# Geometria - Esercizio 4.24

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## Metodo 1: Gauss (rigoroso)

$$A = \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 2 & -1 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & -4 & -3 & -2 & -5 \\ 3 & -2 & 2 & 2 & -2 \end{bmatrix}$$

$$\begin{pmatrix} \\ A_2 - 2A_1 \\ A_3 - A_1 \\ A_4 - A_1 \\ A_5 - 3A_1 \end{pmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 3 & -1 & 3 & 3 \\ 0 & -2 & -6 & 0 & -3 \\ 0 & 4 & -7 & 8 & 4 \end{bmatrix}$$

$$\begin{pmatrix} \\ \\ A_3 - A_2 \\ A_4 + {}^2\!/_3 A_2 \\ A_5 - {}^4\!/_3 A_2 \end{pmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 0 & 4 & -4 & -3 \\ 0 & 0 & {}^{-28}\!/_3 & {}^{14}\!/_3 & 1 \\ 0 & 0 & {}^{-1}\!/_3 & {}^{-4}\!/_3 & -4 \end{bmatrix}$$

$$\begin{pmatrix} \\ \\ \\ A_4 + {}^7\!/_3 A_3 \\ A_5 + {}^1\!/_{12} A_3 \end{pmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 0 & 4 & -4 & -3 \\ 0 & 0 & 0 & {}^{-14}\!/_3 & -6 \\ 0 & 0 & 0 & {}^{-5}\!/_3 & {}^{-17}\!/_4 \end{bmatrix}$$

$$\begin{pmatrix} \\ \\ \\ \\ A_5 - {}^5\!/_{14} A_4 \end{pmatrix} \rightsquigarrow \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 0 & 4 & -4 & -3 \\ 0 & 0 & 0 & {}^{-14}\!/_3 & -6 \\ 0 & 0 & 0 & 0 & {}^{-59}\!/_{28} \end{bmatrix}$$

$$\det A = \det \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 0 & 4 & -4 & -3 \\ 0 & 0 & 0 & {}^{-14}\!/_3 & -6 \\ 0 & 0 & 0 & 0 & {}^{-59}\!/_{28} \end{bmatrix} = 1 \cdot 3 \cdot 4 \cdot \left(-\frac{14}{3}\right) \cdot \left(-\frac{59}{28}\right) = 118$$

## Metodo 1: Operazioni elementari

Come Gauss, ma più efficiente (forse?).

$$
\begin{aligned}
\det \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 2 & -1 & 1 & 3 & 2 \\ 1 & 1 & 2 & 1 & 1 \\ 1 & -4 & -3 & -2 & -5 \\ 3 & -2 & 2 & 2 & -2 \end{bmatrix} &= \det \begin{bmatrix} 1 & -2 & 3 & -2 & -2 \\ 0 & 3 & -5 & 7 & 6 \\ 0 & 3 & -1 & 3 & 3 \\ 0 & -2 & -6 & 0 & -3 \\ 0 & 4 & -7 & 8 & 4 \end{bmatrix} \\
&= \det \begin{bmatrix} 3 & -5 & 7 & 6 \\ 3 & -1 & 3 & 3 \\ -2 & -6 & 0 & -3 \\ 4 & -7 & 8 & 4 \end{bmatrix} \\
&= \det \begin{bmatrix} 0 & -4 & 4 & 3 \\ 3 & -1 & 3 & 3 \\ -2 & -6 & 0 & -3 \\ 0 & -19 & 8 & -2 \end{bmatrix} \\
&= \det \begin{bmatrix} 0 & -4 & 4 & 3 \\ 1 & -7 & 3 & 6 \\ -2 & -6 & 0 & -3 \\ 0 & -19 & 8 & -2 \end{bmatrix} \\
&= \det \begin{bmatrix} 0 & -4 & 4 & 3 \\ 1 & -7 & 3 & 6 \\ 0 & -6-14 & 0+6 & -3+12 \\ 0 & -19 & 8 & -2 \end{bmatrix}
\end{aligned}
$$