)}\ \to I
$$

9a

$$\underbrace{\forall x(\bar{P}(x) \to \bar{Q}(\bar{c}))}_{B_1}, \underbrace{\forall x(\bar{Q}(x) \to \bar{R}(x,\bar{c}))}_{B_2} \overset{?}{\models} \underbrace{\forall x(\bar{P}(x) \to \bar{R}(x,\bar{c}))}_{C}$$

Låt
$$A = \{0,1,2\}$$
$$(\bar{c})^{\mathfrak{A}} = 0$$
$$(\bar{P})^{\mathfrak{A}} = P = \{1\}$$
$$(\bar{Q})^{\mathfrak{A}} = Q = \{0\}$$
$$(\bar{R})^{\mathfrak{A}} = \{(0,0)\}$$

Nu gäller $\mathfrak{A} \models B_1$ eftersom $0 \in Q$

$\mathfrak{A} \models B_2$ eftersom $(0,0) \in Q$

Men $\mathfrak{A} \not\models C$ eftersom $1 \in P$ och $(1,0) \notin Q$.

$\therefore$ $B_1, B_2 \not\models C$. //

9b

$$\underbrace{\exists x \forall y \bar{R}(x,y)}_{B} \models \underbrace{\forall y \exists x \bar{R}(x,y)}_{C}$$

giltig.

$$\cfrac{\exists x \forall y \bar{R}(x,y) \qquad \cfrac{\cfrac{\cfrac{\cancel{\forall y \bar{R}(x,y)}}{\bar{R}(x,y)}\,\forall E}{\exists x \bar{R}(x,y)}\,\exists I}{\forall y \exists x \bar{R}(x,y)}\,\forall I}{\forall y \exists x \bar{R}(x,y)}\,\exists E\ (1)$$

Alltså gäller $B \vdash C$, ~~[illegible]~~ sundhetssatsen medför att $B \models C$.

(10) $\varphi_1 \quad \forall x\ \neg R(x,x)$

$\varphi_2 \quad \forall x \forall y \forall z (R(x,y) \wedge R(y,z) \to R(x,z))$

$\varphi_3 \quad \forall x\ \exists y\ R(x,y)$

(a) $\mathfrak{N} = \langle \mathbb{N}, < \rangle$ är en modell för Γ.

b) $\varphi_1, \varphi_2 \nvdash \varphi_3$

1) $\mathfrak{A}_3 = \langle A_3, R_3 \rangle$ där $A_3 = \{0,1,2\}$

$R_3 = \emptyset$

$\mathfrak{A}_3 \models \varphi_1 \wedge \varphi_2$ men $\mathfrak{A}_3 \not\models \varphi_3$

Sundhetssatsen ger $\varphi_1, \varphi_2 \nvdash \varphi_3$.

2) $\varphi_1, \varphi_3 \nvdash \varphi_2$

$R_2 = \{ (0,1), (1,2), (2,0) \}$ $\quad A_2 = \{0,1,2\}$

$\mathfrak{A}_2 = \langle A_2, R_2 \rangle$

$\mathfrak{A} \models \varphi_1 \wedge \varphi_3$ men $\mathfrak{A} \models \varphi_2$ ty $(0,1)$ och $(1,2)$ $\in R_2$

Men $(0,2) \notin R_2$.

$\therefore \varphi_1, \varphi_3 \not\models \varphi_2$

Sundhet ger $\varphi_1, \varphi_3 \nvdash \varphi_2$.

3) $\varphi_2, \varphi_3 \nvdash \varphi_1$

$\mathfrak{A}_1 = \langle A_1, R_1 \rangle$ $\quad A_1 = \{0,1,2\}$

$R_1 = \{ (0,0), (1,1), (2,2) \}$

$\cancel{\mathfrak{A} \models \langle A_1,}$ $\mathfrak{A}_1 \models \varphi_2 \wedge \varphi_3$ men $\mathfrak{A}_1 \not\models \varphi_1$

$\therefore \varphi_2, \varphi_3 \not\models \varphi_1$

Sundhetssatsen ger $\varphi_2, \varphi_3 \nvdash \varphi_1$. ///