Multi-objective optimization of the Multiple Traveling Salesmen Problem Using a Non-dominated Sorting Genetic Algorithm (NSGA-II)

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*Abstract*— **as.**

1. INTRODUCTION

Solutions to multi-objective optimization problems (MOOPs) often find applications in the real world [], as many real world problems often have multiple conflicting objectives. A few of the domains that benefit from research into such solutions are resource management [], manufacturing [], network coverage [], and various other domains []. A popular example of such problems is the multi-objective multiple traveling salesmen problem (MOmTSP), first studied by Frederickson, Hecht, and Kim [] in 1978.

The MOmTSP, which is NP-Hard, can be briefly formulated as minimizing the total distance travelled and balancing the workload for *m* salesmen visiting *n* cities exactly once, with each salesman originally departing and finally returning to a common origin city or ‘depot’. A particular modern application of the problem that comes to mind would be in the deployment of multiple drones to achieve certain goals (e.g. delivery of packages) at certain locations and return. Due to limitations on battery life, there is a need to ensure all deployed drones are active for similar times so the coverage of locations can be maximized. In addition, many other multi-objective combinatorial optimization problems (MOCOPs) like the vehicle routing problem [] and job shop scheduling problem [] can be transformed into the MOmTSP and solved, which allows the MOmTSP to act as an adequate test problem for MOCOP solvers.

Evolutionary algorithms have been at the forefront of heuristic algorithms for solving MOCOPs and MOOPs for over a decade [], with the non-dominated sorting genetic algorithm NSGA-II, proposed by Deb [] in 2002, being the most cited and used algorithm in literature for solving problems in this class []. Some salient points which make this algorithm so popular are that it incorporates an explicit diversity preservation mechanism, as well as implicit elitism [] to retain the best solutions across generations. Due to the mentioned advantages, proven effectiveness [] as well as the open availability of the NSGA-II [], the algorithm has been used as a sort of gold standard for comparing other (novel) algorithms [], especially since similarly (or more) efficient tools used in industry are inaccessible.

However, the NSGA-II has been criticized for two failings— its poor search efficiency for many (>10) objective problems [] and the non-universality of its diversity preservation mechanism. Disregarding the former case as only two objectives are considered in this paper, specific to combinatorial optimization, NSGA-II when used with conventional crossover operators devised for permutation-based problems— Partially Mapped Crossover (PMX) [], Cyclic Crossover (CX) [], and Ordered Crossover (OX) [], has been noted to converge prematurely []. Although there is a lack of studies as to why this is the case, it may be intuitively attributed to two reasons: first, a lack of evolvability when using the mentioned operators, as the ‘good’ subsequences or arrangement of genes are not preserved during inheritance; and secondly, because in later generations, the population of solutions may lack in diversity (Discussed in more detail in Section 2).

Other prominent EAs and other (non-evolutionary) algorithms used in this problem domain are SPEA, PSO, ACO... . Comparisons of the other algorithms with the NSGA-II yields that while the ..., the NSGA-II with small modifications [] (‘small’ as described in the cited paper), has been found to perform ...

In literature, the problems of insufficient solution diversity in later generations, and ... faced by the NSGA-II have been bypassed or overcome in a myriad of ways. Some examples of methods that have been shown to improve solution diversity and thus performance of the NSGA-II are ... . Contrarily, in [], the authors implemented a hierarchical crossover operator that appears to improve the evolvability of the solutions and thus reach solutions for instances of the MOmTSP from TSPLIB [] comparable to the existing benchmark. Somebody [] proposed .... Some other [] proposed ....

In this project, the main objectives are to implement the NSGA-II algorithm to solve the MOmTSP and test a number of crossover operators (PMX, CX, OX, HGAX), and effects of varying population size and mutation probabilities on instances from TSPLIB. An additional objective is to implement a parallel GA mechanism to try and avoid premature convergence of the NSGA-II when using PMX and OX operators. The main deliverable is the software written for the implementation in Python, submitted alongside the report and also made available in a public access GitHub repository. The repository contains functions to implement the NSGA-II as well as the various crossover operators mentioned, four mutation operators, and a selection operator; and also the main code to solve the MOmTSP for both random instances and select instances (eil51, berlin52, eil76, and rat99) from TSPLIB, which are also included as text files.

The project report beyond this introduction is structured as follows: first, the relevant background information needed for the project, such as terms related to multi-objective optimization and the NSGA-II algorithms are introduced along with the mathematical formulation of the MOmTSP; the next section briefly exposits on related work on NSGA-II as well as its (and other GA) application to the TSP and mTSP (and variants) with specially mentioned proposed solutions to the above discussed issues with EAs; thirdly, the implementation of the algorithm and genetic operators is detailed along with the design of the experiments to test the implementation; and lastly, the obtained results are discussed, with some critical analysis connecting the results to the broader issues in EAs and their usage in solving MOCOPs.

2. BACKGROUND

***2.1. Pareto-Optimality and Dominance***

In multi-objective problems, it is required to find an optima for every one of the objectives, and thus, for conflicting objectives, it becomes impossible to find a singular optimal solution. Instead, the goal of solving the problem is to determine an optimal pareto front, which is the set of optimal solutions to the problem (Pareto-Optimal Solutions or POS), from which appropriate trade-offs can be considered to choose a suboptimal solution.

The pareto-optimal front is composed of solutions that “dominate” all other solutions but not each other. Here, a solution, say X, is said to dominate another, say Y, if X evaluates to better than Y for at least one objective function, and equal to Y for the rest of the objective functions. Graphically, this can be seen in Figure 1, where the round solutions dominate the square ones for two objectives (obj1 and obj 2). Thus, the round solutions constitute the optimal pareto front.

More generally, a pareto front (alternatively pareto frontier) consists of solutions that do not dominate each other. This is the concept of non-dominance which is exploited in the NSGA and NSGA-II.

***2.2. Multiple Traveling Salesmen Problem***

As described earlier, the mTSP involves *m* salesmen visiting *n* cities exactly once. While there are multiple variations that arise from this point, such as multiple origin cities, different origin and final destination cities, and so on, the problem considered is of the conventional closed circuit with all salesmen originating and finally returning to the same depot, called the SD-MTSP.

Considering the problem in graph form, it can be represented by the undirected graph with *V* being the set of vertices representing cities and *A* being the set of arcs between the cities. There is an associated distance matrix C of dimensions , with each element being the distance between *i* and *j*. Each salesman , travels from city *i* to city *j* and is an indicator of whether *k* travelled the arc between *i* and *j* (if yes,=1; else,=0).The set of constraints on the tours can then be mathematically described as:

|  |  |
| --- | --- |
|  | (1) |
|  | (2) |
|  | (3) |
|  | (4) |
|  | (5) |

where ‘0’ is taken to be the depot city.

Constraints (1) and (2) limit the traversal of each arc to just one time across all salesmen; constraints (3) and (4) ensure all salesmen start and end their tours at the depot; and lastly, constraint (5) is to ensure that no sub-tours excluding the depot are created.

The objectives are to minimize the total distance travelled by all salesmen and the difference between the longest and shortest tours, which can be represented by the equations (6) and (7) respectively:

|  |  |
| --- | --- |
|  | (6) |

and,

|  |  |
| --- | --- |
|  | (7) |

where,

***2.3. NSGA-II***

The Non-dominated Sorting Genetic Algorithm was proposed by Deb and Agarwal [] in 2002 as an improvement to the older NSGA, with the addition of elitism. The basic principle behind the titular sorting mechanism is the concept of ‘non-dominance,’ or simply that the solutions in a population be ranked based on their level of non-dominance.

In the algorithm, the solutions which are not dominated by any other solutions are ranked first, with solutions that are dominated by a single other solution being ranked second. Naturally, solutions having the same rank constitute a pareto frontier, which is the mechanism exploited by the algorithm to sort the solution population and determine elite solutions.

There are two main methods in the NSGA-II: fast non-dominated sorting, and crowding distance assignment, of which the former is described above. The crowding distance of a solution, as described by the authors, is the sum of the differences between each objective function evaluation of the solution and its nearest neighbor. Thus for a bi-objective problem, the crowding distance would be the sum of a long side and short side of the rectangle formed if the function values of the solution and its nearest neighbor were plotted on diagonally opposite vertices.

According to the algorithm, the less crowded solutions, i.e., ones that are farther away from any other would be relatively unique and thus worth preserving more than more crowded solutions in the same non-dominance level. Thus the algorithm discriminates between the solutions first on the basis of non-dominance level, and second (if the solutions are on same rank), on the basis of crowding distance. This is the diversity preservation mechanism in the algorithm.

The steps of the algorithm are: 1. Initial population formation (size=*N*); 2. Selection of parents for next generation by binary crowded tournament; 3. Recombination and mutation to form an offspring population of size *N*; 4. Parents and offspring are clumped together in a population of size *2N* from which the top *N* solutions are selected via non-dominated sorting and comparison of crowding distance (elitism); and, 5. The steps 2-5 are repeated until a satisfactory number of iterations or until there is no to little change in the pareto-front (convergence).

***2.4. Issue of the crowding distance operator in NSGA-II***

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3. RELATED WORK

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4. METHODOLOGY

***4.1. Design***

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The.

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To.

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If the.

***4.2. Implementation*—**

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Two.

*4.2.2.*

While.

5. SIMULATION AND RESULTS

‘results’.

***5.1.***

*5.1.1.*

As.

*5.1.2.*

A.

***5.2.***

*5.2.1.*

The.

*5.2.2.*

The results.

***5.3. Discussion on the experiment and results***

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6. CONCLUSION

The project

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REFERENCES

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