

Introducing R: from Your Laptop to HPC and Big Data

Drew Schmidt¹ and George Ostrouchov^{1,2}

July 8, 2013



Affiliations and Support

The pbdR Core Team

<http://r-pbd.org>

Wei-Chen Chen¹, George Ostrouchov^{1,2}, Pragneshkumar Patel², Drew Schmidt¹

Ostrouchov, Patel, and Schmidt were supported in part by the project “NICS Remote Data Analysis and Visualization Center” funded by the Office of Cyberinfrastructure of the U.S. National Science Foundation under Award No. ARRA-NSF-OCI-0906324 for NICS-RDAV center.

Chen and Ostrouchov were supported in part by the project “Visual Data Exploration and Analysis of Ultra-large Climate Data” funded by U.S. DOE Office of Science under Contract No. DE-AC05-00OR22725.

¹Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN

²Remote Data Analysis and Visualization Center, University of Tennessee, Knoxville, TN

About This Presentation

Downloads

This presentation and supplemental materials are available at:

<http://r-pbd.org/handouts>

About This Presentation

Speaking Serial R with a Parallel Accent

The content of this presentation is based in part on the **pbdDEMO** vignette *Speaking Serial R with a Parallel Accent*

<https://github.com/wrathematics/pbdDEMO/blob/master/inst/doc/pbdDEMO-guide.pdf?raw=true>

It contains more examples, and sometimes added detail.

About This Presentation

Installation Instructions

Installation instructions for setting up a pbdB environment are available:

<http://r-pbd.org/install.html>

This includes instructions for installing R, MPI, and pbdB.

About This Presentation

Conventions

We use:

- “.” as a decimal mark
- “,” as order of magnitude separator

Example	Yes	No
One million	1,000,000	1.000.000
One half	0.5	0,5
One thousand and one half	1,000.5	1.000,5

Contents

- 1 Introduction
- 2 pbdR
- 3 Introduction to pbdMPI
- 4 The Generalized Block Distribution
- 5 Brief Intermission
- 6 Basic Statistics Examples
- 7 Introduction to pbdDMAT
- 8 Examples Using pbdDMAT
- 9 Wrapup

Contents

1 Introduction

- A Concise Introduction to Parallelism
- R and Parallelism

What is Parallelism?

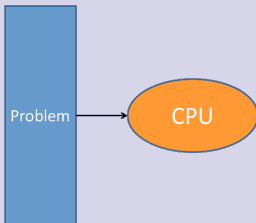
Broadly, *doing more than one thing at a time.*

The simultaneous use of multiple compute resources to solve a computational problem:

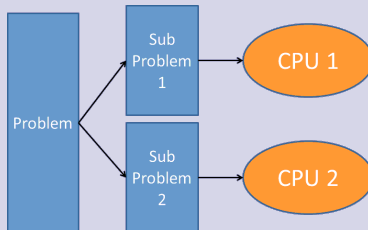
A Concise Introduction to Parallelism

Parallelism

Serial Programming



Parallel Programming

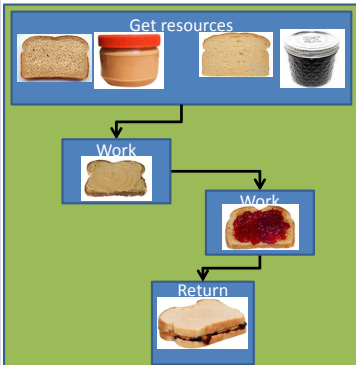


A Concise Introduction to Parallelism

Parallelism

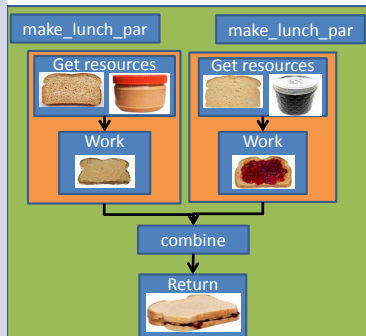
Serial Programming

make_lunch



Parallel Programming

mpirun -np 2 make_lunch_par



A Concise Introduction to Parallelism

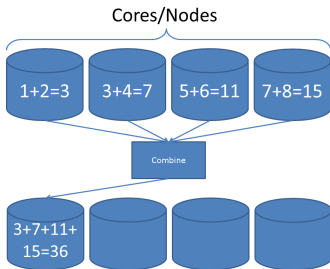
What is Parallelism?

- *Task Parallelism*: Splitting the problem by task
- *Data Parallelism*:

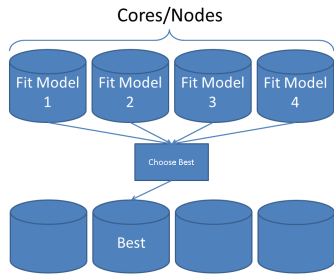
A Concise Introduction to Parallelism

Data vs Task Parallelism

Data Parallelism



Task Parallelism



Common Terms

- ① *Embarrassingly Parallel*: Also called *loosely coupled*. Obvious how to make parallel; lots of independence in computations.
- ② *Tightly Coupled*: Opposite of embarrassingly parallel; lots of dependence in computations.
- ③ *Implicit parallelism*: Parallel details hidden from user
- ④ *Explicit parallelism*: Some assembly required. . .

R and Parallelism

The solution to many of R's problems is parallelism. However ...

What we have

- ① Mostly serial.
- ② Parallelism mostly not distributed (foreach, parallel/snow/multicore, ...)
- ③ Data parallelism mostly explicit (Rmpi, R+Hadoop, ...)

What we want

- ① Mostly parallel.
- ② Mostly distributed.
- ③ Mostly implicit.

Why We Need Parallelism

- ① Saves time (long term).
- ② Data size is skyrocketing.
- ③ Necessary for many problems.
- ④ Like it or not, it's coming.
- ⑤ *It's really cool.*

Problems with R

- ❶ Slow.
- ❷ If you don't know what you're doing, it's *really* slow.
- ❸ Performance improvements usually for small machines.
- ❹ Very ram intensive.
- ❺ Chokes on big data.

Contents

2 pbdR

- The pbdR Project
- pbdR Paradigms

```

ooooo
ooo

```

```

●ooo
ooooo

```

```

oooo
ooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo
ooo

```

```

ooooo
oooooooooooo
ooo

```

```

ooo
ooooo
oo

```

Programming with Big Data in R (pbdR)

Striving for *Productivity, Portability, Performance*



- Series of *free*^a R packages.
- Scalable, big data analytics with high-level syntax.
- Implicit management of distributed data details.
- Methods have syntax *identical* to R.
- Powered by state of the art numerical libraries (MPI, ScaLAPACK, PBLAS, BLACS, LAPACK, BLAS, ...)

^aMPL, BSD, and GPL licensed

```

○○○○○○
○○○

```

```

○●○○
○○○○○

```

```

○○○○
○○○
○○○○○

```

```

○○○
○○○
○○○

```

```

○○○○
○○○
○○○

```

```

○○○○○
○○○○○○○○○
○○○

```

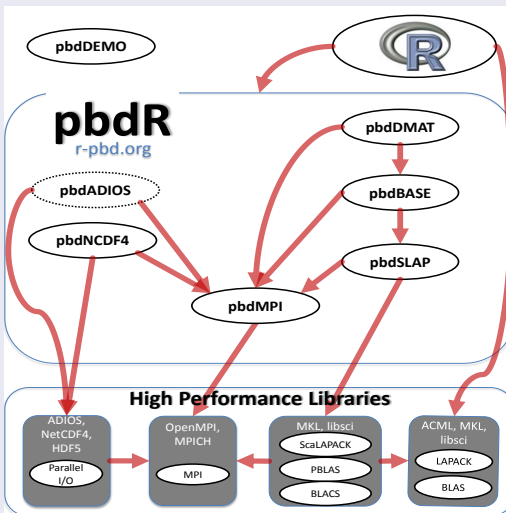
```

○○○
○○○○
○○

```

The pbdR Project

pbdR Packages



```

○○○○○○
○○○

```

```

○○●○
○○○○○

```

```

○○○○
○○○
○○○○○

```

```

○○○
○○○
○○○

```

```

○○○○
○○○
○○○

```

```

○○○○○
○○○○○○○○○
○○○

```

```

○○○
○○○○
○○

```

pbdR Packages — <http://code.r-pbd.org>

Released to CRAN:

- **pbdMPI**: MPI bindings (explicit, low-level)
- **pbdSLAP**: Foreign library (just install it, nothing to use)
- **pbdBASE**: Compiled code (used by DMAT, also for devs)
- **pbdDMAT**: Distributed matrices (mostly implicit, high-level)
- **pbdNCDF4**: Parallel NetCDF4 reader
- **pbdDEMO**: Package demonstrations, examples, vignette written in textbook style

Future Development:

- Profiling tools
- Client/server interface for interactive sessions
- ...

oooooo
ooo

ooo●
oooooo

oooo
ooo
ooooo

ooo
ooo
ooo

oooo
ooo
ooo

ooooo
oooooooooooo
ooo

ooo
oooo
oo

The pbdR Project

Example Syntax

```
1 x <- x[-1, 2:5]
2 x <- log(abs(x) + 1)
3 xtx <- t(x) %*% x
4 ans <- svd(solve(xtx))
```

Look familiar?

The above runs on 1 core with R or 10,000 cores with pbdR

pbdR Focus: Distributed Machines

Shared Memory Machines

Thousands of cores

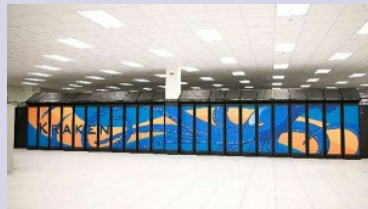


Nautilus, University of Tennessee

1024 cores

Distributed Memory Machines

Hundreds of thousands of cores



Kraken, University of Tennessee

112,896 cores

pbdR Paradigms

Programs that use pbdR are meant to utilize the:

- Data Parallelism method
- Single Program/Multiple Data (SPMD) style

SPMD

The pbdR Packages enable high-level “Single Program/Multiple Data” (SPMD) programming:

- SPMD is a programming *paradigm*.
- Arguably the simplest extension of serial programming.
- Sort of like trying to explain breathing . . .
- Not to be confused with SIMD.
- SPMD utilizes MIMD architecture computers.
- Only one program is written, executed in batch independently on all processors.
- Different processors are autonomous; there is no manager.

SPMD

SPMD codes are run in batch (non-interactively):

From the Shell

```
1 mpirun -np 4 Rscript my_script.R
```

pbdR Paradigms: Data Parallelism

With data parallelism:

- No one processor/node owns all the data.
- Processors own local pieces of a (conceptually) global object

pbdR Paradigms: SPMD

- Natural extension of writing serial codes.
- Different from Manager/Worker.
- No one processor is in charge. Each thinks it's the boss ("it's like academia").
- One program written, executed independently by all processors.
- Each processor owns a local sub-piece of data from the (conceptual) whole.

Contents

- 3 Introduction to pbdMPI
 - Managing a Communicator
 - Reductions and Gathers
 - Other

Message Passing Interface (MPI)

- *MPI*: Standard for managing communications (data and instructions) between different nodes/computers.
- *Implementations*: OpenMPI, MPICH2, Cray MPT, ...
- Enables parallelism on distributed machines.
- *Communicator*: manages communications between processors.

MPI Operations (1 of 3)

- **Managing a Communicator:** Create and destroy communicators.
`init()` — initialize communicator
`finalize()` — shut down communicator(s)
- **Rank query:** determine the processor's position in the communicator.
`comm.rank()` — “who am I?”
`comm.size()` — “how many of us are there?”
- **Printing:** Printing output from various ranks.
`comm.print(x)`
`comm.cat(x)`
WARNING: only use these functions on *results*, never on yet-to-be-computed things.

○○○○○
○○○○○○○
○○○○○○○●○
○○○
○○○○○○○
○○○
○○○○○○○
○○○
○○○○○○○○
○○○○○○○○○
○○○○○○
○○○○
○○

Quick Example 1

Rank Query: 1_rank.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 my.rank <- comm.rank()
5 comm.print(my.rank, all.rank=TRUE)
6
7 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 1_rank.r
```

Sample Output:

```

1 COMM.RANK = 0
2 [1] 0
3 COMM.RANK = 1
4 [1] 1

```


○○○○○○
○○○○○○○
○○○○○○○○○●
○○○
○○○○○○○
○○○
○○○○○○○
○○○
○○○○○○○○
○○○○○○○○○
○○○○○○
○○○○
○○

Quick Example 2

Hello World: 2_hello.r

```

1 library(pbdMPI, quiet=TRUE)
2 init()
3
4 comm.print("Hello, world")
5
6 comm.print("Hello again", all.rank=TRUE, quiet=TRUE)
7
8 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 2_hello.r
```

Sample Output:

```

1 COMM.RANK = 0
2 [1] "Hello, world"
3 [1] "Hello again"
4 [1] "Hello again"

```

MPI Operations (2 of 3)

- Reduction:** each processor has a number x ; add all of them up, find the largest/smallest,
`reduce(x, op='sum')` — reduce to one
`allreduce(x, op='sum')` — reduce to all
- Gather:** each processor has a number; create a new object on some processor containing all of those numbers.
`gather(x)` — gather to one
`allgather(x)` — gather to all
- Broadcast:** one processor has a number x that every other processor should also have.
`bcast(x)`

○○○○○○
○○○○○○○
○○○○○○○○○○
○●○○
○○○○○○○
○○○
○○○○○○○
○○○
○○○○○○○○
○○○○○○○○○○
○○○○○○
○○○○
○○

Reductions and Gathers

Quick Example 3

Reduce and Gather: 3_gt.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 comm.set.seed(diff=TRUE)
5
6 n <- sample(1:10, size=1)
7
8 gt <- gather(n)
9 comm.print(unlist(gt))
10
11 sm <- allreduce(n, op='sum')
12 comm.print(sm, all.rank=T)
13
14 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 3_gt.r
```

Sample Output:

```

1 COMM.RANK = 0
2 [1] 2 8
3 COMM.RANK = 0
4 [1] 10
5 COMM.RANK = 1
6 [1] 10

```

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
oo●
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
oooooooooooo
ooo

```

```

ooo
ooooo
oo

```

Reductions and Gathers

Quick Example 4

Broadcast: 4_bcast.r

```

1 library(pbdMPI, quiet=T)
2 init()
3
4 if (comm.rank()==0){
5   x <- matrix(1:4, nrow=2)
6 } else {
7   x <- NULL
8 }
9
10 y <- bcast(x, rank.source=0)
11
12 comm.print(y, rank=1)
13
14 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 4_bcast.r
```

Sample Output:

```

1 COMM.RANK = 1
2   [,1] [,2]
3 [1,]   1   3
4 [2,]   2   4

```

MPI Operations (3 of 3)

- **Barrier:** “computation wall”; no processor can proceed until *all* processors can proceed.
`barrier()`
- **Random Seeds:** Random number seeds.
`comm.set.seed(diff=TRUE)` — Independent streams via **rlecuyer**
`comm.set.seed(seed=1234, diff=FALSE)` — All processors use the same seed.
- ***ply:** Functions in the *ply family.
`pbdApply(X, MARGIN, FUN, ...)` — analogue of `apply()`
`pbdLapply(X, FUN, ...)` — analogue of `lapply()`
`pbdSapply(X, FUN, ...)` — analogue of `sapply()`

Quick Example 3

Barrier: 3_barrier.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 for (rank in 1:comm.size()-1){
5   if (comm.rank() == rank){
6     cat(paste("Hello from process", rank+1, "of",
7               comm.size(), "\n"))
8   }
9   barrier()
10 }
11 comm.cat("\n", quiet=TRUE)
12
13 comm.cat(paste("Hello from process", comm.rank()+1,
14               "of", comm.size(), "\n"), all.rank=TRUE, quiet=TRUE)
15 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript barrier.r
```

Sample Output:

```

1 Hello from process 1
   of 2
2 Hello from process 2
   of 2

```

Quick Example 4

Timing: timer.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 .SPMD.CT$msg.barrier <- TRUE
5 .SPMD.CT$print.quiet <- TRUE
6
7 for (rank in 1:comm.size()-1){
8   if (comm.rank() == rank){
9     cat(paste("Hello from process", rank+1, "of",
10              comm.size(), "\n"))
11   }
12   barrier()
13 }
14 comm.cat("\n")
15
16 comm.cat(paste("Hello from process", comm.rank()+1,
17               "of", comm.size(), "\n"), all.rank=TRUE)
18 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 4_timer.r
```

Sample Output:

```

1      min  mean  max
2  1  0.17  0.173  0.176

```

Quick Example 4

Timing: timer.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 .SPMD.CT$msg.barrier <- TRUE
5 .SPMD.CT$print.quiet <- TRUE
6
7 for (rank in 1:comm.size()-1){
8   if (comm.rank() == rank){
9     cat(paste("Hello from process", rank+1, "of",
10              comm.size(), "\n"))
11   }
12   barrier()
13 }
14 comm.cat("\n")
15
16 comm.cat(paste("Hello from process", comm.rank()+1,
17               "of", comm.size(), "\n"), all.rank=TRUE)
18 finalize()

```

Execute this script via:

```
1 mpirun -np 2 Rscript 4_timer.r
```

Sample Output:

```

1      min  mean  max
2 1 0.17 0.173 0.176

```


Quick Comments for Using pbdMPI

- 1 Start by loading the package:

```
1 library(pbdMPI, quiet = TRUE)
```

- 2 Always initialize before starting and finalize when finished:

```
1 init()
2
3 # ...
4
5 finalize()
```

Contents

4 The Generalized Block Distribution

- The GBD Data Structure
- GBD: Example 1
- GBD: Example 2

Distributing a Matrix Across 4 Processors: Block Distribution

Data

Processors

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \\ \hline x_{4,1} & x_{4,2} & x_{4,3} \\ x_{5,1} & x_{5,2} & x_{5,3} \\ x_{6,1} & x_{6,2} & x_{6,3} \\ \hline x_{7,1} & x_{7,2} & x_{7,3} \\ x_{8,1} & x_{8,2} & x_{8,3} \\ x_{9,1} & x_{9,2} & x_{9,3} \\ \hline x_{10,1} & x_{10,2} & x_{10,3} \end{bmatrix}_{10 \times 3}$$

0

1

2

3

Distributing a Matrix Across 4 Processors: Local Load Balance

	Data	Processors
$X =$	$X_{1,1}$ $X_{1,2}$ $X_{1,3}$	0
	$X_{2,1}$ $X_{2,2}$ $X_{2,3}$	1
	$X_{3,1}$ $X_{3,2}$ $X_{3,3}$	2
	$X_{4,1}$ $X_{4,2}$ $X_{4,3}$	3
	$X_{5,1}$ $X_{5,2}$ $X_{5,3}$	
	$X_{6,1}$ $X_{6,2}$ $X_{6,3}$	
	$X_{7,1}$ $X_{7,2}$ $X_{7,3}$	
	$X_{8,1}$ $X_{8,2}$ $X_{8,3}$	
	$X_{9,1}$ $X_{9,2}$ $X_{9,3}$	
	$X_{10,1}$ $X_{10,2}$ $X_{10,3}$	

10×3

The GBD Data Structure

The GBD Data Structure

Throughout the examples, we will make use of the Generalized Block Distribution, or GBD distributed matrix structure.

- 1 GBD is *distributed*. No processor owns all the data.
- 2 GBD is *non-overlapping*. Rows uniquely assigned to processors.
- 3 GBD is *row-contiguous*. If a processor owns one element of a row, it owns the entire row.
- 4 GBD is globally *row-major*, locally *column-major*.
- 5 GBD is often *locally balanced*, where each processor owns (almost) the same amount of data. But this is not required.
- 6 The last row of the local storage of a processor is adjacent (by global row) to the first row of the local storage of next processor (by communicator number) that owns data.
- 7 GBD is (relatively) easy to understand, but can lead to bottlenecks if you have many more columns than rows.

$x_{1,1}$	$x_{1,2}$	$x_{1,3}$
$x_{2,1}$	$x_{2,2}$	$x_{2,3}$
$x_{3,1}$	$x_{3,2}$	$x_{3,3}$
$x_{4,1}$	$x_{4,2}$	$x_{4,3}$
$x_{5,1}$	$x_{5,2}$	$x_{5,3}$
$x_{6,1}$	$x_{6,2}$	$x_{6,3}$
$x_{7,1}$	$x_{7,2}$	$x_{7,3}$
$x_{8,1}$	$x_{8,2}$	$x_{8,3}$
$x_{9,1}$	$x_{9,2}$	$x_{9,3}$
$x_{10,1}$	$x_{10,2}$	$x_{10,3}$

GBD: Example 1

Understanding GBD: Global Matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5

ooooo
ooo

oooo
ooooo

oooo
ooo
ooooo

ooo
ooo
ooo

oooo
ooo
ooo

ooooo
ooooooooo
ooo

ooo
ooooo
oo

GBD: Example 1

Understanding GBD: Load Balanced GBD

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5

GBD: Example 1

Understanding GBD: Local View

[X ₁₁	X ₁₂	X ₁₃	X ₁₄	X ₁₅	X ₁₆	X ₁₇	X ₁₈	X ₁₉]	2×9
[X ₂₁	X ₂₂	X ₂₃	X ₂₄	X ₂₅	X ₂₆	X ₂₇	X ₂₈	X ₂₉]	2×9
[X ₃₁	X ₃₂	X ₃₃	X ₃₄	X ₃₅	X ₃₆	X ₃₇	X ₃₈	X ₃₉]	2×9
[X ₄₁	X ₄₂	X ₄₃	X ₄₄	X ₄₅	X ₄₆	X ₄₇	X ₄₈	X ₄₉]	2×9
[X ₅₁	X ₅₂	X ₅₃	X ₅₄	X ₅₅	X ₅₆	X ₅₇	X ₅₈	X ₅₉]	2×9
[X ₆₁	X ₆₂	X ₆₃	X ₆₄	X ₆₅	X ₆₆	X ₆₇	X ₆₈	X ₆₉]	2×9
[X ₇₁	X ₇₂	X ₇₃	X ₇₄	X ₇₅	X ₇₆	X ₇₇	X ₇₈	X ₇₉]	1×9
[X ₈₁	X ₈₂	X ₈₃	X ₈₄	X ₈₅	X ₈₆	X ₈₇	X ₈₈	X ₈₉]	1×9
[X ₉₁	X ₉₂	X ₉₃	X ₉₄	X ₉₅	X ₉₆	X ₉₇	X ₉₈	X ₉₉]	1×9

Processors = 0 1 2 3 4 5


```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooo
ooooo

```

```

ooo
ooo
●oo

```

```

oooo
ooo
ooo
ooo

```

```

ooooo
oooooooooooo
ooo

```

```

ooo
ooooo
oo

```

GBD: Example 2

Understanding GBD: Non-Balanced GBD

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5

GBD: Example 2

Understanding GBD: Local View

[]									0x9
X ₁₁	X ₁₂	X ₁₃	X ₁₄	X ₁₅	X ₁₆	X ₁₇	X ₁₈	X ₁₉] 4x9
X ₂₁	X ₂₂	X ₂₃	X ₂₄	X ₂₅	X ₂₆	X ₂₇	X ₂₈	X ₂₉	
X ₃₁	X ₃₂	X ₃₃	X ₃₄	X ₃₅	X ₃₆	X ₃₇	X ₃₈	X ₃₉	
X ₄₁	X ₄₂	X ₄₃	X ₄₄	X ₄₅	X ₄₆	X ₄₇	X ₄₈	X ₄₉	
[]									0x9
X ₅₁	X ₅₂	X ₅₃	X ₅₄	X ₅₅	X ₅₆	X ₅₇	X ₅₈	X ₅₉] 2x9
X ₆₁	X ₆₂	X ₆₃	X ₆₄	X ₆₅	X ₆₆	X ₆₇	X ₆₈	X ₆₉	
[]									0x9
X ₇₁	X ₇₂	X ₇₃	X ₇₄	X ₇₅	X ₇₆	X ₇₇	X ₇₈	X ₇₉] 1x9
[]									0x9
X ₈₁	X ₈₂	X ₈₃	X ₈₄	X ₈₅	X ₈₆	X ₈₇	X ₈₈	X ₈₉] 2x9
X ₉₁	X ₉₂	X ₉₃	X ₉₄	X ₉₅	X ₉₆	X ₉₇	X ₉₈	X ₉₉	

Processors = 0 1 2 3 4 5

GBD: Example 2

Quick Comment for GBD

Local pieces of GBD distributed objects will be given the suffix `.gbd` to visually help distinguish them from global objects. This suffix carries no semantic meaning.

Brief Intermission

Brief Intermission

Questions? Comments?

Don't forget to talk to us at our discussion group:

<http://group.r-pbd.org/>

If you have an affiliation at a United States institution (university, research lab, etc.), consider getting an allocation with us:

<http://www.nics.tennessee.edu/getting-an-allocation>

Contents

- 6 Basic Statistics Examples
 - pbdMPI Example: Monte Carlo Simulation
 - pbdMPI Example: Sample Covariance
 - pbdMPI Example: Linear Regression

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

●ooo
ooo
ooo

```

```

ooooo
oooooooooooo
ooo

```

```

ooo
ooooo
oo

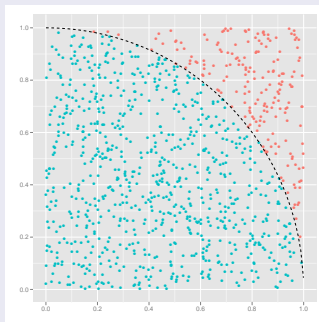
```

pbdMPI Example: Monte Carlo Simulation

Example 1: Monte Carlo Simulation

Sample N uniform observations (x_i, y_i) in the unit square $[0, 1] \times [0, 1]$. Then

$$\pi \approx 4 \left(\frac{\# \text{ Inside Circle}}{\# \text{ Total}} \right) = 4 \left(\frac{\# \text{ Blue}}{\# \text{ Blue} + \# \text{ Red}} \right)$$



Example 1: Monte Carlo Simulation GBD Algorithm

- 1 Let n be big-ish; we'll take $n = 50,000$.
- 2 Generate an $n \times 2$ matrix x of standard uniform observations.
- 3 Count the number of rows satisfying $x^2 + y^2 \leq 1$
- 4 Ask everyone else what their answer is; sum it all up.
- 5 Take this new answer, multiply by 4 and divide by n
- 6 If my rank is 0, print the result.

Example 1: Monte Carlo Simulation Code

Serial Code

```

1 N <- 50000
2 X <- matrix(runif(N * 2), ncol=2)
3 r <- sum(rowSums(X^2) <= 1)
4 PI <- 4*r/N
5 print(PI)

```

Parallel Code

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3 comm.set.seed(diff=TRUE)
4
5 N.gbd <- 50000 / comm.size()
6 X.gbd <- matrix(runif(N.gbd * 2), ncol = 2)
7 r.gbd <- sum(rowSums(X.gbd^2) <= 1)
8 r <- allreduce(r.gbd)
9 PI <- 4*r/(N.gbd * comm.size())
10 comm.print(PI)
11
12 finalize()

```


Note

For the remainder, we will exclude loading, init, and finalize calls.

Example 2: Sample Covariance

$$\text{cov}(x_{n \times p}) = \frac{1}{n-1} \sum_{i=1}^n (x_i - \mu_x)(x_i - \mu_x)^T$$

Example 2: Sample Covariance GBD Algorithm

- 1 Determine the total number of rows N .
- 2 Compute the vector of column means of the full matrix.
- 3 Subtract each column's mean from that column's entries in each local matrix.
- 4 Compute the crossproduct locally and reduce.
- 5 Divide by $N - 1$.

```
oooooo
ooo
```

```
oooo
oooooo
```

```
oooo
ooo
ooooo
```

```
ooo
ooo
ooo
```

```
oooo
ooo
ooo
```

```
ooooo
oooooooooo
ooo
```

```
ooo
oooo
oo
```

Example 2: Sample Covariance Code

Serial Code

```
1 N <- nrow(X)
2 mu <- colSums(X) / N
3
4 X <- sweep(X, STATS=mu, MARGIN=2)
5 Cov.X <- crossprod(X) / (N-1)
6
7 print(Cov.X)
```

Parallel Code

```
1 N <- allreduce(nrow(X.gbd), op="sum")
2 mu <- allreduce(colSums(X.gbd) / N, op="sum")
3
4 X.gbd <- sweep(X.gbd, STATS=mu, MARGIN=2)
5 Cov.X <- allreduce(crossprod(X.gbd), op="sum") / (N-1)
6
7 comm.print(Cov.X)
```

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo
●oo

```

```

ooooo
oooooooooooo
ooo

```

```

ooo
oooo
oo

```

Example 3: Linear Regression

Find β such that

$$\mathbf{y} = \mathbf{X}\beta + \epsilon$$

When \mathbf{X} is full rank,

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$

Example 3: Linear Regression GBD Algorithm

- ① Locally, compute $tx = x^T$
- ② Locally, compute $A = tx * x$. Query every other processor for this result and sum up all the results.
- ③ Locally, compute $B = tx * y$. Query every other processor for this result and sum up all the results.
- ④ Locally, compute $A^{-1} * B$

○○○○○○
○○○

○○○○
○○○○○○

○○○○
○○○
○○○○○

○○○
○○○
○○○

○○○○
○○○
○○●

○○○○○
○○○○○○○○○
○○○

○○○
○○○○
○○

Example 3: Linear Regression Code

Serial Code

```
1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
```

Parallel Code

```
1 tX.gbd <- t(X.gbd)
2 A <- allreduce(tX.gbd %*% X.gbd, op = "sum")
3 B <- allreduce(tX.gbd %*% y.gbd, op = "sum")
4
5 ols <- solve(A) %*% B
```

Contents

- 7 Introduction to pbdDMAT
 - Introduction to Distributed Matrices
 - DMAT Distributions
 - pbdDMAT

Distributed Matrices

Most problems in data science

- Data structure: block-cyclic matrix distributed across a 2-dimensional grid of processors.
- No single processor should hold all of the data.
- Very robust, but very confusing data structure.

Distributed Matrices



(a) Block



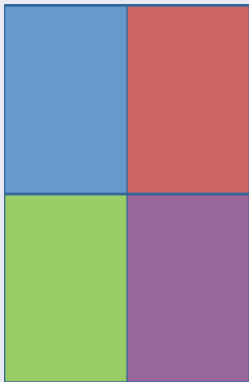
(b) Cyclic



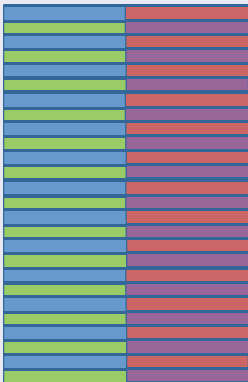
(c) Block-Cyclic

Figure: Matrix Distribution Schemes

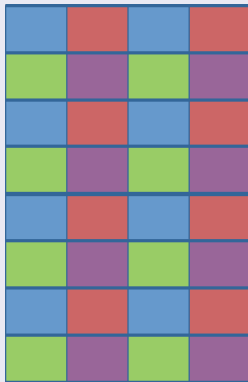
Distributed Matrices



(a) 2d Block



(b) 2d Cyclic



(c) 2d Block-Cyclic

Figure: Matrix Distribution Schemes Onto a 2-Dimensional Grid

Processor Grid Shapes

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}^T$$

(a) 1×6

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

(b) 2×3

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}$$

(c) 3×2

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

(d) 6×1

Table: Processor Grid Shapes with 6 Processors

```
oooooo
ooo
```

```
oooo
oooooo
```

```
oooo
ooo
ooooo
```

```
ooo
ooo
ooo
```

```
oooo
ooo
ooo
```

```
oooo●
ooooooooo
ooo
```

```
ooo
oooo
oo
```

Distributed Matrices

The data structure is a special R class (in the OOP sense) called `ddmatrix`. It is the “under the rug” storage for a block-cyclic matrix distributed onto a 2-dimensional processor grid.

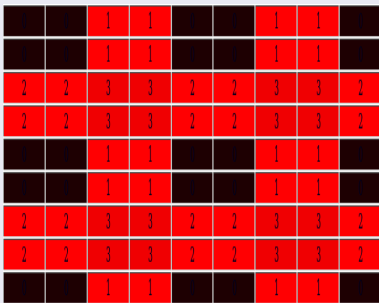
$$\text{ddmatrix} = \left\{ \begin{array}{ll} \text{Data} & \text{S4 local submatrix, an R matrix} \\ \text{dim} & \text{S4 dimension of the global matrix, a numeric pair} \\ \text{ldim} & \text{S4 dimension of the local submatrix, a numeric pair} \\ \text{bldim} & \text{S4 ScaLAPACK blocking factor, a numeric pair} \\ \text{CTXT} & \text{S4 BLACS context, an numeric singleton} \end{array} \right.$$

with prototype

$$\text{new("ddmatrix")} = \left\{ \begin{array}{ll} \text{Data} & = \text{matrix}(0.0) \\ \text{dim} & = \text{c}(1,1) \\ \text{ldim} & = \text{c}(1,1) \\ \text{bldim} & = \text{c}(1,1) \\ \text{CTXT} & = 0.0 \end{array} \right.$$

Distributed Matrices: The Data Structure

Example: an 9×9 matrix is distributed with a “block-cycling” factor of 2×2 on a 2×2 processor grid:



$$= \left\{ \begin{array}{ll} \text{Data} & = \text{matrix}(\dots) \\ \text{dim} & = \text{c}(9, 9) \\ \text{ldim} & = \text{c}(\dots) \\ \text{bldim} & = \text{c}(2, 2) \\ \text{CTXT} & = 0 \end{array} \right.$$

See <http://acts.nersc.gov/scalapack/hands-on/datadist.html>

Understanding Dmat: Global Matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
o●ooooooo
ooo

```

```

ooo
oooo
oo

```

DMAT: 1-dimensional Row Block

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ \hline X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ \hline X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$


```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
oo●ooooooo
ooo

```

```

ooo
oooo
oo

```

DMAT: 2-dimensional Row Block

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ \hline X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
ooo●oooooo
ooo

```

```

ooo
oooo
oo

```

DMAT: 1-dimensional Row Cyclic

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
ooo●ooooo
ooo

```

```

ooo
oooo
oo

```

DMAT: 2-dimensional Row Cyclic

$$X = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
ooooo●ooooo
ooo

```

```

ooo
ooooo
oo

```

DMAT: 2-dimensional Block-Cyclic

$$X = \begin{bmatrix} \begin{array}{cc|cc|cc|cc|c} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ \hline X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ \hline X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ \hline X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ \hline X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{array} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
ooooooo●ooo
ooo

```

```

ooo
ooooo
oo

```

Understanding DMAT: Distributed with bldim = (2,2)

$$X = \begin{bmatrix} \begin{matrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{matrix} & \begin{matrix} x_{13} & x_{14} \\ x_{23} & x_{24} \end{matrix} & \begin{matrix} x_{15} & x_{16} \\ x_{25} & x_{26} \end{matrix} & \begin{matrix} x_{17} & x_{18} \\ x_{27} & x_{28} \end{matrix} & x_{19} \\ \begin{matrix} x_{31} & x_{32} \\ x_{41} & x_{42} \end{matrix} & \begin{matrix} x_{33} & x_{34} \\ x_{43} & x_{44} \end{matrix} & \begin{matrix} x_{35} & x_{36} \\ x_{45} & x_{46} \end{matrix} & \begin{matrix} x_{37} & x_{38} \\ x_{47} & x_{48} \end{matrix} & x_{39} \\ \begin{matrix} x_{51} & x_{52} \\ x_{61} & x_{62} \end{matrix} & \begin{matrix} x_{53} & x_{54} \\ x_{63} & x_{64} \end{matrix} & \begin{matrix} x_{55} & x_{56} \\ x_{65} & x_{66} \end{matrix} & \begin{matrix} x_{57} & x_{58} \\ x_{67} & x_{68} \end{matrix} & x_{59} \\ \begin{matrix} x_{71} & x_{72} \\ x_{81} & x_{82} \end{matrix} & \begin{matrix} x_{73} & x_{74} \\ x_{83} & x_{84} \end{matrix} & \begin{matrix} x_{75} & x_{76} \\ x_{85} & x_{86} \end{matrix} & \begin{matrix} x_{77} & x_{78} \\ x_{87} & x_{88} \end{matrix} & x_{79} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{vmatrix}$$

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
ooooo
ooo

```

```

ooo
oooo
oo

```

Introduction to Distributed Matrices

Understanding DMAT: Local View

$\begin{bmatrix} X_{11} & X_{12} & X_{17} & X_{18} \\ X_{21} & X_{22} & X_{27} & X_{28} \\ X_{51} & X_{52} & X_{57} & X_{58} \\ X_{61} & X_{62} & X_{67} & X_{68} \\ X_{91} & X_{92} & X_{97} & X_{98} \end{bmatrix}$	5×4	$\begin{bmatrix} X_{13} & X_{14} & X_{19} \\ X_{23} & X_{24} & X_{29} \\ X_{53} & X_{54} & X_{59} \\ X_{63} & X_{64} & X_{69} \\ X_{93} & X_{94} & X_{99} \end{bmatrix}$	5×3	$\begin{bmatrix} X_{15} & X_{16} \\ X_{25} & X_{26} \\ X_{55} & X_{56} \\ X_{65} & X_{66} \\ X_{95} & X_{96} \end{bmatrix}$	5×2
$\begin{bmatrix} X_{31} & X_{32} & X_{37} & X_{38} \\ X_{41} & X_{42} & X_{47} & X_{48} \\ X_{71} & X_{72} & X_{77} & X_{78} \\ X_{81} & X_{82} & X_{87} & X_{88} \end{bmatrix}$	4×4	$\begin{bmatrix} X_{33} & X_{34} & X_{39} \\ X_{43} & X_{44} & X_{49} \\ X_{73} & X_{74} & X_{79} \\ X_{83} & X_{84} & X_{89} \end{bmatrix}$	4×3	$\begin{bmatrix} X_{35} & X_{36} \\ X_{45} & X_{46} \\ X_{75} & X_{76} \\ X_{85} & X_{86} \end{bmatrix}$	4×2

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{vmatrix}$$

The DMAT Data Structure

- 1 DMAT is *distributed*. No one processor owns all of the matrix.
- 2 DMAT is *non-overlapping*. Any piece owned by one processor is owned by no other processors.
- 3 DMAT can be row-contiguous or not, depending on the processor grid and blocking factor used.
- 4 DMAT is locally column-major and globally, it depends. . .
- 5 GBD is a generalization of the one-dimensional block DMAT distribution; otherwise there is no relation.
- 6 DMAT is confusing, but very robust.

Pros and Cons of This Data Structure

Pros

- Fast for distributed matrix computations

Cons

- Literally everything else

This is why we hide most of the distributed details.

The details are there if you want them (you don't want them).

Distributed Matrix Methods

pbdDMAT has over 100 methods with *identical* syntax to R:

- ``[, rbind(), cbind(), ...`
- `lm.fit(), prcomp(), cov(), ...`
- ``%*%`, solve(), svd(), norm(), ...`
- `median(), mean(), rowSums(), ...`

Serial Code

```
1 cov(x)
```

Parallel Code

```
1 cov(x)
```

Comparing pbdMPI and pbdDMAT

- **pbdMPI** is MPI + some sugar.
- The GBD data structure is not the only thing **pbdMPI** can handle (just a useful convention).
- **pbdDMAT** is more of a software package.
- The block-cyclic DMAT structure *must* be used for **pbdDMAT**.

Quick Comments for Using pbdDMAT

- 1 Start by loading the package:

```
1 library(pbdDMAT, quiet = TRUE)
```

- 2 Always initialize before starting and finalize when finished:

```
1 init.grid()
2
3 # ...
4
5 finalize()
```

- 3 Distributed DMAT objects will be given the suffix `.dmat` to visually help distinguish them from global objects. This suffix carries no semantic meaning.

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
oooooooooo
ooo

```

```

●ooo
oooo
oo

```

Statistics Examples with pbdDMAT

Sample Covariance

Serial Code

```

1 Cov.X <- cov(X)
2 print(Cov.X)

```

Parallel Code

```

1 Cov.X <- cov(X)
2 print(Cov.X)

```

```

○○○○○○
○○○

```

```

○○○○
○○○○○○

```

```

○○○○
○○○
○○○○○

```

```

○○○
○○○
○○○

```

```

○○○○
○○○
○○○

```

```

○○○○○
○○○○○○○○○○
○○○

```

```

○●○
○○○○
○○

```

Statistics Examples with pbdDMAT

Linear Regression

Serial Code

```

1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
6
7 # or
8 ols <- lm.fit(X, y)

```

Parallel Code

```

1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
6
7 # or
8 ols <- lm.fit(X, y)

```

Quick Example 3

PCA: pca.r

```

1 library(pbdDMAT, quiet=T)
2 init.grid()
3
4 n <- 1e4
5 p <- 250
6
7 comm.set.seed(diff=T)
8 x.dmat <- ddmatrix("rnorm", nrow=n, ncol=p, mean=100, sd=25)
9
10 pca <- prcomp(x=x.dmat, retx=TRUE, scale=TRUE)
11 prop_var <- cumsum(pca$sdev)/sum(pca$sdev)
12 i <- max(min(which(prop_var > 0.9)) - 1, 1)
13
14 y.dmat <- pca$x[, 1:i]
15
16 comm.cat("\nCols: ", i, "\n", quiet=T)
17 comm.cat("%Cols: ", i/dim(x.dmat)[2], "\n\n", quiet=T)
18
19 finalize()

```

Execute this script via:

Sample Output:

```
1 mpirun -np 2 Rscript pca.r
```

```

1 Cols: 221
2 %Cols: 0.884

```

Generating Random Data

Using randomly generated matrices is the best way to “get your feet wet” with the pbd tools. You can do this in 2 ways:

- 1 Generate a global matrix and distribute it.
- 2 Generate locally only what is needed.

```
oooooo
ooo
```

```
oooo
oooooo
```

```
oooo
ooo
ooooo
```

```
ooo
ooo
ooo
```

```
oooo
ooo
ooo
```

```
ooooo
oooooooooo
ooo
```

```
ooo
o●oo
oo
```

pbdDMAT Example: Generating Data

Example 1: Random Distributed Matrix Generation

Generate a global matrix and distribute it

```
1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 # Common global on all processors --> distributed
5 comm.set.seed(diff=FALSE)
6 x <- matrix(rnorm(100), nrow=10, ncol=10)
7 x.dmat <- as.ddmatrix(x)
8
9 # Global on processor 0 --> distributed
10 if (comm.rank()==0){
11   x <- matrix(rnorm(100), nrow=10, ncol=10)
12 } else {
13   x <- NULL
14 }
15 x.dmat <- as.ddmatrix(x)
16
17 finalize()
```



```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
oooooooooo
ooo

```

```

ooo
ooo●o
oo

```

Example 2: Random Distributed Matrix Generation

Generate locally only what is needed

```

1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 comm.set.seed(diff = TRUE) # good seeds via rlecuyer
5 x.dmat <- ddmatrix("rnorm", nrow=10, ncol=10)
6
7 finalize()

```

```

oooooo
ooo

```

```

oooo
oooooo

```

```

oooo
ooo
ooooo

```

```

ooo
ooo
ooo

```

```

oooo
ooo
ooo

```

```

ooooo
oooooooooo
ooo

```

```

ooo
ooo●
oo

```

pbdDMAT Example: Generating Data

Example 3: Random Distributed Matrix Generation

Generate locally only what is needed

```

1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 zero.dmat <- ddmatrix(0, nrow=100, ncol=100)
5 id.dmat <- diag(1, nrow=100, ncol=100)
6
7 finalize()

```

```
oooooo
ooo
```

```
oooo
oooooo
```

```
oooo
ooo
ooooo
```

```
ooo
ooo
ooo
```

```
oooo
ooo
ooo
```

```
ooooo
oooooooooo
ooo
```

```
ooo
oooo
●o
```

pbdDMAT Example: Converting Between GBD and DMAT

Example 4: Random Distributed Matrix Generation

Convert between GBD and DMAT

```
1 library(pbdDEMO, quiet=TRUE)
2 init.grid()
3
4 comm.set.seed(diff = TRUE)
5
6 N.gbd <- 1 + comm.rank()
7 X.gbd <- matrix(rnorm(N.gbd * 3), ncol = 3)
8
9 # convert GBD to DMAT
10 X.dmat <- gbd2dmat(X.gbd)
11
12 # convert DMAT to GBD
13 new.X.gbd <- dmat2gbd(X.dmat)
14
15 # undistribute
16 X <- as.matrix(X.dmat)
17
18 finalize()
```

Distributed Matrices

pbdDEMO contains many other examples of reading and managing GBD and DMAT data

Introduction	pbdR	pbdMPI	GBD	Break	Stats eg's	pbdDMAT	pbdDMAT eg's	Wrapup
oooooo ooo	oooo oooooo	oooo ooo ooooo	ooo ooo ooo		oooo ooo ooo	oooo oooooooooooo ooo	ooo oooo oo	

Contents

9 Wrapup

Where to Learn More

- Our website <http://r-pbd.org/>
- The **pbdDEMO** package
<http://cran.r-project.org/web/packages/pbdDEMO/>
- The **pbdDEMO** Vignette: <http://goo.gl/HZkRt>
- Our Google Group: <http://group.r-pbd.org>

Thanks for coming!

Questions? Comments?

Please help us improve this tutorial by completing a short survey:

<http://www.surveymonkey.com/s/W8VLJSP>