Item 1

Let X be a discrete random variable with PMF:

$$P(X = x) = \begin{cases} 0.2 & \text{if } x = -2 \\ 0.3 & \text{if } x = 0 \\ 0.5 & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$$

Let $Y = 3X + 2$

Compute $E[Y]$ and $var(Y)$. State your answers as decimals with 2 decimal precision. Use dot (.) as a decimal separator.

$E[Y] =$ ☐

Check Answer

$var(Y) =$ ☐

Check Answer

Item 2

Let X and Y be continuous random variables with joint PDF:

$$f_{X,Y}(x,y) = \begin{cases} 3x, & 0 < x < 1, 0 < y < x \\ 0, & \text{otherwise} \end{cases}$$

Find the conditional expectation $E\left[Y|X=x\right]$ for $x \in (0,1)$. State your answer as an irreducible fraction.

$$E\left[Y|X=x\right] = \frac{x}{\square}$$

Check Answer

Item 3

A radioactive particle detector receives on average 5 hits per minute, modeled by a Poisson process.

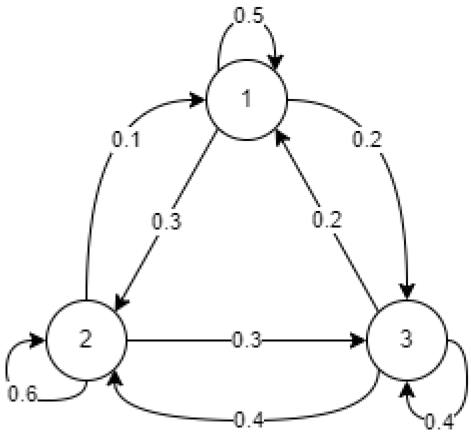
What is the probability that the detector records 5 or more hits in 2 minutes?

State your answers as decimals with 4 decimal precision. Use dot (.) as a decimal separator.

Check Answer

Item 4

Consider the following state transition diagram for a given Markov chain for states $S = \{1,\ 2,\ 3\}$.



Compute the probability of transitioning from state 1 to 3 in two steps. State your answers as decimals with 2 decimal precision. Use dot (.) as a decimal separator.

Check Answer

Item 5

A random sample size of 36 samples from a normal distribution gives $\bar{x} = 105$ and $\sigma = 12$. Test whether the population mean is different from 100 at $\alpha = 0.05$.

State your answers as decimals with 2 decimal precision. Use dot (.) as a decimal separator.

a. Calculate $|z_{test}|$ and $|z_{crit}|$.

$|z_{test}| =$ ☐

$|z_{crit}| =$ ☐

Check Answer

b. Based on these values, the hypothesis is:

A Rejected since $|z_{test}| > |z_{crit}|$

B Rejected since $|z_{test}| < |z_{crit}|$

C Not rejected since $|z_{test}| > |z_{crit}|$

D Not rejected since $|z_{test}| < |z_{crit}|$

Check Answer

Item 6

Let $X, Y \sim \mathcal{N}(0,1)$ be independent.

Let $A = X^2$ and $B = Y^2 - 1$.

Calculate $cov\,(A, B)$. State your answers as integer.

$cov\,(A, B) =$ ☐

Check Answer

Item 7

Let $f(x, y) = 24x^2y$ for $0 \leq x \leq 1, 0 \leq y \leq 1 - x$.

a. Calculate $E[XY]$. State your answers as decimals with 4 decimal precision. Use dot (.) as a decimal separator.

$$E[XY] = \square$$

Check Answer

b. Calculate the marginal PDFs of x and y. Fill in the answers with non-negative integers.

$$f_x(x) = \square x^{\square}(\square - x)^{\square}$$

Check Answer

$$f_y(y) = \square y^{\square}(\square - y)^{\square}$$

Check Answer

Item 8

A quality control engineer is inspecting the weight of cereal boxes. A random sample of 16 boxes is taken, and their weights (in grams) are measured. The sample mean weight is found to be $\bar{x} = 502$ grams, and the sample standard deviation is $s = 4$ grams. Assume the weights of the cereal boxes are normally distributed.

a. Construct a 95% confidence interval for the true mean weight μ of all cereal boxes. State your interval limits with 3 decimal precision. Use dot (.) as a decimal separator. Form your answer as [lower, upper].

[,]

Check Answer

b. Which of the following statements best interprets the 95% confidence interval [lower (L), upper (U)] calculated above?

A There is a 95% probability that the sample mean $\bar{x}$ lies between L and U.

B If we were to take many random samples and construct a 95% confidence interval for each, approximately 95% of those intervals would contain the true population mean μ.

C 95% of all cereal boxes have weights that lie between L and U.

D There is a 95% probability that the true population mean μ is exactly equal to the sample mean $\bar{x}$.

Check Answer

c. To obtain a narrower confidence interval using the same sample data ($n = 16, \bar{x} = 502, s = 4$), the engineer should:

A Use a higher confidence level (e.g., 99%).

B Use a lower confidence level (e.g., 90%).

C Increase the sample standard deviation.

D It's impossible to make the interval narrower without changing the sample data.

Check Answer

Item 9

A basketball player makes free throws with a probability of $P = 0.7$. Assume each free throw attempt is independent. Let X be the number of attempts until the player makes their first successful free throw.

State both your answers with 3 decimal precision. Use dot (.) as a decimal separator.

a. What is the probability that the player makes their first successful free throw on their 3rd attempt?

$P(X = 3) =$ ☐

Check Answer

b. What is the expected number of attempts until the first successful free throw?

$E[X] =$ ☐

Check Answer

Item 10

Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rates $\lambda_1 = 1$ and $\lambda_2 = 1$, respectively. Define the merged process $N(t) = N_1(t) + N_2(t)$. State all inputs as positive integers such that the answers are irreducible fractions.

a. Given that $N(3) = 3$, find

$$P\left(N_1(3) = 1 \mid N(3) = 3\right) = \frac{\square}{\square}$$

Check Answer

b. Given that $N(4) = 5$, find

$$P\left(N_1(4) = 2 \mid N(4) = 5\right) = \frac{\square}{\square}$$

Check Answer

Item 11

Consider a fair (six-sided) die that is rolled repeatedly, with each roll being independent of the others. On average, how many rolls are required until all the numbers from 1 to 6 have been obtained? State your answer as a decimal value with one decimal precision.

Check Answer

Item 12

Let X and Y be independent random variables with

- $X \sim \text{Poisson}(2)$,
- $Y \sim \text{Geometric}(p = 1/3)$.

Let $Z = 2X + 3Y$. Find $E[Z]$ and $\text{Var}(Z)$. State both answers as positive integers

$E[Z] =$ ☐ and $Var(Z) =$ ☐

Check Answer