Multivariate calculus M
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Exercise 5.

5. Multidimensional integrals

We solve the problems together in the exercise sessions. Note that these problems are optional and for learning purposes: solving these does not provide extra points. Actual home assignments (giving you extra points) are given separately.

It is advised to take a look of the problems beforehand. Note that some of the problems might be very challenging, so do not feel bad if you are unable to solve them independently: we will go through the solutions together!

Problems for the session

5.1 Compute the integral $\int\int_D \frac{1}{(1+x+y)^2}dxdy$, where D is a rectangle with corners $(0,0)$, $(1,0)$, $(1,2)$, and $(0,2)$.

5.2 Compute the integral $\int\int_D xe^{-(x^2+y^2)}dxdy$, where $D = \{(x,y) : |x|, |y| \leq 1\}$.

5.3 Compute the integral $\int\int_D e^{-y^2}dxdy$, where $D = \{(x,y) : |x| \leq y, 0 \leq y \leq 1\}$.

5.4 Compute the integral $\int\int_D xy\sqrt{x^2+y^2}dxdy$, where

(a) $D = \{(x,y) : 0 \leq y \leq x, 0 \leq x \leq 1\}$.
(b) $D = \{(x,y) : 0 \leq x \leq 1, 0 \leq y \leq 1\}$.

5.5 Compute the integral $\int\int_D (x^2-y^2)^{10}dxdy$, where $D = \{(x,y) : |x| + |y| \leq 1\}$.

5.6 Compute the integral $\int\int_D \log(1+x^2+y^2)dxdy$, where $D = \{(x,y) : 1 \leq x^2+y^2 \leq 2\}$.

5.7 Compute the integral $\int\int_D (x^4-y^4)dxdy$, where D is defined through $x < 0$, $y > 0$, $1 < x^2 - y^2 < 4$, and $\sqrt{17} < x^2 + y^2 < 5$.

Problems for individual practice

In addition to the problems below, one can get routine by solving similar exercises from the exercise-book "övningar i flerdimensionell analys".

5.1 Compute the integral $\int\int_D y\sin(y+xy)dxdy$, where $D=\{(x,y): 0\leq x\leq 1, -\pi/2\leq y\leq \pi/2\}$.

5.2 Compute the integral $\int\int_D e^{xy}(1+xy)dxdy$, where $D=\{(x,y): 0\leq x\leq 1, 1\leq y\leq 2\}$.

5.3 Compute the integral $\int\int_D \frac{xy}{(1+y^2)^2}dxdy$, where $D=\{(x,y): x,y\geq 0, x^2+y^2\leq 1\}$.

5.4 Compute the integral $\int\int_D x^2e^{x^2+y^2}dxdy$, where $D=\{(x,y): x^2+y^2\leq 25, y\geq |x|\}$.

5.5 Compute the integral $\int\int_D (x^2-y^2)e^{2xy}dxdy$, where $D=\{(x,y): x^2+y^2\leq 1, -x\leq y\leq x, x\geq 0\}$.

5.6 Compute the integral $\int\int_D (x^2+y^2)dxdy$, where $D=\{(x,y): x^2/4+y^2/9\leq 1\}$.