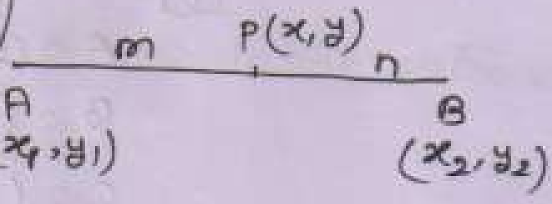
* Section Formula:-

(1) If $P(x, y)$ divides the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m:n$ then.

co-ordinate of $P = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$



where,

$$x = \frac{mx_2+nx_1}{m+n}$$

$$y = \frac{my_2+ny_1}{m+n}$$

*

(2) The mid point of the join of $A(x_1, y_1)$ and $B(x_2, y_2)$ is ~~given by~~ P then,

$m = n$

$\therefore$ co-ordinate of $P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

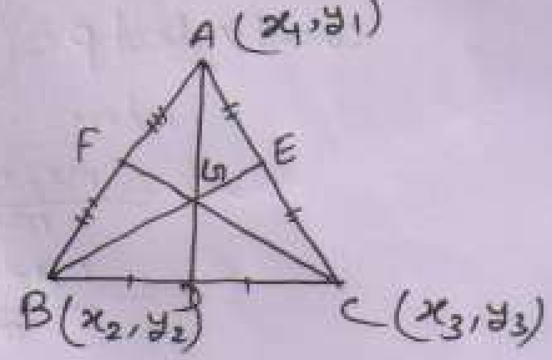
Note:
by midpoint formula -

co-ordinate of $P = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(केन्द्रक)
* Centroid of a triangle:-

The point of intersection of the medians of a triangle is called its centroid of Δ.

let



$A = (x_1, y_1)$

$B = (x_2, y_2)$

$C = (x_3, y_3)$

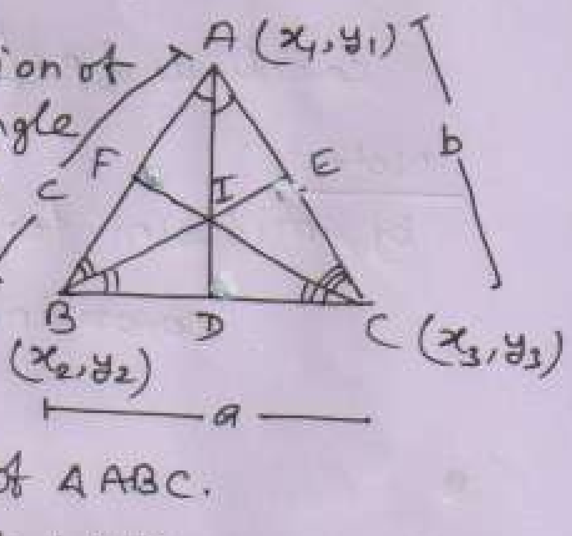
∵ A, B, C be the vertices of ΔABC.

let G be the centroid of ΔABC.

∴ Co-ordinate of $G = \left(\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}\right)$

* Incentre of triangle:-

The point of intersection of the angle bisector of a triangle is called its Incentre of Δ.



let $A = (x_1, y_1)$ and $BC = a$

$B = (x_2, y_2)$ $AB = c$

$C = (x_3, y_3)$ $AC = b$

∵ ∠A, ∠B, ∠C be the angle of ΔABC.

let I be the Incentre of ΔABC.

∴ Co-ordinate of $I = \left(\dfrac{ax_1+bx_2+cx_3}{a+b+c}, \dfrac{ay_1+by_2+cy_3}{a+b+c}\right)$

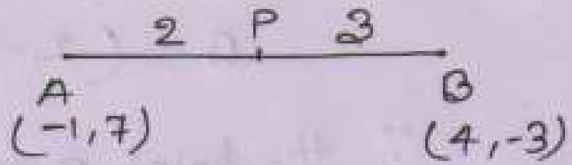
Note:

let required ratio = K : 1 [in the form of]

Exercise - 7.2

1.) let $A = (-1, 7)$

$B = (4, -3)$



Since,

the point P is divides AB in the ratio 2:3

$\therefore \quad m:n = 2:3$

or $m = 2$

$n = 3$

and,

$x_1 = -1 \qquad x_2 = 4$

$y_1 = 7 \qquad y_2 = -3$

by Section formula,

$$P = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

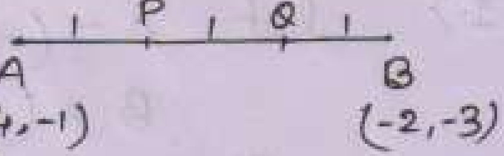
$$= \left(\frac{2\times 4 + 3\times(-1)}{2+3}, \frac{2\times(-3) + 3\times 7}{2+3}\right)$$

$$= \left(\frac{8-3}{5}, \frac{-6+21}{5}\right)$$

$$= \left(\frac{\cancel{5}}{\cancel{5}}, \frac{\cancel{15}^{\,3}}{\cancel{5}}\right)$$

$$= (1, 3) \quad \text{Ans}$$

2. let $A = (4, -1)$
$B = (-2, -3)$



$\because$ the point P and Q of trisection of the line segment AB.

$\therefore$ the point P divides AB in the ratio 1:2.

$\therefore \quad m:n = 1:2$

or $m = 1$
$n = 2$

and,

$x_1 = 4 \qquad x_2 = -2$

$y_1 = -1 \qquad y_2 = -3$

by section formula,

$$P = \left(\frac{1\times(-2) + 2\times 4}{1+2}, \frac{1\times(-3) + 2\times(-1)}{1+2}\right)$$

$$= \left(\frac{-2+8}{3}, \frac{-3-2}{3}\right)$$

$$= \left(\frac{\cancel{6}^{2}}{\cancel{3}}, \frac{-5}{3}\right)$$

$$\Rightarrow \quad P = \left(2, -\frac{5}{3}\right) \text{ Ans}$$

Again,

the point Q divides AB in the ratio 2:1.

$\therefore \quad m:n = 2:1$

or $m = 2$
$n = 1$

by section formula,

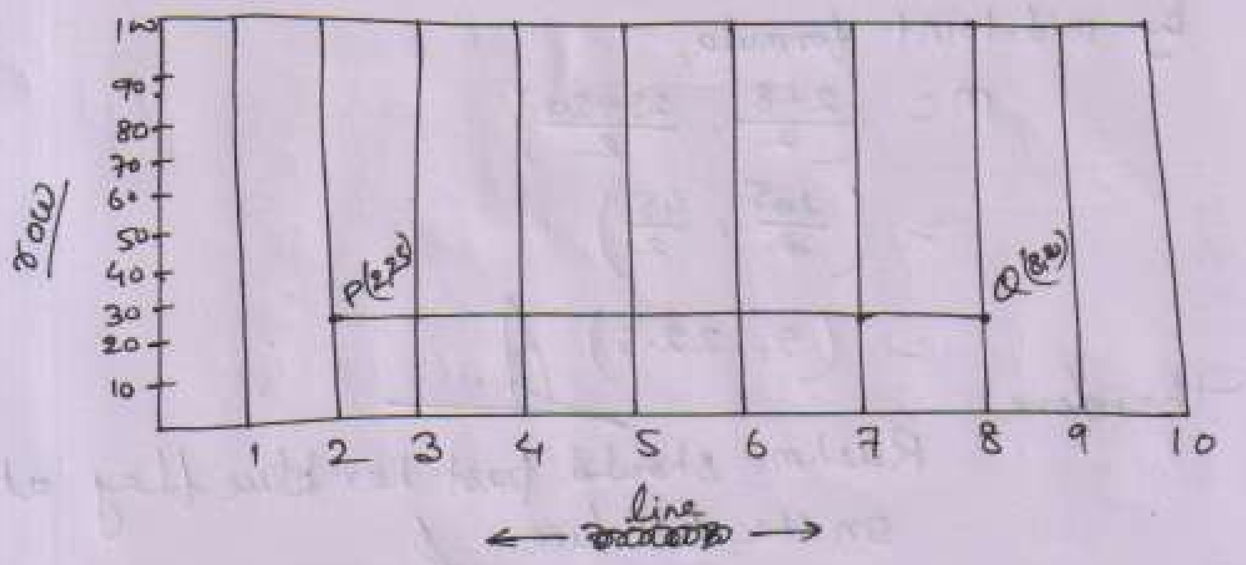
$$\therefore Q = \left(\frac{2\times(-2) + 1\times 4}{2+1}, \frac{2\times(-3) + 1\times(-1)}{2+1}\right)$$

$$= \left(\frac{-4+4}{3}, \frac{-6-1}{3}\right)$$

$$= \left(\frac{0}{3}, \frac{-7}{3}\right)$$

$$\therefore \quad Q = \left(0, \frac{-7}{3}\right) \text{ Ans}$$

3.)



Since,

Niharika posted the green flag $\frac{1}{4}$ of the distance P.
Such that

$\left(\frac{1}{4} \times \overset{25}{\cancel{100}}\right) m = 25\,m$ from the starting point of 2nd line.

$\therefore \quad P = (2, 25)$

and,

Preet posted red flag at $\frac{1}{5}$ of the distance Q
Such that

$\left(\frac{1}{5} \times \overset{20}{\cancel{100}}\right) m = 20\,m$ from the starting point of 8th line.

$\therefore \quad Q = (8, 20)$

by distance formula,

$$PQ = \sqrt{(2-8)^2 + (25-20)^2}$$
$$= \sqrt{(-6)^2 + 5^2}$$
$$= \sqrt{36 + 25}$$
$$= \sqrt{61}$$

Again,
the point at which Rashmi should post her blue flag is the mid-point of the line segment PQ.

$\therefore$ M is the mid-point of PQ.

by mid-point formula,

$$m = \left(\frac{2+8}{2}, \frac{25+20}{2}\right)$$

$$= \left(\frac{10^5}{2}, \frac{45}{2}\right)$$

$$= (5, 22.5)$$

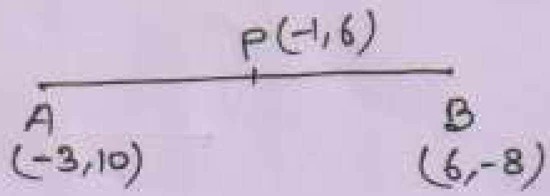
therefore,

Rashmi should post her blue flag at 22.5m on the 5th line.

4) Given that,

$A = (-3, 10)$

$B = (6, -8)$



let required ratio $= K:1$

$\because$ the line segment AB is divided by point P be $K:1$.

$\therefore \quad m:n = K:1$

$\therefore \quad m = K$

$n = 1$

and

$x_1 = -3 \qquad x_2 = 6$

$y_1 = 10 \qquad y_2 = -8$

by section formula,

$$P = \left(\frac{K\times 6 + 1\times(-3)}{K+1}, \frac{K\times(-8) + 1\times 10}{K+1}\right)$$

$$= \left(\frac{6K-3}{K+1}, \frac{-8K+10}{K+1}\right)$$

but, $P = (-1, 6)$

Compare ~~them~~, between both P.

$\therefore \quad \frac{6K-3}{K+1} = -1$ and $\frac{-8K+10}{K+1} = 6$

$\Rightarrow 6K - 3 = -K - 1$

$\Rightarrow 6K + K = -1 + 3$

$\Rightarrow 7K = 2$

$\Rightarrow K = \frac{2}{7}$

$\Rightarrow -8K + 10 = 6K + 6$

$\Rightarrow 10 - 6 = 6K + 8K$

$\Rightarrow 4 = 14K$

$\Rightarrow K = \frac{\cancel{4}^{2}}{\cancel{14}_{7}} = \frac{2}{7}$

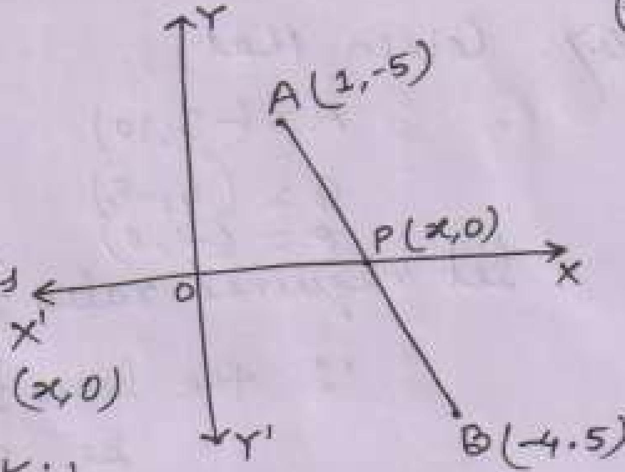
$\therefore$ required ratio $= \left(\frac{2}{7} : 1\right)$

$= \frac{2}{7} \Rightarrow 2:7$

5 Given that —

$A = (1, -5)$

$B = (-4, 5)$



$\because$ P is lie on the x-axis

$\therefore$ co-ordinate of $P = (x, 0)$

let required ratio $= K:1$

$\therefore m:n = K:1$

$m = K$

$n = 1$

and,

$x_1 = 1 \qquad x_2 = -4$

$y_1 = -5 \qquad y_2 = 5$

by section formula,

$$P = \left(\frac{K\times(-4) + 1\times 1}{K+1}, \frac{K\times 5 + 1\times(-5)}{K+1}\right)$$

$$= \left(\frac{-4K+1}{K+1}, \frac{5K-5}{K+1}\right)$$

but,

$P = (x, 0)$

compare both sides,

$$\therefore \frac{5K-5}{K+1} = 0$$

$\Rightarrow 5K - 5 = 0$

$\Rightarrow 5K = 5$

$\Rightarrow K = \frac{\cancel{5}}{\cancel{5}} = 1$

and, $x = \frac{-4K+1}{K+1}$

$= \frac{-4\times 1 + 1}{1+1}$

$= \frac{-4+1}{2}$

$= \frac{-3}{2}$

$\therefore$ required ratio $= K:1$

$= 1:1$

co-ordinate of $P = \left(\frac{-3}{2}, 0\right)$

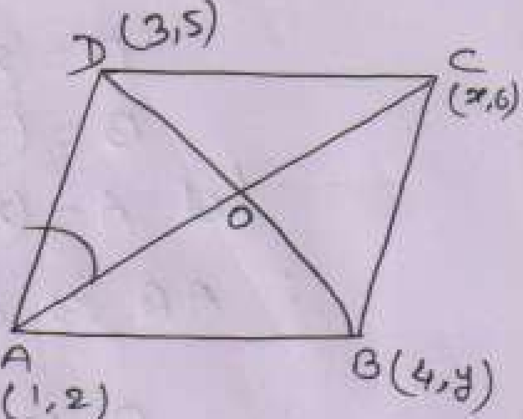
6) In parallelogram ABCD,

$A = (1, 2)$

$B = (4, y)$

$C = (x, 6)$

$D = (3, 5)$



We know that,

the diagonals of $\parallel$gm bisect each other.

So, diagonals AC and BD of $\parallel$gm ABCD bisect each other at the point O.

$\therefore$ O is the mid point of AC, then

$$O = \left(\frac{1+x}{2}, \frac{2+6}{2}\right)$$

$$= \left(\frac{1+x}{2}, \frac{\cancel{8}^{4}}{\cancel{2}}\right)$$

$$= \left(\frac{1+x}{2}, 4\right)$$

Again,

O is the mid point of BD, then

$$\therefore O = \left(\frac{4+3}{2}, \frac{y+5}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{y+5}{2}\right)$$

But, these points coincide at the point O.

$$\therefore \frac{1+x}{\cancel{2}} = \frac{7}{\cancel{2}} \quad \text{and} \quad \frac{y+5}{2} = 4$$

$\Rightarrow 1+x = 7$

$\Rightarrow x = 7-1 = 6$

$\Rightarrow y+5 = 8$

$\Rightarrow y = 8-5 = 3$

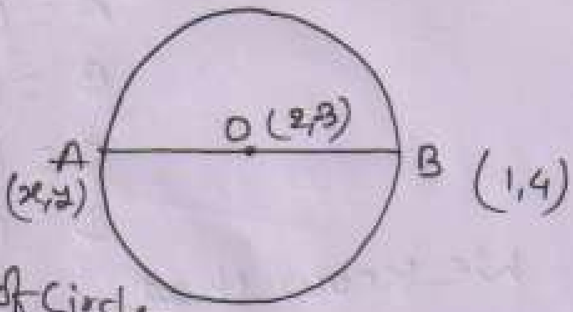
$\therefore x = 6,$
$y = 3$

7) Given that,

$O = (2, -3)$

$B = (1, 4)$

let $A = (x, y)$



$\because$ AB is a diameter of circle.

$\therefore$ O is mid point of AB

by mid point formula,

$$O = \left(\frac{x+1}{2}, \frac{y+4}{2}\right)$$

But,

$O = (2, -3)$

Compare both point O.

$\therefore \frac{x+1}{2} = 2$ and $\frac{y+4}{2} = -3$

$\Rightarrow x + 1 = 4$ $\qquad \Rightarrow y + 4 = -6$

$\Rightarrow x = 4 - 1 = 3$ $\qquad \Rightarrow y = -6 - 4$

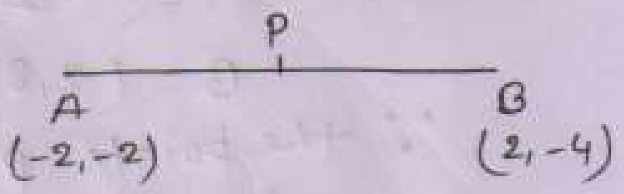
$\Rightarrow y = -10$

$\therefore A = (3, -10)$

8) Given that,

$$A = (-2, -2)$$

$$B = (2, -4)$$



$$AP = \frac{3}{7} AB$$

co-ordinate of P = ?

$\because$ $AP = \frac{3}{7} AB$

$\Rightarrow 7AP = 3AB$

$\Rightarrow 7AP = 3(AP + PB)$

$\Rightarrow 7AP = 3AP + 3PB$

$\Rightarrow 7AP - 3AP = 3PB$

$\Rightarrow 4AP = 3PB$

$\Rightarrow \frac{AP}{PB} = \frac{3}{4}$

$\therefore \Rightarrow AP : PB = 3 : 4$

$\because$ $m : n = 3 : 4$

$m = 3$

$n = 4$

and,

$x_1 = -2 \quad x_2 = 2$

$y_1 = -2 \quad y_2 = -4$

by Section formula,

$$P = \left(\frac{3 \times 2 + 4(-2)}{3+4}, \frac{3 \times (-4) + 4(-2)}{3+4} \right)$$

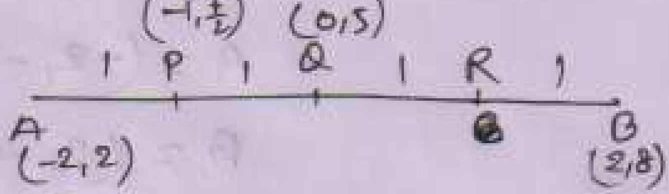
$$= \left(\frac{6 - 8}{7}, \frac{-12 - 8}{7} \right)$$

$$P = \left(-\frac{2}{7}, -\frac{20}{7} \right)$$

9 Given that,

$A = (-2, 2)$

$B = (2, 8)$



$\because$ the points P, Q, R divide the line segment AB into four equal parts.

then, $\therefore$ $AP = PQ = QR = RB$

$\because$ Q is the mid point of AB.

$$\therefore Q = \left(\frac{-2+2}{2}, \frac{2+8}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{\cancel{10}\,5}{\cancel{2}}\right)$$

$$= (0, 5)$$

$\because$ P is the mid-point of AQ

$$\therefore P = \left(\frac{-2+0}{2}, \frac{2+5}{2}\right)$$

$$= \left(\frac{-2}{2}, \frac{7}{2}\right)$$

$$= \left(-1, \frac{7}{2}\right)$$

$\because$ R is the mid point of QB.

$$\therefore R = \left(\frac{0+2}{2}, \frac{5+8}{2}\right)$$

$$= \left(\frac{\cancel{2}}{\cancel{2}}, \frac{13}{2}\right)$$

$$= \left(1, \frac{13}{2}\right)$$

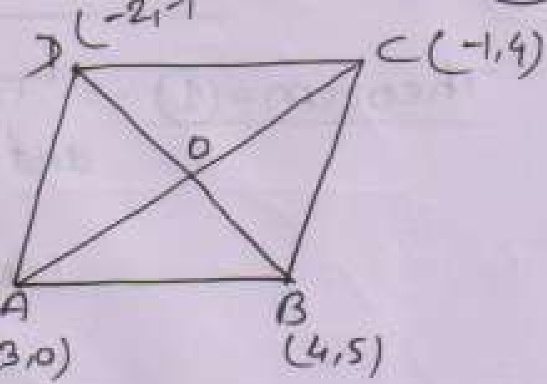
10) In rhombus ABCD,

$A = (3,0)$

$B = (4,5)$

$C = (-1,4)$

$D = (-2,-1)$



∵ AC and BD are the diagonals of rhombus ABCD

∴ by distance formula,

$$AC = \sqrt{[3-(-1)]^2 + (0-4)^2}$$
$$= \sqrt{(3+1)^2 + (-4)^2}$$
$$= \sqrt{4^2 + 16}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$

$$BD = \sqrt{[4-(-2)]^2 + [5-(-1)]^2}$$
$$= \sqrt{(4+2)^2 + (5+1)^2}$$
$$= \sqrt{6^2 + 6^2}$$
$$= \sqrt{36+36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

∴ area of rhombus ABCD $= \frac{1}{2} \times$ Product of diagonals

$$= \frac{1}{2} \times AC \times BD$$
$$= \frac{1}{\cancel{2}} \times \overset{2}{\cancel{4}}\sqrt{2} \times 6\sqrt{2}$$
$$= 12 \times 2$$
$$= 24$$