Last update: Oct 2

List of theorems or properties:

Properties of a probability measure:

- $0 \leq P(A) \leq 1$
- $P(S) = 1$
- If $A \cap B = \emptyset$ then $P(A \cup B) = P(A) + P(B)$
- $P(A^c) = 1 - P(A)$
- $P(A) = P(A \cap B) + P(A \cap B^c)$
- $B \subseteq A$ then $P(A) = P(B) + P(A \cap B^c)$
- $B \subseteq A$ then $P(B) \leq P(A)$
- $A_1, A_2, \ldots$ a partition of S, then $P(B) = \sum_i P(B \cap A_i)$
- $P(A \cup B) = P(A) + P(B) + P(A \cap B)$
- $P(A_1 \cup A_2 \cup \ldots) \leq P(A_1) + P(A_2) + \ldots$

Combinatorics

- $n!$ is the number of permutations of a set of n elements (How many different arrangements)
- $\binom{n}{k} = \frac{n!}{k!(n-k!)}$ $\rightarrow$ choose k elements from a set of n order doesn't matter
- What if order matters? $\binom{n}{k} \times k!$
- $\frac{n!}{(n-k!)}$ Number of ways to pick k items out of n where order does matter.
    - Is the same as $\binom{n}{k} \times k!$ Choosing without order and then calculating permutations of k items.

Conditional probability

- $P(A|B) = \frac{P(A \cap B)}{P(B)}$ if $P(B) > 0$
- $P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$
- $A_1, A_2, \ldots$ a partition of S, then $P(B) = \sum_i P(A_i)P(B|A_i) = \sum_i P(B \cap A_i)$
- $P(A|B) = \frac{P(A)}{P(B)} P(B|A)$

Independence

- A and B are independent if $P(A \cap B) = P(A)P(B)$
- $A_1, A_2, A_3, \ldots$ are independent if
  $P(A_{i_1} \cap A_{i_2} \cap \ldots \cap A_{i_k}) = P(A_{i_1})P(A_{i_2})\ldots P(A_{i_k})$ for any finite subcollection of events.

Continuity

- If $\{A_n\} \nearrow A$ then $\lim_{n\to\infty} P(A_n) = P(A)$
- If $\{A_n\} \searrow A$ then $\lim_{n\to\infty} P(A_n) = P(A)$

Random variables (r.v)

- A function $X : S \to \mathbb{R}$ is a random variable
- The distribution of a random variable is the collection of all of the probabilities
  of the variable being in every possible subset of $\mathbb{R}$.
- A r.v. is discrete if $\sum_{x\in\mathbb{R}} P(X = x) = 1$
- Probability function $p_X(k) := P(X = k)$

Discrete random variables

- $X \sim Bernoulli(\theta)$ if

$$p_X(k) = \theta^k(1-\theta)^{1-k}\,\mathbb{1}_{k\in\{0,1\}}$$

$$p_X(k) = \begin{cases} \theta & \text{if } k = 1 \\ 1-\theta & \text{if } k = 0 \\ 0 & \text{otherwise} \end{cases}$$

- $X \sim Binomial(n, \theta)$ if

$$p_X(k) = \binom{n}{k}\theta^k(1-\theta)^{n-k}\,\mathbb{1}_{k\in\{0,1,\ldots,n\}}$$

- $X \sim Geometric(\theta)$ if

$$p_X(k) = (1-\theta)^k \theta \, \mathbb{1}_{k \in \{0,1,\ldots\}}$$

This one counts # of failures before the first success.
there's a second parametrization of geometric counting the number of trials until first success. Check wikipedia!

$$p_Y(k) = (1-\theta)^{k-1} \theta \, \mathbb{1}_{k \in \{1,2,\ldots\}}$$

Geometric series

Let a_0 be a constant and define $a_n = a_0 r^n$ as a geometric series

Define $S_n := \sum_{k=0}^{n} r^k$ the partial sums of the first $\{r^k\}_{k=0}^{n}$
note that $\sum_{k=0}^{n} a_k = \sum_{k=0}^{n} a_0 r^k = a_0 \sum_{k=0}^{n} r^k = a_0 S_n$

Now $S_n = r^0 + r^1 + \ldots r^n$ and $rS_n = r^1 + r^2 + \ldots r^{n+1}$, then

$S_n - rS_n = r^0 - r^{n+1}$
$S_n(1-r) = r^0 - r^{n+1}$
then

$$S_n = \frac{r^0 - r^{n+1}}{1-r} = \frac{1-r^{n+1}}{1-r} \qquad \text{if } r \neq 1$$

Now if $r \in (0,1)$

$$\sum_{k=0}^{\infty} r^k = \lim_{n \to \infty} \sum_{k=0}^{n} r^k = \lim_{n \to \infty} S_n = \frac{1}{1-r}$$

#probability
#ta