Exercise - 8.4

1.) $\sin A = \frac{1}{\operatorname{cosec} A} = \frac{1}{\sqrt{1+\cot^2 A}}$ Ans

$\sec A = \sqrt{1+\tan^2 A} = \sqrt{1+\frac{1}{\cot^2 A}}$

$= \sqrt{\frac{\cot^2 A+1}{\cot^2 A}}$

$= \frac{\sqrt{\cot^2 A+1}}{\cot A}$

$\tan A = \frac{1}{\cot A}$ A

2.) $\sin A = \sqrt{1-\cos^2 A}$

$= \sqrt{1-\frac{1}{\sec^2 A}}$

$= \sqrt{\frac{\sec^2 A-1}{\sec^2 A}}$

$= \frac{\sqrt{\sec^2 A-1}}{\sec A}$

$\cos A = \frac{1}{\sec A}$

$\tan A = \sqrt{\sec^2 A-1}$

$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{1}{\frac{\sqrt{\sec^2 A-1}}{\sec A}} = \frac{\sec A}{\sqrt{\sec^2 A-1}}$ A

$\cot A = \frac{1}{\tan A} = \frac{1}{\sqrt{\sec^2 A-1}}$ A

3.) (i) $\dfrac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ}$

$= \dfrac{\sin^2 63^\circ + \sin^2(90^\circ - 63^\circ)}{\cos^2(90^\circ - 73^\circ) + \cos^2 73^\circ}$

$= \dfrac{\sin^2 63^\circ + \cos^2 63^\circ}{\sin^2 73^\circ + \cos^2 73^\circ}$

$= \dfrac{\cancel{1}}{\cancel{1}} \quad [\sin^2\theta + \cos^2\theta]$

$= 1$ Ans

(ii) $\sin 25^\circ \cos 65^\circ + \cos 25^\circ \sin 65^\circ$

$= \sin 25^\circ \cos(90^\circ - 25^\circ) + \cos 25^\circ \cdot \sin(90^\circ - 25^\circ)$

$= \sin 25^\circ \cdot \sin 25^\circ + \cos 25^\circ \cdot \cos 25^\circ$

$= \sin^2 25^\circ + \cos^2 25^\circ$

$= 1 \quad [\sin^2\theta + \cos^2\theta]$

Ans

4) (i) $9\sec^2 A - 9\tan^2 A$

$= 9(\sec^2 A - \tan^2 A)$

$= 9 \times 1 \quad [\sec^2\theta - \tan^2\theta]$

$= 9$

(B) Ans

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

$= \left(1+\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta}\right)\left(1+\frac{\cos\theta}{\sin\theta}-\frac{1}{\sin\theta}\right)$

$= \left(\frac{\cos\theta+\sin\theta+1}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$

$= \frac{(\sin\theta+\cos\theta)+1}{\cos\theta} \times \frac{(\sin\theta+\cos\theta)-1}{\sin\theta}$

$= \frac{(\sin\theta+\cos\theta)^2-1^2}{\sin\theta\cdot\cos\theta}$

$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cdot\cos\theta-1}{\sin\theta\cdot\cos\theta}$

$= \frac{\cancel{1}+2\sin\theta\cdot\cos\theta-\cancel{1}}{\sin\theta\cdot\cos\theta}$

$= \frac{2\cancel{\sin\theta}\cdot\cancel{\cos\theta}}{\cancel{\sin\theta}\cdot\cancel{\cos\theta}}$

$= 2$ (C) Ans

(iii) $(\sec A + \tan A)(1 - \sin A)$

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1^2 - \sin^2 A}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos A}$$

$$= \frac{\cos^2 A}{\cos A}$$

$$= \frac{\cos A \cdot \cancel{\cos A}}{\cancel{\cos A}}$$

$$= \cos A$$ D

(iv) $\dfrac{1 + \tan^2 A}{1 + \cot^2 A}$

$$= \frac{\sec^2 A}{\operatorname{cosec}^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$ D

5) (i) $\left(\operatorname{cosec}\theta - \cot\theta\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

L.H.S,

$\left(\operatorname{cosec}\theta - \cot\theta\right)^2$

$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$

$= \left(\frac{1-\cos\theta}{\sin\theta}\right)^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{1-\cos^2\theta}$

$= \frac{\cancel{(1-\cos\theta)}(1-\cos\theta)}{\cancel{(1-\cos\theta)}(1+\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta} = R.H.S$

सिद्ध

(ii) $\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=2\sec A$

L.H.S,

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}$$

$$=\frac{\cos^2 A+(1+\sin A)^2}{(1+\sin A)\cos A}$$

$$=\frac{\cos^2 A+1+\sin^2 A+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{\sin^2 A+\cos^2 A+1+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{1+1+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{2+2\sin A}{(1+\sin A)\cos A}$$

$$=\frac{2\cancel{(1+\sin A)}}{\cancel{(1+\sin A)}\cos A}$$

$$=\frac{2}{\cos A}$$

$$=2\times\frac{1}{\cos A}$$

$$=2\sec A$$

R.H.S

सिद्ध

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\,\mathrm{cosec}\,\theta$

L.H.S,

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta-\cos\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta-\sin\theta}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta} + \frac{\cos^2\theta}{-\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$

$$= \frac{\sin^3\theta - \cos^3\theta}{\sin\theta\cdot\cos\theta\,(\sin\theta-\cos\theta)}$$

$$= \frac{\cancel{(\sin\theta-\cos\theta)}\,(\sin^2\theta+\cos^2\theta+\sin\theta\cdot\cos\theta)}{\sin\theta\cdot\cos\theta\,\cancel{(\sin\theta-\cos\theta)}}$$

$$= \frac{1+\sin\theta\cdot\cos\theta}{\sin\theta\cdot\cos\theta}$$

$$= \frac{1}{\sin\theta\cdot\cos\theta} + \frac{\cancel{\sin\theta}\cdot\cancel{\cos\theta}}{\cancel{\sin\theta}\cdot\cancel{\cos\theta}}$$

$$= \mathrm{cosec}\,\theta\cdot\sec\theta + 1$$

$$= 1+\sec\theta\cdot\mathrm{cosec}\,\theta = R.H.S$$ सिद्ध

(iv) $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos A}$

L.H.S,

$$\frac{1+\sec A}{\sec A} = \frac{1+\frac{1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\frac{\cos A+1}{\cos A}}{\frac{1}{\cos A}}$$

$$= \frac{\cos A+1}{\cancel{\cos A}} \times \frac{\cancel{\cos A}}{1}$$

$$= 1+\cos A$$

R.H.S,

$$\frac{\sin^2 A}{1-\cos A} = \frac{1-\cos^2 A}{1-\cos A}$$

$$= \frac{\cancel{(1-\cos A)}(1+\cos A)}{\cancel{(1-\cos A)}}$$

$$= 1+\cos A$$

$\therefore$ L.H.S = R.H.S

Proved

(V) $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A$

L.H.S,

$$\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$$

अंश तथा हर में $\sin A$ से भाग देने पर

$$= \frac{\frac{\cos A}{\sin A} - \frac{\cancel{\sin A}}{\cancel{\sin A}} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\cancel{\sin A}}{\cancel{\sin A}} - \frac{1}{\sin A}}$$

$$= \frac{\cot A - 1 + \operatorname{cosec} A}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - 1}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec}^2 A - \cot^2 A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A) - (\operatorname{cosec} A \; \cot A)(\operatorname{cosec} A - \cot A)}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\left[1 - (\operatorname{cosec} A - \cot A)\right]}{\cot A + 1 - \operatorname{cosec} A}$$

$$= \frac{(\operatorname{cosec} A + \cot A)\cancel{(1 - \operatorname{cosec} A + \cot A)}}{\cancel{(1 - \operatorname{cosec} A + \cot A)}}$$

$= \operatorname{cosec} A + \cot A$

$=$ R.H.S

(vi) $\sqrt{\dfrac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S,

$$\sqrt{\frac{1+\sin A}{1-\sin A}} = \sqrt{\frac{1+\sin A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{1-\sin^2 A}}$$

$$= \sqrt{\frac{(1+\sin A)(1+\sin A)}{\cos^2 A}}$$

$$= \frac{1+\sin A}{\cos A}$$

$$= \frac{1}{\cos A} + \frac{\sin A}{\cos A}$$

$$= \sec A + \tan A$$

$$= \text{R.H.S}$$

प्रमाणित

(vii) $\dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$

L.H.S,

$$\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \frac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$$

$$= \tan\theta \times \frac{1-2\sin^2\theta}{2(1-\sin^2\theta)-1}$$

$$= \tan\theta \times \frac{1-2\sin^2\theta}{2-2\sin^2\theta-1}$$

$$= \tan\theta \times \frac{\cancel{(1-2\sin^2\theta)}}{\cancel{(1-2\sin^2\theta)}}$$

$$= \tan\theta$$

$$= \text{R.H.S}$$

प्रमाणित

(viii) $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

L.H.S,

$(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2$

$= \sin^2 A + \operatorname{cosec}^2 A + 2\sin A \cdot \operatorname{cosec} A + \cos^2 A + \sec^2 A + 2 \cdot \cos A \cdot \sec A$

$= \sin^2 A + \cos^2 A + 1 + \cot^2 A + 2 \times \cancel{\sin A} \times \frac{1}{\cancel{\sin A}} + 1 + \tan^2 A + 2 \times \cancel{\cos A} \times \frac{1}{\cancel{\cos A}}$

$= 1 + 1 + \cot^2 A + 2 + 1 + \tan^2 A + 2$

$= 7 + \tan^2 A + \cot^2 A$

$=$ R.H.S

सिद्ध

(IX) $(\operatorname{cosec} A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

L.H.S,

$(\operatorname{cosec} A - \sin A)(\sec A - \cos A)$

$= \left(\frac{1}{\sin A} - \sin A\right)\left(\frac{1}{\cos A} - \cos A\right)$

$= \left(\frac{1 - \sin^2 A}{\sin A}\right)\left(\frac{1 - \cos^2 A}{\cos A}\right)$

$= \frac{\cos^2 A}{\sin A} \times \frac{\sin^2 A}{\cos A}$

$= \frac{\cos A \cdot \cancel{\cos A}}{\cancel{\sin A}} \times \frac{\cancel{\sin A} \cdot \sin A}{\cancel{\cos A}}$

$= \sin A \cdot \cos A$

R.H.S,

$$\frac{1}{\tan A + \cot A}$$

$$= \frac{1}{\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}}$$

$$= \frac{1}{\frac{\sin^2 A + \cos^2 A}{\sin A \cdot \cos A}}$$

$$= \frac{1}{\frac{1}{\sin A \cdot \cos A}}$$

$$= \frac{\sin A \cdot \cos A}{1}$$

$$= \sin A \cdot \cos A$$

$\therefore$ L.H.S = R.H.S

सिद्ध

(x) $$\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$$

L.H.S,

$$\frac{1+\tan^2 A}{1+\cot^2 A} = \frac{\sec^2 A}{\csc^2 A}$$

$$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{1}{\cos^2 A} \times \frac{\sin^2 A}{1}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$= R.H.S$$

M.H.S,

$$\left(\frac{1-\tan A}{1-\cot A}\right)^2 = \left(\frac{1-\frac{\sin A}{\cos A}}{1-\frac{\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\frac{\cos A-\sin A}{\cos A}}{\frac{\sin A-\cos A}{\sin A}}\right)^2$$

$$= \left(\frac{\cos A-\sin A}{\cos A} \times \frac{\sin A}{\sin A-\cos A}\right)^2$$

$$= \frac{(\cos A-\sin A)^2 \cdot \sin^2 A}{\cos^2 A \cdot (\sin A-\cos A)^2}$$

$$= \frac{\cancel{(\sin A-\cos A)^2} \cdot \sin^2 A}{\cos^2 A \cdot \cancel{(\sin A-\cos A)^2}}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$= R.H.S$$

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