Waveforms as Qubits

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Abstract

As scientist and companies struggles to overcome problems in quantum computing, some novel approaches have became more popular. Based on the mathematical formalism of quantum computing and linear algebra, analog solutions may be a viable and cheaper solution to build a emulator for a quantum system. In this paper, is suggested a way to encode qubits as complex analog signals and discussed how it relates to a true quantum system.

I. Introduction

As classical computational power reaches Moore's Law limit, quantum computing has emerged as a viable alternative. Despite the efforts from many researchers, the implementation and scalability of those systems are a troubling problem. This problems are mostly related to short coherence times [xxx]. To increase it, it is necessary to build systems completely isolated and at cryogenic temperatures. To overcome this problems and considering the mathematical foundations of quantum computing, it is proposed a way of encode the qubit information on complex analog signals.

The organization of the paper is as follows. In section II we describe the basic formulation of complex and real signals and how they are related with each other. In section III we introduce the basic concepts and properties of quantum computing. It is introduced the qubit and multi-qubit formalism and relevant properties, like entanglement. At last, in section IV, it is introduced a way of encoding qubit information based in signal processing techniques.

II. From complex number to waveform

Consider the phasor z defined such that

$$z = a + jb = Ae^{j\theta_A} \tag{1}$$

where $j = \sqrt{-1}$. This phasor can be represented a complex-valued signal such that

$$x_a(t) = ze^{jwt} \tag{2}$$

called the analytic signal. In order to represent the phasor from 1 as a sinusoid, we must add to equation 2 the negative counterpart such that

$$x(t) = ze^{jwt} + ze^{-jwt} \tag{3}$$

$$z(t) = \begin{bmatrix} Ae^{j(wt+\theta_A)} \\ Be^{j(-wt+\theta_B)} \end{bmatrix} \tag{4}$$

We define that the first complex number gets a positive frequency and the second the negative conterpart, as shown in equation 4.

As we are interested in representing the complex signal as a real one, lets now consider the real part of complex signal $z(t)$, in equation 2 such that

$$u(t) = Re[z(t)] = \frac{z(t) + z^*(t)}{2} \tag{5}$$

In the frequency domain we have

$$U(f) = \frac{Z(f) + Z^*(-f)}{2} \tag{6}$$

From equations 5 and 6 we can see that the real part of a complex signal is the sum of the signal itself and its complex conjugate, multiplied by some constant. The problem in this representation comes when w is too small causing signals $Z(f)$ and $Z^*(-f)$ to overlap. In order to overcome this problem we need to move the complex signal to higer frequency. To do so, $z(t)$ needs to be multiplied by a carrier signal such that

$$y(t) = z(t)e^{jw_c} \tag{7}$$

where w_c represents the carrier frequency such that $w_c >> w$. From equations 6 and 7 we can say that signal $u(t)$ is now defined such that

$$u(t) = Re[y(t)] = \frac{y(t) + y^*(t)}{2} \tag{8}$$

In the frequency domain comes

$$U(f) = \frac{Y(f) + Y^*(-f)}{2} \tag{9}$$

In conclusion, as long as $Y(f)$ and $Y^*(-f)$ do not overlap, the signal $u(t)$ contains all information about $z(t)$. It is then possible to represent a complex number in a form of a real signal.

III. Introducing the Qubit

A quantum bit, or qubit, is the fundamental unit of information in quantum computing. In a mathematical point of view, a qubit is nothing more that a linear combination of two vectors, each one of them representing the probability amplitude of each state. Let $|\psi\rangle$ denote a qubit such that

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle \tag{10}$$

With the condition

$$|\alpha|^2 + |\beta|^2 = 1 \tag{11}$$

Where α and β are complex numbers called probability amplitudes. Besides the basis states $|0\rangle$ and $|1\rangle$, a qubit can be defined, according to equation 10, in a superposition of both states. The probability of observing states $|0\rangle$ and $|1\rangle$ are $|\alpha|^2$ and $|\beta|^2$, restricted by the condition in equation 11.

The general state of an N-qubit system, also known as qubit register, $|\psi\rangle$, composed of N different qubits, $|\psi_0\rangle, \ldots, |\psi_{N-1}\rangle$, can be obtained by applying the tensor product between all qubits, such that

$$|\psi\rangle = |\psi_0\rangle \otimes |\psi_1\rangle \otimes \cdots \otimes |\psi_{N-1}\rangle \tag{12}$$

The state $|\psi\rangle$, from equation 12, will then be defined in a superposition of all 2^N possible states.

When working with N-qubit systems, properties like quantum entanglement must be of maximal concern. Entanglement occurs when groups of qubits interact in ways such that the quantum state of each qubit cannot be described individually. Instead the quantum state of the systems must be described as a whole. This property is at the base of major quantum computing applications, like Shor and Groover algorithms.

IV. Qubit as a Complex Signal

Taking into consideration the properties of complex signal, in this section is introduced a way of representing the qubit with complex signals.

Considering the qubit from equation 10, lets define two baseband complex signals, $\alpha(t)$ and $\beta(t)$, such that

$$\alpha(t) = \alpha e^{jwt}$$

$$\beta(t) = \beta e^{-jwt}$$

Where α and β represent the complex numbers from equation 10. A qubit signal $\psi(t)$ is defined such that

$$\psi(t) = \alpha(t) + \beta(t) = \alpha e^{jwt} + \beta e^{-jwt} \tag{13}$$

In order to avoid overlapping, the signal must be modulated to high frequency. In this sense, $\psi(t)$ is multiplied by a carrier signal of frequency w_c, such that

$$y(t) = \psi(t)e^{jw_c t}$$

Given the results from II, it is possible to recover all information about $\psi(t)$ from the real part of $y(t)$. In this case

$$u(t) = Re[y(t)] = Re[\psi(t)e^{jw_c t}] \quad (14)$$

The signal $u(t)$ can fully describe any qubit defined as a signal $\psi(t)$. Lets now consider a 2-qubit system such that

$$\psi_0(t) = \alpha_0 e^{jw_0 t} + \beta_0 e^{-jw_0 t} \quad (15)$$

$$\psi_1(t) = \alpha_1 e^{jw_1 t} + \beta_1 e^{-jw_1 t} \quad (16)$$

For each qubit is assigned a different frequency where, for convenience, $0 < w_0 < w_1 < w_c$. For a general N-qubit system, each qubit is assigned with a frequency such that $0 < w_0 < w_1 < \cdots < w_N < w_c$. The general state of the N-qubit register $\psi(t)$ will be defined such that

$$\psi(t) = \sum_{n=0}^{2^N-1} a_n e^{j\epsilon t} \quad (17)$$

With

$$\epsilon = \sum_{x=0}^{N-1} w_x(-1)^{n_x}$$

Where N represents the number of qubits in the system and n_x represents the binary position x of the number n such that $n = n_0 n_1 \ldots n_{N-1}$.

From equation 14 and 13 it can be concluded that, for representing a N-qubit system, it is only necessary $n+1$ frequencies, one for each qubit and one for the carrier. Although, when comes to represent the general state of the system, equation 17, it is necessary to use $2^N - 1$ different frequencies.

References