i Information sheet 1TD395

Exam Scientific Computing II

The exam is divided into two parts, part A and part B. Part A is related to grade 3, and part B is to grades 4 and 5.

Part A (grade 3)

The tasks in Part A are directly linked to one of the four course objectives: *Key Concepts, Algorithms, Analysis,* and *Argumentation*. In this part, there are 2 questions related to each objective (questions 1 to 8) and maximum 2 points per question.

Questions in part A are either multiple-choice questions or require entering a numerical value in a provided box. For these types of questions, you may need to solve the problem on paper and then select or enter the correct answer. You are able to answer this part in the Inspera only. As an option, you can write down your detailed solutions on paper and hand them in for review. We can then correct the information if you have entered the wrong answer due to something that we judge to be a simple careless error.

Part B (grades 4 and 5)

In part B there are 3 questions (questions 9, 10, 11). You can either type the detailed solution directly into Inspera or hand it in on paper to the invigilators. If you hand in paper-answers, make a note about it in Inspera in the corresponding question.

Questions 9 and 10 can give 0 or 10 points and question 11 can give 0, 10, or 20 points.

In both parts, if you hand in paper-answers please write in English and in neat and legible handwriting.

Grades

- **Grade 3:** At least 8 points. You must answer at least one question on each objective of part A. This corresponds to minimum 4 questions (i.e. 8 points) distributed among the four objectives. If you fail to meet a course objective entirely, it will result in a failing grade on the exam.
- **Grade 4:** At least 20 points. You must fulfill Grade 3 + solve one more question from part A (at least 5 correct answers) + fulfill one of the following items:
  -- solve either question 9 or question 10 completely
  -- solve either part (1) or part (2) of question 11 completely.
- **Grade 5:** At least 32 points. You must fulfill Grade 3 + solve two more questions from part A (at least 6 correct answers) + fulfill one of the following items:
  -- solve both questions 9 and 10 completely
  -- solve question 11 (both parts) completely
  -- solve question 9 and part (2) of question 11 completely
  -- solve question 10 and part (1) of question 11 completely.

Allowed aid

- Calculator
- The formula sheet (is available as a link in Inspera. Sections 3 and 7 are related to this course)

Good Luck!

1 Algorithm1_20230819

Compute $y(0.1)$ using the backward Euler's (Implicit Euler's) method for differential equation
$y'(t) = -10(y(t) - t^2), \;\; t \geq 0, \; y(0) = 1$
with steplength $h = 0.1$.
You can find the backward Euler's method in the formula sheet.
Perform the calculation on paper and choose your answer by selecting one of the options below (only one item is correct).

Select one alternative:

- 0.5500
- 0.6734
- 0.5050
- 0.0000
- 0.3367

Maximum marks: 2

2 Algorithm2_20230819

The task is to approximate the value of integral

$$\int_0^\infty (1+x)\exp(-2x)\,dx$$

using the Monte Carlo method on five random points

$$0.0108,\ 0.0602,\ 0.3568,\ 0.8921,\ 1.7759$$

which are exponentially distributed according to probability density function (pdf) $f(x) = 2\exp(-2x)$. What is the approximate value?

Select one alternative:

- 0.29911
- 0.80958
- 0.59823
- 1.61916

Maximum marks: 2

3 Concept1_20230819

Classify the algorithms and methods

	Deterministic Model	Stochastic Model	Deterministic Method	Stochastic Method
Gillespies algorithm (SSA)	○	○	○	○
Monte Carlo Integration	○	○	○	○
$\begin{aligned} dD_A/dt &= \theta_A D'_A - \gamma_A D_A A \\ dD_R/dt &= \theta_R D'_R - \gamma_R D_R A \\ dD'_A/dt &= \gamma_A D_A A - \theta_A D'_A \\ dD'_R/dt &= \gamma_R D_R A - \theta_R D'_R \\ dM_A/dt &= \alpha'_A D'_A + \alpha_A D_A - \delta_{M_A} M_A \\ dA/dt &= \beta_A M_A + \theta_A D'_A + \theta_R D'_R \\ &\quad - A(\gamma_A D_A + \gamma_R D_R + \gamma_C R + \delta_A) \\ dM_R/dt &= \alpha'_R D'_R + \alpha_R D_R - \delta_{M_R} M_R \\ dR/dt &= \beta_R M_R - \gamma_C AR + \delta_A C - \delta_R R \\ dC/dt &= \gamma_C AR - \delta_A C, \end{aligned}$	○	○	○	○
$I = \int_{-1}^{1} x^2 e^{\frac{-x^2}{2}} dx$	○	○	○	○
Assume all quantities to be integers. $\left.\begin{aligned} A + R &\xrightarrow{\gamma_c} C \\ A &\xrightarrow{\delta_a} \emptyset \\ C &\xrightarrow{\delta_a} R \\ R &\xrightarrow{\delta_r} \emptyset \\ D_a + A &\xrightarrow{\gamma_a} D'_a \\ D_r + A &\xrightarrow{\gamma_r} D'_r \end{aligned}\right\}$	○	○	○	○
$\begin{aligned} \frac{dx}{dt} &= \alpha x - \beta xy, \\ \frac{dy}{dt} &= \delta xy - \gamma y, \end{aligned}$	○	○	○	○
Classical Runge Kutta	○	○	○	○
Trapezoidal rule for integration	○	○	○	○

Maximum marks: 2

4 Concept2_20230819

When numerically solving differential equations, it is important to consider whether the equation is stiff. What are the characteristics of a stiff ODE?

One or more options may be correct, tick all correct items.

Select one or more alternatives:

- ☐ Implicit methods are appropriate to solve it
- ☐ There may be some fast transients (rapid variations) in the solution
- ☐ It requires a small steplength for stability in explicit methods
- ☐ The coefficients on the right-hand side of the ODE are significantly different in magnitude.
- ☐ Explicit methods are appropriate to solve it
- ☐ The RK4 method can solve it with a relatively large steplength

---

Maximum marks: 2

5 Analysis1_20230819

Assume that the solution of an initial value problem (IVP) is calculated using a second order method (say Heun's method) with step length $h = 0.05$. It is estimated that the global error is ≈ 0.4. How small does h need to be for obtaining the approximate error ≈ 0.025?

Calculate the new h and enter it here to four decimal places*: ☐ .

* for example: 0.1234

Maximum marks: 2

6 Analysis2_20230819

Suppose that we have estimated the integral $\int_a^b f(x)\,dx$ using the Monte Carlo method with $N = 1000$ random numbers. Alongside, we have assessed the error through the length of the confidence interval (or standard deviation), which we found to be $\varepsilon = 0.2$. Now, if we seek to enhance the accuracy by increasing the number of random points to $N = 16000$, what would the resulting length of the confidence interval (or standard deviation) be?

Calculate and enter it here: ______ .

Maximum marks: 2

7 Argumentation1_20230819

Specify the more suitable method (column) for each application (row).

	Implicit method	Explicit method
Stability is crucial	○	○
Stiff equation	○	○
Non-stiff equation	○	○
$y_1' = y_2$ $y_2' = (1 - y_1^2)y_2 - y_1$	○	○
Reactions with fast transients	○	○
$y_1' = y_2$ $y_2' = 1000(1 - y_1^2)y_2 - y_1$	○	○
Low complexity per timestep is important	○	○

Maximum marks: 2

8 Argumentation2_20230819

Specify the more suitable method (column) for each application (row).

Please match the values:

	Stochastic method	Deterministic method
2D integral	○	○
10D Integral	○	○
ODE	○	○
Scenarios in epidemic models with limited number of individuals	○	○
Solution is continuous (concentration, velocity, ...)	○	○
Solution is discrete (individuals, number of molecules, ...)	○	○
Stochastic differential equation (SDE)	○	○

Maximum marks: 2

9 Grade45_1_20230819

Consider the following linear system of ODEs:

$$\begin{bmatrix} y_1'(t) \\ y_2'(t) \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad t > 0,$$

with initial conditions $y_1(0) = 5,\ y_2(0) = 2$. Apply the forward Euler's method with a steplength of h to solve this ODE, and determine the permissible range of values for h that guarantee the absolute stability of the method for this particular ODE.

Fill in your answer here or write in the answer sheet and hand it in.

Format

Words: 0

Maximum marks: 10

10 Grade45_2_20230819

Assume that you are running a lumber mill in Krokom and you are trying to **estimate the production price** of a single piece of your standard framing timber. The cost to produce your standard framing timber includes labor, energy and trees. Assume the cost of labor is constant at 3 SEK per piece of framing timber. The cost of energy needed to make a single piece of framing timber is normally distributed with mean $\mu_E = 0.5$ SEK and standard deviation $\sigma_E = 0.1$ SEK. The price of tree needed to make a single piece of timber is distributed according to Weibull pdf $f_T(x) = 5x^4 \exp(-x^5)$ for $x \in [0, \infty)$.
Design a Monte Carlo algorithm to estimate the mean production price p_{mean} and variance $p_{variance}$ of standard framing timber.
Assume the function `randn()` exists and that it returns one standard normal number (with mean 0 and variance 1) every time that it is called. However, it is necessary to provide a detailed formulation of how we can generate a random number from the given Weibull distribution and incorporate it into the Monte Carlo algorithm.

Fill in your answer here or write in the answer sheet and hand it in.

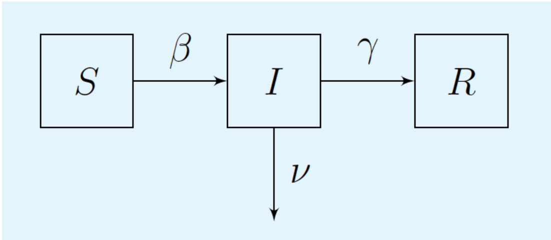
Format B I U x_2 x^2 I_x Ω Σ

Words: 0

Maximum marks: 10

11 Grade45_3_20230819

Consider the following SIR diagram for spread of a virus within a population. The population is divided into three groups S, I and R standing for **Susceptible**, **Infected** and **Recovered** individuals, respectively. Here, β is the infection rate, γ is the recovery rate, and ν is the pathogen-induced mortality rate (death due to the virus).



Furthermore, we assume that the infection rate depends on the proportion of infected individuals relative to the entire population, while the recovery and mortality rates are constants.

Part (1): By assuming that S, I and R are continuous and deterministic variables, write down a system of ODEs with initial conditions that governs this phenomenon. Then write a Matlab code for solving and plotting the solutions of the resulting ODE in time interval $[0, T]$, utilizing a suitable build in function (ODE solver), provided that all constant rates and initial values are given. It should be a detailed and executable program with justification for the choice of methods, e.g. why you choose a particular ODE solver.

Part (2): Consider a scenario where S, I and R are integer and stochastic variables. Write down all processes (reactions), propensity functions, probabilities, and state-change (stoichiometry) vectors pertaining to the model. Finally, provide a Matlab code that implements the Gillespies (SSA) algorithm for solving the model iteratively, and plot the solutions. Assume that the SSA subroutine has been given. It should be a detailed and executable program.

Note: minor errors in codes are acceptable.

Fill in your answer here or write in the answer sheet and hand it in.

Format

Words: 0

Maximum marks: 20