

A5

Consider the heat equation with constant coefficients in the interval $\mathcal{I} = (0, L)$:

$$\begin{aligned}u_t &= au_{xx}, & x \in \mathcal{I}, & \quad t > 0, \\u &= g, & x = 0, & \quad t > 0, \\u_x &= 0, & x = L, & \quad t > 0, \\u &= f, & x \in \mathcal{I}, & \quad t = 0,\end{aligned}$$

where $g, a > 0$, and $L > 0$ are real constants. We assume that the solution is real.

Let V be the space of functions $v(x, t)$ such that v and v_x are square-integrable over \mathcal{I} for any t :

$$V = \{v(x, t) : \|v(\cdot, t)\| + \|v_x(\cdot, t)\| < \infty\}.$$

Further, define two spaces of functions that additionally satisfy Dirichlet boundary conditions at $x = 0$:

$$V_0 = \{v \in V : v(0, t) = 0\}$$

$$V_g = \{v \in V : v(0, t) = g\}.$$

Below is the weak form of the IBVP, but there are three gaps (X1, X2, and X3) that you need to fill in! Choose from the 7 alternatives listed below. To score 1 point, you need to get all gaps (X1, X2, and X3) right.

Find X1 such that

X2

for all X3.

Alt. 1: $u \in V_g$

Alt. 2: $u \in V_0$

Alt. 3: $v \in V_g$

Alt. 4: $v \in V_0$

Alt. 5: $u_t = au_{xx}$

Alt. 6: $(v, u_t) = -a(v_x, u_x)$

Alt. 7: $(v, u_t) = -a(v_x, u_x) + agv(L, t)$