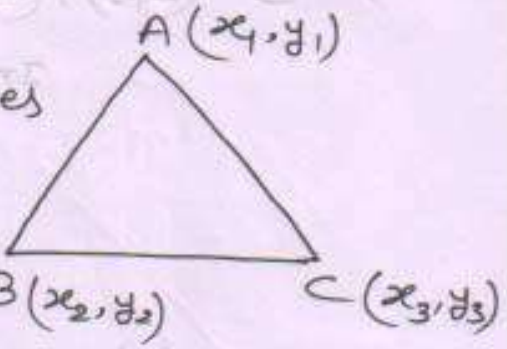
Area of Triangle

* The area of ΔABC with vertices A, B and C. Then

$A = (x_1, y_1)$

$B = (x_2, y_2)$

$C = (x_3, y_3)$

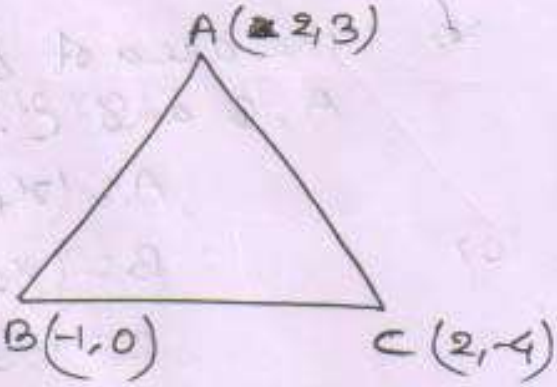


$$\therefore \quad ar(\Delta ABC) = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

* Three points A, B, C are collinear only when area of Δ is 0.

Exercise - 7.3

1.) (i) (2,3), (−1,0), (2,−4)



In ΔABC,

$A = (2,3)$

$B = (-1,0)$

$C = (2,-4)$

let

$x_1 = 2$ $\quad y_1 = 3$

$x_2 = -1$ $\quad y_2 = 0$

$x_3 = 2$ $\quad y_3 = -4$

$\therefore ar(\Delta ABC) = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

$= \frac{1}{2}\left| 2(0+4) + (-1)(-4-3) + 2(3-0) \right|$

$= \frac{1}{2}\left| 2\times 4 - 1\times(-7) + 2\times 3 \right|$

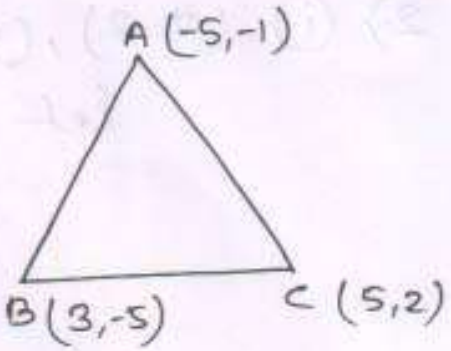
$= \frac{1}{2}\left| 8+7+6 \right|$

$= \frac{1}{2}|21|$

$= \frac{1}{2}\times 21$

$= \frac{21}{2}$ square unit.

1) (ii) $(-5,-1), (3,-5), (5,2)$



In ΔABC,

$A = (-5,-1)$

$B = (3,-5)$

$C = (5,2)$

let $x_1 = -5$, $y_1 = -1$

$x_2 = 3$, $y_2 = -5$

$x_3 = 5$, $y_3 = 2$

area of $\Delta ABC = \frac{1}{2} \left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$

$= \frac{1}{2} \left| -5(-5-2) + 3(2+1) + 5(-1+5) \right|$

$= \frac{1}{2} \left| -5(-7) + 3\times 3 + 5\times 4 \right|$

$= \frac{1}{2} \left| 35 + 9 + 20 \right|$

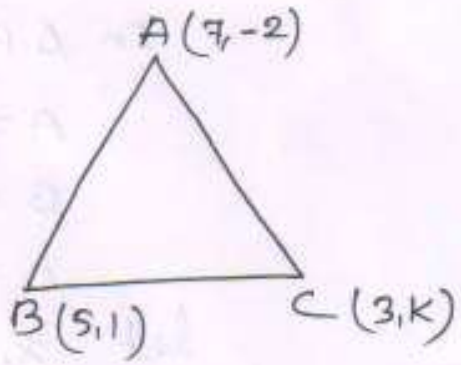
$= \frac{1}{2} |64|$

$= \frac{1}{2} \times 64^{32}$

$= 32$ Square unit

Q2.

2.) (i) $(7,-2), (5,1), (3,K)$



let In ΔABC,

$A = (7,-2)$

$B = (5,1)$

$C = (3,K$

and

$x_1 = 7$, $y_1 = -2$

$x_2 = 5$, $y_2 = 1$

$x_3 = 3$, $y_3 = K$

area of $\Delta ABC = \frac{1}{2} | x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) |$

$= \frac{1}{2} | 7(1-K) + 5(K+2) + 3(-2-1) |$

$= \frac{1}{2} | 7 - 7K + 5K + 10 + 3x(-3) |$

$= \frac{1}{2} | 7 - 2K + 10 - 9 |$

$= \frac{1}{2} | 8 - 2K |$

$= \frac{1}{2} x (8-2K)$

$= \frac{1}{2} x 2(4-K)$

$= 4-K$

$\because$ the point are collinear. then

area of $\Delta ABC = 0$

$\therefore 4-K = 0$

$\Rightarrow 4 = K$

$\Rightarrow K = 4$

2) (ii) $(8,1), (K,-4), (2,-5)$

let In ΔABC,

$A = (8,1)$

$B = (K,-4)$

$C = (2,-5)$

and,

$x_1 = 8 \quad y_1 = 1$

$x_2 = K \quad y_2 = -4$

$x_3 = 2 \quad y_2 = -5$

$$ar(\Delta ABC) = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

$$= \frac{1}{2}\left| 8(-4+5) + K(-5-1) + 2(1+4) \right|$$

$$= \frac{1}{2}\left| 8\times 1 + K(-6) + 2\times 5 \right|$$

$$= \frac{1}{2}\left| 8 - 6K + 10 \right|$$

$$= \frac{1}{2}\left| 18 - 6K \right|$$

$$= \frac{1}{2} \times (18-6K)$$

$$= \frac{1}{2} \times 2(9-3K)$$

$$= 9-3K$$

∵ the points are collinear. Then,

$ar(\Delta ABC) = 0$

$\therefore 9-3K = 0$

$\Rightarrow 9 = 3K$

$\Rightarrow K = \frac{9}{3}$

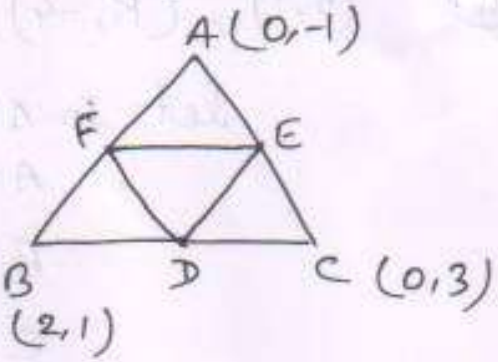
$\therefore K = 3$

3.) In ΔABC,

$A = (0,-1)$

$B = (2,1)$

$C = (0,3)$



let $x_1 = 0$ $y_1 = -1$

$x_2 = 2$ $y_2 = 1$

$x_3 = 0$ $y_3 = 3$

$$ar(\Delta ABC) = \frac{1}{2}\left| x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2) \right|$$

$$= \frac{1}{2}\left| 0(1-3) + 2(3+1) + 0(-1-1) \right|$$

$$= \frac{1}{2}\left| 0 + 2\times 4 + 0 \right|$$

$$= \frac{1}{2}|8|$$

$$= \frac{1}{2}\times 8^{4}$$

$$= 4$$

Now,

$\because$ D, E, F are the midpoint of sides BC, AC, AB of ΔABC respectively.

$\therefore$ D is the mid point of BC.

$$\therefore D = \left(\frac{2+0}{2}, \frac{1+3}{2}\right)$$

$$= \left(\frac{2}{2}, \frac{4^{2}}{2}\right)$$

$$D = (1, 2)$$

and, E is the mid point of AC.

$$\therefore E = \left(\frac{0+0}{2}, \frac{-1+3}{2}\right)$$

$$= \left(\frac{0}{2}, \frac{2}{2}\right)$$

$$E = (0, 1)$$

and, F is the mid point of AB.

$\therefore F = \left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$

$= \left(\frac{2}{2}, \frac{0}{2}\right)$

$F = (1, 0)$

Now,

In ΔDEF,

$D = (1, 2)$

$E = (0, 1)$

$F = (1, 0)$

let $x_1 = 1 \quad y_1 = 2$

$x_2 = 0 \quad y_2 = 1$

$x_3 = 1 \quad y_3 = 0$

$ar(\Delta DEF) = \frac{1}{2} \left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$

$= \frac{1}{2} \left| 1(1-0) + 0(0-2) + 1(2-1) \right|$

$= \frac{1}{2} \left| 1 \times 1 + 0 + 1 \times 1 \right|$

$= \frac{1}{2} \left| 1 + 0 + 1 \right|$

$= \frac{1}{2} |2|$

$= \frac{1}{2} \times 2$

$= 1$

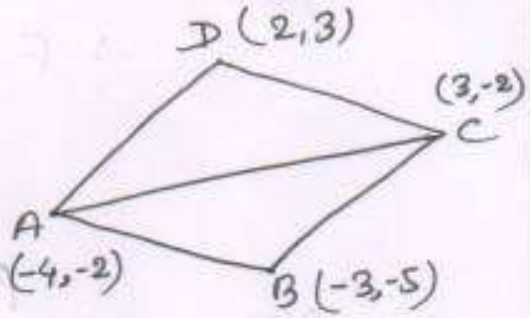
$\therefore$ ratio $= \frac{ar(\Delta DEF)}{ar(\Delta ABC}$

$= \frac{1}{4}$

$= 1:4$

4) let,

In quadrilateral ABCD,



$A = (-4,-2)$

$B = (-3,-5)$

$C = (3,-2)$

$D = (2,3)$

∴ join AC.

for ΔABC,

$A = (-4,-2)$

$B = (-3,-5)$

$C = (3,-2)$

Here,

$x_1 = -4 \quad y_1 = -2$.

$x_2 = -3 \quad y_2 = -5$

$x_3 = 3 \quad y_3 = -2$

$$ar(\Delta ABC) = \frac{1}{2}\left|x_1(y_2-y_3) + x_2(y_3-y_1) + x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|-4(-5+2) + (-3)(-2+2) + 3(-2+5)\right|$$

$$= \frac{1}{2}\left|-4(-3) + (-3)\times 0 + 3\times 3\right|$$

$$= \frac{1}{2}\left|12 + 0 + 9\right|$$

$$= \frac{1}{2}\left|21\right|$$

$$= \frac{1}{2}\times 21$$

$$= \frac{21}{2}$$

for ΔACD,

$A = (-4, -2)$

$C = (3, -2)$

$D = (2, 3)$

Here,

$x_1 = -4 \quad y_1 = -2$

$x_2 = 3 \quad y_2 = -2$

$x_3 = 2 \quad y_3 = 3$

$$ar(\Delta ACD) = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|-4(-2-3)+3(3+2)+2(-2+2)\right|$$

$$= \frac{1}{2}\left|-4(-5)+3\times5+2\times0\right|$$

$$= \frac{1}{2}\left|20+15+0\right|$$

$$= \frac{1}{2}\left|35\right|$$

$$= \frac{1}{2}\times35$$

$$= \frac{35}{2}$$

$\therefore ar(\text{quad. } ABCD) = ar(\Delta ABC) + ar(\Delta ACD)$

$$= \frac{21}{2}+\frac{35}{2}$$

$$= \frac{21+35}{2}$$

$$= \frac{56}{2}$$

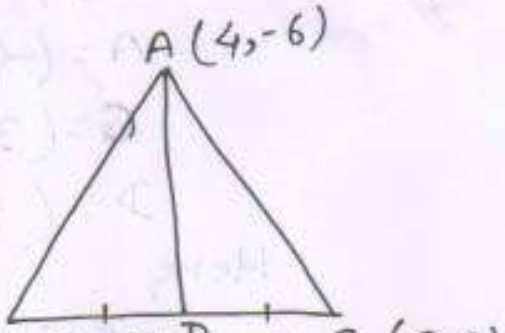
$$= 28 \text{ Square unit}$$

5) Let In ΔABC,

$A = (4, -6)$

$B = (3, -2)$

$C = (5, 2)$



$\because$ D is the mid point of BC.

$\therefore D = \left(\frac{3+5}{2}, \frac{-2+2}{2}\right)$

$= \left(\frac{8}{2}, \frac{0}{2}\right)$

$D = (4, 0)$

In ΔABD,

$A = (4, -6)$ and $x_1 = 4$ $y_1 = -6$

$B = (3, -2)$ $x_2 = 3$ $y_2 = -2$

$D = (4, 0)$ $x_3 = 4$ $y_3 = 0$

$ar(\Delta ABD) = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \frac{1}{2}\left|4(-2-0) + 3(0+6) + 4(-6+2)\right|$

$= \frac{1}{2}\left|4(-2) + 3 \times 6 + 4(-4)\right|$

$= \frac{1}{2}\left|-8 + 18 - 16\right|$

$= \frac{1}{2}\left|-24 + 18\right|$

$= \frac{1}{2}|-6|$

$= \frac{1}{2} \times 6$

$= 3$ square unit

5) In ΔACD,

$A = 4, -6$ and $x_1 = 4$ $y_1 = -6$

$C = 5, 2$ $x_2 = 5$ $y_2 = 2$

$D = 4, 0$ $x_3 = 4$ $y_3 = 0$

$$ar(\Delta ACD) = \frac{1}{2}\left| x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) \right|$$

$$= \frac{1}{2}\left| 4(2-0) + 5(0+6) + 4(-6-2) \right|$$

$$= \frac{1}{2}\left| 4 \times 2 + 5 \times 6 + 4(-8) \right|$$

$$= \frac{1}{2}\left| 8 + 30 - 32 \right|$$

$$= \frac{1}{2}\left| 38 - 32 \right|$$

$$= \frac{1}{2} * 6|$$

$$= \frac{1}{2} \times 6^3$$

$$= 3 \text{ Square unit}$$

$\therefore$ $ar(\Delta ABD) = ar(\Delta ACD)$

Proved