Coordinate Geometry

* The distance between the points A and B.

Here,

$$A = (x_1, y_1)$$
$$B = (x_2, y_2)$$
$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$\text{or } \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

* The distance between of the points $P(x, y)$ and origin $O(0,0)$ is given then

$$OP = \sqrt{x^2 + y^2}$$

* Properties of Various types of Quadrilaterals –

A quadrilateral is —

(i) rectangle if its opposite sides are equal and the diagonals are equal.

(ii) Square if all sides are equal and the diagonals are equal.

(iii) Parallelogram if its opposite sides are equal.

(iv) rhombus if its all its sides are equal and diagonals are equal.

* <u>Collinear points:-</u> Three points A, B, C are said to be collinear if they lie on the same straight line.

So,

$$AC + BC = AB$$

Hence,
A, B, C are collinear

Exercise - 7.1

1.) (i) (2,3), (4,1)

let $A = (2,3)$

$B = (4,1)$

by distance formula,

$$AB = \sqrt{(2-4)^2 + (3-1)^2}$$
$$= \sqrt{(-2)^2 + 2^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2} \text{ units}$$

(ii) (-5,7), (-1,3)

let $A = (-5,7)$

$B = (4,1)$

by distance formula,

$$AB = \sqrt{(-5-4)^2 + (7-1)^2}$$
$$= \sqrt{(-9)^2 + 6^2}$$
$$= \sqrt{81+36}$$
$$= \sqrt{117} \text{ units}$$

1.> (iii) $(a, b), (-a, -b)$

let $A = a, b$

$B = -a, -b$

by distance formula,

$$AB = \sqrt{[a-(-a)]^2 + [b-(-b)]^2}$$

$$= \sqrt{(a+a)^2 + (b+b)^2}$$

$$= \sqrt{(2a)^2 + (2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= \sqrt{4(a^2+b^2)}$$

$$= 2\sqrt{a^2+b^2} \text{ units}$$

2.> let $O = (0,0)$

$A = (36, 15)$

by distance formula,

$$OA = \sqrt{(36-0)^2 + (15-0)^2}$$

$$= \sqrt{(36)^2 + (15)^2}$$

$$= \sqrt{1296 + 225}$$

$$= \sqrt{1521}$$

$$= 39 \text{ unit}$$

5.

3) let A, B, C are three points

So, $A = (1,5)$

$B = (2,3)$

$C = (-2,-11)$

by distance formula,

$AB = \sqrt{(1-2)^2 + (5-3)^2}$

$= \sqrt{(-1)^2 + 2^2}$

$= \sqrt{1+4}$

$= \sqrt{5}$

$BC = \sqrt{[2-(-2)]^2 + [3-(-11)]^2}$

$= \sqrt{(2+2)^2 + (3+11)^2}$

$= \sqrt{4^2 + 14^2}$

$= \sqrt{16+196}$

$= \sqrt{212}$

$AC = \sqrt{[1-(-2]^2 + [5-(-11)]^2}$

$= \sqrt{(1+2)^2 + (5+11)^2}$

$= \sqrt{3^2 + 16^2}$

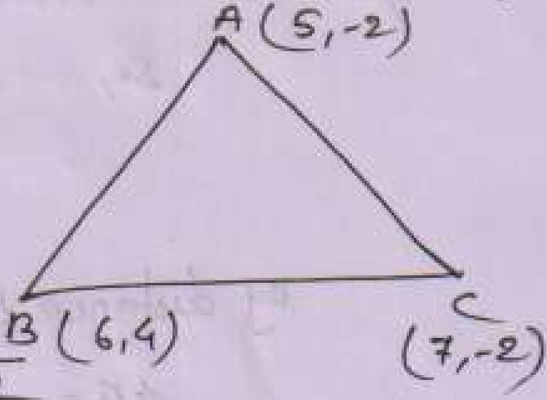
$= \sqrt{9+256}$

$= \sqrt{265}$

$\therefore AB + BC \neq AC$

$\therefore$ A,B, C are not in collinear.

4.) let A, B, C are the vertices of an isosceles triangle.



$A = (5,-2)$

$B = (6,4)$

$C = (7,-2)$

by distance formula,

$AB = \sqrt{(5-6)^2 + (-2-4)^2}$

$= \sqrt{(-1)^2 + (-6)^2}$

$= \sqrt{1 + 36}$

$= \sqrt{37}$

$BC = \sqrt{(6-7)^2 + [4-(-2)]^2}$

$= \sqrt{(-1)^2 + (4+2)^2}$

$= \sqrt{1 + 6^2}$

$= \sqrt{1+36}$

$= \sqrt{37}$

$AC = \sqrt{(5-7)^2 + [-2-(-2)]^2}$

$= \sqrt{(-2)^2 + (-2+2)^2}$

$= \sqrt{4+0}$

$= \sqrt{4}$

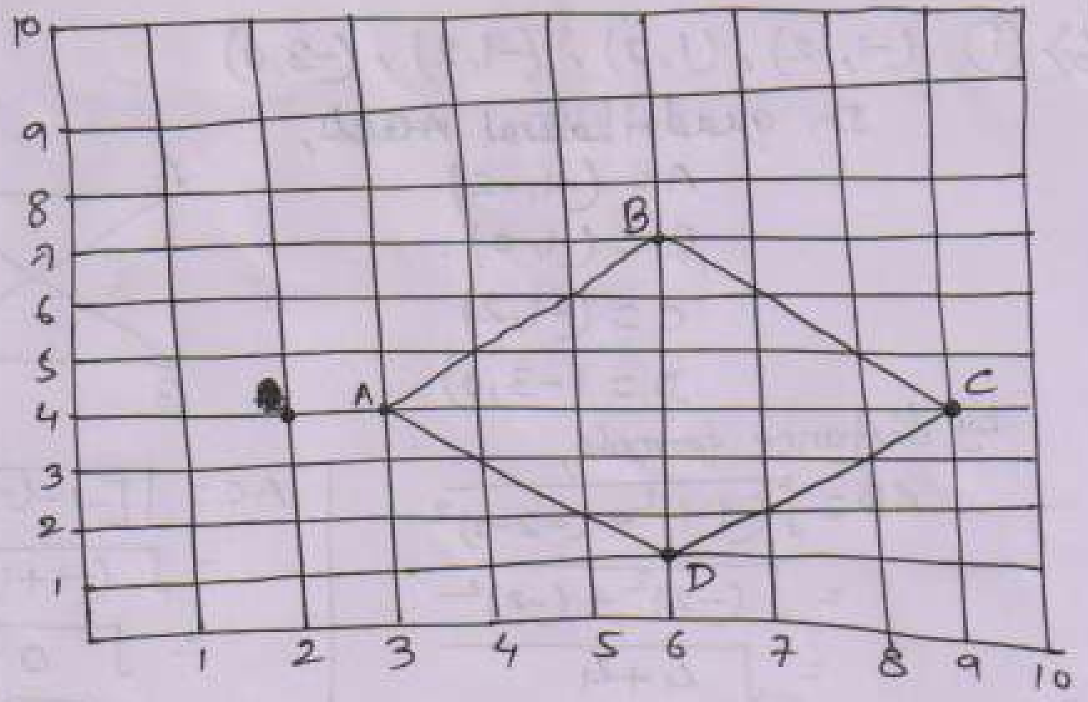
$= 2$

$\therefore AB = AC.$

$\therefore$ ΔABC is an isosceles triangle.

$\therefore$ A, B, C are the vertices of A isosceles Δ. R

5>



let $A = (3,4)$

$B = (6,7)$

$C = (9,4)$

$D = (6,1)$

by distance formula,

$AB = \sqrt{(3-6)^2 + (4-7)^2}$

$= \sqrt{(-3)^2 + (-3)^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$BC = \sqrt{(6-9)^2 + (7-4)^2}$

$= \sqrt{(-3)^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$CD = \sqrt{(9-6)^2 + (4-1)^2}$

$= \sqrt{3^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$AD = \sqrt{(3-6)^2 + (4-1)^2}$

$= \sqrt{(-3)^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Since,

$AB = BC = CD = AD$

So,

ABCD is a square.

$\therefore$ Champa is correct.

6.> (i) $(-1,-2), (1,0), (-1,2), (-3,0)$

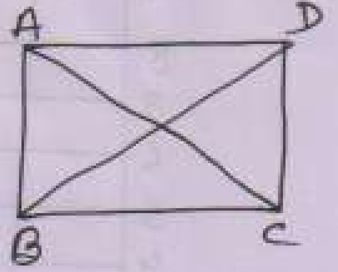
In quadrilateral ABCD,

$A = (-1,-2)$

$B = (1,0)$

$C = (-1,2)$

$D = (-3,0)$



by distance formula,

$$AB = \sqrt{(-1-1)^2 + (-2-0)^2}$$
$$= \sqrt{(-2)^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$$BC = \sqrt{[1-(-1)]^2 + (0-2)^2}$$
$$= \sqrt{(1+1)^2 + (-2)^2}$$
$$= \sqrt{2^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$$CD = \sqrt{[-1-(-3)]^2 + (2-0)^2}$$
$$= \sqrt{(-1+3)^2 + 2^2}$$
$$= \sqrt{2^2 + 2^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$$AD = \sqrt{[-1-(-3)]^2 + [-2-0]^2}$$
$$= \sqrt{(-1+3)^2 + (-2)^2}$$
$$= \sqrt{2^2 + (-2)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$$AC = \sqrt{[-1-(-1)]^2 + (-2-2)^2}$$
$$= \sqrt{(-1+1)^2 + (-4)^2}$$
$$= \sqrt{0+16}$$
$$= \sqrt{16}$$
$$= 4$$

$$BD = \sqrt{[1-(-3)]^2 + (0-0)^2}$$
$$= \sqrt{(1+3)^2 + 0}$$
$$= \sqrt{4^2}$$
$$= \sqrt{16}$$
$$= 4$$

Since,

$AB = BC = CD = AD$

and $AC = BD$

So, ABCD is a square.

6> (ii) $(-3,5), (3,1), (0,3), (-1,-4)$

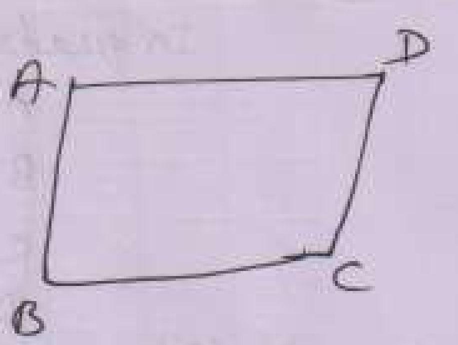
In quadrilateral ABCD,

$A = (-3,5)$

$B = (3,1)$

$C = (0,3)$

$D = (-1,-4)$



by distance formula,

$$AB = \sqrt{(-3-3)^2 + (5-1)^2}$$
$$= \sqrt{(-6)^2 + (4)^2}$$
$$= \sqrt{36+16}$$
$$= \sqrt{52}$$

$$BC = \sqrt{(3-0)^2 + (1-3)^2}$$
$$= \sqrt{3^2 + (-2)^2}$$
$$= \sqrt{9+4}$$
$$= \sqrt{13}$$

$$CD = \sqrt{[0-(-1)]^2 + [3-(-4)]^2}$$
$$= \sqrt{(0+1)^2 + (3+4)^2}$$
$$= \sqrt{1^2 + 7^2}$$
$$= \sqrt{1+49}$$
$$= \sqrt{50}$$

$$AD = \sqrt{[-3-(-1)]^2 + [5-(-4)]^2}$$
$$= \sqrt{(-3+1)^2 + (5+4)^2}$$
$$= \sqrt{(-2)^2 + 9^2}$$
$$= \sqrt{4+81}$$
$$= \sqrt{85}$$

Since,

$AB \neq BC \neq CD \neq AD$

$\therefore$ It is not quadrilateral.

6) (iii) (4,5), (7,6), (4,3), (1,2)

In quadrilateral ABCD,

$A = (4,5)$

$B = (7,6)$

$C = (4,3)$

$D = (1,2)$

by distance formula,

$AB = \sqrt{(4-7)^2 + (5-6)^2}$

$= \sqrt{(-3)^2 + (-1)^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$BC = \sqrt{(7-4)^2 + (6-3)^2}$

$= \sqrt{3^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

$CD = \sqrt{(4-1)^2 + (3-2)^2}$

$= \sqrt{3^2 + 1^2}$

$= \sqrt{9+1}$

$= \sqrt{10}$

$AD = \sqrt{(4-1)^2 + (5-2)^2}$

$= \sqrt{3^2 + 3^2}$

$= \sqrt{9+9}$

$= \sqrt{18}$

$= 3\sqrt{2}$

Since,

$AB = CD = \sqrt{10}$

$BC = AD = 3\sqrt{2}$

$\therefore$ ABCD is parallelogram.

7.) let the point P on the x-axis.

$\therefore \quad P = (a, 0)$

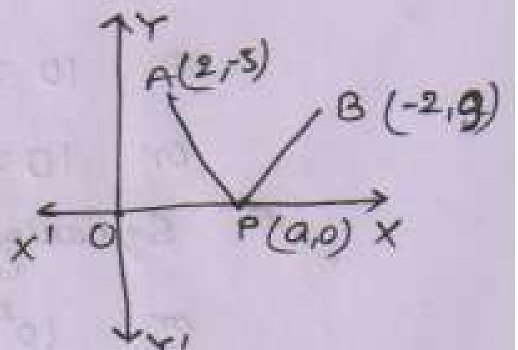
and

$A = (2, -5)$

$B = (-2, 9)$

Since,

the point A and B is equidistance from the point P on the x-axis.



$\because \quad PA = PB \quad — ①$

$\therefore \quad PA = \sqrt{(a-2)^2 + [0-(-5)]^2}$

$= \sqrt{a^2 - 4a + 4 + (0+5)^2}$

$= \sqrt{a^2 - 4a + 4 + 5^2}$

$= \sqrt{a^2 - 4a + 4 + 25}$

$= \sqrt{a^2 - 4a + 29}$

$PB = \sqrt{[a-(-2)]^2 + (0-9)^2}$

$= \sqrt{(a+2)^2 + (-9)^2}$

$= \sqrt{a^2 + 4a + 4 + 81}$

$= \sqrt{a^2 + 4a + 85}$

from equ^n ①, we get

$PA = PB$

$\Rightarrow \sqrt{a^2 - 4a + 29} = \sqrt{a^2 + 4a + 85}$

Squaring both sides

$\Rightarrow \left(\sqrt{a^2 - 4a + 29}\right)^2 = \left(\sqrt{a^2 + 4a + 85}\right)^2$

$\Rightarrow \cancel{a^2} - 4a + 29 = \cancel{a^2} + 4a + 85$

$\Rightarrow -4a - 4a = 85 - 29$

$\Rightarrow -8a = 56$

$\Rightarrow a = \frac{\cancel{56}\,7}{\cancel{-8}}$

$\Rightarrow a = -7$

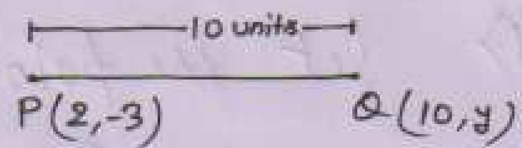
$\therefore$ Co-ordinate of $P = (-7, 0)$

8) Given that :-

$P = (2, -3)$

$Q = (10, y)$

$PQ = 10$



by distance formula,

$PQ = \sqrt{(2-10)^2 + (-3-y)^2}$

or $10 = \sqrt{(-8)^2 + [-(3+y)]^2}$

or $10 = \sqrt{64 + (3+y)^2}$

or $10 = \sqrt{64 + 9 + 6y + y^2}$

or $10 = \sqrt{y^2 + 6y + 73}$

Squaring both sides,

or $10^2 = \left(\sqrt{y^2+6y+73}\right)^2$

or $100 = y^2 + 6y + 73$

or $y^2 + 6y + 73 = 100$

or $y^2 + 6y + 73 - 100 = 0$

or $y^2 + 6y - 27 = 0$

or $y^2 + 9y - 3y - 27 = 0$

or $y(y+9) - 3(y+9) = 0$

or $(y-3)(y+9) = 0$

or $y - 3 = 0$ or $y + 9 = 0$

or $y = 3$ $\Rightarrow y = -9$

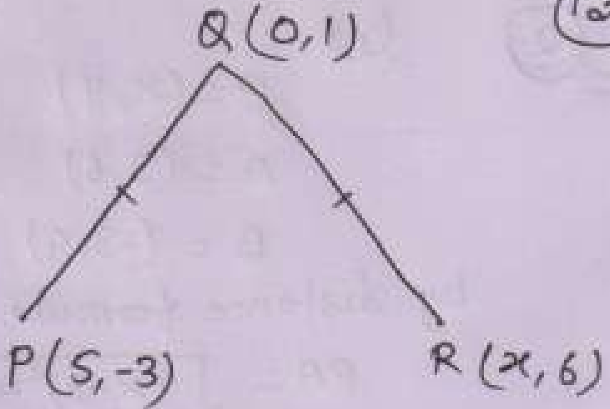
$\therefore y = 3, -9$ Ans

9.) Given that:-

$Q = (0,1)$

$P = (5,-3)$

$R = (x,6)$



by distance formula,

$PQ = \sqrt{(5-0)^2+(-3-1)^2}$

$= \sqrt{5^2+(-4)^2}$

$= \sqrt{25+16}$

$= \sqrt{41}$

$RQ = \sqrt{(x-0)^2+(6-1)^2}$

$= \sqrt{x^2+5^2}$

$= \sqrt{x^2+25}$

Since,

Q is the equidistance from P and R.

$\therefore PQ = RQ$

$\Rightarrow \sqrt{41} = \sqrt{x^2+25}$

Squaring both sides,

$\Rightarrow (\sqrt{41})^2 = (\sqrt{x^2+25})^2$

$\Rightarrow 41 = x^2+25$

$\Rightarrow x^2+25 = 41$

$\Rightarrow x^2 = 41-25 = 16$

$\Rightarrow x = \sqrt{16}$

$\Rightarrow x = \pm 4$

$\therefore QR = \sqrt{x^2+25}$

$= \sqrt{4^2+25}$

$= \sqrt{16+25}$

$= \sqrt{41}$

if $x=4$

$PR = \sqrt{(5-4)^2+(-3-6)^2}$

$= \sqrt{1^2+(-9)^2}$

$= \sqrt{1+81}$

$= \sqrt{82}$

if $x=-4$

$PR = \sqrt{[5-(-4)]^2+(-3-6)^2}$

$= \sqrt{(5+4)^2+(-9)^2}$

$= \sqrt{9^2+81}$

$= \sqrt{81+81}$

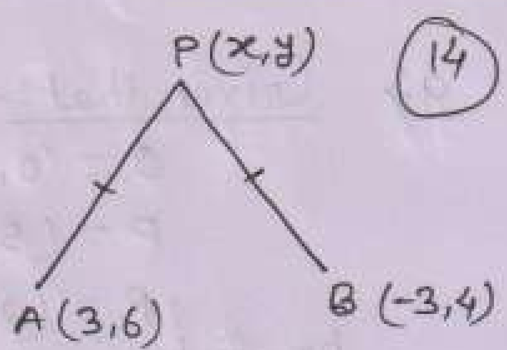
$= \sqrt{162}$

(10) let

$P = (x, y)$

$A = (3, 6)$

$B = (-3, 4)$



by distance formula,

$$PA = \sqrt{(x-3)^2 + (y-6)^2}$$

$$= \sqrt{x^2 - 6x + 9 + y^2 - 12y + 36}$$

$$= \sqrt{x^2 + y^2 - 6x - 12y + 45}$$

$$PB = \sqrt{[x-(-3)]^2 + (y-4)^2}$$

$$= \sqrt{(x+3)^2 + (y-4)^2}$$

$$= \sqrt{x^2 + 6x + 9 + y^2 - 8y + 16}$$

$$= \sqrt{x^2 + y^2 + 6x - 8y + 25}$$

A/Q,

$$PA = PB$$

$$\Rightarrow \sqrt{x^2 + y^2 - 6x - 12y + 45} = \sqrt{x^2 + y^2 + 6x - 8y + 25}$$

Squaring both sides,

$$\Rightarrow \left(\sqrt{x^2 + y^2 - 6x - 12y + 45}\right)^2 = \left(\sqrt{x^2 + y^2 + 6x - 8y + 25}\right)^2$$

$$\Rightarrow \cancel{x^2} + \cancel{y^2} - 6x - 12y + 45 = \cancel{x^2} + \cancel{y^2} + 6x - 8y + 25$$

$$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$$

$$\Rightarrow -6x - 12y - 6x + 8y = 25 - 45$$

$$\Rightarrow -12x - 4y = -20$$

$$\Rightarrow \cancel{-}\cancel{4}(3x + y) = \cancel{-}\cancel{20}\,5$$

$$\Rightarrow 3x + y = 5$$

$$\Rightarrow 3x + y - 5 = 0$$