Improved wave-current modeling with the SeaState module of OpenFAST

The improvements to the modeling of combined waves and current in the SeaState module of OpenFAST consist of two parts:

- **Doppler effect:** Instead of simply adding the wave and current velocities together, as was done in previous versions of OpenFAST, the improved wave-current model allows the waves to “ride” a uniform current. This introduces a Doppler effect that is missing from previous implementations. This change primarily affects the loads on strip-theory members, which are computed using the flow-field velocity, material acceleration, and dynamic pressure.
- **Wave spectrum correction:** Optionally, a wave-current interaction model can be enabled to correct the incident wave spectrum measured in a region without current. This is limited to colinear (aligned or opposing) waves and currents in deep water. The change to the incident wave spectrum affects both strip-theory members and potential-flow bodies. However, with the latter, the solution is not fully corrected for current unless the potential-flow hydrodynamic coefficients solved with the current are also used. For small current velocity/forward speed, corrected potential-flow hydrodynamic coefficients can be obtained using, e.g., Capytaine [1].

The improvements to wave-current modeling are limited to constant and uniform mean currents without shear (at least in the near-surface region where waves are present). Turbulent fluctuations of the current about the mean can be included through simple superposition; however, the fluctuations do not directly affect the waves, nor the potential-flow hydrodynamic loads. Furthermore, the improved wave-current modeling can only be used with first-order linear wave theory without directional spreading.

1. Doppler effect

In previous versions of OpenFAST, the velocities and accelerations (if using dynamic current from InflowWind for MHK turbine simulations) of the current and the waves are simply added together to construct the combined flow field. This is, however, not a correct description of waves in the presence of a current. In the case of a constant and uniform current, waves should simply ride the current, such that the zero-current wave solutions are applicable in a steadily translating frame of reference that moves with the current.

Consider a constant and uniform current velocity $\vec{U}_c = (u_c, v_c)$ with long-crested waves. The relation between a point in the earth-fixed coordinate system $\vec{X} = (X, Y)$ and a point fixed in the current-attached frame of reference $\vec{x} = (x, y)$ is given by

$$\vec{X} = \vec{x} + \vec{U}_c t. \tag{1}$$

For a linear wave component in the current-attached frame, the phase of the wave component at a point in time and space is given by

$$\phi(t, \vec{x}) = \omega t - \vec{k} \cdot \vec{x}, \tag{2}$$

where ω is the *intrinsic* or *relative* angular frequency of the wave component. $\vec{k} = (k \cos\theta\,, k \sin\theta)$ is the directional wave number based on the wave heading θ (the direction perpendicular to the crestline). Note that the scalar wave number, k, remains the same irrespective of the frame of reference. k and ω are related by the standard linear dispersion relation without current:

$$\omega^2 = gk \tanh(kh), \tag{3}$$

where h is the water depth and g is the gravitational acceleration.

Substituting Eq. (1) into Eq. (2), we get the phase of the wave component as observed in the earth-fixed frame:

$$\phi_e(t, \vec{X}) = \phi\left(t, \underbrace{\vec{X} - \vec{U}_c t}_{\vec{x}}\right) = \underbrace{\left(\omega + \vec{k} \cdot \vec{U}_c\right)}_{\omega_e} t - \vec{k} \cdot \vec{X}, \tag{4}$$

where ω_e is the *total* or *absolute* angular frequency.

The associated complex amplitudes of wave elevation, velocity, material acceleration, and hydrodynamic pressure remain unchanged from those given by the linear wave theory without current provided the intrinsic frequency ω is used instead of the absolute frequency ω_e, which only appears in the wave phase, ϕ_e. The resulting wave velocity and material acceleration can be added to the current velocity and acceleration to obtain the combined flow field. The contribution to hydrodynamic pressure from turbulent current is neglected as before because this information is generally not available.

Following the above discussions, we only need to compute ω from ω_e to construct the wave field in the presence of a uniform mean current. The modified dispersion relation in terms of the absolute frequency is obtained by substituting the expression of ω_e in Eq. (4) into Eq. (3):

$$\omega_e - \vec{k} \cdot \vec{U}_c = \sqrt{gk \tanh(kh)}. \tag{5}$$

When $\vec{k} \cdot \vec{U}_c > 0$, we have a simple monotonic relation between ω and ω_e with $\omega_e(\omega) > \omega$. With $\vec{k} \cdot \vec{U}_c < 0$, the relation is more complex. An example is shown in Figure 1. In this example, we consider a simple scenario with colinear but opposite wave and current headings. We have wave group velocity $V_g > 0$, wave phase velocity $V_p > 0$, and current velocity $U_c = -1$ m/s.

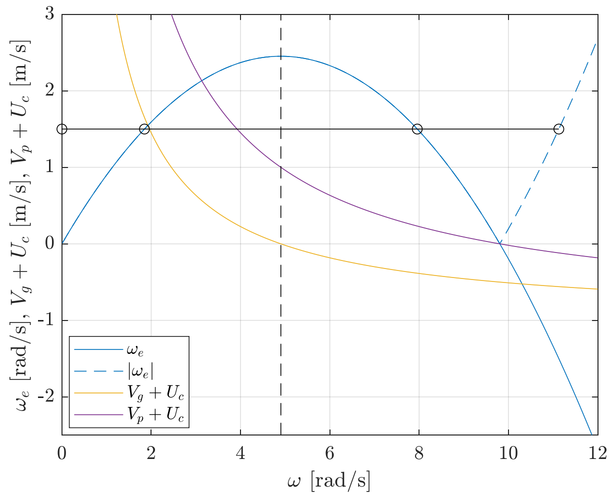

Figure 1. Example relation between ω_e and ω with opposing wave and current directions. The horizontal line shows a possible one-to-three mapping from ω_e to ω. The vertical dashed line marks the critical frequency ω_{crit}, at which the wave group velocity and current velocity cancels.

For long waves (small ω), we have a simple monotonic relation between ω_e and ω until approximately $\omega = 4.9$ rad/s, at which point the wave group velocity and current velocity cancel each other. This is a critical frequency of $\omega = \omega_{crit}$ that corresponds to $\omega_e = \omega_{e,crit}$, where wave energy can no longer propagate against the current. In reality, if waves of this frequency propagate from a region without current into a region with opposite current, wave breaking will occur because waves cannot grow indefinitely. Beyond $\omega = 4.9$ rad/s, ω_e starts to decrease with increasing ω. At approximately $\omega = 9.8$ rad/s, the wave phase velocity and current velocity cancel. Beyond this frequency, a hypothetical wave will appear to be propagating “backward” to an earth-fixed observer. Due to the nonmonotonic relation between ω_e and ω, up to three different intrinsic frequencies ω (if we consider the absolute value) can map to the same absolute frequency ω_e, as shown in Figure 1.

In SeaState, we shall assume, by default, the input wave spectrum/time series, irrespective of the WaveMod option selected (except for WaveMod = 6 with full externally defined flow field, for which the present discussion does not apply), is that observed at an earth-fixed point. In other words, the input wave spectrum (or the Fourier transform of the input wave time series) is a function of ω_e instead of ω. Generally, we cannot have significant waves with $\omega \geq \omega_{crit}$ when $\vec{k} \cdot \vec{U}_c < 0$ due to wave breaking. Below ω_{crit}, we have a simple one-to-one mapping from ω_e to ω. Therefore, we neglect the possibility of $\omega \geq \omega_{crit}$ to avoid the indeterminant one-to-three mapping of wave contents. Note that this also means SeaState will neglect any wave content in the input wave spectrum or time series with $\omega_e \geq \omega_{e,crit}$.

2. Wave spectrum correction for wave-current interaction

Up to this point, we have assumed that the wave spectrum based on the absolute frequency, ω_e, is directly available from user input wave frequency components (WaveMod = 7), wave time series (WaveMod = 5), or built-in/user-defined wave spectrum (WaveMod = 1 [regular], 2 [JONSWAP], 3 [White-noise], or 4 [user-defined]).

When waves propagate from a region without current to a region with current, the wave spectrum will change due to wave-current interactions. Therefore, if only the wave spectrum without current is known, SeaState needs to modify the wave spectrum to account for wave-current interactions. This is an optional functionality that can be enabled through a new user input switch WvCrntMod in the SeaState input file. If this option is enabled, SeaState will interpret the input wave spectrum/time series to be those of the waves before encountering the current. SeaState will compute the wave spectrum after the waves encounter the current internally. Due to the simplified model used, we limit this option to colinear (aligned or opposing) waves and current in deep water without wave directional spreading.

When waves propagate from a region without current to a region with current, the absolute wave frequency is invariant [2]. Given the wave phase, ϕ_e, observed in the earth-fixed frame of reference, the wave number is more generally defined as

$$\vec{k}(t,\vec{X}) = -\nabla\phi_e, \tag{6}$$

and the wave angular frequency is

$$\omega_e(t,\vec{X}) = \frac{\partial\phi_e}{\partial t}. \tag{7}$$

Therefore, we have the following conservation law

$$\frac{\partial\vec{k}}{\partial t} + \nabla\omega_e = 0. \tag{8}$$

At steady state, this leads to $\nabla\omega_e = 0$, i.e., the absolute frequency ω_e is constant.

In the region without current, the intrinsic frequency and the absolute frequency are one and the same. In the region with current, the two are different, which implies that the intrinsic frequency of the waves, ω, must change going from a region without current to a region with current. As a result, the wavelength/wave number, k, must change as well.

Furthermore, wave action is conserved (not wave energy due to energy exchange between waves and current). Wave action, $\mathcal{A}$, is defined as

$$\mathcal{A} = \frac{E}{\omega}, \tag{9}$$

where E is the wave energy density (energy/area) and ω is the intrinsic frequency. With linear waves, we have

$$E = \frac{1}{8}\rho g H^2. \tag{10}$$

For small amplitude waves, the conservation of wave action is in the following form (see, e.g., Ref. [3]):

$$\frac{\partial}{\partial t}\left(\frac{E}{\omega}\right) + \nabla \cdot \left[\left(\vec{U}_c + \vec{V}_g\right)\frac{E}{\omega}\right] = 0, \tag{11}$$

where $\vec{V}_g$ is the group velocity of the waves. For steady state (constant wave and current properties), the conservation law reduces to

$$\nabla \cdot \left[\left(\vec{U}_c + \vec{V}_g\right)\frac{E}{\omega}\right] = 0. \tag{12}$$

If we further assume colinear long crested waves and current, the conservation law further reduces to

$$\frac{\partial}{\partial x_w}\left[\left(U_c + V_g\right)\frac{E}{\omega}\right] = 0, \tag{13}$$

where x_w is the distance in the wave direction. The current and group velocities are replaced with scalar quantities giving the component in the direction of wave propagation. As a result, we have the following relation between the wave properties in the region with current and in the region without [3, 4]:

$$\left(U_c + V_g\right)\frac{E}{\omega} = \text{constant} = \frac{V_g^0 E^0}{\omega^0}, \tag{14}$$

where the superscript “0” indicates quantities in the region without current. The ratio of wave energy density with and without current is then

$$\frac{E}{E^0} = \left(\frac{A}{A^0}\right)^2 = \frac{\omega}{\omega^0}\frac{V_g^0}{U_c + V_g}. \tag{15}$$

Leveraging the fact that the absolute wave frequency, ω_e, is invariant, we have the following relation:

$$\omega^0 = \omega_e = \omega + kU_c, \tag{16}$$

which, if substituted into Eq. (15), leads to

$$\left(\frac{A}{A^0}\right)^2 = \frac{\omega}{\omega + kU_c}\frac{V_g(\omega + kU_c)}{U_c + V_g(\omega)}. \tag{17}$$

As a reminder, ω here is the intrinsic frequency in the current region, and V_g is the relative group velocity (as observed by an observer that moves with the current).

In deep water, $V_g = \frac{1}{2}V_p = \frac{1}{2}\frac{g}{\omega}$. We can simplify the amplitude ratio of Eq. (17) to

$$\left(\frac{A}{A^0}\right)^2 = \frac{4}{\left(1 + \sqrt{1 + \frac{4U_c\omega_e}{g}}\right)^2 \sqrt{1 + \frac{4U_c}{g}\omega_e}}. \tag{18}$$

In irregular waves, we have

$$A = \sqrt{2S(\omega_e, U)\Delta\omega_e}, \tag{19}$$

and

$$A^0 = \sqrt{2S(\omega^0)\Delta\omega^0}. \tag{20}$$

Note that $\omega_e = \omega^0$ because the absolute wave frequency is invariant going from a region without current to a region with current. Therefore, we have the following relation between the wave spectra with and without current [2, 3, 4]:

$$\frac{S(\omega_e, U)}{S(\omega_e)} = \frac{4}{\left(1 + \sqrt{1 + \frac{4U_c\omega_e}{g}}\right)^2 \sqrt{1 + \frac{4U_c\omega_e}{g}}} = \frac{\omega^2}{\omega_e^2}\frac{1}{1 + \frac{2U\omega}{g}}. \tag{21}$$

To arrive at the above relation, we have assumed that the wave and current are colinear and the water is deep. This limits the applicability of the above model.

When $1 + \frac{4U_c\omega_e}{g} = 0$, which happens at $\omega_{e,crit}$ with $U_c = -V_g$, wave energy cannot propagate against the current, and $S(\omega_e, U) \rightarrow \infty$. Wave breaking will occur before reaching this point, dissipating the wave energy. Based on this argument, SeaState will truncate the wave spectrum to just below $\omega_{e,crit}$. It is further suggested that the user should set an appropriate high wave cutoff frequency that is less than $\omega_{e,crit}$.

3. Implementation details

The improved wave-current modeling affects all WaveMod options of SeaState, except for WaveMod = 0 (no waves) and WaveMod = 6 (full externally defined flow field). A new input WvCrntMod is added to the SeaState module primary input file. It can be set to 0, 1, or 2:

- WvCrntMod = 0: No enhanced wave-current modeling. Simple superposition of wave and current velocity and acceleration will be used as in previous versions of OpenFAST. No additional restrictions.
- WvCrntMod = 1: Doppler effect included in wave-current modeling. Irrespective of WaveMod (except WaveMod = 0 or 6), the input wave spectrum/time series is interpreted as that measured with the current. Any wave content with $\omega_e \geq \omega_{e,crit}$ is neglected. Only long-crested waves without directional spreading are allowed, but oblique wave and current headings are supported.
- WvCrntMod = 2: Doppler effect and wave spectrum correction for wave-current interactions are both enabled. The input wave spectrum/time series is interpreted as that measured without current, and SeaState will correct the wave spectrum internally for wave-current interactions. The waves and current must be colinear without wave directional spreading. Deep water condition is assumed when correcting the wave spectrum, although this is not checked by SeaState.

When WvCrntMod>0, wave directional spreading and second-order wave kinematics will be disabled for now. The constrained NewWave model can still be applied.

The enhanced wave-current modeling described in this document assumes shear-free mean current over the near-surface region where the waves are present. If the SeaState current model is used, the total current velocity at the free surface will be used in the wave-current modeling (SeaState only supports steady current). If the dynamic turbulent current from InflowWind is used instead, the time-averaged current velocity at the free surface will be computed and used in the wave-current modeling. In either case, the user is responsible for ensuring that the current from the free surface down to an adequate depth is approximately shear free, and that the current velocity at the free surface reasonably approximates the current velocity over this near-surface shear-free region.

References

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