A TIGHTNESS CRITERION FOR WEINSTEIN CONVEX HYPERSURFACES

1. WHAT DO WE WANT TO PROVE?

Our goal is to give a tightness criterion for (neighborhoods of) Weinstein convex hypersurfaces [HH19] in analogy with "a contact manifold is overtwisted iff it is the negative stabilization of some open book".

Applying [BHH, GP17], any $\dim = 2n + 2$ Weinstein manifold can be described by the "combinatorial" data of a Lefschetz fibration, that is, an ordered sequence of (framed) Lagrangian spheres $\boldsymbol{L} = (L_1, \ldots, L_k)$ in some $\dim = 2n$ Weinstein manifold which are the boundaries of Lefschetz thimbles. Then the boundary is described as an open book with monodromy a product of the Dehn twists along the spheres, $\phi = \tau_{L_k} \circ \cdots \circ \tau_{L_1}$. An open book with *two* positive Dehn twist factorizations of its monodromy ϕ given by some $\boldsymbol{L}^\pm$ yields two Lefschetz fibrations with their boundaries identified, and so a Weinstein convex hypersurface $G(\boldsymbol{L}^\pm)$.

Definition 1.1. $\boldsymbol{L}^\pm$ *is a* double positive factorization *(DPF) of* ϕ *and the convex hypersurface* $G(\boldsymbol{L}^\pm)$ *is its* geometric realization. *Write* W *for the smooth* $2n + 2$ *manifold underlying* $G(\boldsymbol{L}^\pm)$.

By [BHH] all Weinstein convex hypersurfaces arise in this way and we know that they are all related by "WLF moves". This is a "Giroux correspondence" for $\dim > 2$ Weinstein convex hypersurfaces.

Example 1.2. *Suppose that* μ *and* λ *are meridian and longitude circles on a punctured torus* $T \setminus \{pt\}$. *Then* $\boldsymbol{L}^+ = (\mu, \lambda, \mu)$ *and* $\boldsymbol{L}^- = (\lambda, \mu, \lambda)$ *give a DPF, determining a* $\dim = 4$ *convex hypersurface. Indeed* $\tau_\mu \tau_\lambda \tau_\mu = \tau_\lambda \tau_\mu \tau_\lambda$ *is the classical braid relation. This can be generalized to higher dimensions by looking at matching paths in* A_3 *Milnor fibers.*

In §4 we describe bypass and antibypass operations on the "combinatorial" DPFs, replacing $\boldsymbol{L}^\pm$ with $\boldsymbol{L}_b^\pm$ and $\boldsymbol{L}_a^\pm$, respectively when L_1^+ intersects L_1^- transversely in a single point, as in the above example. These operations apply bypasses and antibypasses to the geometric realization as it's defined in [HH18]. In §4, I still have to give an explanation of why the PDF bypass corresponds to the geometric bypass, but in the current state you can see what the definition is and the proof that the operations satisfy

$$\boldsymbol{L}_{bbb}^\pm = \boldsymbol{L}_{aaa}^\pm = \boldsymbol{L}^\pm.$$

This is in analogy with the 3-dimensional case in which an application of three consecutive bypasses on a convex surface has trivial effect. Cf. the bypass exact triangle of [HT22, §2.3]. When we apply a bypass to some $\boldsymbol{L}^\pm$ it gives us a contact structure on $[0, 1] \times W$ with convex boundary the union of $\{0\} \times G(\boldsymbol{L}^\pm)$ and $\{1\} \times G(\boldsymbol{L}_b^\pm)$. Likewise the antibypass gives a contact structure on $[-1, 0] \times W$ with convex boundary the union of $\{-1\} \times G(\boldsymbol{L}_a^\pm)$ and $\{0\} \times G(\boldsymbol{L}^\pm)$

Conjecture 1.3. *A Weinstein convex hypersurface has an overtwisted neighborhood iff it is the geometric realization of some* $\boldsymbol{L}^\pm$ *for which* L_1^+ *and* L_1^- *intersect transversely at a single point.*

Here is the strategy I have in mind for one direction: Suppose the $L_1^\pm$ intersect at a single point. We want to prove that the geometric realization has an overtwisted neighborhood. So we apply three anti-bypasses and three bypasses. This gives us a contact manifold $[-3, 3] \times W$ for which

$$\{-3\} \times W = G(\boldsymbol{L}^\pm), \quad \{-2\} \times W = G(\boldsymbol{L}_{aa}^\pm), \quad \{-1\} \times W = G(\boldsymbol{L}_a^\pm), \quad \{0\} \times W = G(\boldsymbol{L}^\pm)$$

$$\{1\} \times W = G(\boldsymbol{L}_b^\pm), \quad \{2\} \times W = G(\boldsymbol{L}_{bb}^\pm), \quad \{3\} \times W = G(\boldsymbol{L}_{bbb}^\pm) = G(\boldsymbol{L}^\pm)$$

Applying an antibypass to one side of a convex hypersurface and then a bypass to the other side gives an overtwisted neighborhood [HH18, Proposition 6.3.1.]. So $[-1, 1] \times W$ is overtwisted and $[-3, 3] \times W$ has the same convex boundary as the trivial $[-3, 3] \times G(\boldsymbol{L}^\pm)$. We want to show that the (boundary-relative) *almost contact structure* on $[-3, 3] \times W$ is the same as that on the trivial $[-3, 3] \times G(\boldsymbol{L}^\pm)$. If true, the contact structures will be boundary-relative isotopic by [BEM15] and $[-3, 3] \times G(\boldsymbol{L}^\pm)$ is overtwisted.

For the other direction of the "proof", I have no idea what to do. My intuition is that if W has an overtwisted neighborhood, then there should be lots available bypasses and anti-bypasses by an h-principle. So we should be able to find a bypass/anti-bypass pair which looks like the model situation.

2. DOUBLE POSITIVE FACTORIZATIONS AND THEIR GEOMETRIC REALIZATIONS

Let $\boldsymbol{\Sigma} = (\Sigma, \beta, f)$ be a Weinstein domain. That is,

(1) Σ is a compact manifold with boundary,
(2) $\beta \in \Omega^1(\Sigma)$ is such that $d\beta$ is symplectic,
(3) the Liouville vector field X_β defined $d\beta(X_\beta, *) = \beta$ points transversely out of $\partial\Sigma$, and
(4) X_β is gradient-like for f, meaning that there is a $C > 0$ for which $df(X_\beta) > C(|df|^2 + |X_\beta|^2)$ with respect to some metric on Σ.

Consequently Σ has even dimension $2n$, and we assume throughout that $n \geq 1$.

A $\mathbb{D}$-*framed Lagrangian sphere* is a Lagrangian sphere $L \subset \Sigma$ with an identification $L = \mathbb{S}^n = \partial\mathbb{D}^{n+1}$ which we call a $\mathbb{D}$-framing. Two $\mathbb{D}$ framings are equivalent if they differ by a diffeomorphism of $\mathbb{S}^n$ which extends to a diffeomorphism of $\mathbb{D}^{n+1}$. In particular a $\mathbb{D}$-framing encodes an orientation of L once we have fixed an orientation of $\mathbb{D}^{n+1}$. Note that an orientation is equivalent to a homotopy class of $\mathbb{D}$-framing for $n < 6$. To simplify notation, $\mathbb{D}$-framed Lagrangian spheres will simply denoted L with a $\mathbb{D}$-framing implicitly specified. We assume that $\mathbb{D}$-framed Lagrangian spheres are *Legendrian realizable*, meaning that L does not bound a codimension 0 submanifold of Σ. This is only a non-trivial constraint when $n > 1$.

Write $\mathrm{Symp}_c(\boldsymbol{\Sigma})$ for the space of compactly supported symplectomorphisms of $(\Sigma, d\beta)$. For $\phi, \psi \in \mathrm{Symp}_c(\boldsymbol{\Sigma})$, we write $\phi \sim_0 \psi$ if they live in the same connected component. Clearly $\sim_0$ defines an equivalence relation and we write $[\phi]$ for the equivalence class of ϕ. Note that $\phi \in \mathrm{Symp}_c(\boldsymbol{\Sigma})$ applied to a $\mathbb{D}$-framed Lagrangian is also a $\mathbb{D}$-framed Lagrangian. The β and f parts of a Weinstein structure are transformed by pullback ϕ^* so that compactly supported symplectomorphisms take Weinstein domains to Weinstein domains.

A *positive factorization of* $\phi \in \mathrm{Symp}_c(\boldsymbol{\Sigma})$ is an ordered collection of $\mathbb{D}$-framed Lagrangian spheres $\boldsymbol{L} = (L_1, \ldots, L_k)$ in Σ such that

$$\tau_{\boldsymbol{L}} = \tau_{L_k} \circ \cdots \circ \tau_{L_1} \sim_0 \phi$$

where τ_L is a positive symplectic Dehn twist about a Lagrangian sphere L. Positive Dehn twists are only defined up to $\sim_0$. We abbreviation PF for positive factorization. An *isotopy* of a PF is a modification of one of the $L_i \in \boldsymbol{L}$ by a symplectic isotopy. Isotopies leave $[\tau_{\boldsymbol{L}}]$ unaffected. Observe that $\emptyset$ is a positive factorization of Id_Σ.

2.1. Positive factorizations.

A positive factorization $\boldsymbol{L}$ of a $\phi \in \mathrm{Symp}_c(\boldsymbol{\Sigma})$ determines a Lefschetz fibration $\pi : W \to \mathbb{D}$ whose non-singular fiber is $\boldsymbol{\Sigma} = \pi^{-1}(1)$, with $\#\boldsymbol{L}$ singular fibers. Here $\mathbb{D}$ is the unit disk in $\mathbb{C}$. At the kth single fiber $\pi^{-1}(k\epsilon i)$ (with $\epsilon > 0$ small), we have a Lefschetz thimble whose projection to $\mathbb{D}$ is the straight arc from $k\epsilon i$ to 1 and whose boundary is L_k. We say that (W, π) is the *geometric realization* of $\boldsymbol{L}$. According to [BHH, GP17] every Weinstein manifold is the geometric realization of some positive factorization.

The total space of W then has a Weinstein structure which depends only on the symplectic isotopy class of $\boldsymbol{L}$ and the constant ϵ. Then ∂W is naturally a contact manifold and is supported by an open book decomposition with page (Σ, β) and monodromy $\tau_{\boldsymbol{L}}$.

2.2. Double positive factorizations.

A *double positive factorization* (DPF) is a triple $(\boldsymbol{\Sigma}, \boldsymbol{L}^-, \boldsymbol{L}^+)$ consisting of a Weinstein manifold $\boldsymbol{\Sigma}$ and a pair ordered tuples of $\mathbb{D}$-framed Lagrangian spheres $\boldsymbol{L}^\pm = (L_i^\pm)_{i=1}^{k^\pm}$ for which

$$\tau_{\boldsymbol{L}^-} \sim_0 \tau_{\boldsymbol{L}^+}.$$

There are many famous examples of DPFs: Consider the lantern or chain relations for Σ an oriented surface with boundary.

The *geometric realization* of a DPF is a convex hypersurface W of dimension $\dim \Sigma + 2$ whose positive region W^+ is the geometric realization of $\boldsymbol{L}^+$ and whose negative region W^- is $\boldsymbol{L}^-$. The boundaries of the $W^\pm$ are identified using the open book decompositions with page Σ and monodromy $\phi = \tau_{\boldsymbol{L}^\pm}$.

Theorem 2.1. *Every convex hypersurface of dimension* ≥ 4 *whose positive and negative regions are Weinstein is the geometric realization of some DPF.*

Sketch of the proof. Suppose we're given a Weinstein convex hypersurface. Apply [BHH, GP17] to describe the positive and negative regions as geometric realizations of positive factorizations. This gives two open book decompositions of the dividing set Γ. Applying sufficiently many positive stabilizations to the two open books, they will eventually become isotopic [BHH]. The positive stabilization of the open book can be realized as stabilizations of the Lefschetz fibrations on the positive and negative regions, leaving the homotopy classes of their Weinstein structures unmodified. Once the open books are isotopic, we have realized the convex hypersurface as a DPF. $\square$

3. GEOMETRICALLY TRIVIAL OPERATIONS ON DFPs

Here we describe some basic operations on PFs and DPFs which do not modify their geometric realizations by [BHH, Corollary 1.3.3]. By the same result, the following result should follow:

Theorem 3.1. *Two DPFs have the same geometric realization iff they are related by the moves described in this section.*

3.1. **Change of orientation.** For any DPF, we can swap the roles of the $L^{\pm}$ to obtain another DPF. We call this modification a *change of orientation*. Clearly this operation squares to the identity. On the level of geometric realization, this corresponds to swapping the positive and negative regions, which amounts to changing the orientation of the convex hypersurface.

3.2. **Handle slide.** A *handle slide* of a $\boldsymbol{L}$ is a replacement

$$(\boldsymbol{L} = (L_1, \ldots, L_k)) \mapsto (\boldsymbol{L}_h = (L_1, \ldots, L_{i-1}, \tau_{L_i}(L_{i+1}), L_i, L_{i+2}, \ldots, L_k))$$

for some $i = 1, \ldots, k-1$. The conjugacy relation

$$(1) \qquad \qquad \psi \circ \tau_L \circ \psi^{-1} = \tau_{\psi(L)}$$

holds for any $\psi \in \mathrm{Symp}_c(\boldsymbol{\Sigma})$ and L, implying the relation

$$\tau_{L_{i+1}} \circ \tau_{L_i} = \tau_{L_i} \circ \tau_{\tau_{L_i}(L_{i+1})}.$$

Hence a handle slide leaves the class $[\tau_{\boldsymbol{L}}]$ unchanged. Therefore applying a handle slide to one of the $\boldsymbol{L}^{\pm}$ in a DPF $(\boldsymbol{\Sigma}, \boldsymbol{L}^-, \boldsymbol{L}^+)$ yields another DPF.

3.3. **Rotation.** If $(\boldsymbol{\Sigma}, \boldsymbol{L}^-, \boldsymbol{L}^+)$ is a DPF, then so it $(\boldsymbol{\Sigma}, \boldsymbol{L}_c^-, \boldsymbol{L}_r^+)$, where it is defined

$$\begin{aligned} \boldsymbol{L}_r^- &= (L_{k^+}^-, L_1^-, \ldots, L_{k^--1}^-), \\ \boldsymbol{L}_r^+ &= (\tau_{L_{k^+}^-}(L_1^-), \ldots, \tau_{L_{k^+}^-}(L_{k^+}^+)). \end{aligned}$$

This follows from the following computation using the conjugacy relation

$$\tau_{\boldsymbol{L}_c^-} = \tau_{L_{k^-}^-} \circ \tau_{\boldsymbol{L}^-} \circ \tau_{L_{k^-}^-}^{-1} = \tau_{L_{k^-}^-} \circ \tau_{\boldsymbol{L}^+} \circ \tau_{L_{k^-}^-}^{-1} = \tau_{\tau_{L_{k^-}^-}(L_1^-)} \circ \cdots \circ \tau_{L_{k^-}^-(L_{k^+}^+)} = \tau_{\boldsymbol{L}_c^+}.$$

A replacement $(\boldsymbol{\Sigma}, \boldsymbol{L}^-, \boldsymbol{L}^+) \mapsto (\boldsymbol{\Sigma}, \boldsymbol{L}_c^-, \boldsymbol{L}_c^+)$ is a *rotation*. We use the same name for an inverse of a rotation or an orientation change followed by a rotation followed by another orientation change.

3.4. **Stabilization.** Let D be a Lagrangian disk in Σ with $\partial D \subset \partial \Sigma$ a Legendrian sphere. A *positive stabilization of (Σ, ϕ) by D* is a pair (Σ', ϕ') obtained as follows. Attach a critical-index Weinstein handle to ∂D to obtain a new Weinstein manifold $\Sigma_s = (\Sigma_s, \beta_s, f_s)$ which contains Σ as a Weinstein subdomain of codimension 0. The union of D with the core of the handle determines a Lagrangian sphere L_D and we define $\phi_s = \phi \circ \tau_{L_D}$.

If ϕ has a positive factorization $\boldsymbol{L} = (L_i)$, then $\boldsymbol{L}_s = (L_D, L_1, \ldots, L_k)$ is a positive factorization of ϕ_s and we say that $\boldsymbol{L}_s$ is the *positive stabilization of $\boldsymbol{L}$*. A Lagrangian disk $D \subset \Sigma$ with Legendrian boundary in $\partial \Sigma$ may be used to stabilize the $\boldsymbol{L}^{\pm}$ in a DPF simultaneously. We say that the result $(\boldsymbol{\Sigma}, \boldsymbol{L}_s^-, \boldsymbol{L}_s^+)$ is a *stabilization of $(\boldsymbol{\Sigma}, \boldsymbol{L}^-, \boldsymbol{L}^+)$*. In the notation of the preceding paragraph, a stabilization of a DPF is then a DPF with underlying Weinstein domain $\boldsymbol{\Sigma}'$.

Let L_i be a $\mathbb{D}$-framed Lagrangian in some $\boldsymbol{L}$ associated to a $\boldsymbol{\Sigma}$. We say that L_i is an *unknot* if there is some $(\Sigma', \boldsymbol{L}')$ with Σ' containing a D such that $\Sigma = \Sigma_s'$ and $\boldsymbol{L}_s'$ is related to $\boldsymbol{L}$ by a sequence of handle slides and conjugacies sending $L_D \in \boldsymbol{L}_s'$ to $L_i \in \boldsymbol{L}$.

4. BYPASSES AND ANTI-BYPASSES ON DFPS

Here we describe bypass and anti-bypass operations on DFPs. These operations induce bypasses on their geometric realizations according to the definition of bypass in [HH18].

4.1. **Bypass.** Suppose that we have a DPF $(\mathbf{\Sigma}, \mathbf{L}^-, \mathbf{L}^+)$ with the L_1^+ intersecting L_1^- transversely in a single point, implying that

$$\tag{2} \tau_{L_1^+}(L_1^-) = \tau_{L_1^-}^{-1}(L_1^+)$$

Indeed, both sides of the equation are described by the same Lagrangian connected sum $L_1^- \# L_1^+$, also known as a Polterovich surgery. Consequently, we have the braid relations

$$\tau_{L_1^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+}^{-1} = \tau_{L_1^-}^{-1} \circ \tau_{L_1^+} \circ \tau_{L_1^-} \iff \tau_{L_1^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+} = \tau_{L_1^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-}.$$

We then see that $(\mathbf{\Sigma}, \mathbf{L}_b^-, \mathbf{L}_b^+)$ is also a DPF where it is defined

$$\mathbf{L}_b^- = (\tau_{L_1^+} L_1^-, L_2^-, \dots, L_{k^+}^-),$$
$$\mathbf{L}_b^+ = (L_1^-, L_2^+, \dots, L_{k^+}^+).$$

Indeed we compute

$$\tau_{\mathbf{L}_b^-} = \tau_{L_{k^-}^-} \circ \cdots \circ \tau_{L_2^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+}^{-1}$$

$$= \tau_{L_{k^-}^-} \circ \cdots \circ \tau_{L_2^-} \circ \tau_{L_1^-} \circ \tau_{\tau_{L_1^-}(L_1^+)} \circ \tau_{L_1^+}^{-1} \qquad \text{(handle slide)}$$

$$= \tau_{\mathbf{L}^-} \circ \tau_{\tau_{L_1^-}(L_1^+)} \circ \tau_{L_1^+}^{-1}$$

$$= \tau_{\mathbf{L}^-} \circ \tau_{L_1^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-}^{-1} \circ \tau_{L_1^+}^{-1} \qquad \text{(conjugacy)}$$

$$= \tau_{\mathbf{L}^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-}^{-1} \circ \tau_{L_1^+}^{-1} \qquad (\tau_{\mathbf{L}^-} = \tau_{\mathbf{L}^+})$$

$$= \tau_{L_{k^+}^+} \circ \cdots \circ \tau_{L_2^+} \circ \tau_{L_1^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-}^{-1} \circ \tau_{L_1^+}^{-1}$$

$$= \tau_{L_{k^+}^+} \circ \cdots \circ \tau_{L_2^+} \circ \tau_{L_1^-} \circ \tau_{L_1^+} \circ \tau_{L_1^-} \circ \tau_{L_1^-}^{-1} \circ \tau_{L_1^+}^{-1} \qquad \text{(braid relation)}$$

$$= \tau_{L_{k^+}^+} \circ \cdots \circ \tau_{L_2^+} \circ \tau_{L_1^-} = \tau_{\mathbf{L}_b^+}. \qquad \text{(cancellation)}$$

We say that $(\mathbf{\Sigma}, \mathbf{L}_b^-, \mathbf{L}_b^+)$ is obtained from $(\mathbf{\Sigma}, \mathbf{L}^-, \mathbf{L}^+)$ by a *bypass*.

A bypass results in a DPF whose first Lagrangians also intersect in a single point. Therefore the bypass operation may be iterated. The square is given

$$\mathbf{L}_{b^2}^- = (L_1^+, L_2^-, \dots, L_{k^+}^-),$$
$$\mathbf{L}_{b^2}^+ = (\tau_{L_1^+} L_1^-, L_2^+, \dots, L_{k^+}^+).$$

using the fact that $\tau_{L_1^-} \circ \tau_{L_1^+} L_1^- = \tau_{L_1^-} \circ \tau_{L_1^-}^{-1} L_1^+ = L_1^+$ to simplify the first Lagrangian in $\mathbf{L}_{b^2}^-$. For the cube,

$$\mathbf{L}_{b^3}^- = (L_1^-, L_2^-, \dots, L_{k^+}^-) = \mathbf{L}^-,$$
$$\mathbf{L}_{b^3}^+ = (L_1^+, L_2^+, \dots, L_{k^+}^+) = \mathbf{L}^+.$$

We have simplified the first Lagrangian in $\mathbf{L}_{b^2}^-$ via the calculation

$$\tau_{\tau_{L_1^+} L_1^-} L_1^+ = \tau_{L_1^+} \tau_{L_1^-} \tau_{L_1^+}^{-1} L_1^+ = \tau_{L_1^+} \tau_{L_1^-} L_1^+ = \tau_{L_1^+} \tau_{L_1^+}^{-1} L_1^- = L_1^-$$

using the conjugacy relation of Equation (1), the fact that a Lagrangian is invariant under a Dehn twist along itself, and then Equation (2). So we have proved the following.

Lemma 4.1. *The cube of the bypass operation is the identity.*

The bypass operation can more generally be applied to a pair $L_i^- \in \mathbf{L}^-, L_j^+ \in \mathbf{L}^+$ intersecting transversely in a single point. We say that all such operations are also bypasses.

4.2. **Anti-bypass.** Again suppose that (Σ, L^-, L^+) is a DPF such that L_1^- and L_1^+ intersect transversely in a single point. We define an *anti-bypass* to be the DPF obtained from (Σ, L^-, L^+) by performing a change of orientation, a bypass, and finally another change of orientation. Writing the result of an anti-bypass as (Σ, L_a^-, L_a^+), we see that

$$L_a^- = (L_1^+, L_2^-, \ldots, L_{k^+}^-),$$
$$L_a^+ = (\tau_{L_1^-} L_1^+, L_2^+, \ldots, L_{k^+}^+).$$

By the above lemma and the definition of the antibypass we have the following.

Lemma 4.2. *The cube of the anti-bypass is the identity.*

Now we want to calculate the effect of two anti-bypasses, followed by a bypass. The relevance of the calculation is the geometrically, an anti-bypass followed by a bypass will be overtwisted. Starting with a (Σ, L^-, L^+), we calculate

$$L_{a^2}^- = (\tau_{L_1^-} L_1^+, \ldots), \quad L_{a^2}^+ = (L_1^-, L_2^+, \ldots)$$
$$L_{a^2 b}^- = (\tau_{L_1^-}^2 L_1^+, \ldots), \quad L_{a^2 b}^+ = (\tau_{L_1^-} L_1^+, \ldots)$$

REFERENCES

[BEM15] M. S. Borman, Y. Eliashberg, and E. Murphy, *Existence and classification of overtwisted contact structures in all dimensions*, Acta Math. 215, p.281–361, 2015. 1

[BHH] J. Breen, K. Honda, and Y. Huang, *The Giroux correspondence in arbitrary dimensions*, arXiv:2307.02317, 2023. 1, 2, 3

[GP17] E. Giroux and J. Pardon, *Existence of Lefschetz fibrations on Stein and Weinstein domains*, Geom. Topol., 21, no. 2, 963–997, 2017. 1, 2, 3

[HH18] K. Honda and H. Huang, *Bypass attachments in higher-dimensional contact topology*, preprint, arXiv:1803.09142, 2018. 1, 4

[HH19] K. Honda and H. Huang, *Convex hypersurface theory in contact topology*, preprint, arXiv:1907.06025, 2019. 1

[HT22] Ko Honda and Yin Tian, *Contact categories of disks*, J. Symp. Geom., Vol. 20, 2022. 1