



Programming with Big Data in R

Drew Schmidt and George Ostrouchov

July 8, 2013





Affiliations and Support

The pbDR Core Team

<http://r-pbd.org>

Wei-Chen Chen¹, George Ostrouchov^{1,2}, Pragneshkumar Patel², Drew Schmidt¹

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¹ Computer Science and Mathematics Division, Oak Ridge National Laboratory, Oak Ridge, TN

² Remote Data Analysis and Visualization Center, University of Tennessee, Knoxville, TN





About This Presentation

Downloads

This presentation and supplemental materials are available at:

<http://r-pbd.org/user2013>



Break

Stats eg's



Wrapup

About This Presentation

Speaking Serial R with a Parallel Accent

The content of this presentation is based in part on the
pbdDEMO vignette *Speaking Serial R with a Parallel Accent*

<https://github.com/wrathematics/pbdDEMO/blob/master/inst/doc/pbdDEMO-guide.pdf?raw=true>

It contains more examples, and sometimes added detail.



About This Presentation

Installation Instructions

Installation instructions for setting up a pbdR environment are available:

<http://r-pbd.org/install.html>

This includes instructions for installing R, MPI, and pbdR.



About This Presentation

Conventions

We use:

- “.” as a decimal mark
- “,” as order of magnitude separator

Example	Yes	No
One million	1,000,000	1.000.000
One half	0.5	0,5
One thousand and one half	1,000.5	1.000,5



Contents

- 1 Introduction
- 2 pbdR
- 3 Introduction to pbdMPI
- 4 The Generalized Block Distribution
- 5 Brief Intermission
- 6 Basic Statistics Examples
- 7 Introduction to pbdDMAT
- 8 Examples Using pbdDMAT
- 9 Wrapup





Contents

1 Introduction

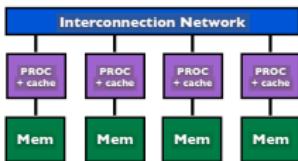
- Quick Overview of Parallel Hardware
- A Concise Introduction to Parallelism
- R and Parallelism



Quick Overview of Parallel Hardware

Three Basic Flavors of Hardware

Distributed Memory



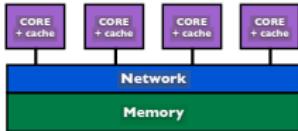
Co-Processor



GPU: Graphical Processing Unit

MIC: Many Integrated Core

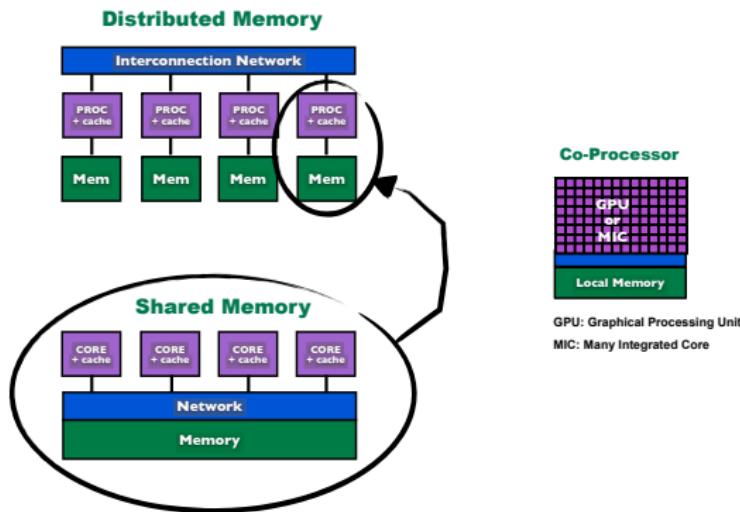
Shared Memory





Quick Overview of Parallel Hardware

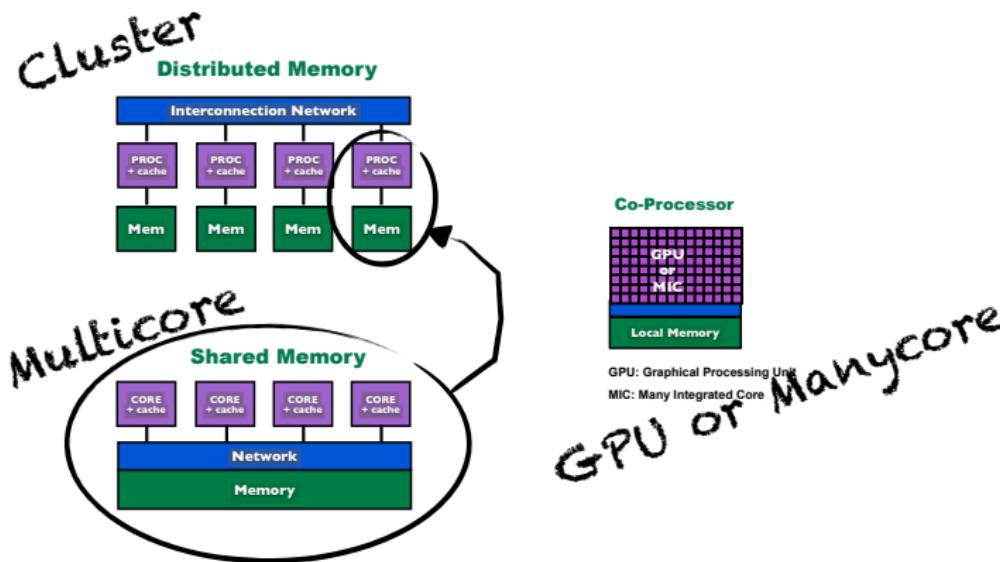
Usually Mixed





Quick Overview of Parallel Hardware

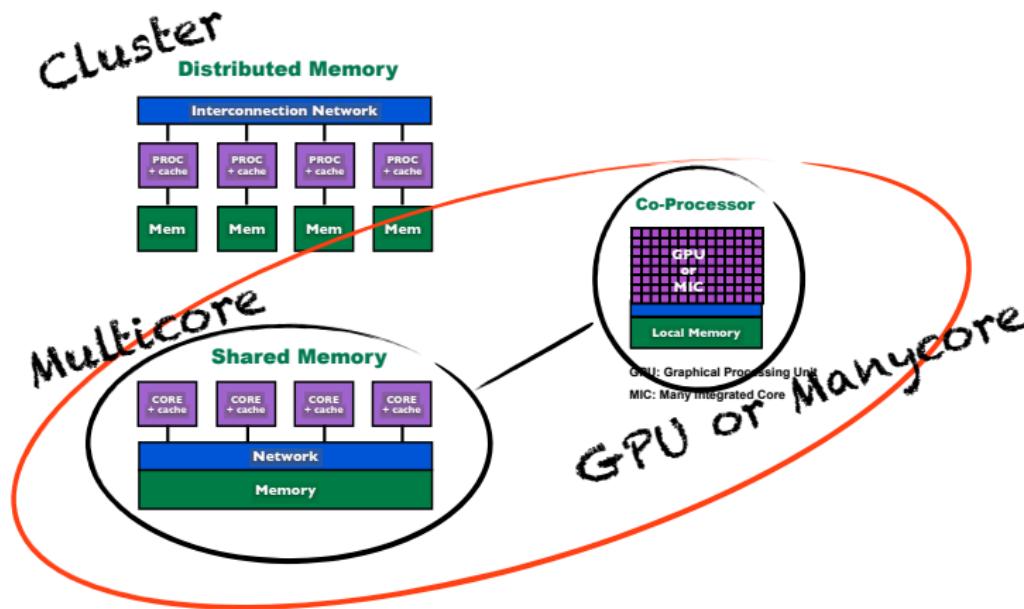
Knowing the Right Words





Quick Overview of Parallel Hardware

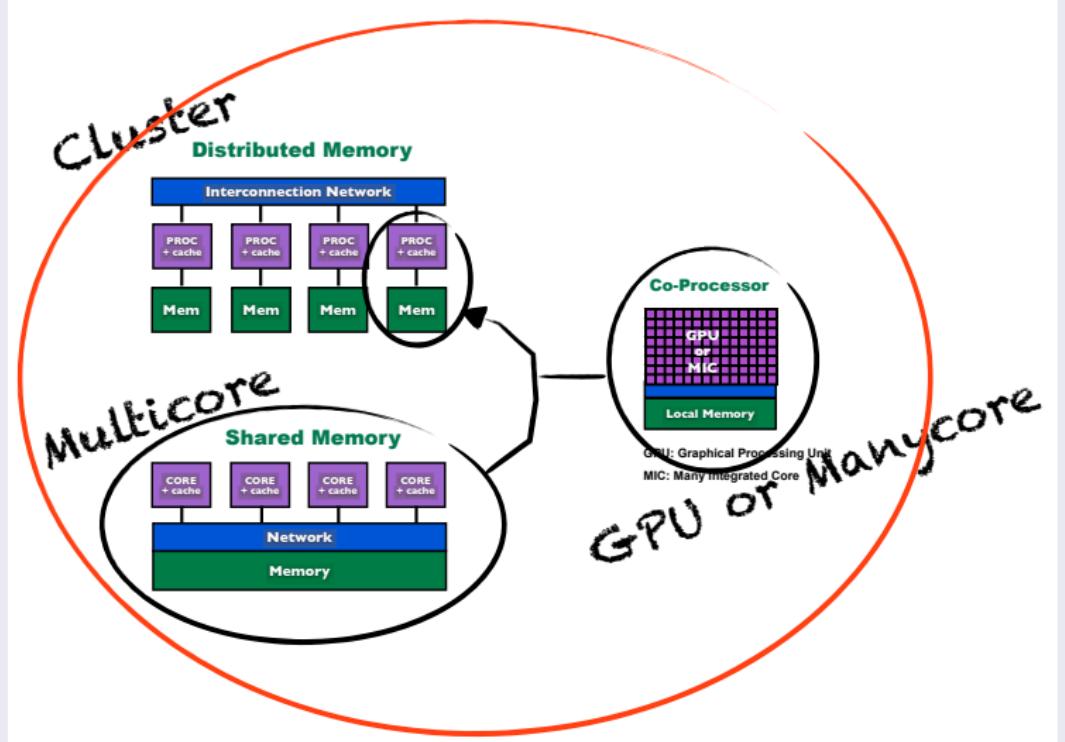
Your Laptop or Desktop





Quick Overview of Parallel Hardware

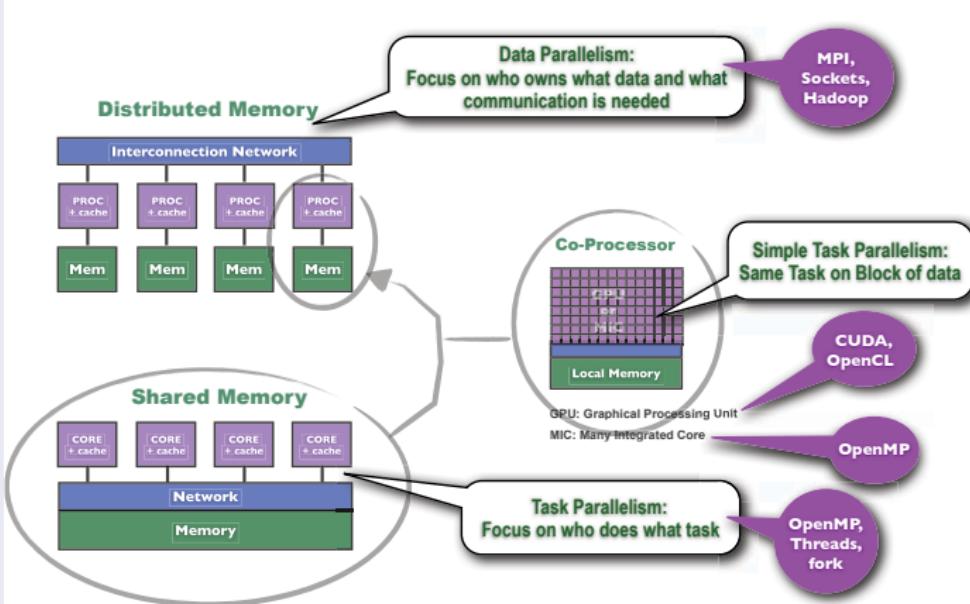
Supercomputers Mix All Flavors





Quick Overview of Parallel Hardware

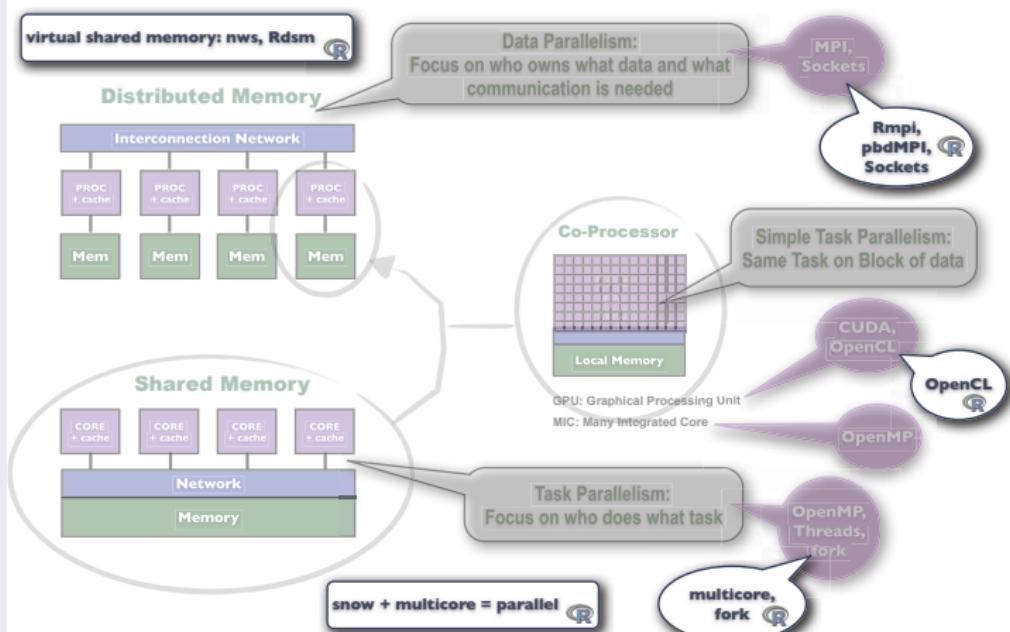
“Native” Tools





Quick Overview of Parallel Hardware

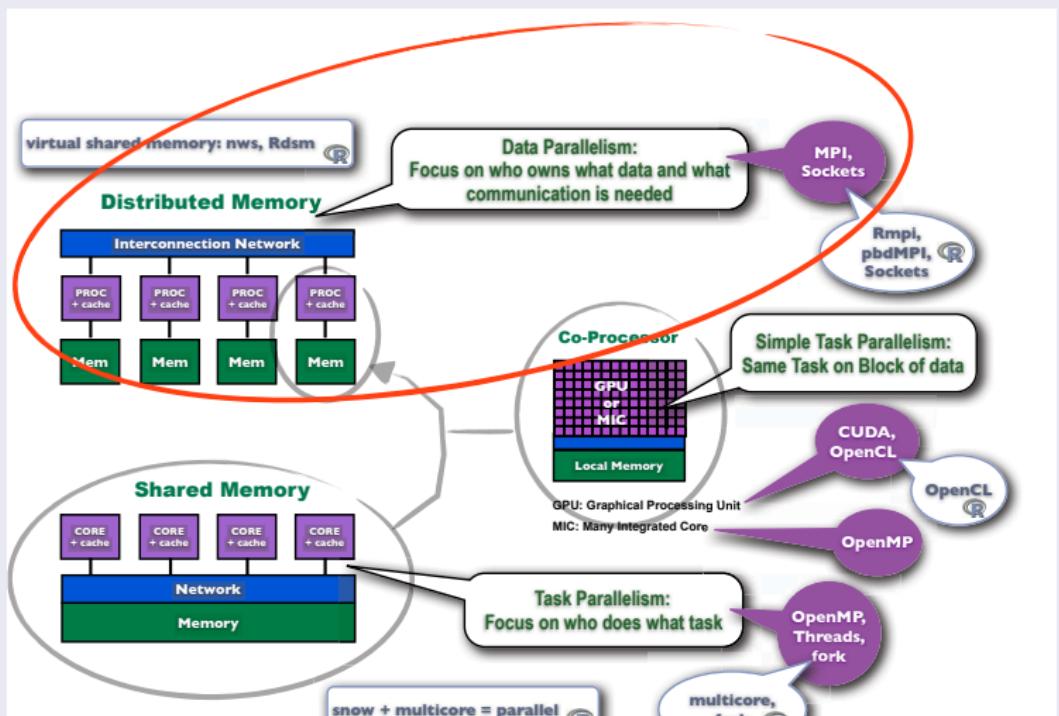
R Interfaces to Native Tools





Quick Overview of Parallel Hardware

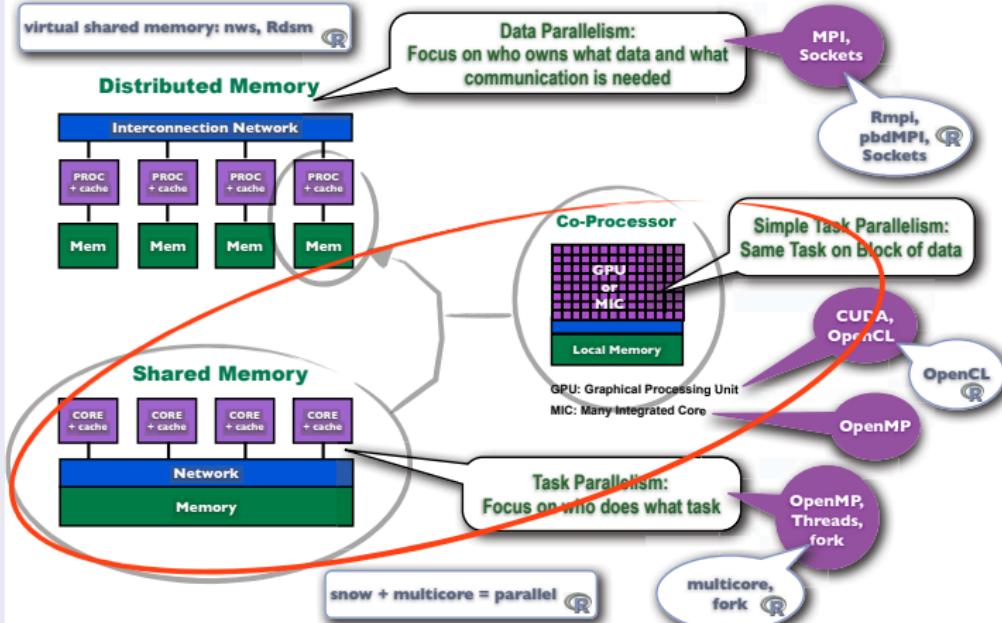
30+ Years of Parallel Computing Research





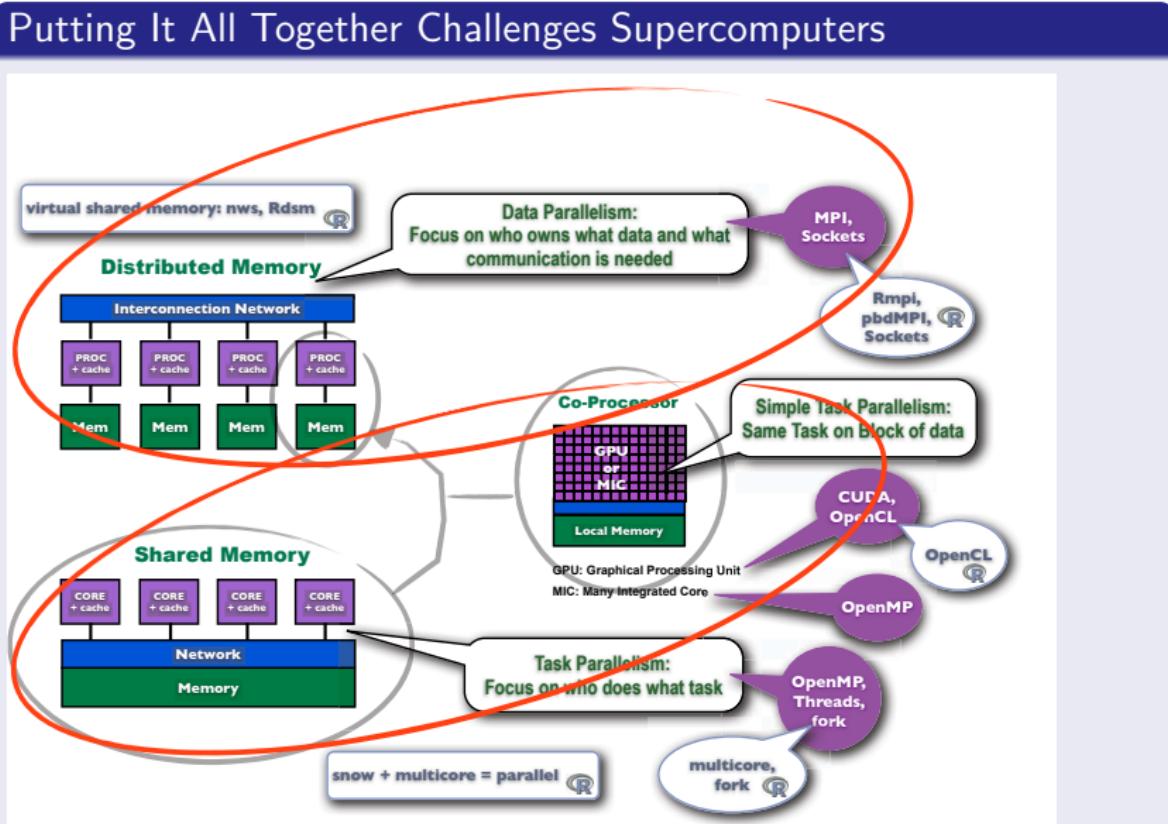
Quick Overview of Parallel Hardware

Last 10 years of Advances



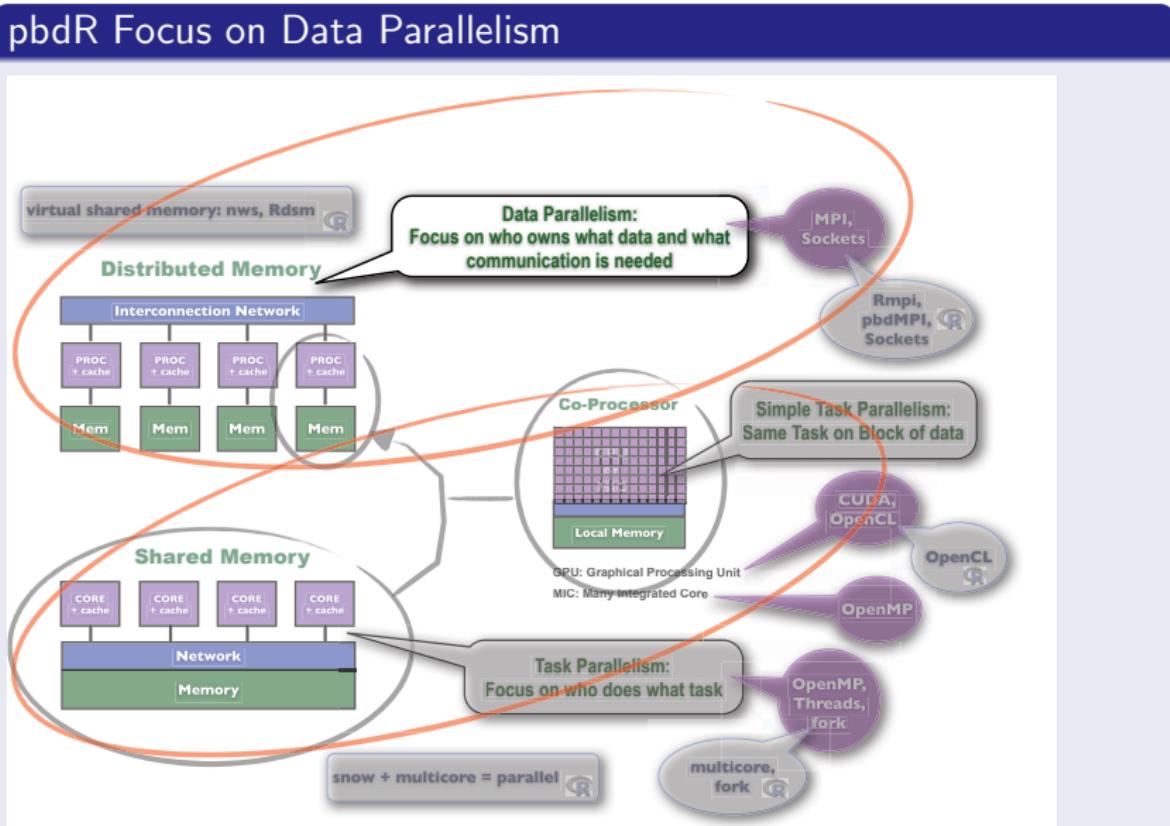


Quick Overview of Parallel Hardware





Quick Overview of Parallel Hardware





A Concise Introduction to Parallelism

What is Parallelism?

Broadly, *doing more than one thing at a time.*

The simultaneous use of multiple compute resources to solve a computational problem:

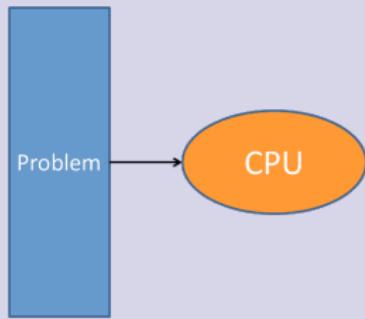




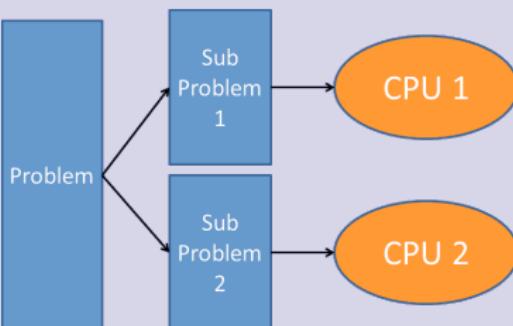
A Concise Introduction to Parallelism

Parallelism

Serial Programming



Parallel Programming

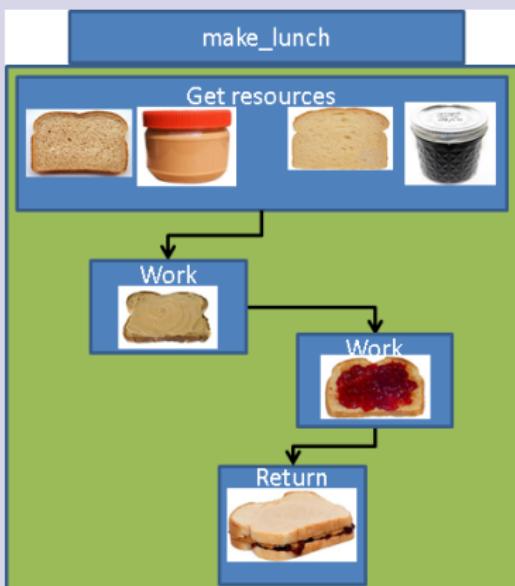




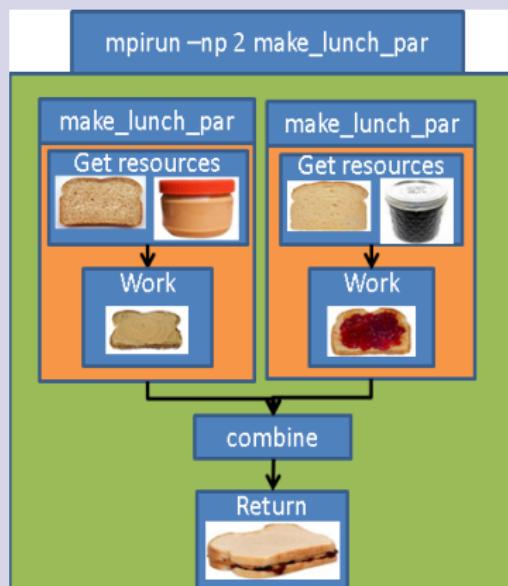
A Concise Introduction to Parallelism

Parallelism

Serial Programming



Parallel Programming





Break

Stats eg's



A Concise Introduction to Parallelism

Kinds of Parallelism

- *Data Parallelism*: Data is distributed
- *Task Parallelism*: Tasks are distributed



A Concise Introduction to Parallelism

pbdR Paradigms: Data Parallelism

With data parallelism:

- No one processor/node owns all the data.
- Processors own local pieces of a (conceptually) global object

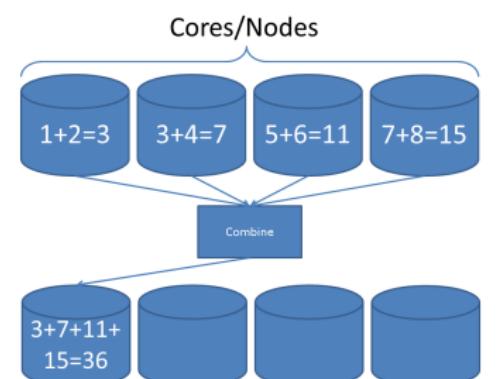




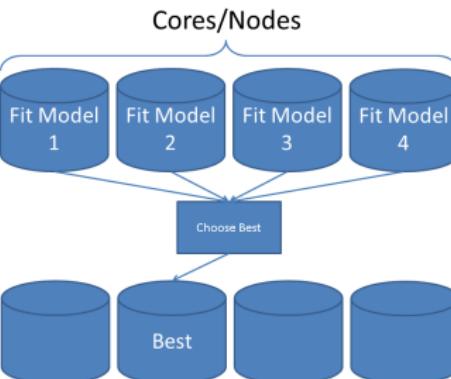
A Concise Introduction to Parallelism

Data vs Task Parallelism

Data Parallelism



Task Parallelism



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A Concise Introduction to Parallelism

Difficulty

- ① *Implicit parallelism*: Parallel details hidden from user
- ② *Explicit parallelism*: Some assembly required...
- ③ *Embarrassingly Parallel*: Also called *loosely coupled*. Obvious how to make parallel; lots of independence in computations.
- ④ *Tightly Coupled*: Opposite of embarrassingly parallel; lots of dependence in computations.





A Concise Introduction to Parallelism

Scalability

Scalability: unitless measure of performance;

$$\frac{\tau_i}{\tau_0}$$

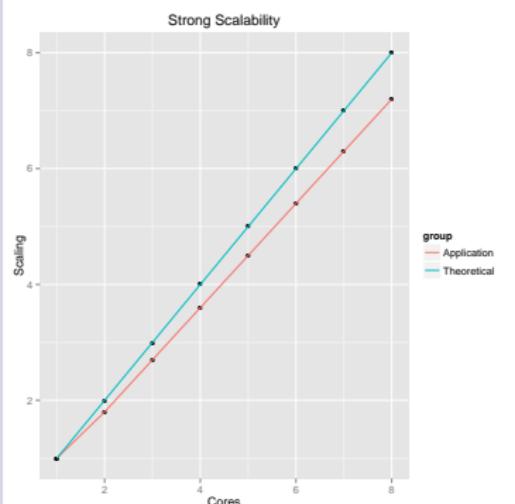




Types of Scalability: Strong and Weak

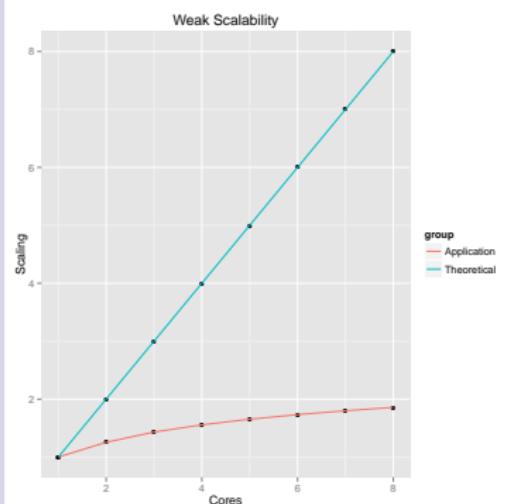
Strong

Fix *total* data size



Weak

Fix *local* data size



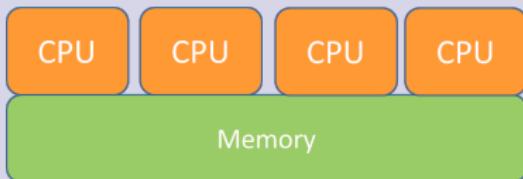


A Concise Introduction to Parallelism

Shared and Distributed Memory Machines

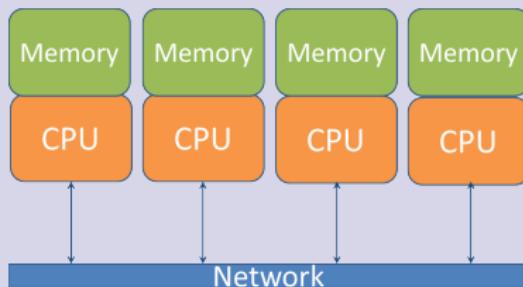
Shared Memory

Different processors can directly access and modify each others' memory. There is only one node.



Distributed

Different processors/nodes can not directly access/modify different processors'/nodes' memory.





A Concise Introduction to Parallelism

Shared and Distributed Memory Machines

Shared Memory Machines

Thousands of cores



Nautilus, University of Tennessee

1024 cores

Distributed Memory Machines

Hundreds of thousands of cores



Kraken, University of Tennessee

112,896 cores



R and Parallelism

R and Parallelism

What about R?





R and Parallelism



Problems with Serial R

- ➊ Slow.
- ➋ If you don't know what you're doing, it's *really* slow.
- ➌ Performance improvements usually for small machines.
- ➍ Very ram intensive.
- ➎ Chokes on big data.



R and Parallelism

Parallel R Packages

Shared Memory

- ① **foreach**
- ② **parallel**
- ③ **snow**
- ④ **multicore**

Distributed

- ① **Rmpi**
- ② **R+Hadoop**
- ③ **pbdR**





R and Parallelism

R and Parallelism

The solution to many of R's problems is parallelism. However . . .

What we have

- ① Mostly serial.
- ② Mostly not distributed
- ③ Data parallelism mostly explicit

What we want

- ① Mostly parallel.
- ② Mostly distributed.
- ③ Mostly implicit.





R and Parallelism

Why We Need Parallelism

- ① Saves time (long term).
- ② Data size is skyrocketing.
- ③ Necessary for many problems.
- ④ Like it or not, it's coming.
- ⑤ *It's really cool.*



Contents

2 pbdR

- The pbdR Project
- pbdR Paradigms

Programming with Big Data in R (pbdr)

Striving for *Productivity, Portability, Performance*



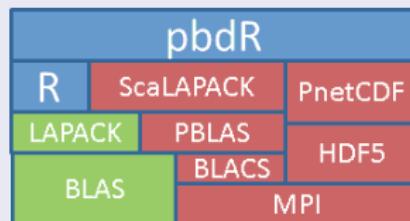
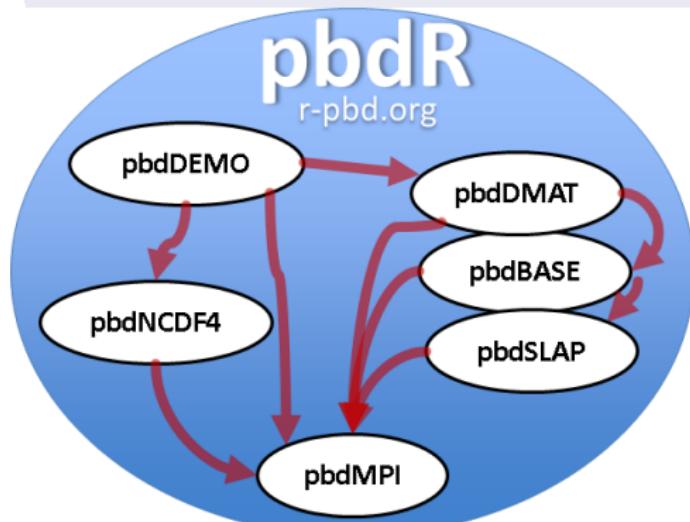
- *Free^a* R packages.
- Bridging high-performance C with high-productivity of R
- Scalable, big data analytics.
- Distributed data details implicitly managed.
- Methods have syntax *identical* to R.
- Powered by state of the art numerical libraries (MPI, ScaLAPACK, ...)

^aMPL, BSD, and GPL licensed



The pbdR Project

pbdR Packages





The pbdR Project

pbdR Packages — <http://code.r-pbd.org>

Released to CRAN:

- **pbdMPI**: MPI bindings (explicit, low-level)
- **pbdSLAP**: Foreign library (just install it, nothing to use)
- **pbdBASE**: Compiled code (used by DMAT, also for devs)
- **pbdDMAT**: Distributed matrices (mostly implicit, high-level)
- **pbdNCFD4**: Parallel NetCDF4 reader
- **pbdDEMO**: Package demonstrations, examples, vignette written in textbook style

Future Development:

- Updates and expansions
- Profiling Tools for Parallel Computing with R
- ...



Example Syntax

```
1 x <- x[-1, 2:5]
2 x <- log(abs(x) + 1)
3 xtx <- t(x) %*% x
4 ans <- svd(solve(xtx))
```

Look familiar?

The above runs on 1 core with R or 10,000 cores with pbdR



pbdR Paradigms

pbdR Paradigms

Programs that use pbdR are utilize:

- Batch execution
- Single Program/Multiple Data (SPMD) style

And generally utilize:

- Data Parallelism



pbdR Paradigms



Batch Execution

- Non-interactive
- Use

```
1 Rscript my_script.r
```

or

```
1 R CMD BATCH my_script.r
```

- In parallel:

```
1 mpirun -np 2 Rscript my_par_script.r
```



pbdR Paradigms

Single Program/Multiple Data (SPMD)

- SPMD is a programming *paradigm*.
- Not to be confused with SIMD.
- SPMD utilizes MIMD architecture computers.
- Arguably the simplest extension of serial programming.
- Difficult to describe, easy to do...
- Only one program is written, executed in batch on all processors.
- Different processors are autonomous; there is no manager.
- The dominant programming model for large machines.





Break



pbdr Paradigms

SPMD

Manager/Worker

SPMD





Break

Stats eg's

pbddMAT



Wrapup

Contents

3 Introduction to pbddMPI

- Managing a Communicator
- Reduce, Gather, Broadcast, and Barrier
- Other pbddMPI Tools





Managing a Communicator

Message Passing Interface (MPI)

- *MPI*: Standard for managing communications (data and instructions) between different nodes/computers.
- *Implementations*: OpenMPI, MPICH2, Cray MPT, ...
- Enables parallelism (via communication) on distributed machines.
- *Communicator*: manages communications between processors.





Managing a Communicator

MPI Operations (1 of 2)

- **Managing a Communicator:** Create and destroy communicators.

`init()` — initialize communicator

`finalize()` — shut down communicator(s)

- **Rank query:** determine the processor's position in the communicator.

`comm.rank()` — “who am I?”

`comm.size()` — “how many of us are there?”

- **Printing:** Printing output from various ranks.

`comm.print(x)`

`comm.cat(x)`

WARNING: only use these functions on *results*, never on yet-to-be-computed things.



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Managing a Communicator

Quick Example 1

Rank Query: 1_rank.r

```
1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 my.rank <- comm.rank()
5 comm.print(my.rank, all.rank=TRUE)
6
7 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 1_rank.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] 0
3 COMM.RANK = 1
4 [1] 1
```



Managing a Communicator

Quick Example 2

Hello World: 2_hello.r

```
1 library(pbDMPI, quiet=TRUE)
2 init()
3
4 comm.print("Hello, world")
5
6 comm.print("Hello again", all.rank=TRUE, quiet=TRUE)
7
8 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 2_hello.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] "Hello, world"
3 [1] "Hello again"
4 [1] "Hello again"
```

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Break

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Reduce, Gather, Broadcast, and Barrier

MPI Operations

- ① Reduce
- ② Gather
- ③ Broadcast
- ④ Barrier



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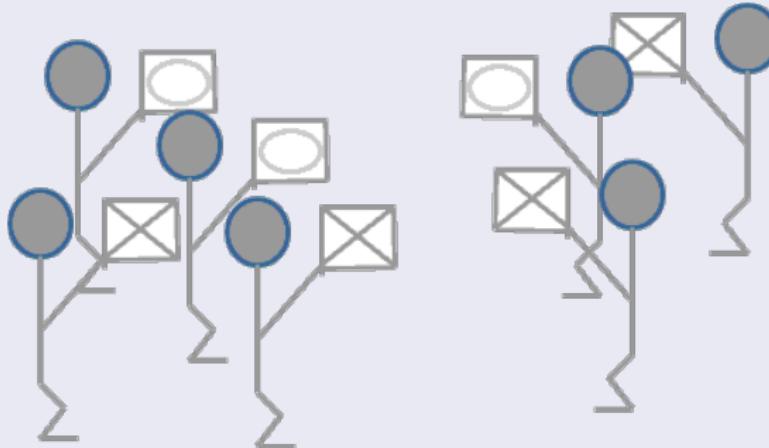
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Reduce, Gather, Broadcast, and Barrier

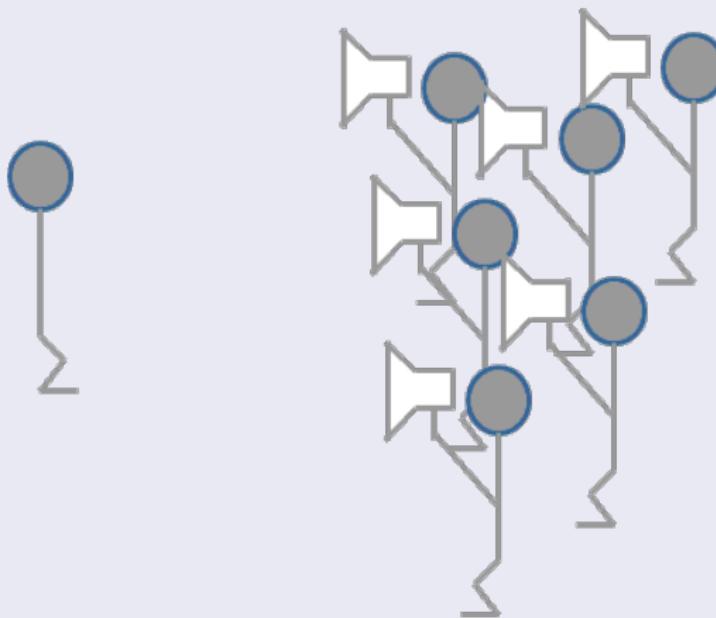
Reductions — Combine results into single result





Reduce, Gather, Broadcast, and Barrier

Gather — Many-to-one





Break

Stats eg's

Break

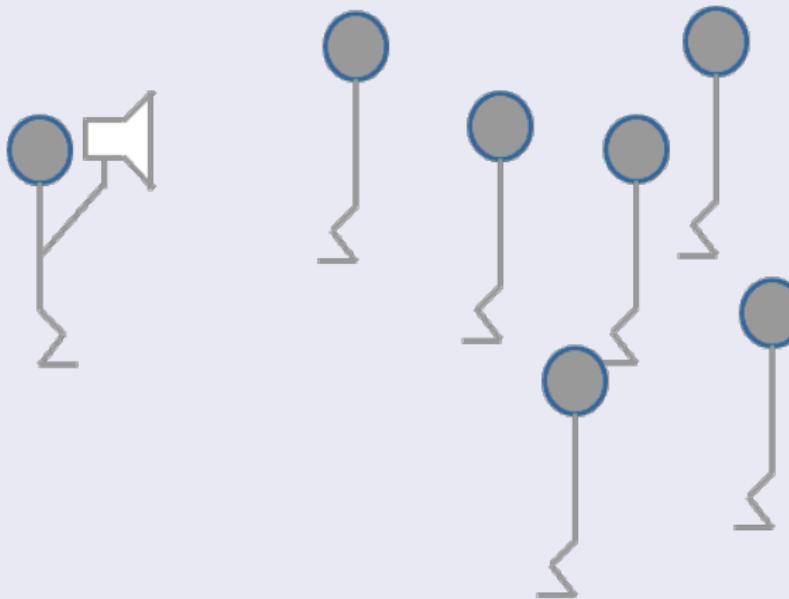
Stats eg's



Break

Reduce, Gather, Broadcast, and Barrier

Broadcast — One-to-many



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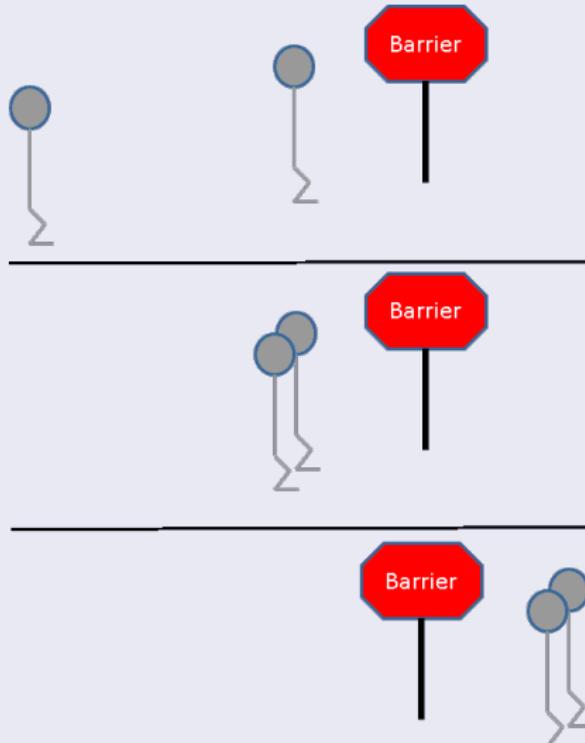
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Reduce, Gather, Broadcast, and Barrier

Barrier — Synchronization



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Reduce, Gather, Broadcast, and Barrier

MPI Operations (2 of 2)

- **Reduction:** each processor has a number x ; add all of them up, find the largest/smallest,
`reduce(x, op='sum')` — reduce to one
`allreduce(x, op='sum')` — reduce to all
- **Gather:** each processor has a number; create a new object on some processor containing all of those numbers.
`gather(x)` — gather to one
`allgather(x)` — gather to all
- **Broadcast:** one processor has a number x that every other processor should also have.
`bcast(x)`
- **Barrier:** “computation wall”; no processor can proceed until *all* processors can proceed.
`barrier()`



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Reduce, Gather, Broadcast, and Barrier

Quick Example 3

Reduce and Gather: 3_gt.r

```
1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 comm.set.seed(diff=TRUE)
5
6 n <- sample(1:10, size=1)
7
8 gt <- gather(n)
9 comm.print(unlist(gt))
10
11 sm <- allreduce(n, op='sum')
12 comm.print(sm, all.rank=T)
13
14 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 3_gt.r
```

Sample Output:

```
1 COMM.RANK = 0
2 [1] 2 8
3 COMM.RANK = 0
4 [1] 10
5 COMM.RANK = 1
6 [1] 10
```



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Reduce, Gather, Broadcast, and Barrier

Quick Example 4

Broadcast: 4_bcstr.r

```
1 library(pbdMPI, quiet=T)
2 init()
3
4 if (comm.rank() == 0) {
5   x <- matrix(1:4, nrow=2)
6 } else {
7   x <- NULL
8 }
9
10 y <- bcast(x, rank.source=0)
11
12 comm.print(y, rank=1)
13
14 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript 4_bcstr.r
```

Sample Output:

```
1 COMM.RANK = 1
2      [,1] [,2]
3 [1,]     1     3
4 [2,]     2     4
```



Other pbdMPI Tools

MPI Package Controls

The `.SPMD.CT` object allows for setting different package options with **pbdMPI**. See the entry *SPMD Control* of the **pbdMPI** manual for information about the `.SPMD.CT` object:

<http://cran.r-project.org/web/packages/pbdMPI/pbdMPI.pdf>



Other pbdMPI Tools

Quick Example 5

Barrier: 5_barrier.r

```

1 library(pbdMPI, quiet = TRUE)
2 init()
3
4 .SPMD.CT$msg.barrier <- TRUE
5 .SPMD.CT$print.quiet <- TRUE
6
7 for (rank in 1:comm.size()-1){
8   if (comm.rank() == rank){
9     cat(paste("Hello", rank+1, "of", comm.size(), "\n"))
10  }
11  barrier()
12 }
13
14 comm.cat("\n")
15
16 comm.cat(paste("Hello", comm.rank()+1, "of",
17   comm.size(), "\n"), all.rank=TRUE)
18 finalize()
```

Execute this script via:

```
mpirun -np 2 Rscript 5_barrier.r
```

Sample Output:

```
1 Hello 1 of 2
2 Hello 2 of 2
```



Other pbdMPI Tools

Random Seeds

pbdMPI offers a simple interface for managing random seeds:

- `comm.set.seed(diff=TRUE)` — Independent streams via the **rlecuyer** package.
- `comm.set.seed(seed=1234, diff=FALSE)` — All processors use the same seed `seed=1234`
- `comm.set.seed(diff=FALSE)` — All processors use the same seed, determined by processor 0 (using the system clock and PID of processor 0).



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Other pbDR Tools

Quick Example 6

Timing: 6_timer.r

```
1 library(pbdMPI, quiet=TRUE)
2 init()
3
4 comm.set.seed(diff=T)
5
6 test <- function(timed)
7 {
8   ltime <- system.time(timed)[3]
9
10  mintime <- allreduce(ltime, op='min')
11  maxtime <- allreduce(ltime, op='max')
12  meantime <- allreduce(ltime, op='sum')/comm.size()
13
14  return(data.frame(min=mintime, mean=meantime,
15                  max=maxtime))
16}
17
18 times <- test(rnorm(1e6)) # ~7.6MiB of data
19 comm.print(times)
20 finalize()
```

Execute this script via:

```
mpirun -np 2 Rscript 6_timer.r
```

Sample Output:

	min	mean	max
1	0.17	0.173	0.176



Other pbdMPI Tools

Other Helper Tools

pbdMPI Also contains useful tools for Manager/Worker and task parallelism codes:

- **Task Subsetting:** Distributing a list of jobs/tasks

`get.jid(n)`

- ***ply:** Functions in the *ply family.

`pbdApply(X, MARGIN, FUN, ...)` — analogue of `apply()`

`pbdLapply(X, FUN, ...)` — analogue of `lapply()`

`pbdSapply(X, FUN, ...)` — analogue of `sapply()`



Other pbdMPI Tools

Quick Comments for Using pbdMPI

- ① Start by loading the package:

```
1 library(pbdMPI, quiet = TRUE)
```

- ② Always initialize before starting and finalize when finished:

```
1 init()  
2  
3 # ...  
4  
5 finalize()
```





Contents

4 The Generalized Block Distribution

- The GBD Data Structure
- GBD: Example 1
- GBD: Example 2

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The GBD Data Structure

Distributing a Matrix Across 4 Processors: Block Distribution

Data	Processors
$x_{1,1}$	0
$x_{2,1}$	1
$x_{3,1}$	2
$x_{4,1}$	3
$x_{5,1}$	
$x_{6,1}$	
$x_{7,1}$	
$x_{8,1}$	
$x_{9,1}$	
$x_{10,1}$	
$x_{1,2}$	
$x_{2,2}$	
$x_{3,2}$	
$x_{4,2}$	
$x_{5,2}$	
$x_{6,2}$	
$x_{7,2}$	
$x_{8,2}$	
$x_{9,2}$	
$x_{10,2}$	
$x_{1,3}$	
$x_{2,3}$	
$x_{3,3}$	
$x_{4,3}$	
$x_{5,3}$	
$x_{6,3}$	
$x_{7,3}$	
$x_{8,3}$	
$x_{9,3}$	
$x_{10,3}$	
	10×3



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○○
```

The GBD Data Structure

Distributing a Matrix Across 4 Processors: Local Load Balance

	Data	Processors
$x =$	$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \\ \hline x_{4,1} & x_{4,2} & x_{4,3} \\ x_{5,1} & x_{5,2} & x_{5,3} \\ x_{6,1} & x_{6,2} & x_{6,3} \\ \hline x_{7,1} & x_{7,2} & x_{7,3} \\ x_{8,1} & x_{8,2} & x_{8,3} \\ \hline x_{9,1} & x_{9,2} & x_{9,3} \\ x_{10,1} & x_{10,2} & x_{10,3} \end{bmatrix}$	0 1 2 3
		10×3



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```

The GBD Data Structure

Throughout the examples, we will make use of the Generalized Block Distribution, or GBD distributed matrix structure.

- ➊ GBD is *distributed*. No processor owns all the data.
- ➋ GBD is *non-overlapping*. Rows uniquely assigned to processors.
- ➌ GBD is *row-contiguous*. If a processor owns one element of a row, it owns the entire row.
- ➍ GBD is globally *row-major*, locally *column-major*.
- ➎ GBD is often *locally balanced*, where each processor owns (almost) the same amount of data. But this is not required.
- ➏ The last row of the local storage of a processor is adjacent (by global row) to the first row of the local storage of next processor (by communicator number) that owns data.
- ➐ GBD is (relatively) easy to understand, but can lead to bottlenecks if you have many more columns than rows.

$X_{1,1}$	$X_{1,2}$	$X_{1,3}$
$X_{2,1}$	$X_{2,2}$	$X_{2,3}$
$X_{3,1}$	$X_{3,2}$	$X_{3,3}$
$X_{4,1}$	$X_{4,2}$	$X_{4,3}$
$X_{5,1}$	$X_{5,2}$	$X_{5,3}$
$X_{6,1}$	$X_{6,2}$	$X_{6,3}$
$X_{7,1}$	$X_{7,2}$	$X_{7,3}$
$X_{8,1}$	$X_{8,2}$	$X_{8,3}$
$X_{9,1}$	$X_{9,2}$	$X_{9,3}$
$X_{10,1}$	$X_{10,2}$	$X_{10,3}$



GBD: Example 1

Understanding GBD: Global Matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5



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GBD: Example 1

Understanding GBD: Load Balanced GBD

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5

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GBD: Example 1

Understanding GBD: Local View

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \end{bmatrix}_{2 \times 9}$$

$$\begin{bmatrix} x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \end{bmatrix}_{2 \times 9}$$

$$\begin{bmatrix} x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \end{bmatrix}_{2 \times 9}$$

$$\begin{bmatrix} x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \end{bmatrix}_{1 \times 9}$$

$$\begin{bmatrix} x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \end{bmatrix}_{1 \times 9}$$

$$\begin{bmatrix} x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{1 \times 9}$$

Processors = 0 1 2 3 4 5





GBD: Example 2

Understanding GBD: Non-Balanced GBD

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ \hline x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ \hline x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ \hline x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

Processors = 0 1 2 3 4 5

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GBD: Example 2

Understanding GBD: Local View

$$\left[\begin{array}{ccccccccc} & & & & & & & & \\ & & & & & & & &]_{0 \times 9} \\ \left[\begin{array}{ccccccc} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \end{array} \right]_{4 \times 9} \\ \left[\begin{array}{ccccccc} x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \end{array} \right]_{2 \times 9} \\ \left[\begin{array}{ccccccc} x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \end{array} \right]_{1 \times 9} \\ \left[\quad \right]_{0 \times 9} \\ \left[\begin{array}{ccccccc} x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{array} \right]_{2 \times 9} \end{array} \right]$$

Processors = 0 1 2 3 4 5



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GBD: Example 2

Quick Comment for GBD

Local pieces of GBD distributed objects will be given the suffix .gbd to visually help distinguish them from global objects. This suffix carries no semantic meaning.





Brief Intermission

Questions? Comments?

Don't forget to talk to us at our discussion group:

<http://group.r-pbd.org/>

If you have an affiliation at a United States institution (university, research lab, etc.), consider getting an allocation with us:

<http://www.nics.tennessee.edu/getting-an-allocation>

Come to the talk *Elevating R to Supercomputers*, Friday, July 12th at 10:00 at the High Performance Computing session





Contents

6 Basic Statistics Examples

- pbdMPI Example: Monte Carlo Simulation
- pbdMPI Example: Sample Covariance
- pbdMPI Example: Linear Regression

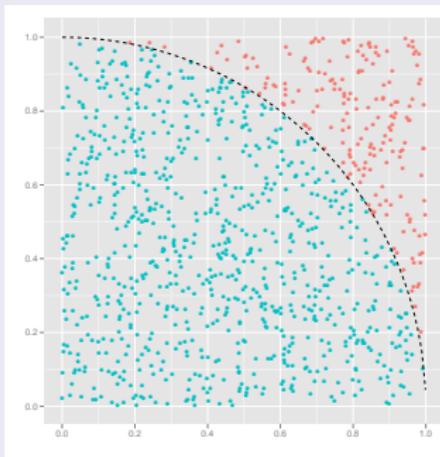


pbdMPI Example: Monte Carlo Simulation

Example 1: Monte Carlo Simulation

Sample N uniform observations (x_i, y_i) in the unit square $[0, 1] \times [0, 1]$. Then

$$\pi \approx 4 \left(\frac{\# \text{ Inside Circle}}{\# \text{ Total}} \right) = 4 \left(\frac{\# \text{ Blue}}{\# \text{ Blue} + \# \text{ Red}} \right)$$





pbdMPI Example: Monte Carlo Simulation

Example 1: Monte Carlo Simulation GBD Algorithm

- ① Let n be big-ish; we'll take $n = 50,000$.
- ② Generate an $n \times 2$ matrix x of standard uniform observations.
- ③ Count the number of rows satisfying $x^2 + y^2 \leq 1$
- ④ Ask everyone else what their answer is; sum it all up.
- ⑤ Take this new answer, multiply by 4 and divide by n
- ⑥ If my rank is 0, print the result.

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pbdMPI Example: Monte Carlo Simulation

Example 1: Monte Carlo Simulation Code

Serial Code

```
1 N <- 50000
2 X <- matrix(runif(N * 2), ncol=2)
3 r <- sum(rowSums(X^2) <= 1)
4 PI <- 4*r/N
5 print(PI)
```

Parallel Code

```
1 library(pbdMPI, quiet = TRUE)
2 init()
3 comm.set.seed(diff=TRUE)
4
5 N.gbd <- 50000 / comm.size()
6 X.gbd <- matrix(runif(N.gbd * 2), ncol = 2)
7 r.gbd <- sum(rowSums(X.gbd^2) <= 1)
8 r <- allreduce(r.gbd)
9 PI <- 4*r/(N.gbd * comm.size())
10 comm.print(PI)
11
12 finalize()
```



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pbdMPI Example: Monte Carlo Simulation

Note

For the remainder, we will exclude loading, init, and finalize calls.



pbdMPI Example: Sample Covariance

Example 2: Sample Covariance

$$\text{cov}(x_{n \times p}) = \frac{1}{n-1} \sum_{i=1}^n (x_i - \mu_x)(x_i - \mu_x)^T$$





pbdMPI Example: Sample Covariance

Example 2: Sample Covariance GBD Algorithm

- ① Determine the total number of rows N .
- ② Compute the vector of column means of the full matrix.
- ③ Subtract each column's mean from that column's entries in each local matrix.
- ④ Compute the crossproduct locally and reduce.
- ⑤ Divide by $N - 1$.





pbdMPI Example: Sample Covariance

Example 2: Sample Covariance Code

Serial Code

```
1 N <- nrow(X)
2 mu <- colSums(X) / N
3
4 X <- sweep(X, STATS=mu, MARGIN=2)
5 Cov.X <- crossprod(X) / (N-1)
6
7 print(Cov.X)
```

Parallel Code

```
1 N <- allreduce(nrow(X.gbd), op="sum")
2 mu <- allreduce(colSums(X.gbd) / N, op="sum")
3
4 X.gbd <- sweep(X.gbd, STATS=mu, MARGIN=2)
5 Cov.X <- allreduce(crossprod(X.gbd), op="sum") / (N-1)
6
7 comm.print(Cov.X)
```

pbdMPI Example: Linear Regression

Example 3: Linear Regression

Find β such that

$$\mathbf{y} = \mathbf{X}\beta + \epsilon$$

When \mathbf{X} is full rank,

$$\hat{\beta} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y}$$





pbdMPI Example: Linear Regression

Example 3: Linear Regression GBD Algorithm

- ① Locally, compute $tx = x^T$
- ② Locally, compute $A = tx * x$. Query every other processor for this result and sum up all the results.
- ③ Locally, compute $B = tx * y$. Query every other processor for this result and sum up all the results.
- ④ Locally, compute $A^{-1} * B$



pbdMPI Example: Linear Regression

Example 3: Linear Regression Code

Serial Code

```
1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
```

Parallel Code

```
1 tX.gbd <- t(X.gbd)
2 A <- allreduce(tX.gbd %*% X.gbd, op = "sum")
3 B <- allreduce(tX.gbd %*% y.gbd, op = "sum")
4
5 ols <- solve(A) %*% B
```



Contents

7 Introduction to pbdDMAT

- Introduction to Distributed Matrices
- DMAT Distributions
- pbdDMAT





Introduction to Distributed Matrices

Distributed Matrices

Most problems in data science are matrix algebra problems

- Data structure: block-cyclic matrix distributed across a 2-dimensional grid of processors.
- No single processor should hold all of the data.
- Very robust, but very confusing data structure.

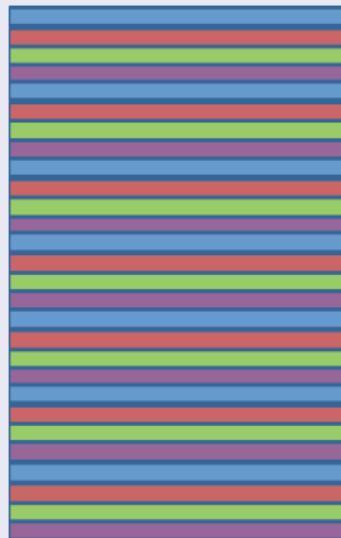


Introduction to Distributed Matrices

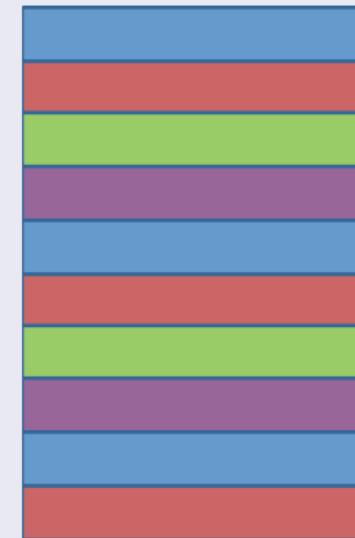
Distributed Matrices



(a) Block



(b) Cyclic



(c) Block-Cyclic

Figure: Matrix Distribution Schemes

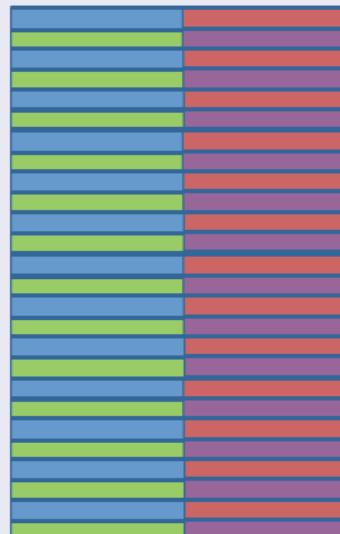


Introduction to Distributed Matrices

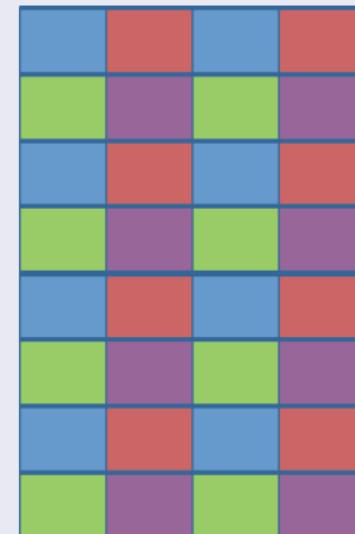
Distributed Matrices



(a) 2d Block



(b) 2d Cyclic



(c) 2d Block-Cyclic

Figure: Matrix Distribution Schemes Onto a 2-Dimensional Grid



Introduction to Distributed Matrices

Processor Grid Shapes

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}^T$$

$$\begin{bmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 \\ 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$$

(a) 1×6 (b) 2×3 (c) 3×2 (d) 6×1

Table: Processor Grid Shapes with 6 Processors



Introduction to Distributed Matrices

Distributed Matrices

The data structure is a special R class (in the OOP sense) called `ddmatrix`. It is the “under the rug” storage for a block-cyclic matrix distributed onto a 2-dimensional processor grid.

$$\text{ddmatrix} = \begin{cases} \textbf{Data} & \text{S4 local submatrix, an R matrix} \\ \textbf{dim} & \text{S4 dimension of the global matrix, a numeric pair} \\ \textbf{Idim} & \text{S4 dimension of the local submatrix, a numeric pair} \\ \textbf{bldim} & \text{S4 ScaLAPACK blocking factor, a numeric pair} \\ \textbf{CTXT} & \text{S4 BLACS context, an numeric singleton} \end{cases}$$

with prototype

$$\text{new("ddmatrix")} = \begin{cases} \textbf{Data} & = \text{matrix}(0.0) \\ \textbf{dim} & = \text{c}(1,1) \\ \textbf{Idim} & = \text{c}(1,1) \\ \textbf{bldim} & = \text{c}(1,1) \\ \textbf{CTXT} & = 0.0 \end{cases}$$

Introduction to Distributed Matrices

Distributed Matrices: The Data Structure

Example: an 9×9 matrix is distributed with a “block-cycling” factor of 2×2 on a 2×2 processor grid:

1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
2	2	3	3	2	2	3	3	2
2	2	3	3	2	2	3	3	2
1	1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1	1
2	2	3	3	2	2	3	3	2
2	2	3	3	2	2	3	3	2
1	1	1	1	1	1	1	1	1

$$= \begin{cases} \textbf{Data} & = \text{matrix}(\dots) \\ \textbf{dim} & = c(9, 9) \\ \textbf{ldim} & = c(\dots) \\ \textbf{bldim} & = c(2, 2) \\ \textbf{CTXT} & = 0 \end{cases}$$

See <http://acts.nersc.gov/scalapack/hands-on/datadist.html>



Introduction to Distributed Matrices

Understanding Dmat: Global Matrix

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$





Introduction to Distributed Matrices

DMAT: 1-dimensional Row Block

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ \hline x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ \hline x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{array}{c|c} 0 & (0,0) \\ 1 & (0,1) \\ 2 & (1,0) \\ 3 & (1,1) \end{array}$$



Introduction to Distributed Matrices

DMAT: 2-dimensional Row Block

$$X = \left[\begin{array}{cc|cc|cc|cc|cc} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ \hline X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ \hline X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{array} \right]_{9 \times 9}$$

$$\text{Processor grid} = \left| \begin{matrix} 0 & 1 \\ 2 & 3 \end{matrix} \right| = \left| \begin{matrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{matrix} \right|$$



Introduction to Distributed Matrices

DMAT: 1-dimensional Row Cyclic

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} & X_{19} \\ X_{21} & X_{22} & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} & X_{29} \\ X_{31} & X_{32} & X_{33} & X_{34} & X_{35} & X_{36} & X_{37} & X_{38} & X_{39} \\ X_{41} & X_{42} & X_{43} & X_{44} & X_{45} & X_{46} & X_{47} & X_{48} & X_{49} \\ X_{51} & X_{52} & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} & X_{59} \\ X_{61} & X_{62} & X_{63} & X_{64} & X_{65} & X_{66} & X_{67} & X_{68} & X_{69} \\ X_{71} & X_{72} & X_{73} & X_{74} & X_{75} & X_{76} & X_{77} & X_{78} & X_{79} \\ X_{81} & X_{82} & X_{83} & X_{84} & X_{85} & X_{86} & X_{87} & X_{88} & X_{89} \\ X_{91} & X_{92} & X_{93} & X_{94} & X_{95} & X_{96} & X_{97} & X_{98} & X_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{array}{|c|c|} \hline 0 & (0,0) \\ \hline 1 & (0,1) \\ \hline 2 & (1,0) \\ \hline 3 & (1,1) \\ \hline \end{array}$$



Introduction to Distributed Matrices

DMAT: 2-dimensional Row Cyclic

$$x = \left[\begin{array}{ccccc|ccccc} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{array} \right]_{9 \times 9}$$

$$\text{Processor grid} = \left| \begin{array}{cc} 0 & 1 \\ 2 & 3 \end{array} \right| = \left| \begin{array}{cc} (0,0) & (0,1) \\ (1,0) & (1,1) \end{array} \right|$$



Introduction to Distributed Matrices

DMAT: 2-dimensional Block-Cyclic

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) \\ (1,0) & (1,1) \end{vmatrix}$$

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pbdDMAT

The DMAT Data Structure

The more complicated the processor grid, the more complicated the distribution.





pbdDMAT

DMAT: 2-dimensional Block-Cyclic with 6 Processors

$$x = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} & x_{15} & x_{16} & x_{17} & x_{18} & x_{19} \\ x_{21} & x_{22} & x_{23} & x_{24} & x_{25} & x_{26} & x_{27} & x_{28} & x_{29} \\ x_{31} & x_{32} & x_{33} & x_{34} & x_{35} & x_{36} & x_{37} & x_{38} & x_{39} \\ x_{41} & x_{42} & x_{43} & x_{44} & x_{45} & x_{46} & x_{47} & x_{48} & x_{49} \\ x_{51} & x_{52} & x_{53} & x_{54} & x_{55} & x_{56} & x_{57} & x_{58} & x_{59} \\ x_{61} & x_{62} & x_{63} & x_{64} & x_{65} & x_{66} & x_{67} & x_{68} & x_{69} \\ x_{71} & x_{72} & x_{73} & x_{74} & x_{75} & x_{76} & x_{77} & x_{78} & x_{79} \\ x_{81} & x_{82} & x_{83} & x_{84} & x_{85} & x_{86} & x_{87} & x_{88} & x_{89} \\ x_{91} & x_{92} & x_{93} & x_{94} & x_{95} & x_{96} & x_{97} & x_{98} & x_{99} \end{bmatrix}_{9 \times 9}$$

$$\text{Processor grid} = \begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{vmatrix}$$

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pbdDMAT

Understanding DMAT: Local View

$$\begin{bmatrix} X_{11} & X_{12} & | & X_{17} & X_{18} \\ X_{21} & X_{22} & | & X_{27} & X_{28} \\ \hline X_{51} & X_{52} & | & X_{57} & X_{58} \\ X_{61} & X_{62} & | & X_{67} & X_{68} \\ \hline X_{91} & X_{92} & | & X_{97} & X_{98} \end{bmatrix}_{5 \times 4}$$

$$\begin{bmatrix} X_{13} & X_{14} & | & X_{19} \\ X_{23} & X_{24} & | & X_{29} \\ \hline X_{53} & X_{54} & | & X_{59} \\ X_{63} & X_{64} & | & X_{69} \\ \hline X_{93} & X_{94} & | & X_{99} \end{bmatrix}_{5 \times 3}$$

$$\begin{bmatrix} X_{15} & X_{16} \\ X_{25} & X_{26} \\ \hline X_{55} & X_{56} \\ X_{65} & X_{66} \\ \hline X_{95} & X_{96} \end{bmatrix}_{5 \times 2}$$

$$\begin{bmatrix} X_{31} & X_{32} & | & X_{37} & X_{38} \\ X_{41} & X_{42} & | & X_{47} & X_{48} \\ \hline X_{71} & X_{72} & | & X_{77} & X_{78} \\ X_{81} & X_{82} & | & X_{87} & X_{88} \end{bmatrix}_{4 \times 4}$$

$$\begin{bmatrix} X_{33} & X_{34} & | & X_{39} \\ X_{43} & X_{44} & | & X_{49} \\ \hline X_{73} & X_{74} & | & X_{79} \\ X_{83} & X_{84} & | & X_{89} \end{bmatrix}_{4 \times 3}$$

$$\begin{bmatrix} X_{35} & X_{36} \\ X_{45} & X_{46} \\ \hline X_{75} & X_{76} \\ X_{85} & X_{86} \end{bmatrix}_{4 \times 2}$$

Processor grid = $\begin{vmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{vmatrix} = \begin{vmatrix} (0,0) & (0,1) & (0,2) \\ (1,0) & (1,1) & (1,2) \end{vmatrix}$

The DMAT Data Structure

- ➊ DMAT is *distributed*. No one processor owns all of the matrix.
- ➋ DMAT is *non-overlapping*. Any piece owned by one processor is owned by no other processors.

- ➌ DMAT can be row-contiguous or not, depending on the processor grid and blocking factor used.
- ➍ DMAT is locally column-major and globally, it depends...
- ➎ GBD is a generalization of the one-dimensional block DMAT distribution. Otherwise there is no relation.
- ➏ DMAT is confusing, but very robust.

X_{11}	X_{12}	X_{13}	X_{14}	X_{15}
X_{21}	X_{22}	X_{23}	X_{24}	X_{25}
X_{31}	X_{32}	X_{33}	X_{34}	X_{35}
X_{41}	X_{42}	X_{43}	X_{44}	X_{45}
X_{51}	X_{52}	X_{53}	X_{54}	X_{55}
X_{61}	X_{62}	X_{63}	X_{64}	X_{65}
X_{71}	X_{72}	X_{73}	X_{74}	X_{75}
X_{81}	X_{82}	X_{83}	X_{84}	X_{85}
X_{91}	X_{92}	X_{93}	X_{94}	X_{95}



pbdDMAT

Pros and Cons of This Data Structure

Pros

- Fast for distributed matrix computations

Cons

- Literally everything else

This is why we hide most of the distributed details.

The details are there if you want them (you don't want them).



Distributed Matrix Methods

pbdDMAT has over 100 methods with *identical* syntax to R:

- `[, rbind(), cbind(), ...]
- lm.fit(), prcomp(), cov(), ...
- `%*%`, solve(), svd(), norm(), ...
- median(), mean(), rowSums(), ...

Serial Code

```
1 cov(x)
```

Parallel Code

```
1 cov(x)
```



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pbdDMAT

Comparing pbdMPI and pbdDMAT

- **pbdMPI** is MPI + some sugar.
- The GBD data structure is not the only thing **pbdMPI** can handle (just a useful convention).
- **pbdDMAT** is more of a software package.
- The block-cyclic DMAT structure *must* be used for **pbdDMAT**.





pbdDMAT

Quick Comments for Using pbdDMAT

- 1 Start by loading the package:

```
1 library(pbdDMAT, quiet = TRUE)
```

- 2 Always initialize before starting and finalize when finished:

```
1 init.grid()  
2  
3 # ...  
4  
5 finalize()
```

- 3 Distributed DMAT objects will be given the suffix .dmat to visually help distinguish them from global objects. This suffix carries no semantic meaning.



Statistics Examples with pbdDMAT

Sample Covariance

Serial Code

```
1 Cov.X <- cov(X)
2 print(Cov.X)
```

Parallel Code

```
1 Cov.X <- cov(X)
2 print(Cov.X)
```





Statistics Examples with pbdDMAT

Linear Regression

Serial Code

```
1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
6
7 # or
8 ols <- lm.fit(X, y)
```

Parallel Code

```
1 tX <- t(X)
2 A <- tX %*% X
3 B <- tX %*% y
4
5 ols <- solve(A) %*% B
6
7 # or
8 ols <- lm.fit(X, y)
```



Statistics Examples with pbdDMAT

Quick Example 3

PCA: pca.r

```

1 library(pbdDMAT, quiet=T)
2 init.grid()
3
4 n <- 1e4
5 p <- 250
6
7 comm.set.seed(diff=T)
8 x.dmat <- ddmatrix("rnorm", nrow=n, ncol=p, mean=100, sd=25)
9
10 pca <- prcomp(x=x.dmat, retx=TRUE, scale=TRUE)
11 prop_var <- cumsum(pca$sdev)/sum(pca$sdev)
12 i <- max(min(which(prop_var > 0.9)) - 1, 1)
13
14 y.dmat <- pca$x[, 1:i]
15
16 comm.cat("\nCols: ", i, "\n", quiet=T)
17 comm.cat("%Cols:", i/dim(x.dmat)[2], "\n\n", quiet=T)
18
19 finalize()
```

Execute this script via:

```
1 mpirun -np 2 Rscript pca.r
```

Sample Output:

1	Cols: 221
2	%Cols: 0.884

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pbdDMAT Example: Generating Data

Generating Random Data

Using randomly generated matrices is the best way to “get your feet wet” with the pbd tools. You can do this in 2 ways:

- ① Generate a global matrix and distribute it.
- ② Generate locally only what is needed.





pbdDMAT Example: Generating Data

Example 1: Random Distributed Matrix Generation

Generate a global matrix and distribute it

```
1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 # Common global on all processors --> distributed
5 comm.set.seed(diff=FALSE)
6 x <- matrix(rnorm(100), nrow=10, ncol=10)
7 x.dmat <- as.ddmatrix(x)
8
9 # Global on processor 0 --> distributed
10 if (comm.rank()==0){
11   x <- matrix(rnorm(100), nrow=10, ncol=10)
12 } else {
13   x <- NULL
14 }
15 x.dmat <- as.ddmatrix(x)
16
17 finalize()
```



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pbdDMAT Example: Generating Data

Example 2: Random Distributed Matrix Generation

Generate locally only what is needed

```
1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 comm.set.seed(diff = TRUE) # good seeds via rlecuyer
5 x.dmat <- ddmatrix("rnorm", nrow=10, ncol=10)
6
7 finalize()
```



pbdDMAT Example: Generating Data

Example 3: Random Distributed Matrix Generation

Generate locally only what is needed

```
1 library(pbdDMAT, quiet=TRUE)
2 init.grid()
3
4 zero.dmat <- ddmatrix(0, nrow=100, ncol=100)
5 id.dmat <- diag(1, nrow=100, ncol=100)
6
7 finalize()
```





pbdDMAT Example: Converting Between GBD and DMAT

Example 4: Random Distributed Matrix Generation

Convert between GBD and DMAT

```
1 library(pbdDEMO, quiet=TRUE)
2 init.grid()
3
4 comm.set.seed(diff = TRUE)
5
6 N.gbd <- 1 + comm.rank()
7 X.gbd <- matrix(rnorm(N.gbd * 3), ncol = 3)
8
9 # convert GBD to DMAT
10 X.dmat <- gbd2dmat(X.gbd)
11
12 # convert DMAT to GBD
13 new.X.gbd <- dmat2gbd(X.dmat)
14
15 # undistribute
16 X <- as.matrix(X.dmat)
17
18 finalize()
```

pbdDMAT Example: Converting Between GBD and DMAT

Distributed Matrices

pbdDEMO contains many other examples of reading and managing
GBD and DMAT data



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pbdMPI

GBD

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pbdDMAT

pbdDMAT eg's

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Break



Where to Learn More

- Our website <http://r-pbd.org/>
- The **pbdDEMO** package
<http://cran.r-project.org/web/packages/pbdDEMO/>
- The **pbdDEMO** Vignette: <http://goo.gl/HZkRt>
- Our Google Group: <http://group.r-pbd.org>





Break

Stats eg's

pbdDMAT



Wrapup

Thanks for coming!

Questions? Comments?

Don't forget to come to the talk:

Elevating R to Supercomputers

Friday, July 12th at 10:00

at the High Performance Computing session!