Journal of Experimental Child Psychology 199 (2020) 104931

Contents lists available at ScienceDirect

Journal of Experimental Child Psychology

journal homepage: www.elsevier.com/locate/jecp

Children's discrete proportional reasoning is related to inhibitory control and enhanced by priming continuous representations

Roberto A. Abreu-Mendoza[a,*], Linsah Coulanges[a,1], Kendell Ali[b], Arthur B. Powell[c], Miriam Rosenberg-Lee[a,d]

[a] Department of Psychology, Rutgers University–Newark, Newark, NJ 07103, USA
[b] Graduate School of Education, Rutgers University, New Brunswick, NJ 08901, USA
[c] Department of Urban Education, Rutgers University–Newark, Newark, NJ 07103, USA
[d] Center for Molecular and Behavioral Neuroscience, Rutgers University-Newark

ARTICLE INFO

Article history:
Received 18 February 2020
Revised 12 June 2020

Keywords:
Proportional reasoning
Inhibitory control
Mathematical development
Continuous and discrete magnitudes
Individual differences
Numerical cognition

ABSTRACT

Children can successfully compare continuous proportions as early as 4 years of age, yet they struggle to compare discrete proportions at least to 10 years of age, especially when the discrete information is misleading. This study examined whether inhibitory control contributes to individual differences in discrete proportional reasoning and whether reasoning could be enhanced by priming continuous information. A total of 49 second-graders completed two tasks. In the Hearts and Flowers (H&F) task, a measure of inhibition, children pressed on either the corresponding or opposite side, depending on the identity of the displayed figure. In the Spinners task, a measure of proportional reasoning, children chose the spinner with the proportionally larger red area across continuous and two discrete formats. In the discrete adjacent format, the continuous stimuli were segmented into sections, which could be compatible with the proportional information or misleading; the discrete mixed format interspersed the colored sections from the discrete adjacent conditions. Finally, two priming groups were formed. Children who saw the continuous format immediately before the discrete adjacent format formed the continuous priming group (n = 26). Children who saw the discrete mixed format immediately before the discrete adjacent format formed the discrete priming group (n = 23). Our results showed that children who performed better

* Corresponding author.
E-mail address: roberto.abreu@rutgers.edu (R.A. Abreu-Mendoza).
[1] Current address: Department of Psychology, University of Pittsburgh, Pittsburgh, PA 15260, USA.

https://doi.org/10.1016/j.jecp.2020.104931

on the H&F task also had better performance on the discrete counting misleading trials. Furthermore, children in the continuous priming group outperformed children in the discrete priming group, specifically in contexts where discrete information was misleading. These results suggest that children's proportional reasoning may be improved by fostering continuous representations of discrete stimuli and by enhancing inhibitory control skills.

Introduction

Children can reason about nonsymbolic ratios several years before they can work easily with the same quantities represented symbolically. Before starting elementary school, children can reliably indicate which of two figures has the proportionally larger target area (Hurst & Cordes, 2016), and kindergarteners can identify objects of differing sizes that have the same ratio of colored areas (Boyer & Levine, 2015; Boyer, Levine, & Huttenlocher, 2008). These tasks are thought to require *proportional reasoning,* that is, "understanding the multiplicative relationships between rational quantities" (Boyer et al., 2008, p. 1478).

Despite these capacities, when proportions are segmented into discrete elements, children often ignore multiplicative relationships and instead tend to focus on the absolute number of pieces. This bias is especially problematic when discrete information and proportional information are at odds with each other. Children's performance plummets when counting information contradicts proportional information (Jeong, Levine, & Huttenlocher, 2007), and even adults resort to counting strategies in these cases (Plummer, DeWolf, Bassok, Gordon, & Holyoak, 2017). Thus, a critical step in correctly accessing nonsymbolic proportional information is inhibiting misleading discrete information, suggesting that individuals who are better at resolving interferences in general may show enhanced proportional reasoning abilities. Yet, to date, there is no empirical evidence for the link between individual differences in proportional reasoning and the executive function capacity of inhibitory control. In this study, we examined the contributions of inhibitory control to proportional reasoning in second-grade children, who had yet to receive any formal fraction instruction, particularly in those cases where there is misleading discrete information.

Contributions of inhibitory control to proportional reasoning

Emerging evidence suggests that inhibitory control is involved in successful processing of symbolic ratios (Avgerinou & Tolmie, 2020; Coulanges et al., 2020; Gomez, Jimenez, Bobadilla, Reyes, & Dartnell, 2015). Avgerinou and Tolmie (2020) found that 8- to 10-year-old children's inhibitory control skills were positively related to their ability to compare rational numbers, both fractions and decimals. Importantly, children's inhibitory control skills were specifically related to their performance on comparisons that had misleading information, for example, trials where the largest fraction had the smallest denominator. These results add to the growing evidence suggesting that inhibitory control plays a role in reducing the impact of misleading information in numerosity processing more generally (Gilmore et al., 2013; Norris & Castronovo, 2016) and even in scientific reasoning when content contradicts learners' prior knowledge (Bascandziev, Tardiff, Zaitchik, & Carey, 2018; Brookman-Byrne, Mareschal, Tolmie, & Dumontheil, 2018). Here, we aimed to investigate, for the first time in children, whether inhibitory control is related to nonsymbolic proportional reasoning, especially in contexts with misleading discrete information.

Distinguishing between contexts with and without misleading discrete information may account for the mixed patterns of results when studies examine relations between inhibitory control and fraction understanding in adults. Matthews, Lewis, and Hubbard (2016) found that performance on a nonsymbolic proportional reasoning task significantly predicted symbolic fraction skills even after controlling for performance on a flanker task, their measure of inhibitory control. However, they combined continuous and discrete stimuli and did not consider the independent contributions of consistent and inconsistent trials in their predictors or outcomes. Nor did they examine the effects of segmenting the nonsymbolic continuous proportions.

The lack of consensus on how best to measure inhibitory control (Lee & Lee, 2019) may also contribute to the mixed evidence for the contribution of inhibitory control to proportional reasoning. For instance, Gomez et al. (2015) found a positive relationship between performance on a numerical Stroop task and performance on a fraction comparison task, a measure of symbolic proportional reasoning. However, the authors reported that children's mathematical performance mediated this relationship, suggesting that inhibitory control did not uniquely drive fraction comparison performance. Still, the mediation effect could have been due in part to the fact that their measure of inhibitory control involved numerical magnitude understanding skills, which might overestimate the relationship between inhibition and mathematical skills. To avoid potential confounds between measures, in the current study we employed a domain-general inhibitory control measure, the Hearts and Flowers (H&F) task (Davidson, Amso, Anderson, & Diamond, 2006; Wright & Diamond, 2014). This task not only shows a clear developmental trajectory from 4 to 26 years of age (Davidson et al., 2006) but also has appropriate concurrent validity with a latent variable of inhibition that comprises performance on the color–word and animal Stroop tasks (Brocki & Tillman, 2014).

Priming effects in nonsymbolic proportional reasoning

These individual difference studies of symbolic proportional reasoning suggest that successful nonsymbolic proportional reasoning of discrete stimuli may also require inhibitory control to overcome interfering counting information, yet they do not identify the appropriate representation for processing discrete stimuli. One possibility is suggested by studies reporting reduced interference from counting information when children are primed with nonsymbolic continuous representations of proportions. For example, Boyer and Levine (2015) examined whether presenting proportions in a continuous format could prompt proportional reasoning in discrete format trials that were presented after the continuous ones. They found that 10-year-olds, but not 6- or 8-year-olds, benefited from being prompted with continuous proportions.

Similarly, Hurst and Cordes (2018) reported that when 5-year-old children saw continuous proportional stimuli before discrete stimuli, they successfully compared trials with and without discrete misleading information. However, this benefit of priming was not observed in 3- or 7-year-old children. These results suggest that working with continuous stimuli can prompt proportional reasoning in subsequent discrete stimuli; however, the evidence is mixed about what age children benefit from continuous priming. The discrepancies between these studies may stem from the difficulty of the tasks employed. In both studies, only age groups not at ceiling or floor (chance level) benefited from the continuous priming, suggesting that it may be beneficial only when there is sufficient variation within an age group. In the current study, we employed a proportion comparison task that is simpler than the match-to-sample task employed by Boyer and Levine (2015) but used a more challenging set of stimuli than those used by Hurst and Cordes (2018). Thus, this task may better afford detection priming effects in 7- and 8-year-olds.

The current study

The goals of the study were threefold. First, we aimed to provide empirical evidence for the presumed relationship between inhibitory control and proportional reasoning in contexts where proportional and whole-number counting information were at odds. Thus, we hypothesized that better inhibitory control skills, measured by the H&F task, would correlate with better proportional reasoning in misleading contexts, whereas such correlation would be absent in contexts where there is no discrete information (i.e., comparisons of continuous stimuli) and when discrete information is congruent with proportional information. Second, although it was not originally designed to test this hypothesis, we leveraged our counterbalanced study design to further examine whether proportional reasoning could be enhanced by priming continuous information. For this exploratory analysis, we hypothesized that seeing continuous information in a nonsymbolic proportion comparison task would facilitate performance on discrete comparisons. Finally, we examined whether the facilitating effects of continuous information were related to, or independent of, individual differences in inhibitory control. If these two effects are independent, it would suggest that improving proportional reasoning in children could be achieved either by providing the right representation or by helping children to overcome misleading counting strategies. Alternatively, if the facilitating effects of continuous priming mitigate effects of poor inhibitory control, we might expect an interaction between these factors.

Method

Participants

A total of 49 second-grade children aged 6.9 to 8.1 years (M_{age} = 7.53 years, *SD* = 0.30; 28 boys and 21 girls) from a public school in Newark, New Jersey participated in this study. Fully 89% of students at this school were eligible for free or reduced-price lunch. Originally, the full sample consisted of 52 children; however, 3 children were excluded either because their age suggested that they were likely to have failed a grade (n = 1) or because they did not have the minimum number of usable trials in the experimental tasks (n = 2). All children's parents gave written informed consent, and children gave oral assent for their participation. All protocols were approved by the Rutgers University institutional review board.

This sample was part of a larger study that aims to evaluate the effects of an after-school enrichment program on children's proportional reasoning skills. Because a subset of second-graders in the after-school program formed the training group, we invited all students in that grade to participate in the current study so that the remainder would form the control group. From the total population of 57 second-graders at the school, 52 parents (91%) provided consent. Sensitivity power analyses indicated that the final sample size (N = 49) enables detecting medium to large correlations (Pearson's r > .38), medium to large within-participant differences (Cohen's d > .61), and large between-participants differences (Cohen's d > .82) using alpha = .05 and power = .80.

Materials

Standardized mathematics assessment

To evaluate children's mathematical achievement, we used the Math Fluency–Addition and Math Fluency–Subtraction subtests from the Wechsler Individual Achievement Test–Third Edition (WIAT-III; Wechsler, 2009). In each test, children answered as many addition (subtraction) problems as they could in 1 min. The dependent variable for these subtests was the Math Fluency composite score, which combined the grade-normed scores of each subtest.

Proportional reasoning

We implemented a computerized version of the Spinners task (Jeong et al., 2007) to measure children's proportional reasoning. In this task, children indicated which of two spinners had the proportionally larger red area. The task started with on-screen instructions that the experimenter read aloud to the children. Then, the experimenter presented children with a set of three stickers and told them that they could win more stickers during the task. Next, the experimenter explained that children would see two spinners, one on the left and one on the right, which would spin and that they needed to select one. After spinning, if the arrow pointed to the red part of the spinner, children would win a sticker, but if the arrow stopped on the blue part, they would lose a sticker. Children were instructed that their task was to pick the spinner they wanted to play with to win another sticker. Next, children saw a video of two spinners of equal size spinning and coming to stop with the arrow pointed to the red part of the spinner with the proportionally larger red area and the arrow pointed to the blue part of the other spinner that had a proportionally smaller red area. Children were reminded that they would have won a sticker if they had chosen the spinner where the arrow landed on the red part but would have lost one if they had chosen the other spinner, which landed on the blue part. Before the experimental trials began, children performed a practice trial to familiarize themselves with pressing the corresponding key for each side. In the experimental trials, the spinners did not spin; however, at the end of each of the three blocks, there was an additional trial in which the spinners spun and the arrow always landed in the red portion on the side that children selected regardless of the proportion presented. These trials were not included in the analyses, and all children received six stickers at the end of the task because they had always made the "correct" choice on the 3 sticker trials. After completing a sticker trial, children were reminded of the instructions before starting the next block.

The 12 proportions used by Jeong et al. (2007) were presented in three different format blocks for a total of 36 experimental trials. In the *continuous* format, each spinner had only two continuous sections, one red and one blue (Fig. 1A). In the *discrete adjacent* format, the two continuous parts were broken in discrete but adjacent sections of red and blue segments (Fig. 1B). In the *discrete mixed* format, the red and blue segments were interspersed (Fig. 1C). In the discrete blocks, the number of segments was manipulated so that in half of the trials the spinner with the larger number of red pieces was also the one with the proportionally larger red area (*counting consistent trials*), whereas in the other half the spinner with the fewer red pieces was the one with the proportionally larger red area (*counting misleading trials*). Although "counting" information could not be meaningfully assessed in the continuous format, trials that had the same proportions as the counting consistent trials of the discrete formats were considered continuous "counting consistent" trials by convention; similarly, continuous trials that showed the same proportions as the counting misleading trials were considered continuous "counting misleading" trials. Fig. 1A and B illustrate the correspondence between continuous and discrete "counting consistent" stimuli (i.e., 2/7 vs. 1/5 in left panels) and "counting misleading" stimuli (i.e., 3/5 vs. 4/8 in right panels).

For all formats, we also manipulated the size of the individual spinners to prevent children from relying on the absolute size of the red area in making their selections. Thus, on half of the trials of each format the physically larger spinner also had the proportionally larger red area (*size congruent trials*), whereas on the other half the opposite pattern held, with the smaller spinner being the one with the proportionally larger red area (*size incongruent trials*). Spinners could be 6, 9, or 12 cm in diameter. For size congruent trials the proportionally larger spinner was always the 12-cm spinner and the other spinner could be 6 or 9 cm, whereas for size incongruent trials the proportionally larger spinner was the 6-cm spinner and the other spinner was 9 or 12 cm. Proportion pairs (size congruent or size incongruent) were counterbalanced across participants.

For all participants, the continuous condition was presented first and the presentation order of the two discrete blocks was counterbalanced across participants, generating the priming conditions. Size congruency and counting consistency were counterbalanced within each block; however, due to an experimental error, the four condition combinations (size congruent–counting consistent, size congruent–counting misleading, size incongruent–counting consistent, and size incongruent–counting misleading) were unevenly distributed within each block. Across participants, some blocks had 3 trials in each condition (3–3–3–3), whereas others were 2–4–4–2 or 4–2–2–4, respectively. For further details, see Table S1 in the online supplementary material, where we provide summary details of the three different trial distribution groups (hereafter *design group factor*), which we use to assess any effects of this experimental error. Importantly, none of the reported effects was due to this error.

For all conditions, trials started with a blank screen presented for 500 ms, followed by the pair of spinners. Spinners remained on the screen until the children responded by pressing one of two possible keys, "z" for the left spinner or "m" for the right one. Within each block, half of the correct responses were presented on the left and the other half were presented on the right.

The dependent variable for this task was the proportion of correct responses (accuracy). Following Wright and Diamond, (2014), and for consistency with the inhibitory control measure, when computing accuracy we excluded anticipatory responses (reaction times [RT] shorter than 250 ms) and outlier responses (RTs at least 3 standard deviations above the individual's mean). After applying these criteria, 1 participant from the full sample did not have at least 1 trial from each type and was excluded from the final sample. Thus, among the 49 children of the final sample, we analyzed 1713 (97.11%) of 1764 trials.

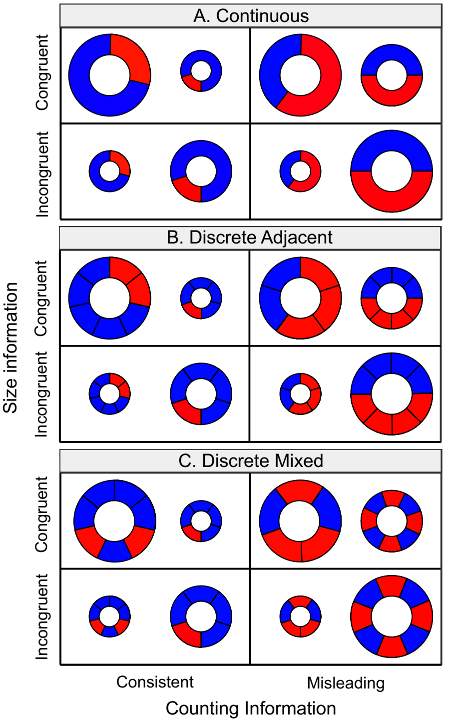

Fig. 1. Examples of the stimuli used in the Spinners task, where participants needed to choose the spinner with the proportionally larger red area: (A) continuous format; (B) discrete adjacent format; (C) discrete mixed format. All spinner pairs in the left column show a 2/7 versus 1/5 comparison, whereas pairs in the right column show a 3/5 versus 4/8 comparison. For each of the three formats, the upper row presents pairs where the larger spinner is the one with the proportionally larger red area (size congruent trials); the lower row shows pairs where the opposite pattern holds (size incongruent trials). Finally, for the two discrete formats, columns also represent the counting information. In the left column, there are pairs of spinners where the spinner with the larger number of pieces is the correct answer (counting consistent trials; 2/7 vs. 1/5). In the right column, there are pairs where the spinner with the larger number of pieces is the incorrect answer (counting misleading trials; 3/5 vs. 4/8). (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

In the priming manipulation, participants were randomly assigned to one of two groups that showed the formats of the Spinners task in different orders. Children who saw the discrete adjacent block immediately after the continuous format formed the continuous priming group (n = 26). This order might promote proportional reasoning in the discrete adjacent trials. Children who saw the continuous format, but then the discrete mixed format, before ending with the discrete adjacent format formed the discrete priming group (n = 23). This order might promote the use of counting strategies.

Inhibitory control measure

Children's inhibitory control was assessed with the H&F task (Brocki & Tillman, 2014; Davidson et al., 2006; Wright & Diamond, 2014). This computerized task consisted of three blocks presented in the following fixed order: congruent, incongruent, and mixed. The experimenter read aloud the on-screen instructions to the children. In the congruent block, children were instructed to press the key on the same side as where the target (hearts) appeared, using the same keys as in the Spinners task (the letter "z" for the left side and the letter "m" for the right side. In the incongruent block, children were instructed to press the key on the opposite side from where the target (flowers) appeared. In the mixed block, children saw interspersed hearts and flowers and were asked to respond according to the previously learned rules. At the beginning of the congruent and incongruent blocks, there were 2 example trials. The corresponding figure (heart or flower) appeared first on the right and then on the left. The target images remained on the screen until children pressed the correct key. The first two blocks comprised 12 trials each, which randomly presented the corresponding figure on each side 6 times. The third (mixed) block comprised 33 trials, and the first trial of this block was always a heart presented on the right side. Subsequent trials randomly presented each figure 16 times, 8 times on each side. We considered this last block as the measure of inhibitory control because prior research found that performance in the mixed block was strongly correlated with a latent variable of inhibition ($r = .71$), whereas performance in the congruent block and performance in the incongruent block were negative ($r = -.03$) and weakly associated ($r = .17$), respectively (Brocki & Tillman, 2014).

Before the experimental blocks, there were 4 practice trials for the congruent and incongruent blocks, whereas there were 8 practice trials for the mixed block. All trials started with a fixation cross presented for 500 ms, followed by a blank screen for 500 ms. Then, within a horizontal rectangle presented at the center of the screen, children saw the target image (a heart or flower, depending on the block). After 750 ms, the rectangle and the image disappeared and were replaced by a blank screen that lasted 500 ms before the next trial began. Children needed to respond during this 1250-ms period.

Following the same procedure as in the Spinners task, the dependent variable was the proportion of correct responses (accuracy), excluding anticipatory and outlier responses. One participant of the full sample lost more than 20% of the trials and was excluded from the final sample. Thus, we analyzed 2742 (98.17%) of 2793 trials from the 49 participants.

Positive Attitude toward Math questionnaire

As part of the larger training study, we also administered the Positive Attitude toward Math (PAM) questionnaire to evaluate the relationship between children's attitudes toward math and learning gains. Following Chen et al. (2018), we used a 5-point Likert-type scale but employed emojis to help connote the response options. The measure comprises 12 questions; the first 6 questions evaluate children's attitude toward math, and the other 6 evaluate their general attitude toward academics.

General procedure

Children performed the tasks in the following fixed order: Math Fluency–Addition subtest, Math Fluency–Subtraction subtest, the Spinners task, the H&F task, and the PAM questionnaire (Chen et al., 2018), which is not reported here. Both the Spinners and H&F tasks were presented using PsychoPy2 Experiment Builder Version 1.90.3 (Pierce, 2007). Children were evaluated individually by trained experimenters in quiet corners of a large room at the children's school (maximum of 4 children at a time). The evaluation lasted approximately 25 min.

Statistical analyses

Statistical analyses were conducted in R 3.5.3 (R Core Team, 2019). As measures of effect sizes, we reported the absolute values of Cohen's d for t tests and generalized eta squared (η_g^2) for analyses of variance (ANOVAs) and analyses of covariance (ANCOVAs). For ANOVAs, generalized eta squared is preferred over partial eta squared because it allows comparing between-participants and within-participant designs (Bakeman, 2005). For this measure of effect size, a value of .020 is considered small, a value of .130 is considered medium, and a value of .260 is considered large.

Results

Mathematical performance

Children's mean WIAT Math Fluency composite score, which combines the Addition and Subtraction subtests, was 80.65 ($SD = 9.96$, range = 63–106), indicating that children performed below average relative to their grade.

Proportional reasoning

To evaluate how children's performance was affected by the spinners' format, size, and counting information, we performed a repeated-measures ANOVA with format (continuous, discrete adjacent, or discrete mixed), size (congruent or incongruent), and counting (consistent or misleading) as within-participant factors and accuracy as the dependent variable. This analysis yielded main effects of size, $F(1, 48) = 31.95$, $p < .001$, $\eta^2_g = .066$, and counting, $F(1, 48) = 35.54$, $p < .001$, $\eta^2_g = .096$, which were qualified by an interaction between these factors, $F(1, 48) = 5.58$, $p = .022$, $\eta^2_g = .005$ (Fig. 2A). Specifically, children's performance was particularly low on trials where the smaller spinner, with fewer pieces, had the proportionally larger red area. Children's performance on these trials was not better than chance, $t(48) = 0.93$, $p = .354$, Cohen's $d = .13$. Although there was no main effect of format, $F(2, 96) = 0.05$, $p = .955$, $\eta^2_g < .001$, because performance across conditions was similar when collapsing both types of counting trials (consistent and misleading), there was an interaction between format and counting, $F(2, 96) = 14.79$, $p < .001$, $\eta^2_g = .049$ (Fig. 2B). Counting strategies are possible only when the sections of the spinners are broken into segments; thus, this effect should be absent in the continuous format but present in the discrete formats. Consistently, children's performance was impaired in the misleading conditions of the discrete formats but not in the continuous condition.

Inhibitory control

Next, to examine the effect of cognitive control demands on children's performance on the H&F task, we conducted a repeated-measures ANOVA with block (congruent, incongruent, or mixed) as a within-participant factor and accuracy as the dependent variable. This analysis yielded a main effect of block, $F(2, 96) = 46.10$, $p < .001$, $\eta^2_g = .335$. Children had higher accuracy in the congruent block than in the incongruent block, which was in turn higher than accuracy in the mixed block. In fact, performance in the mixed block was not better than chance, $t(48) = 1.64$, $p = .11$, Cohen's $d = .23$ (Fig. 3).

Relations between proportional reasoning and inhibitory control

To investigate whether better inhibitory control results in better proportional reasoning skills, we examined Pearson correlations between the mixed trials of the H&F task and performance on the counting misleading trials of the discrete adjacent format. In our sample, children on average were around chance level (50%) in the mixed block, with only 1 child being at an extreme level (>90%). In contrast, in the incongruent block, there were 15 children above 90% and another 5 children below 10%, suggesting that in this condition many children were not engaging inhibition but rather just applying a single response rule. These results further support using the mixed block as the measure of children's inhibitory control. For the Spinners task, we were particularly interested in the counting misleading trials of the discrete adjacent format because the only difference between the continuous trials and these trials was the segmentation lines, allowing a more direct exploration of the effect of discrete information on proportional reasoning. Furthermore, as opposed to the counting misleading trials of the discrete mixed format, these trials did not require a mental manipulation to bring the red pieces together.

As expected, children who performed better on these taxing comparisons also had higher accuracy on mixed trials, $r(47) = .32$, $p = .023$ (Fig. 4, upper panel). Furthermore, if this correlation is driven by the contribution of inhibitory control to avoiding inappropriate counting strategies, inhibitory control should not be associated with comparison performance in contexts where these strategies are not feasible (continuous trials) or where using them leads to the correct answer (discrete adjacent and counting consistent trials). Accordingly, the Pearson correlation between performance in the mixed block of H&F and the misleading trials of the continuous format was not significant, $r(47) = .13$, $p = .374$ (Fig. 4, left lower panel), nor was it significantly related to performance on the consistent trials of the discrete adjacent format, $r(47) = .16$, $p = .262$ (Fig. 4, right lower panel). However, the Steiger's Z test (Steiger, 1980) for nonindependent group values indicated that the two latter correlation coefficients were not different from the one between performance in the mixed block and counting misleading discrete

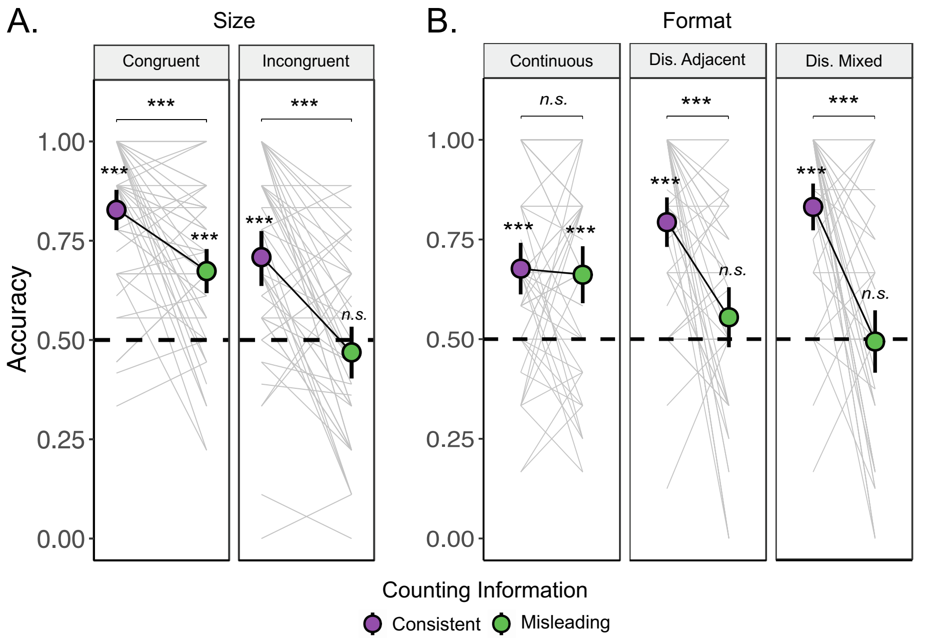

Fig. 2. Performance on the Spinners task. (A) The effect of counting information differed between the two size conditions. Children's performance was particularly low on trials where the smaller spinner with fewer pieces had the proportionally larger red area (size incongruent and counting misleading trials); on these trials, children's performance was no better than chance. In contrast, performance was highest on trials where the larger spinner with the larger number of pieces had the proportionally larger red area (size congruent and counting consistent trials). (B) Counting information affected children's performance differently across formats. In the continuous format, because there was no counting information, there was no difference between the two types of counting trials. In contrast, for the discrete (Dis.) adjacent and discrete mixed formats, misleading counting information impaired performance and was no better than chance. $^{***}p < .001$; n.s., not significant. (For interpretation of the references to color in this figure legend, the reader is referred to the Web version of this article.)

adjacent trials (*ps* > .30). These results were not driven by the distribution of trials seen (see Table S1 in supplementary material).

Exploratory analyses

Effects of continuous and discrete priming

The two priming groups did not differ in age, $t(47) = 1.32$, $p = .19$, Cohen's $d = .38$, math performance, $t(47) = 0.98$, $p = .33$, Cohen's $d = .28$, or inhibitory control skills, $t(47) = 0.13$, $p = .90$, Cohen's $d = .04$ (see Table 1).

To explore whether comparing continuous proportions had a short-term facilitating effect on children's discrete proportional reasoning, we added priming group as a between-participants factor to the original repeated-measures ANOVA, which considered the effects of format, size, and counting on Spinners task accuracy. In addition to the same main effects and interactions of the previous ANOVA, this mixed-design repeated-measures ANOVA yielded a significant interaction between priming group and counting, $F(1, 47) = 4.90$, $p = .032$, $\eta^2_g = .014$, which was qualified by a marginal three-way interaction among priming group, counting, and format, $F(2, 94) = 2.73$, $p = .071$, $\eta^2_g = .009$. Importantly, there was no significant interaction between priming group and format, $F(2, 94) = 2.23$, $p = .11$, $\eta^2_g = .008$, suggesting that performance in each format was not affected by presentation order. Next, we performed separate follow-up ANOVAs for each of the formats with priming group as a between-participants factor and counting consistency as a within-participant factor. The ANOVA for the discrete adjacent format showed no main effect of priming group, $F(1, 47) = 0.84$, $p = .36$, $\eta^2_g = .001$, indicating

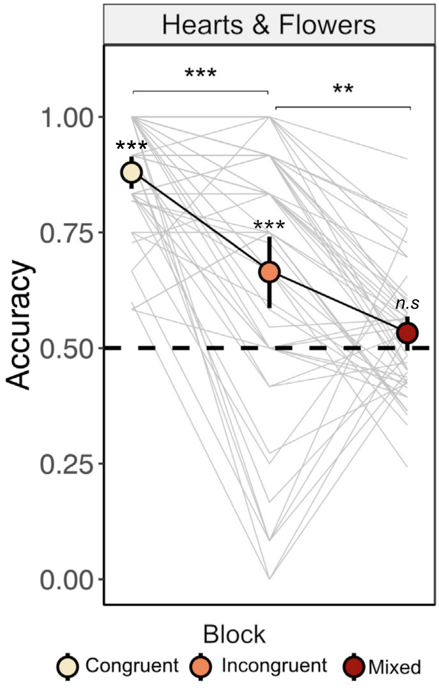

Fig. 3. Performance on the Hearts and Flowers task. Children's performance varied depending on the inhibitory control demands of each block. Their performance was the highest in the congruent block, where they needed to press a key on the same side as where the figures appeared, followed by the incongruent block, where they needed to press a key on the opposite side. Children's performance was the lowest in the mixed block, where they saw interspersed congruent and incongruent trials. **$p < .01$; ***$p < .001$; n.s., not significant.

that even though children in the continuous priming group saw the discrete adjacent trials as the second block, whereas children in the discrete priming group saw it as the third block, differences between their performance were not likely due to fatigue leading to overall lower performance later in the experiment. Instead, we found the expected main effect of counting, $F(1, 47) = 27.83$, $p < .001$, $\eta^2_g = .20$, but it was qualified by an interaction between priming group and counting, $F(1, 47) = 11.24$, $p = .002$, $\eta^2_g = .094$ (Fig. 5A). Specifically, children in the continuous priming group performed significantly better on the discrete misleading trials, $t(47) = 2.67$, $p = .010$, Cohen's $d = .76$, but performed marginally worse on the discrete consistent trials, $t(47) = 1.67$, $p = .101$, Cohen's $d = .48$, relative to children in the discrete priming group. Furthermore, in contrast to the discrete group, children in the continuous group performed significantly above chance on the misleading trials (65%), $t(25) = 3.46$, $p = .002$, Cohen's $d = .68$. Taken together, these results indicate that priming continuous representations may allow children to extend their continuous strategies to discrete comparisons, especially for misleading stimuli. Finally, the imbalanced number of trials did not drive these effects; children who saw the three different trial distributions (design group factor) were evenly distributed across the two priming groups, $\chi^2(2, N = 49) = 0.40$, $p = .820$ (see Table S2 in supplementary material), and there was no main effect of design group or significant interactions (all $ps > .12$) (Table S3).

The ANOVAs for the other two formats showed only the expected effects: for the discrete mixed format, only a main effect of counting, $F(1, 47) = 35.48$, $p < .001$, $\eta^2_g = .298$; for the continuous format, no main effects or interactions. Furthermore, there were no main effects or interactions with priming group (all $ps > .20$) for either format. Coupled with the lack of difference in age, math ability, and inhibitory control capacity between the priming groups, these results suggest that facilitation effects of proportional priming were not driven by confounding individual differences between the groups.

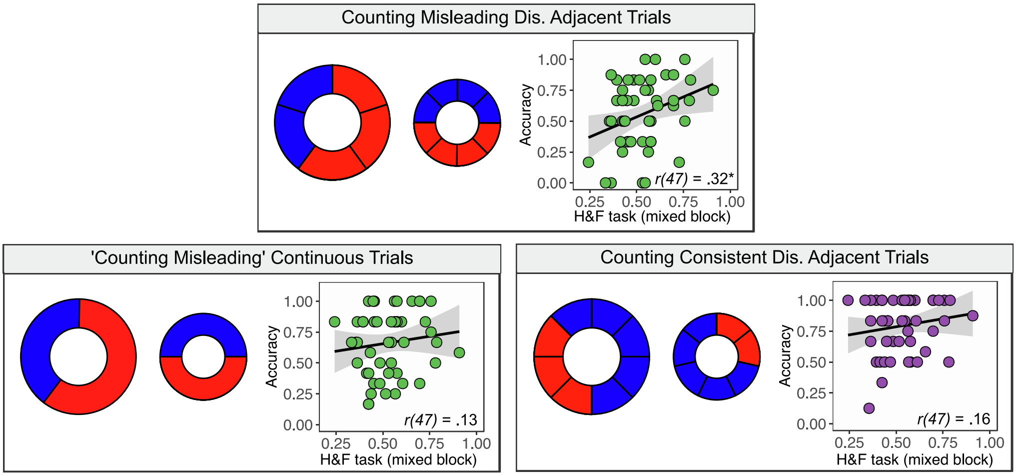

Fig. 4. Associations between proportional reasoning and inhibitory control. Children with greater inhibitory control skills, measured by the mixed block of the Hearts and Flowers (H&F) task, also performed better on the counting misleading discrete (Dis.) adjacent trials. In contrast, there was no significant correlation with same ratios in continuous counting misleading trials or with counting consistent discrete trials. *p < .05.

Table 1
Groups' mean age and control measures.

	Continuous priming group (n = 26) M (SD)	Discrete priming group (n = 23) M (SD)	t	p
Age (years)	7.48 (0.31)	7.60 (0.27)	1.32	.19
WIAT Math Fluency	79.34 (11.23)	82.13 (8.30)	0.98	.33
H&F mixed trials	0.53 (0.12)	0.53 (0.16)	0.13	.90

Note. WIAT, Wechsler Individual Achievement Test; H&F, Hearts and Flowers.

Relations between priming and inhibitory control

Finally, to examine whether the facilitation effect of continuous information was related to or independent of individual differences in inhibitory control, we performed Pearson correlations between performance on the counting misleading trials of the discrete adjacent format and performance on the mixed trials of the H&F task for each group separately. These analyses revealed a significant correlation for the discrete priming group, $r(21)$ = .48, p = .022 (Fig. 5B, left panel), but not for the continuous priming group, $r(24)$ = .16, p = .428 (Fig. 5B, right panel). However, an ANCOVA with group as a between-participants factor, performance in the mixed block of the H&F task as a covariate, and performance on the counting misleading trials of the discrete adjacent format as the dependent variable showed the expected main effect of group, $F(1, 45)$ = 8.30, p = .006, η_g^2 = .156, and the H&F task as a significant covariate, $F(1, 45)$ = 6.42, p = .015, η_g^2 = .129, but no interaction between group and performance on the H&F task, $F(1, 45)$ = 1.35, p = .252, η_g^2 = .029, suggesting that both priming and inhibitory control demonstrated independent contributions to proportional reasoning.

Discussion

In this study, we investigated the relationship between inhibitory control and proportional reasoning in contexts where whole-number information and proportional information contradict each other. Although research has shown that inhibitory control is related to incongruent information in other numerical and mathematical domains (Gilmore et al., 2013; Gilmore, Keeble, Richardson, & Cragg, 2015), to the best of our knowledge, our results are the first to demonstrate that children who had

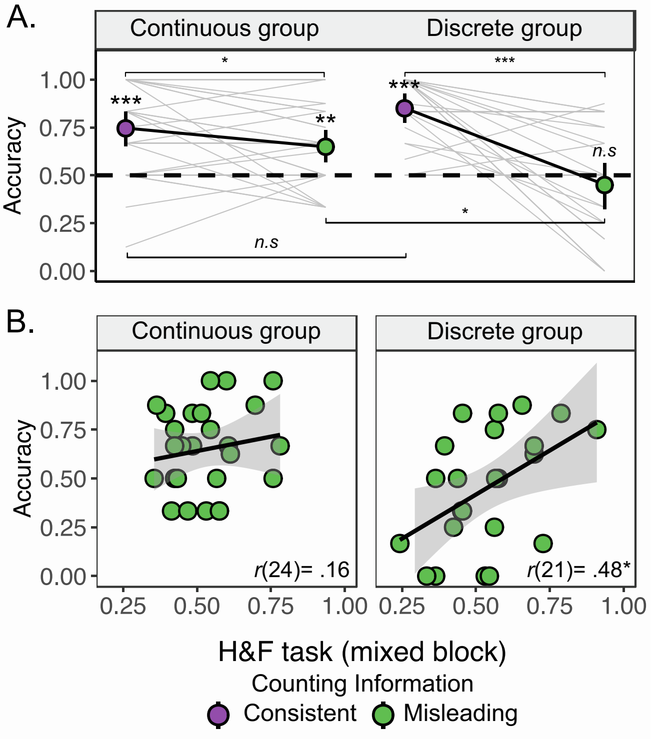

Fig. 5. Priming groups' performance in the discrete adjacent trials. (A) For the discrete misleading trials, children in the continuous priming group performed above chance and significantly better than their peers in the discrete priming group. (B) Inhibitory control, as measured by the Hearts and Flowers (H&F) task, was associated with performance in the counting misleading trials for children in the discrete priming group but not for children in the continuous priming group. $^{*}p < .05$; $^{**}p < .01$; $^{***}p < .001$; n.s., not significant.

greater inhibitory control skills also had better proportional reasoning skills specifically when required to overcome misleading discrete information. As part of our exploratory analyses, we also examined whether continuous information could prime proportional processing and consequently enhance second-graders' discrete proportional reasoning. Consistent with previous studies (Boyer & Levine, 2015; Hurst & Cordes, 2018), we found that children who saw the continuous format immediately before the discrete adjacent format outperformed children who saw the discrete mixed format immediately before the discrete adjacent format, specifically on discrete misleading trials.

Inhibitory control supports overcoming misleading discrete information

Which abilities help children to suppress the interfering effects of misleading discrete information on proportional reasoning? A recent theoretical framework proposed that inhibitory control helps leaners to avoid using intuitive incorrect responses and instead promotes analyzing problems more carefully (Van Dooren & Inglis, 2015). Growing evidence supports this framework; inhibitory control contributes to adolescents' reasoning about counterintuitive scientific facts (Brookman-Byrne et al., 2018), children's performance on numerical tasks with misleading visual information (Gilmore et al., 2013), and children's avoidance of incorrect strategies in measurement tasks (Ren, Lin, & Gunderson, 2019). In the context of discrete proportional reasoning, one possible intuitive response strategy is counting the segments. At least during adulthood, eye-tracking studies have shown that when proportions are presented in a discrete format, they elicit counting strategies (Plummer et al., 2017). Because performance on the H&F task showed a moderate relationship with the discrete adjacent counting misleading trials of the Spinners task that might elicit a counting strategy, we suggest that the role of inhibitory control in proportional reasoning is to help children avoid using this strategy and instead focus on the proportional information. Consistently, children's inhibitory control was weakly and not significantly associated with proportional reasoning when there was no misleading information (continuous trials) and when both discrete information and proportional information were consistent (discrete adjacent consistent trials). However, differences between correlation coefficients did not reach significance, suggesting that inhibitory control may also contribute to these other trial types. Future studies with larger samples are needed to provide definitive evidence of either a general role for inhibition in proportional reasoning or a specific role in misleading discrete cases.

This framework also helps to reconcile our results with previous studies where inhibitory control did not play a role in symbolic proportional reasoning (Gomez et al., 2015; Matthews et al., 2016). For example, Matthews et al. (2016) reported that the relation between nonsymbolic proportional reasoning and fraction comparison skills was independent of inhibitory control. However, their measure of proportional reasoning combined continuous and discrete representations, and it did not distinguish between those trials in which the proportional information and discrete information were consistent from those in which they were not, potentially diluting the role of inhibitory control. Furthermore, in studies of fractions and decimals that did distinguish these types of trials and did find correlations with measures of inhibitory control (Avgerinou & Tolmie, 2020; Coulanges et al., 2020; Gomez et al., 2015), the authors used a measure of inhibitory control that required symbolic number comparison, the numerical Stroop task, confounding the contribution of inhibitory control with numerical skills. Recent studies suggest that numerical and non-numerical inhibition tasks have differential contributions in other math domains (e.g., whole-number arithmetic; Gilmore et al., 2015). Future studies should collect both types of measures to elucidate whether these two forms of inhibition have independent or overlapping contributions to proportional reasoning.

Continuous priming bolsters discrete proportional reasoning

As opposed to discrete representations, continuous representations of proportions do not elicit counting strategies (Plummer et al., 2017). Thus, studies have investigated whether these representations could prime proportional reasoning skills when children make judgments of discrete proportions (Boyer & Levine, 2015; Hurst & Cordes, 2018), finding that continuous representations are successful in improving children's performance. However, the evidence is mixed about the age at which children are sensitive to this priming. Whereas one study found it at 5 years but not at 3 or 7 years (Hurst & Cordes, 2018), another study found it at 10 years but not at 6 or 8 years (Boyer & Levine, 2015). One reason for this discrepancy might be the different difficulty levels of the tasks employed. Whereas there was a ceiling effect in the oldest group's performance of the former study, there was a floor effect in the younger groups' performance of the latter study.

Here, we found that when there is enough room for improvement, 7-year-olds' discrete proportional reasoning benefits from continuous priming. Notably, the two groups were matched on age, math performance, and inhibitory control, and they performed comparably in the continuous and discrete mixed formats. Yet, seeing the continuous information was sufficient to improve performance above chance for discrete misleading problems among 7-year-olds, a level of performance not seen previously even among 10-year-olds (Jeong et al., 2007). Furthermore, the priming effect was independent of the contributions of inhibitory control, suggesting that children's proportional reasoning could be improved either by fostering the appropriate strategies through continuous representations or by inhibiting the inappropriate ones. Importantly, because our study was not initially designed to examine the effects of priming, presentation order of the three formats was confounded with the type of priming that children received. Nevertheless, the specificity of our results for the misleading trials in the discrete adjacent block, rather than poorer performance for all trials in the block, argues against a fatigue interpretation of these results. Still, future studies of priming effects should present the block of interest in the same position across groups, as was done previously (Hurst & Cordes, 2018), and include larger samples. And conversely, studies primarily focused on continuous versus discrete nonsymbolic formats must be cognizant of these order effects.

Experimental and educational considerations

Although the Spinners task has been widely used to measure children's proportional reasoning (Hurst & Cordes, 2018; Jeong et al., 2007), one alternative strategy to solve this task is to compare the angles formed by the red sections of the spinners rather than the proportions. The spinners with the proportionally larger red areas are also those with larger angles. However, based on how the size of the spinners and counting information impaired children's performance, it is unlikely that children were using this strategy. Future eye-tracking studies could provide deeper insight into the kind of strategies that children use while comparing nonsymbolic proportions.

Another concern is that, in contrast to previous reports, children in this study came from a disadvantaged socioeconomic background and had lower math and inhibitory control skills (Davidson et al., 2006). Timed measures of addition and subtraction fluency were particularly low (M = 80.65), below the cutoff of 1 standard deviation (standard score = 85) used in some studies to identify students with mathematical disabilities (Iuculano et al., 2015; Swanson, 2012). However, their socioeconomic background and the time of year when evaluated likely account for their low math scores. We assessed children during the first weeks of their second-grade year. Prior work on the academic losses over the summer, a phenomenon called "summer dip," shows that these losses are especially pronounced in children with lower socioeconomic status relative to their peers with higher socioeconomic status (Alexander, Entwisle, & Olson, 2001). Interestingly, despite these limitations, they showed performance comparable to that of the group of 8-year-olds in Jeong et al.'s (2007) study. This pattern of results suggests that proportional reasoning may be less affected by socioeconomic factors, making it a promising target for educational interventions. Future research should investigate proportional reasoning in samples that include the complete spectrum of math ability and could consider more comprehensive assessments of math knowledge. Given the role of multiplicative understanding in proportional reasoning (Kouba, 1989), assessing multiplication and division might be more informative than addition and subtraction, especially in older children with more familiarity with these arithmetic operations.

Our results also speak to an ongoing debate about how best to teach fractions, suggesting advantages for continuous representations over discrete ones. Instruction programs that introduce fractions with area models (e.g., segmented pie charts) have been criticized as misleading children to think of fractions as parts of a whole and prompting them to use counting strategies (Obersteiner, Dresler, Bieck, & Moeller, 2019). As an alternative to the part–whole approach, a measurement perspective has been proposed (Davydov & Tsvetkovich, 1991; Powell, 2019; Wong & Evans, 2008). From this perspective, fractions serve as a more accurate measurement of a reference, specifically in those cases where the multiplicative relation between the reference and the measuring unit does not result in a whole number. In one practical example of this perspective (Powell, 2019), children start learning about proportions with physical continuous representations such as Cuisenaire rods. Using these rods, children observe the multiplicative relation between a reference and the measuring units; for instance, they can observe that the length of one black rod equals the length of three red rods plus one white rod, which is half of the size of the red rod. As a result, children can observe that one black rod equals seven halves of a red rod. After being familiarized with the rods, children start working with symbolic representations of proportions.

Further support for the importance of continuous representations in educational contexts comes from correlational studies reporting that better acuity in placing fraction magnitudes on numbers is related to better understanding of fractions (Siegler, Thompson, & Schneider, 2011). Finally, there is also evidence from brief intervention studies (Hamdan & Gunderson, 2017) showing that children who go through a 30-min training period on how to represent fractions in a number line improve their fraction comparison skills, whereas children who use area models do not. Our results suggest that these approaches may be effective because they facilitate proportional representation of symbolic fractions. Future studies should examine this contention using more intensive interventions based on the measurement perspective.

Conclusions

Our study provides the first evidence from individual difference designs for the relationship between inhibitory control and discrete proportional reasoning. Moreover, our results add to the growing body of evidence demonstrating the short-term facilitating effect of continuous proportions on discrete proportional reasoning, extending for the first time to 7-year-old children. Finally, we found that facilitating effects of continuous priming and individual differences in inhibitory control both are independent contributors to discrete proportional reasoning.

Acknowledgments

We thank the principal, Simone Rose, as well as Shirlene Crockett and all the teachers and staff at Thirteenth Avenue/Dr. MLK, Jr. School who helped in this study. We also thank Blessing Eluwa, Milagros Grados, Jessica Amato, Dima Karim, Isabel Paucar, Kainaat Anwar, and Catalina Guzman for their assistance with data collection as well as Elisee Doh for stimuli generation. A. Powell was supported by the National Council of Teacher of Mathematics under the Mathematics Education Trust (MET) 2018-2019 Classroom Research Grant. Elisee Doh gratefully acknowledges receiving a research stipend from the National Science Foundation through the Garden State-LSAMP (NSF Award 1909824).

Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jecp.2020.104931.

References

Alexander, K. L., Entwisle, D. R., & Olson, L. S. (2001). Schools, achievement, and inequality: A seasonal perspective. *Educational Evaluation and Policy Analysis, 23*, 171–191.

Avgerinou, V. A., & Tolmie, A. (2020). Inhibition and cognitive load in fractions and decimals. *British Journal of Educational Psychology, 90*(Suppl. 1), 240–256.

Bakeman, R. (2005). Recommended effect size statistic. *Behavior Research Methods, 37*, 379–384.

Bascandziev, I., Tardiff, N., Zaitchik, D., & Carey, S. (2018). The role of domain-general cognitive resources in children's construction of a vitalist theory of biology. *Cognitive Psychology, 104*, 1–28.

Boyer, T. W., & Levine, S. C. (2015). Prompting children to reason proportionally: Processing discrete units as continuous amounts. *Developmental Psychology, 51*, 615–620.

Boyer, T. W., Levine, S. C., & Huttenlocher, J. (2008). Development of proportional reasoning: Where young children go wrong. *Developmental Psychology, 44*, 1478–1490.

Brocki, K. C., & Tillman, C. (2014). Mental set shifting in childhood: The role of working memory and inhibitory control. *Infant and Child Development, 604*, 588–604.

Brookman-Byrne, A., Mareschal, D., Tolmie, A. K., & Dumontheil, I. (2018). Inhibitory control and counterintuitive science and maths reasoning in adolescence. *PLoS One, 13*(6) e198973.

Chen, L., Bae, S. R., Battista, C., Qin, S., Chen, T., Evans, T. M., & Menon, V. (2018). Positive attitude toward math supports early academic success: Behavioral evidence and neurocognitive mechanisms. *Psychological Science, 29*, 390–402.

Coulanges, L., Abreu-Mendoza, R. A., Varma, S., Uncapher, M., Gazzaley, A., Anguera, J., & Rosenberg-Lee, M. (2020). Linking inhibitory control to math achievement via comparison of conflicting decimal numbers. PsyArXiv. Retrieved June 5, 2020, from psyarxiv.com/qsuz9.

Davidson, M. C., Amso, D., Anderson, L. C., & Diamond, A. (2006). Development of cognitive control and executive functions from 4 to 13 years: Evidence from manipulations of memory, inhibition, and task switching. *Neuropsychologia, 44*, 2037–2078.

Davydov, V. V., & Tsvetkovich, Z. H. (1991). The object sources of the concept of fractions. In V. V. Davydov & L. P. Steffe (Eds.). *Psychological abilities of primary school children in learning mathematics (Soviet Studies in Mathematics Education* (Vol. 6, pp. 86–147). Reston, VA: National Council of Teachers of Mathematics.

Gilmore, C., Attridge, N., Clayton, S., Cragg, L., Johnson, S., Marlow, N., ... Inglis, M. (2013). Individual differences in inhibitory control, not non-verbal number acuity, correlate with mathematics achievement. *PLoS One, 8*(6) e67374.

Gilmore, C., Keeble, S., Richardson, S., & Cragg, L. (2015). The role of cognitive inhibition in different components of arithmetic. *ZDM, 47*, 771–782.

Gomez, D. M., Jimenez, A., Bobadilla, R., Reyes, C., & Dartnell, P. (2015). The effect of inhibitory control on general mathematics achievement and fraction comparison in middle school children. *ZDM, 47*, 801–811.

Hamdan, N., & Gunderson, E. A. (2017). The number line is a critical spatial-numerical representation: Evidence from a fraction intervention. *Developmental Psychology, 53*, 587–596.

Hurst, M. A., & Cordes, S. (2016). Rational-number comparison across notation: Fractions, decimals, and whole numbers. *Journal of Experimental Psychology: Human Perception and Performance, 42*, 281–293.

Hurst, M. A., & Cordes, S. (2018). Attending to relations: Proportional reasoning in 3- to 6-year-old children. *Developmental Psychology, 54*, 428–439.

Iuculano, T., Rosenberg-Lee, M., Richardson, J., Tenison, C., Fuchs, L., Supekar, K., & Menon, V. (2015). Cognitive tutoring induces widespread neuroplasticity and remediates brain function in children with mathematical learning disabilities. *Nature Communications, 6*, 8453.

Jeong, Y., Levine, S. C., & Huttenlocher, J. (2007). The development of proportional reasoning: Effect of continuous versus discrete quantities. *Journal of Cognition and Development, 8*, 237–256.

Kouba, V. L. (1989). Children's solution strategies for equivalent set multiplication and division word problems. *Journal for Research in Mathematics Education, 20*, 147–158.

Lee, K., & Lee, H. W. (2019). Inhibition and mathematical performance: Poorly correlated, poorly measured, or poorly matched? *Child Development Perspectives, 13*, 28–33.

Matthews, P. G., Lewis, M. R., & Hubbard, E. M. (2016). Individual differences in nonsymbolic ratio processing predict symbolic math performance. *Psychological Science, 27*, 191–202.

Norris, J. E., & Castronovo, J. (2016). Dot display affects approximate number system acuity and relationships with mathematical achievement and inhibitory control. *PLoS One, 11*(5) e155543.

Obersteiner, A., Dresler, T., Bieck, S. M., & Moeller, K. (2019). Understanding fractions: Integrating results from mathematics education, cognitive psychology, and neuroscience. In A. Norton & M. W. Alibali (Eds.), *Constructing number: Merging perspectives from psychology and mathematics education* (pp. 135–162). Cham, Switzerland: Springer.

Pierce, J. W. (2007). PsychoPy—Psychophysics software in Python. *Journal of Neuroscience Methods, 162*, 8–13.

Plummer, P., DeWolf, M., Bassok, M., Gordon, P. C., & Holyoak, K. J. (2017). Reasoning strategies with rational numbers revealed by eye tracking. *Attention, Perception, & Psychophysics, 79*, 1426–1437.

Powell, A. B. (2019). How does a fraction get its name?. *Revista Brasileira de Educação em Ciências e Educação Matemática, 3*, 700–713.

R Core Team (2019). *R: A language and environment for statistical computing (Version 3.5.3)*. Vienna, Austria: R Foundation for Statistical Computing.

Ren, K., Lin, Y., & Gunderson, E. A. (2019). The role of inhibitory control in strategy change: The case of linear measurement. *Developmental Psychology, 55*, 1389–1399.

Siegler, R. S., Thompson, C. A., & Schneider, M. (2011). An integrated theory of whole number and fractions development. *Cognitive Psychology, 62*, 273–296.

Steiger, J. H. (1980). Tests for comparing elements of a correlation matrix. *Psychological Bulletin, 87*, 245–251.

Swanson, H. L. (2012). Cognitive profile of adolescents with math disabilities: Are the profiles different from those with reading disabilities? *Child Neuropsychology, 18*, 125–143.

Van Dooren, W., & Inglis, M. (2015). Inhibitory control in mathematical thinking, learning and problem solving: A survey. *ZDM, 47*, 713–721.

Wechsler, D. (2009). *Wechsler individual achievement test-third edition*. San Antonio, TX: Pearson.

Wong, M., & Evans, D. (2008). Fractions as a measure. In M. Goos, R. Brown, & K. Makar (Eds.), *Navigating currents and charting* (pp. 597–603). Adelaide, SA, Australia: Mathematics Education Reserach Group of Australasia.

Wright, A., & Diamond, A. (2014). An effect of inhibitory load in children while keeping working memory load constant. *Frontiers in Psychology, 5*. https://doi.org/10.3389/fpsyg.2014.00213.