Exercise - (5.3) के लिए

Important Point :-

1) यदि किसी A.P का पहला पद 'a' तथा सार्व-अन्तर 'd' हो, तो

प्रथम n-पदों का योग = $S_n = \frac{n}{2}[2a+(n-1)d]$

2) यदि किसी A.P का पहला पद 'a' तथा अन्तिम पद a_n या l/t_n हो, तो

n-पदो का कुल योग = $S_n = \frac{n}{2}(a+a_n)$

or

$S_n = \frac{n}{2}(a+l)$

or

$S_n = \frac{n}{2}(a+t_n)$

3) यदि n-पदों का योग S_n है, तब,

t_n (nवाँ पद) = $S_n - S_{n-1}$ (जहाँ $n \geq 2$)

4) यदि a, n, d और S_n में से किन्हीं तीन का मान दिया हो, तब चौथे का मान ज्ञात किया जा सकता है।

* समान्तर श्रेढ़ी के महत्वपूर्ण सूत्र :-

(i) n वाँ पद $= a_n = a + (n-1)d$

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

या

$S_n = \frac{n}{2}[a + a_n]$

या

$S_n = \frac{n}{2}[a + l]$

या

$S_n = \frac{n}{2}[a + t_n]$

(iii) $t_n = S_n - S_{n-1}$ [जहाँ $n \geq 2$]

Exercise - 53

1) निम्नलिखित समांतर श्रेढ़ियों का योग ज्ञात कीजिए -

(i) 2, 7, 12, ... 10 पदों तक

$\because a = 2$

$d = 7 - 2 = 5$

$n = 10$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{5\cancel{10}}{\cancel{2}}[2 \times 2 + (10-1) \times 5]$

$= 5[4 + 9 \times 5]$

$= 5[4 + 45]$

$= 5 \times 49$

$= 245$ Ans

(ii) $-37, -33, -29, \ldots\ldots\ldots$ 12 पदों तक

$\because a = -37$

$d = -33 - (-37)$

$= -33 + 37$

$= 4$

$n = 12$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{12} = \frac{6\cancel{12}}{\cancel{2}}[2 \times (-37) + (12-1) \times 4]$

$= 6[-74 + 11 \times 4]$

$= 6[-74 + 44]$

$= 6 \times (-30) = -180$ Ans

(iii) 0·6, 1·7, 2·8, ... 100 पदों तक

$\because a = 0·6$

$d = 1·7 - 0·6 = 1·1$

$n = 100$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{100} = \frac{\cancel{100}^{50}}{\cancel{2}}[2 \times 0·6 + (100-1) \times 1·1]$

$= 50[1·2 + 99 \times 1·1]$

$= 50[1·2 + 1089]$

$= 50 \times 110·1$

$= 5505·0$

$= 5505$ Ans

(iv) $\frac{1}{15}, \frac{1}{12}, \frac{1}{10}$, ... 11 पदों तक

$\because a = \frac{1}{15}$

$d = \frac{1}{12} - \frac{1}{15}$

$= \frac{5-4}{60}$

$= \frac{1}{60}$

$n = 11$

$\therefore S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{11} = \frac{11}{2}[2 \times \frac{1}{15} + (11-1) \times \frac{1}{60}]$

(iv) $S_{11} = \frac{11}{2}\left[\frac{2}{15} + 10 \times \frac{1}{60}\right]$

$= \frac{11}{2}\left[\frac{2}{15} + \frac{1}{6}\right]$

$= \frac{11}{2}\left[\frac{4+5}{30}\right]$

$= \frac{11}{2} \times \frac{9}{30}_{10}^{3}$

$= \frac{33}{20}$ Ans

(2.) नीचे दिए हुए योगफलों को ज्ञात कीजिए —

(i) $7 + 10\frac{1}{2} + 14 + \cdots\cdots\cdots + 84$

$\because a = 7$

$d = 10\frac{1}{2} - 7$

$= \frac{21}{2} - 7$

$= \frac{21-14}{2}$

$= \frac{7}{2}$

$a_n = 84$

$\because a_n = a + (n-1)d$

$\Rightarrow 84 = 7 + (n-1) \times \frac{7}{2}$

$\Rightarrow 84 - 7 = (n-1) \times \frac{7}{2}$

$\Rightarrow 77 = (n-1) \times \frac{7}{2}$

$\Rightarrow n-1 = \frac{{}^{11}77 \times 2}{7}$

$\Rightarrow n-1 = 22$

$\Rightarrow n = 22+1$

$\Rightarrow n = 23$

$\therefore S_n = \frac{n}{2}[a+a_n]$

$= \frac{23}{2}[7+84]$

$= \frac{23}{2} \times 91$

$= \frac{2093}{2}$

$= 1046\frac{1}{2}$ Ans

(ii) $34+32+30+\cdots\cdots+10$

$\because a = 34$

$d = 32-34 = -2$

$a_n = 10$

$\because a_n = a+(n-1)d$

$\Rightarrow 10 = 34+(n-1)\times(-2)$

$\Rightarrow 10-34 = (n-1)\times(-2)$

$\Rightarrow -24 = (n-1)\times(-2)$

$\Rightarrow n-1 = \frac{+24}{+2}^{12}$

$\Rightarrow n-1 = 12$

$\Rightarrow n = 12+1$

$\Rightarrow n = 13$

$\therefore S_n = \frac{n}{2}[a+a_n]$

$= \frac{13}{2}[34+10]$

$= \frac{13}{2} \times 44^{22}$

$= 13 \times 22$

$= 286$ Ans

(iii) $-5+(-8)+(-11)+\cdots\cdots+(-230)$

$\because a = -5$

$d = -8-(-5)$

$= -8+5$

$= -3$

$a_n = -230$

$\because a_n = a+(n-1)d$

$\Rightarrow -230 = -5+(n-1)\times(-3)$

$\Rightarrow -230 = -5-3n+3$

$\Rightarrow -230 = -2-3n$

$\Rightarrow 3n = -2+230$

$\Rightarrow 3n = 228$

$\Rightarrow n = \frac{228}{3}^{76}$

$\Rightarrow n = 76$

$\therefore S_n = \frac{n}{2}[a+a_n]$

$= {}^{38}\frac{76}{2}[-5+(-230)]$

$= 38[-5-230]$

$= 38 \times (-235) = -8930$ Ans

(3) एक A.P में,

(i) $a = 5$

$d = 3$

$a_n = 50$

$n = ?$

$S_n = ?$

$\because a_n = a + (n-1)d$

$\Rightarrow 50 = 5 + (n-1) \times 3$

$\Rightarrow 50 = 5 + 3n - 3$

$\Rightarrow 50 = 3n + 2$

$\Rightarrow 50 - 2 = 3n$

$\Rightarrow 48 = 3n$

$\Rightarrow n = \frac{48}{3}^{16}$

$\Rightarrow n = 16$

और

$\because S_n = \frac{n}{2}[a + a_n]$

$= \frac{8\cancel{16}}{\cancel{2}}[5 + 50]$

$= 8 \times 55$

$= 440$ Ans

(ii) $a = 7$, $a_{13} = 35$, $d = ?$, $S_{13} = ?$

$\because \quad a_{13} = a + 12d$

$\Rightarrow \quad 35 = 7 + 12d$

$\Rightarrow \quad 35 - 7 = 12d$

$\Rightarrow \quad 28 = 12d$

$\Rightarrow \quad d = \frac{\cancel{28}^{7}}{\cancel{12}_{3}} = \frac{7}{3}$ Ans

फिर,

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$\Rightarrow S_{13} = \frac{13}{2}\left[2 \times 7 + (13-1) \times \frac{7}{3}\right]$$

$$= \frac{13}{2}\left[14 + \cancel{12}^{4} \times \frac{7}{\cancel{3}}\right]$$

$$= \frac{13}{2}\left[14 + 28\right]$$

$$= \frac{13}{\cancel{2}} \times \cancel{42}^{21}$$

$$= 273 \text{ Ans}$$

(iii) $a_{12} = 37$, $d = 3$, $a = ?$, $S_{12} = ?$

$\because \quad a_{12} = 37$

$\Rightarrow a + 11d = 37$

$\Rightarrow a + 11 \times 3 = 37$

$\Rightarrow a + 33 = 37$

$\Rightarrow a = 37 - 33$

$\Rightarrow a = 4$ Ans

फिर,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{12} = \frac{6\cancel{12}}{\cancel{2}}[2 \times 4 + (12-1) \times 3]$$

$$= 6[8 + 11 \times 3]$$

$$= 6[8 + 33]$$

$$= 6 \times 41$$

$$= 246 \text{ Ans}$$

(iv) $a_3 = 15$, $S_{10} = 125$, $d = ?$, $a_{10} = ?$

$\because a_3 = 15$

$\Rightarrow a + 2d = 15$

$\Rightarrow a = 15 - 2d$ — (1)

फिर,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{10} = \frac{10}{2}[2 \times (15-2d) + (10-1)d]$$

$$\Rightarrow 125 = 5[30 - 4d + 9d]$$

$$\Rightarrow \frac{125}{5} = 30 + 5d$$

$$\Rightarrow 25 = 30 + 5d$$

$$\Rightarrow 25 - 30 = 5d$$

$$\Rightarrow -5 = 5d$$

$$\Rightarrow d = \frac{-5}{5}$$

$$\Rightarrow d = -1$$ Ans

फिर, $\therefore a = 15 - 2d = 15 - 2 \times (-1) = 15 + 2 = 17$

$$a_{10} = a + 9d$$

$$= 17 + 9 \times (-1)$$

$$= 17 - 9$$

$$= 8$$ Ans

(v) $d = 5,\ S_9 = 75,\ a = ?,\ a_9 = ?$

$\because\ S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2a + (9-1)\times 5]$

$\Rightarrow 75 = \frac{9}{2}[2a + 8\times 5]$

$\Rightarrow \frac{75\times 2}{9} = 2a + 40$

$\Rightarrow \frac{75\times 2}{9} = 2(a+20)$

$\Rightarrow \frac{75}{9} = a + 20$

$\Rightarrow \frac{75}{9} - 20 = a$

$\Rightarrow \frac{75 - 180}{9} = a$

$\Rightarrow \frac{-105^{35}}{9_3} = a$

$\Rightarrow a = -\frac{35}{3}$ Ans

फिर,

$a_9 = a + 8d$

$= -\frac{35}{3} + 8\times 5$

$= -\frac{35}{3} + 40$

$= \frac{-35 + 120}{3}$

$= \frac{85}{3}$ Ans

(vi) $a=2,\ d=8,\ S_n=90,\ n=?,\ a_n=?$

$\because S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 90 = \frac{n}{2}[2\times2+(n-1)\times8]$

$\Rightarrow 90 = \frac{n}{2}[4+8n-8]$

$\Rightarrow 90 = \frac{n}{2}[8n-4]$

$\Rightarrow 90 = \frac{n}{2}\times 4^{2}(2n-1)$

$\Rightarrow 90^{45} = 2n(2n-1)$

$\Rightarrow 45 = 2n^2-n$

$\Rightarrow 0 = 2n^2-n-45$

$\Rightarrow 2n^2-n-45=0$

$\Rightarrow 2n^2-10n+9n-45=0$

$\Rightarrow 2n(n-5)+9(n-5)=0$

$\Rightarrow (2n+9)(n-5)=0$

$\Rightarrow 2n+9=0$ और $n-5=0$

$\Rightarrow 2n=-9$ $\Rightarrow n=5$

$\Rightarrow n=\frac{-9}{2}$

$\because$ n का मान ऋणात्मक नहीं होता है।

$\therefore n=5$ Ans

फिर,

$a_n = a+(n-1)d$

$= 2+(5-1)\times8$

$= 2+4\times8$

$= 2+32 = 34$ Ans

(vii) $a=8,\ a_n=62,\ S_n=210,\ n=?,\ d=?$

$\because\ S_n = \frac{n}{2}[a+a_n]$

$\Rightarrow 210 = \frac{n}{2}[8+62]$

$\Rightarrow 210 = \frac{n}{2} \times 70^{35}$

$\Rightarrow 210 = 35n$

$\Rightarrow n = \frac{210}{35}^{6}$

$\Rightarrow n = 6$ Ans

फिर,

$a_n = 62$

$\Rightarrow a+(n-1)d = 62$

$\Rightarrow 8+(6-1)d = 62$

$\Rightarrow 8+5d = 62$

$\Rightarrow 5d = 62-8$

$\Rightarrow 5d = 54$

$\Rightarrow d = \frac{54}{5}$ Ans

(viii) $a_n = 4$, $d = 2$, $S_n = -14$, $n = ?$, $a = ?$

$\because \quad a_n = 4$

$\Rightarrow a + (n-1)d = 4$

$\Rightarrow a + (n-1) \times 2 = 4$

$\Rightarrow a + 2n - 2 = 4$

$\Rightarrow a + 2n = 4 + 2$

$\Rightarrow a + 2n = 6$

$\Rightarrow a = 6 - 2n \quad — (1)$

फिर,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow -14 = \frac{n}{2}[2(6-2n) + (n-1) \times 2]$$

$$\Rightarrow -14 = \frac{n}{2}[12 - 4n + 2n - 2]$$

$$\Rightarrow -14 = \frac{n}{2}[10 - 2n]$$

$$\Rightarrow -14 = \frac{n}{2} \times 2(5-n)$$

$$\Rightarrow -14 = n(5-n)$$

$$\Rightarrow -14 = 5n - n^2$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$\Rightarrow n(n-7) + 2(n-7) = 0$$

$$\Rightarrow (n+2)(n-7) = 0$$

$\Rightarrow n+2=0$ और $n-7=0$

$\Rightarrow n=-2$ $\Rightarrow n=7$

$\because$ n का मान ऋणात्मक नहीं होता है

$\therefore n=7$ Ans

समी० (1) से,

$a = 6-2n$

$= 6-2\times7$

$= 6-14$

$= -8$ Ans

(IX) $a=3,\ n=8,\ S=192,\ d=?$

$\because S = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow 192 = \frac{8}{2}[2\times3+(8-1)d]$

$\Rightarrow 192 = 4(6+7d)$

$\Rightarrow \frac{192}{4} = 6+7d$

$\Rightarrow 48 = 6+7d$

$\Rightarrow 48-6 = 7d$

$\Rightarrow 42 = 7d$

$\Rightarrow d = \frac{42}{7}$

$\Rightarrow d = 6$ Ans

(x) $l = 28,\ S = 144,\ n = 9,\ a = ?$

$\because S = \frac{n}{2}[a + l]$

$\Rightarrow 144 = \frac{9}{2}[a + 28]$

$\Rightarrow \frac{\overset{16}{\cancel{144}} \times 2}{\cancel{9}} = a + 28$

$\Rightarrow 32 = a + 28$

$\Rightarrow 32 - 28 = a$

$\Rightarrow 4 = a$

$\Rightarrow a = 4$ Ans

⟨4⟩ 9, 17, 25, ...

$\because$ $a = 9$

$d = 17 - 9 = 8$

$S_n = 636$

$n = ?$

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 636 = \frac{n}{2}[2\times 9 + (n-1)\times 8]$

$\Rightarrow 636 = \frac{n}{2}[18 + 8n - 8]$

$\Rightarrow 636 = \frac{n}{2}[10 + 8n]$

$\Rightarrow 636 = \frac{n}{2} \times 2(5 + 4n)$

$\Rightarrow 636 = n(5 + 4n)$

$\Rightarrow 636 = 5n + 4n^2$

$\Rightarrow 0 = 4n^2 + 5n - 636$

$\Rightarrow 4n^2 + 5n - 636 = 0$

माना कि,

$a = 4$

$b = 5$

$c = -636$

$\theta = b^2 - 4ac$

$= 5^2 - 4\times 4 \times (-636)$

$= 25 + 10176$

$= 10201 > 0$

$$\therefore n = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-5 \pm \sqrt{10201}}{2 \times 4}$$

$$= \frac{-5 \pm 101}{8}$$

$$= \frac{-5+101}{8}, \frac{-5-101}{8}$$

$$= \frac{96}{8}, \frac{-106}{8}$$

$$= 12, \frac{-106}{8}$$

∵ n का मान ऋणात्मक नहीं होता है।

$$\therefore n = 12 \quad \underline{\text{Ans}}$$

(5) ∵ प्रथम पद 'a' और सार्व-अन्तर 'd' है।

$$\therefore a = 5$$

$$a_n = 45$$

$$S_n = 400$$

$$n = ?$$

$$d = ?$$

$$\because S_n = \frac{n}{2}[a + a_n]$$

$$\Rightarrow 400 = \frac{n}{2}[5+45]$$

$$\Rightarrow 400 = \frac{n}{2} \times 50^{25}$$

$$\Rightarrow 400 = n \times 25$$

$$\Rightarrow n = \frac{400^{16}}{25}$$

$$\Rightarrow n = 16$$

फिर,

$$a_n = a+(n-1)d$$

$$\Rightarrow 45 = 5+(16-1)d$$

$$\Rightarrow 45-5 = 15d$$

$$\Rightarrow 40 = 15d$$

$$\Rightarrow d = \frac{40^{8}}{15_{3}}$$

$$\Rightarrow d = \frac{8}{3}$$ Ans

(6.) माना कि प्रथम पद 'a' तथा सार्व-अन्तर 'd' है।

$\therefore a = 17,\ a_n = 350,\ d = 9,\ n = ?,\ S_n = ?$

$\because a_n = 350$

$\Rightarrow a+(n-1)d = 350$

$\Rightarrow 17+(n-1)\times 9 = 350$

$\Rightarrow 17+9n-9 = 350$

$\Rightarrow 9n+8 = 350$

$\Rightarrow 9n = 350-8$

$\Rightarrow 9n = 342$

$\Rightarrow n = \frac{342^{38}}{9}$

$\Rightarrow n = 38$ Ans

$\therefore S_n = \frac{n}{2}[a+a_n]$

$= \frac{19^{38}}{2}[17+350]$

$= 19\times 367$

$= 6973$ Ans