Linear Algebra III, 1MA026
Spring 2023, period 3
Course Information

Uppsala University
Department of Mathematics

Luís Diogo
luis.diogo@math.uu.se

1 Introduction

Welcome to Linear Algebra III at Uppsala University. This 5 credit course is more abstract than Linear Algebra II (which is a prerequisite), covering:

- general theory of vector spaces over arbitrary fields;
- bilinear forms and inner products;
- linear operators on real and complex vector vector spaces, including the Jordan normal form, the spectral theorem and polar decomposition.

The entire course, including the homework assignments and the exam, will be in **English**.

2 Course Literature

We will mostly follow:

- [A] Linear Algebra Done Right, Sheldon Axler, Springer UTM, Third edition (in English).

You may also find the following useful:

- [L] Linjär algebra, Lars-Åke Lindahl (in Swedish),
  Available on Studium and at http://katalog.uu.se/empinfo/?id=XX1292

3 Course Plan

There will be 15 lectures. You can find the course schedule on TimeEdit.
Below is a tentative schedule for the course (it may change a little during the period).

Lectures	Subjects	Course Literature
1	Introduction, fields, vector spaces, subspaces	[A] 1.A-C; [L] 3.1, 3.3
2	Bases and dimension, sums of subspaces, direct sums	[A] 1.C, 2; [L] 3.3, 3.5-7
3	Linear maps, matrices, isomorphisms. Quotients.	[A] 3.A-E; [L] 2, 3.2, 3.4, 3.8-3.11
4	Tensor products	[A] 3.E; [L] 3.11
5	Problem session	
6	Functionals, dual spaces	[A] 3.F; [L] 4
7	Bilinear and quadratic forms	[L] 5
8	Inner product spaces and orthogonality	[A] 6; [L] 6
9	Operators, eigenvalues, generalized eigenvectors and eigenspaces	[A] 5, 8.A; [L] 8.1-3
10	Problem session	
11	Jordan normal form	[A] 8.B-D; [L] 8.4
12	More on Jordan normal form	[A] 8.B-D; [L] 8.4
13	Spectral theorem	[A] 7.A-B; [L] 9.1-4
14	Polar decomposition	[A] 7.D; [L] 9.5-6
15	Problem session, review	

4 Course Material on Studium

The course material will be available on Studium. There you will find:

- Course information (this document, possibly updated during the course),
- Exercises for practicing (updated during the course),
- Linjär algebra, Lars-Åke Lindahl (notes in Swedish),
- Lecture notes (updated during the course),
- Home assignments,

- Exams for practicing,
- A text about the linear algebra behind the Google PageRank algorithm.

Note that you will have to register to the course before you get access to the material in Studium. Also, some of the content might be updated during the course.

5 Home Assignments

There are two **optional** home assignments. Do not confuse them with the list of exercises (which are for your practice and for the problem sessions).

- The due dates are as follows:

  Home assignment 1: 2023-02-19 (Sunday)
  Home assignment 2: 2023-03-11 (Saturday)

- The assignments can increase the exam grade by a maximum of 3 bonus points. Let P be the sum of the points you get in the two assignments and let T be the sum of the total number of points available in the two assignments. You get:

  1 bonus point: if $P \geq 40\%$ of T;
  2 bonus points: if $P \geq 60\%$ of T;
  3 bonus points: if $P \geq 75\%$ of T.

- The bonus points can only be used on the ordinary exam, and can only increase your exam grade to 31 points (so they cannot raise your course grade to 5).
- You are allowed to cooperate but not to copy from each other.
- The solutions to the assignments are submitted in Studium in pdf-format, either scanned or written in LaTeX. Please make sure that the hand-ins are clearly readable!
- You need to write your answers in **English**.

6 Exam

- The exam will take place on 2023-03-16. Look up the time and place on Ladok.
- **Do not forget to register for the exam.** You can do so on Ladok, until 2 weeks before the exam.
- The exam will have a total of 40 points. The grade limits for grades 3, 4 and 5 are 18, 25 and 32 respectively.

7 Syllabus

The course plan can be found here.

7.1 Course Content

Linear spaces over arbitrary fields, sums and direct sums of subspaces, the dimension formula, quotient spaces. Tensor product. Linear transformations. Linear functionals, the dual space, dual bases. The canonical isomorphism between a linear space and its bidual. Forms: bilinear, Hermitian, symmetric, alternating, quadratic. Inner product spaces: unitary, Euclidean, orthogonal projection, the method of least squares. Linear operators: Hermitian, symmetric, unitary, orthogonal, normal, polynomial, the spectral theorem (complex and real), simultaneous diagonalisation, eigenspaces and generalised eigenspaces, the characteristic polynomial and the minimal polynomial, Jordan's normal form (complex and real). Polar decomposition. Orientation about matrix groups: the general linear group, the orthogonal groups, the unitary group.

7.2 Learning Outcomes

On completion of the course, the student should be able to:

- give an account of important concepts and definitions in the theory of linear spaces over arbitrary fields;
- exemplify and interpret important concepts in specific cases;
- formulate important results and theorems covered by the course;
- describe the main features of the proofs of important theorems;
- express problems from relevant areas of applications in a mathematical form suitable for further analysis;
- use the theory, methods and techniques of the course to solve mathematical problems;
- present mathematical arguments to others.