ML HW2

25 June 2023 13:04

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Question 1

(a) Euclidean distance

Let us assume $z = (z_1, z_2)$

$$d(x,z) = \sqrt{(0.6-z_1)^2 + (0.8-z_2)^2} + \sqrt{(0.8-z_1)^2 + (0.6-z_2)^2} + \sqrt{(-0.8-z_1)^2 + (0.6-z_2)^2}$$

We then minimise this equation, taking a partial derivative with z_1 and z_2 & equating to 0. As this is not possible to compute manually, we use code (can be found in the python notebook)

We get

$z_1 \approx 0.6$

$z_2 \approx 0.8$

$\therefore z = (0.6, 0.8)$

(b) Squared Euclidean distance

Let us assume $z = (z_1, z_2)$

$$d(x,z) = (0.6-z_1)^2 + (0.8-z_2)^2 + (0.8-z_1)^2 + (0.6-z_2)^2 + (-0.8-z_1)^2 + (0.6-z_2)^2$$

we need to minimize this by taking partial derivative with z_1 and z_2 & equating to 0

$$\frac{\partial d}{\partial z_1} = 2(0.6-z_1)(-1) + 2(0.8-z_1)(-1) + 2(-0.8-z_1)(-1) = 0$$

$$-1.2 + 2z_1 \cancel{-1.6} + 2z_1 + \cancel{1.6} + 2z_1 = 0$$

$$6z_1 = 1.2$$

$$z_1 = \frac{1.2}{6} = 0.2$$

$$\frac{\partial d}{\partial z_2} = 2(0.8-z_2)(-1) + 2(0.6-z_2)(-1) + 2(0.6-z_2)(-1)$$

$$-1.6 + 2z_2 - 1.2 + 2z_2 - 1.2 + 2z_2 = 0$$

$$-4.0 + 6z_2 = 0$$

$$z_2 = \frac{4}{6} \approx 0.6666\ldots$$

$\therefore z = (0.2, 0.67)$

(c) Manhattan distance

Let us assume $z = (z_1, z_2)$

$$d(x,z) = |0.6-z_1| + |0.8-z_2| + |0.8-z_1| + |0.6-z_2| + |-0.8-z_1| + |0.6-z_2|$$

we need to minimise d.

As this is the modulus, we need the median of the values as each term will be positive

$z_1 \rightarrow$ Median $(-0.8, 0.6, 0.8) = 0.6$

$z_2 \rightarrow$ Median $(0.6, 0.6, 0.8) = 0.6$

$\therefore z = (0.6, 0.6)$

Question 2

Code can be found in the python notebook

we see that the graph has an elbow at $k=3$

(i) Number of Clusters: 3

(ii) Final Centroids for each cluster:

Cluster 0: (28.23155325, 39.63790052, 59.96814159)

Cluster 1: (126.43443045, 147.84986378, 164.74729401)

Cluster 2: (74.47777884, 95.47497617, 120.07602479)

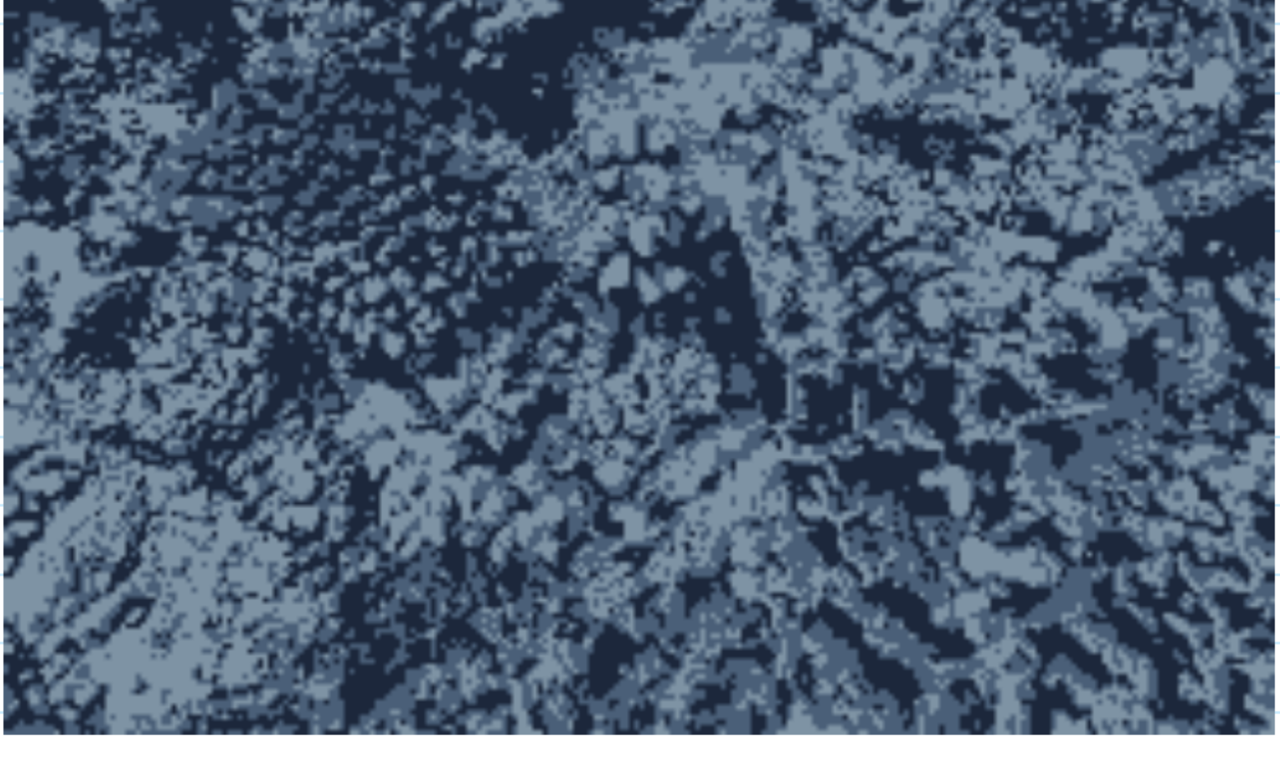
(iii) Number of pixels associated to each cluster:

Cluster 0: 16385

Cluster 1: 13581

Cluster 2: 16784

(iv) Resulting image after replacing the points in each cluster with their centroid is as follows:



The image generated, cluster points, centroids for all other k values ranging from 1 to 8 can be found in the python notebook.

Question 3

$$Q^*_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma \max_{a'} Q^*_i(s',a')\right]$$

(a) $Q^*_1(\text{Uncertain}, A) \leftarrow 0.9(0 + 0.5(0)) + 0.1(1 + 0.5(0)) = 0.1$

$Q^*_1(\text{Certain}, A) \leftarrow 1(1 + 0.5(0)) = 1$

$Q^*_1(\text{Lose}, A) \leftarrow 0.8(0 + 0.5(0)) + 0.2(1 + 0.5(0)) = 0.2$

$Q^*_1(\text{Win}, A) \leftarrow 1(2 + 0.5(0)) = 2$

$Q^*_1(\text{Uncertain}, G) \leftarrow 0.9(-2 + 0.5(0)) + 0.1(2 + 0.5(0)) = -1.6$

$Q^*_1(\text{Certain}, G) \leftarrow 0.3(-2 + 0.5(0)) + 0.7(2 + 0.5(0)) = 0.8$

$Q^*_1(\text{Lose}, G) \leftarrow 0.8(-2 + 0.5(0)) + 0.2(2 + 0.5(0)) = -1.2$

$Q^*_1(\text{Win}, G) \leftarrow 1(2 + 0.5(0)) = 2$

	s = Uncertain	s = Certain	s = Lose	s = Win
A	0.1	1	0.2	2
G	-1.6	0.8	-1.2	2

(b) Policy $\pi^*(s) = \arg\max_a Q^*(s,a)$

	s = Uncertain	s = Certain	s = Lose	s = Win
	A	A	A	A / G

(c) $V^*(s) = \max_a Q^*(a,s)$

	s = Uncertain	s = Certain	s = Lose	s = Win
	0.1	1	0.2	2

(d) $Q^*_2(\text{Uncertain}, A) \leftarrow 0.9(0 + 0.5(0.1)) + 0.1(1 + 0.5(1)) = 0.195$

$Q^*_i(\text{Certain}, A) \leftarrow 1(1 + 0.5(1)) = 1.5$

$Q^*_i(\text{Lose}, A) \leftarrow 0.8(0 + 0.5(0.1)) + 0.2(1 + 0.5(1)) = 0.34$

$Q^*_i(\text{Win}, A) \leftarrow 1(2 + 0.5(2)) = 3$

$Q^*_i(\text{Uncertain}, G) \leftarrow 0.9(-2 + 0.5(0.2)) + 0.1(2 + 0.5(2)) = -1.41$

$Q^*_i(\text{Certain}, G) \leftarrow 0.3(-2 + 0.5(0.2)) + 0.7(2 + 0.5(2)) = 1.53$

$Q^*_i(\text{Lose}, G) \leftarrow 0.8(-2 + 0.5(0.2)) + 0.2(2 + 0.5(2)) = -0.92$

$Q^*_i(\text{Win}, G) \leftarrow 1(2 + 0.5(2)) = 3$

	s = Uncertain	s = Certain	s = Lose	s = Win
A	0.195	1.5	0.34	3
G	-1.41	1.53	-0.92	3

(e)

	s = Uncertain	s = Certain	s = Lose	s = Win
	A	G	A	A / G

(f)

	s = Uncertain	s = Certain	s = Lose	s = Win
	0.195	1.53	0.34	3

Question 4

(a) $$V^{\pi_i}_{k+1}(s) \leftarrow \sum_{s'} T(s, \pi_i(s), s')\left[R(s, \pi_i(s), s') + \gamma V^{\pi_i}_k(s')\right]$$

$$V^{\pi}_{k+1}(A) \leftarrow 0.4(1.0 + 0.5(-0.840)) + 0.6(0 + 0.5(-1.080))$$
$$= -0.092$$

(b) $$Q^*_{i+1}(s,a) \leftarrow \sum_{s'} T(s,a,s')\left[R(s,a,s') + \gamma V^*_i(s')\right]$$

$$Q^{\pi}_{\infty}(A, \text{clockwise}) \leftarrow 0.8(0 + 0.5(-1.114)) + 0.2(2 + 0.5(-1.266))$$
$$= -0.1722$$

(c) $$Q^{\pi}_{\infty}(A, \text{counterclockwise}) \leftarrow 0.4(1 + 0.5(-1.114)) + 0.6(0 + 0.5(-1.266))$$
$$= -0.2026$$

(d) Updated action for A is clockwise as it has a higher Q value

Question 5

Code can be found in the python notebook.

Final Image: