Financial Theory – Lecture 14

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Agenda

- Empirical aspects.

The lecture is based on

- Sections 5.7 and 6.5 in the course book.

Stock market returns

We'll start looking at data for the stocks in the S&P 500 index.

When looking at the stocks from an index, we can construct a value weighted (VW) or equally weighted (EW) portfolio.

- VW:
$$\pi_i = \frac{\text{Asset market cap}}{\text{Total market cap}}, \ \ i = 1, 2, \ldots, n.$$
- EW:
$$\pi_i = \frac{1}{n}, \ \ i = 1, 2, \ldots, n.$$

Stocks market returns

From p. 229 in the course book:

	Period	Mean	Std dev	Skew	Kurt	Min	Max
			Annual returns				
VW nominal	1927-2019	11.9%	19.9%	-0.446	0.098	-45.5%	53.3%
VW nominal	1946-2019	12.4%	17.0%	-0.351	0.109	-36.6%	52.8%
VW nominal	1990-2019	11.5%	17.5%	-0.767	0.623	-36.6%	37.7%
EW nominal	1927-2019	14.3%	24.1%	0.042	1.081	-53.0%	95.7%
EW nominal	1946-2019	13.9%	19.0%	-0.176	0.193	-40.1%	58.0%
EW nominal	1990-2019	13.1%	18.5%	-0.637	1.236	-40.1%	47.4%
Inflation	1946-2019	3.69%	3.36%	1.942	4.960	-2.07%	18.1%
VW real	1946-2019	8.60%	17.4%	-0.301	0.144	-36.7%	53.9%
EW real	1946-2019	10.1%	19.1%	-0.204	0.146	-40.2%	55.2%
1Y riskfree	1946-2019	3.99%	3.12%	0.919	0.886	0.02%	14.7%
VW excess	1946-2019	8.41%	17.4%	-0.320	0.126	-38.2%	51.9%
EW excess	1946-2019	9.96%	19.3%	-0.125	0.267	-41.7%	56.4%
			Monthly returns				
VW nominal	1927-2019	0.94%	5.41%	0.356	9.794	-28.7%	41.4%
VW nominal	1946-2019	0.96%	4.13%	-0.425	1.654	-21.6%	16.8%
VW nominal	1990-2019	0.88%	4.09%	-0.612	1.260	-16.7%	11.4%
EW nominal	1927-2019	1.14%	6.74%	1.472	18.574	-31.0%	68.0%
EW nominal	1946-2019	1.08%	4.69%	-0.325	2.508	-25.6%	23.1%
EW nominal	1990-2019	1.01%	4.65%	-0.492	2.125	-20.9%	18.5%
Inflation	1946-2019	0.30%	0.45%	2.577	29.203	-1.92%	5.88%
VW real	1946-2019	0.66%	4.17%	-0.421	1.458	-21.8%	15.6%
EW real	1946-2019	0.78%	4.73%	-0.320	2.310	-25.8%	22.7%
1M riskfree	1946-2019	0.32%	0.25%	0.976	1.174	0.00%	1.35%
VW excess	1946-2019	0.64%	4.15%	-0.446	1.627	-22.2%	16.3%
EW excess	1946-2019	0.76%	4.70%	-0.345	2.485	-26.2%	22.5%

Table 6.2: Summary statistics for the S&P 500 index.
Data on the value-weighted (VW) and equally-weighted (EW) returns on the stocks in the S&P 500 index as well as the inflation rate were downloaded from CRSP through WRDS on June 22, 2020. Data on the 1 month and the 1 year riskfree rate were downloaded from the homepage of Kenneth French on June 22, 2020. The mean shown in the table is the arithmetic average.

Stocks market returns

From p. 235 in the course book:

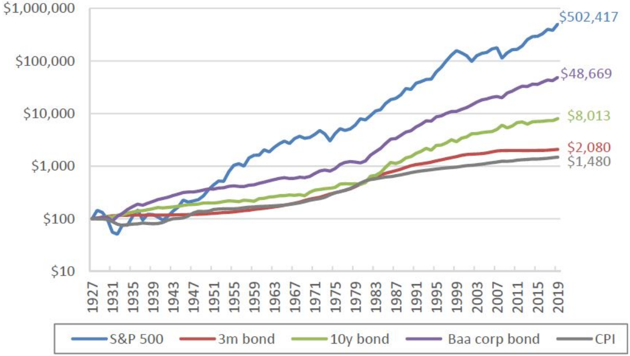


Figure 6.5: Stocks look great in the long run.
The graph shows cumulative returns on U.S. asset classes from 1927 to 2019. The data were downloaded on June 24, 2020 from the homepage of Professor Aswath Damodaran at New York University, see `http://pages.stern.nyu.edu/~adamodar/`.

Stocks market returns

From p. 236 in the course book:

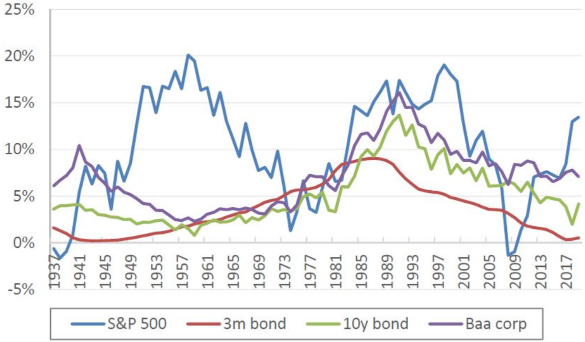


Figure 6.6: Trailing 10-year average returns.
The graphs show the trailing 10-year geometric average rate of return for the S&P 500 stock market index, 3-month Treasury bills, 10-year Treasury bonds, and Baa-rated corporate bonds over the period 1937-2019. The data were downloaded on June 24, 2020 from the homepage of Professor Aswath Damodaran at New York University, see http://pages.stern.nyu.edu/~adamodar/.

Stocks market returns

Are returns predictable?

On a larger scale, there are empirical studies indicating that average stock market returns are counter-cyclical: they are higher in bad times than in good times.

Again, we see this from

$$r_{t+1} = \frac{D_{t+1} + P_{t+1} - P_t}{P_t} \quad \Leftrightarrow \quad P_t = \frac{P_{t+1} + D_{t+1}}{1 + r_{t+1}}.$$

Stocks market returns

Several studies also show momentum in the short run and reversal in the long run. This observation lead to the introduction of the momentum factor.

Can some other variables predict stock returns?

The price-dividend ratio and the dividend yield has been suggested.

Stocks market returns

We can write

$$
\begin{aligned}
\ln\left(1+r_{t+1}\right) &= \ln\left(P_{t+1}+D_{t+1}\right)-\ln P_t \\
&= \ln\left(P_{t+1}+D_{t+1}\right)-\ln P_{t+1}+\ln P_{t+1}-\ln P_t \\
&= \ln\left(1+\frac{D_{t+1}}{P_{t+1}}\right)+\ln P_{t+1}-\ln P_t \\
&= \ln\left(1+e^{d_{t+1}-p_{t+1}}\right)+p_{t+1}-p_t,
\end{aligned}
$$

where $p=\ln P$ and $d=\ln D$. Using an approximation here (the Campbell-Shiller approximation) and iterating we get decompositions that can be used in testing for prediction.

Stocks market returns

There have been several other suggestions, but there seems not to be any strong indication of specific variables being able to, in general, predict stock returns.

Stocks market returns

The cross-sectional investigation of returns focus on if we can predict the return of an asset by looking at the return(s) of other assets.

Typically this includes factor models and testing CAPM or some APT model by using the two-stage regression or the Fama-MacBeth method discussed in Lecture 9.

Asset returns

Moving from the general to the specific, we often make assumptions such as:

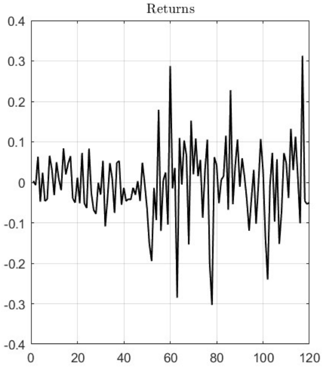
- The rate of returns $r_1, r_2, \ldots, r_n$ are IID.
- Each r_i, $i = 1, 2, \ldots, n$ has a distribution depending on some parameters.

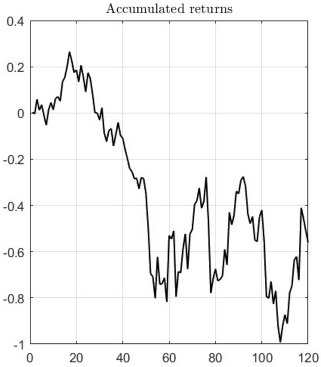
We want to estimate parameters and test hypotheses.

We will see an example of 10 years monthly data for a stock.

Asset returns

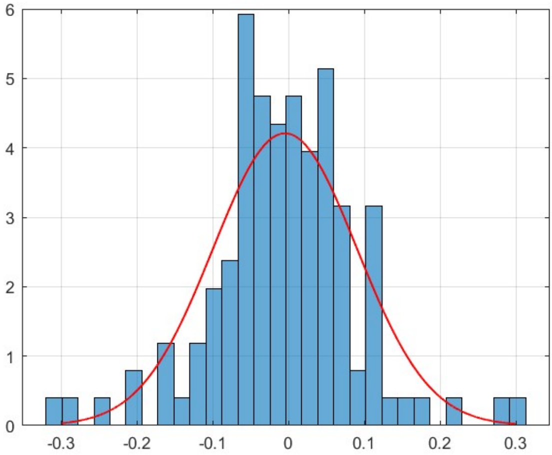
A time series description of the data.



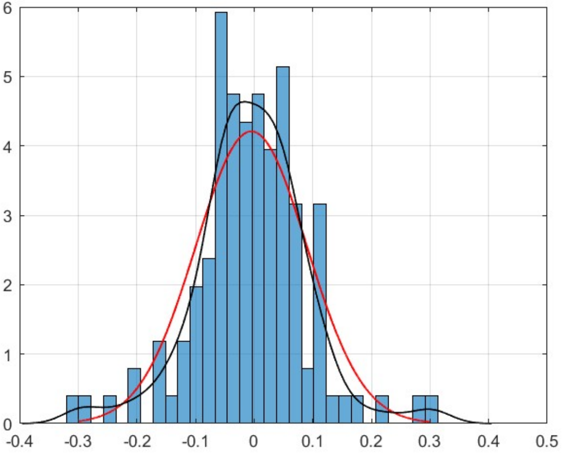


Asset returns

A histogram with an estimated normal density $\hat{\mu} = -0.0047$ and $\hat{\sigma} = 0.0948$.



A histrogram with an estimated normal density together with a kernal estimator.



Asset returns

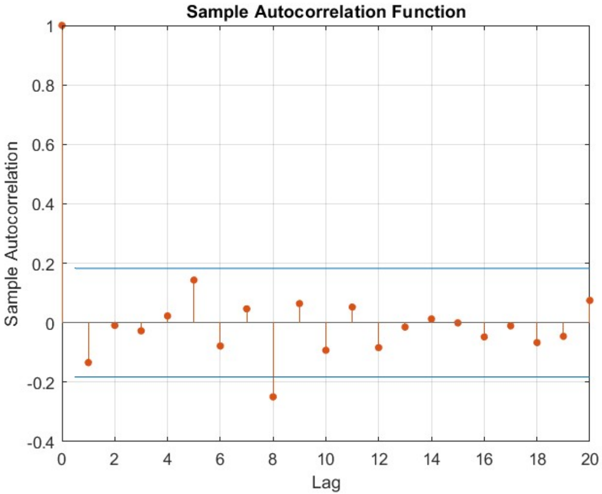
Are asset returns correlated over time?

The autocorrelation function is given by

$$\rho_k = \mathrm{Corr}[r_t, r_{t+k}].$$

If the returns are IID, then they also are uncorrelated, and we should have $\rho_k = 0,\ k = 1, 2, \ldots$.

The autocorrelation function for $k = 1, 2, \dots, 20$.



Asset returns

How do we estimate parameters?

"Old method": Use maximum likelihood (ML).

"Modern method": Use generalised method of moments (GMM).

Method of moments

Assume that $x_1, x_2 \ldots, x_n$ are observations from a random variable X with mean μ.

Then

$$E\left[X - \mu\right] = 0.$$

Now replace E with $\frac{1}{n}\sum_{i=1}^{n}$ to get an estimator:

$$\frac{1}{n}\sum_{i=1}^{n}(x_i - \hat{\mu}_n) = 0 \quad \Leftrightarrow \quad \hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} x_i.$$

The (weak) law of large numbers implies that

$$\hat{\mu}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \xrightarrow{P} \mu.$$

GMM

Let $r_1, r_2, \ldots, r_T$ be a time series of rate of returns (random variables).

We need some technical assumptions on the time series of returns for the following method to work (stationary and ergodic).

Assume that there exists a function g such that for each t

$$E\left[g(r_t, b)\right] = 0.$$

We want to estimate the parameter b.

Again, replace E with $\frac{1}{T}\sum_{i=1}^{T}$:

$$\frac{1}{T}\sum_{i=1}^{T} g(r_i^{\text{obs}}, \hat{b}_T) = 0.$$

GMM

If the parameter is one-dimensional, then we have 1 equation and 1 unkown $\longrightarrow$ we can (in theory) calculate $\hat{b}_T$.

In general, we have N conditions:

$$E\left[g_i(r_t, b)\right] = 0,\ \ i = 1, 2, \ldots, N,$$

and a parameter vector with $K < N$ number of elements.

We then choose K linear combinations (this can be done in several ways) to get estimates, and use the rest $N - K$ equations to get goodness-of-fit tests.

GMM

In general, the estimator is unbiased:

$$E\left[\hat{b}_n\right] = b$$

and consistent:

$$\hat{b}_n \xrightarrow{P} b \text{ as } n \to \infty.$$

There are also typically some asymptotic distribution result(s) which we can use to test hypothesis.

GMM

If the parameter vector b is K-dimensional, then (from a version of the CLT)

$$\frac{1}{\sqrt{T}} \sum_{t=1}^{T} g(r_t, b) \xrightarrow{d} N(\mathbf{0}, S),$$

where

$$S = \sum_{s=-\infty}^{\infty} E\left[g(r_t, b) g(r_{t-s}, b)^\top\right]$$

is a $K \times K$-dimensional asymptotic variance-coraviance matrix.

How do we estimate S?

Assume that the the r_t's are uncorrelated (easy, but not realistic) or use a few time lags á la Newey-West.

Bond returns and interest rates

> *Average returns and volatilities increase with maturity, whereas Sharpe ratios decrease with maturity.*

(Munk, p. 174.)

From p. 175 in the course book:

	Inflation	1M	3M	1Y	2Y	5Y	7Y	10Y	20Y	30Y
Avg return	3.67%	3.84%	4.25%	4.72%	4.95%	5.50%	5.82%	5.59%	6.18%	6.16%
Standard dev	1.59%	0.92%	1.05%	1.79%	2.75%	4.99%	6.14%	7.36%	9.86%	11.35%
Sharpe ratio			0.398	0.492	0.406	0.334	0.323	0.238	0.238	0.204

Table 5.2: **Return statistics for U.S. Treasury bonds.**
The statistics are based on monthly observations over the period from January 1946 to December 2021 downloaded from CRSP U.S. Treasury and Inflation Indexes on July 4, 2022. The returns are nominal. The statistics shown are annualized from monthly statistics. For the average return and standard deviation for both bond returns and the inflation rate, the annualization follows Eqs. (3.83) and (3.84). The annualized Sharpe ratio for a given maturity is calculated as the difference of the annualized average return for that maturity minus the annualized average return on 1-month bills, divided by the annualized standard deviation for the given maturity.

Bond returns and interest rates

We can see the same feature in risky bond (p. 176 in the course book):

	Intermediate maturity ($\approx$ 5 years)				Long maturity ($\approx$ 10 years)			
	AAA	AA	A	BAA	AAA	AA	A	BAA
Average excess return	2.38%	2.53%	2.76%	3.44%	3.12%	3.80%	3.75%	4.60%
Standard deviation	5.02%	4.99%	5.28%	5.48%	10.45%	9.74%	9.67%	9.82%
Sharpe ratio	0.47	0.51	0.52	0.63	0.30	0.39	0.39	0.47

Table 5.3: **Return statistics for U.S. corporate bonds.**
The statistics are annualized and based on data from Barclays corporate bond indexes over the period from January 1973 to August 2014. Intermediate maturity corresponds to a duration of about 5 years, and long maturity to a duration of about 10 years. Source: Table 5 in van Binsbergen and Koijen (2017).

Bond returns and interest rates

Bonds with close maturities are highly correlated (p. 177 in the course book):

	1M	3M	6M	1Y	2Y	3Y	5Y	7Y	10Y	20Y	30Y
1M	1.00	0.97	0.93	0.88	0.72	0.60	0.46	0.37	0.36	0.30	0.30
3M	0.97	1.00	0.98	0.93	0.78	0.67	0.53	0.45	0.43	0.37	0.36
6M	0.93	0.98	1.00	0.97	0.83	0.73	0.58	0.49	0.46	0.39	0.38
1Y	0.88	0.93	0.97	1.00	0.91	0.82	0.67	0.58	0.54	0.46	0.44
2Y	0.72	0.78	0.83	0.91	1.00	0.97	0.86	0.77	0.72	0.63	0.59
3Y	0.60	0.67	0.73	0.82	0.97	1.00	0.95	0.87	0.82	0.73	0.68
5Y	0.46	0.53	0.58	0.67	0.86	0.95	1.00	0.98	0.94	0.87	0.83
7Y	0.37	0.45	0.49	0.58	0.77	0.87	0.98	1.00	0.99	0.94	0.90
10Y	0.36	0.43	0.46	0.54	0.72	0.82	0.94	0.99	1.00	0.97	0.95
20Y	0.30	0.37	0.39	0.46	0.63	0.73	0.87	0.94	0.97	1.00	0.99
30Y	0.30	0.36	0.38	0.44	0.59	0.68	0.83	0.90	0.95	0.99	1.00

Table 5.4: **Bond correlations.**
The table shows correlations between monthly changes in yields of U.S. Treasury bonds of different maturities in the period January 2012 to December 2021. Source: http://www.federalreserve.gov/releases/h15/data.htm, data retrieved on July 4, 2022.

Implication: There is little diversification effect in investing in bonds with close maturities.

Bond returns and interest rates

Other features include:

- High autocorrelation for short-term interest rates.
- Interest rates, especially short-term, tend to be mean reverting.

Yield curves

Take as given the yields

$$(y_1, y_2, \ldots, y_n)^\top.$$

We can think of this as an n-dimensional random vector. One way of analysing the yields is to use principal component analysis (PCA).

This is a way to get $k < n$ number of vectors – principal components – that "explains" the major part of the movement of the yields.

Yield curves

For PCA to be a successful method, we want two conditions to be satisfied.

- The number k of principal components should be low.
- We should be able to give an interpretation to the principal components.

Yield curves

If we do a PCA on the yields, we typically get three components that explains a large part of the yield curve.

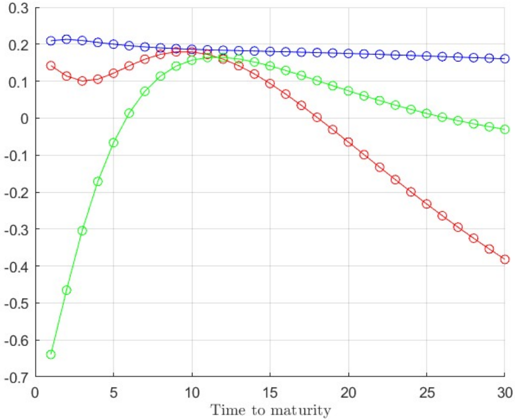
We can also give them an interpretation:

- Level.
- Slope.
- Curvature.

Yield curves

A PCA performed on monthly US nominal bonds for about 35 years.

By using the three first principal components, we take account of 99.52% of the variation.



Yield curves

Number of principal components	Cumulative sum of variation
1	0.938827060103672
2	0.970281483447598
3	0.995221012006218
4	0.999255414190555
5	0.999924874885835
6	0.999990576439362
7	0.999998952231860
8	0.999999923342092
9	0.999999994210718
10	0.999999999522157