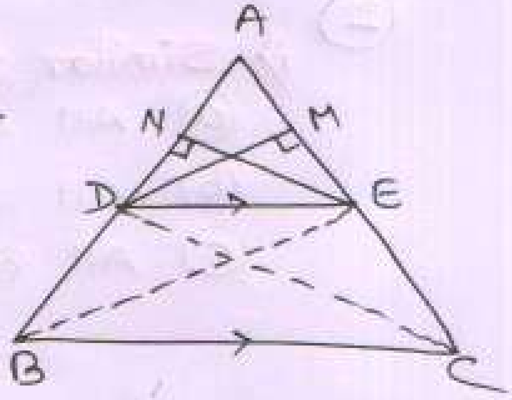
Theorem – (6.1) → Thales' Theorem

or

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct point, the other two sides are divided in the same ratio.



Given that:- In $\triangle ABC$,

$DE \parallel BC$

To prove:- $\frac{AD}{DB} = \frac{AE}{EC}$

Const. → Draw $EN \perp AB$

$DM \perp AC$

join BE and CD

Proof:- We have

$$ar(\triangle ADE) = \frac{1}{2} \times AD \times EN$$

$$ar(\triangle BDE) = \frac{1}{2} \times DB \times EN$$

$$\therefore \frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{\cancel{\frac{1}{2}} \times AD \times \cancel{EN}}{\cancel{\frac{1}{2}} \times DB \times \cancel{EN}} = \frac{AD}{DB} \quad \text{—— (I)}$$

Again,

$$ar(\triangle ADE) = \frac{1}{2} \times AE \times DM$$

$$ar(\triangle CED) = \frac{1}{2} \times EC \times DM$$

$$\therefore \frac{ar(\triangle ADE)}{ar(\triangle CED)} = \frac{\cancel{\frac{1}{2}} \times AE \times \cancel{DM}}{\cancel{\frac{1}{2}} \times EC \times \cancel{DM}} = \frac{AE}{EC} \quad \text{—— (II)}$$

Now,

$\triangle BDE$ and $\triangle CED$ being on the same base DE and between the same parallel DE and BC,

$$\therefore \cancel{ar(\triangle ADE)}\ ar(\triangle BDE) = ar(\triangle CED)$$

from equn (II)

$$\frac{ar(\triangle ADE)}{ar(\triangle CED)} = \frac{AE}{EC}$$

$$\Rightarrow \frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{AE}{EC} \quad \text{—— (III)}$$

From equn (I) and (III)

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Proved

Theorem - 6.2 → Converse of Thale's Theorem

or

If a line divides any two sides of a triangle in the same ratio then the line must be parallel to the third side.

Given that:- A $\triangle ABC$ and a line segment DE intersecting AB at D and AC at E, Such that $\frac{AD}{DB} = \frac{AE}{EC}$ — (1)

To Prove :- $DE \parallel BC$

Const. → Draw $DF \parallel BC$ which intersecting AC at F.

Proof:- In $\triangle ABC$,

$DF \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AF}{FC}$ — (11) [by thales' theorem]

from equ^n (1) and (11), we have

$$\frac{AE}{EC} = \frac{AF}{FC}$$

Adding 1 in both sides,

$$\Rightarrow \frac{AE}{EC} + 1 = \frac{AF}{FC} + 1$$

$$\Rightarrow \frac{AE+EC}{EC} = \frac{AF+FC}{FC}$$

$$\Rightarrow \frac{AC}{EC} = \frac{AC}{FC}$$

Compare both sides,

$\therefore EC = FC$

This is possible only when E and F coincide.

Hence,

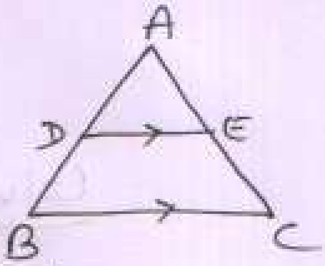
$DE \parallel BC$.

Proved

Important Notes:-

(I) In ΔABC,

$DE \parallel BC$



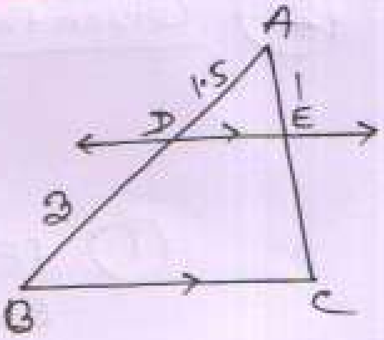
$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ — [Thales' Theorem]

(II) In ΔABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore DE \parallel BC$ — [Converse of Thales' Theorem]

Exercise - 6.2



(1) i) Given that :-

$DE \parallel BC$

$AD = 1.5$ cm

$DB = 3$ cm

$AE = 1$ cm

$EC = ?$

$\because$ In ΔABC,

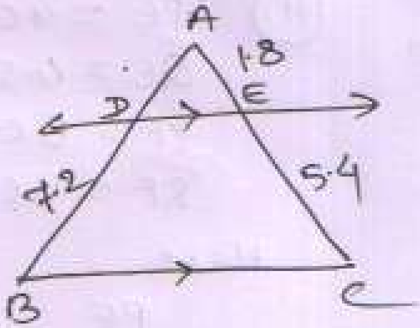
$DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ — [Thales' Theorem]

$\Rightarrow \frac{1.5}{3} = \frac{1}{EC}$

$\Rightarrow \frac{\cancel{15}^{5}}{\cancel{3} \times \cancel{10}_{2}} = \frac{1}{EC}$

$\Rightarrow EC = 2$ cm Ans



(ii) Given that :-

$DE \parallel BC$

$AD = ?$

$DB = 7.2$ cm

$AE = 1.8$ cm

$EC = 5.4$ cm

$\because$ In ΔABC, $DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ — [Thales' Theorem]

$\Rightarrow \frac{AD}{7.2} = \frac{1.8}{5.4}$

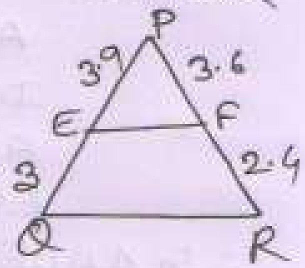
$\Rightarrow \frac{AD \times 10}{\cancel{72}_{24}} = \frac{\cancel{18}^{1} \times \cancel{10}}{\cancel{10} \times \cancel{54}_{3}}$

$\Rightarrow AD \times 10 = 24$

$\Rightarrow AD = \frac{24}{10} = 2.4$ cm Ans

2 Given that:- E and F are points on the sides PQ and PR respectively of a ΔPQR.
State whether $EF \parallel QR$

(i) $PE = 3.9$ cm
$EQ = 3$ cm
$PF = 3.6$ cm
$FR = 2.4$ cm



Here,

$$\frac{PE}{EQ} = \frac{\cancel{3.9}}{\cancel{3}} = 1.3$$

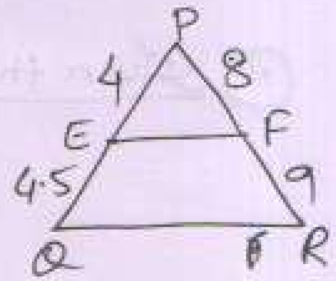
and,

$$\frac{PF}{FR} = \frac{3.6}{2.4} = \frac{\cancel{36}^{3} \times \cancel{10}}{\cancel{10} \times \cancel{24}_{2}} = \frac{3}{2} = 1.5$$

$\because \frac{PE}{EQ} \neq \frac{PF}{FR}$

So, EF is not parallel to QR. NO

(ii) $PE = 4$ cm
$QE = 4.5$ cm
$PF = 8$ cm
$RF = 9$ cm



Here,

$$\frac{PE}{QE} = \frac{4}{4.5} = \frac{4 \times \cancel{10}^{2}}{\cancel{45}_{9}} = \frac{8}{9}$$

and,

$$\frac{PF}{RF} = \frac{8}{9}$$

$\because \frac{PE}{QE} = \frac{PF}{RF}$

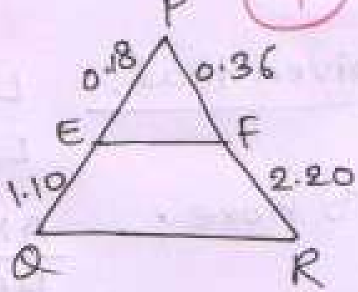
So, $EF \parallel QR$ (yes)

(iii) $PQ = 1.28$ c.m

$PR = 2.56$ c.m

$PE = 0.18$ c.m

$PF = 0.36$ c.m



$\therefore$ $EQ = PQ - PE$

$= 1.28 - 0.18$

$= 1.10$ c.m

$FR = PR - PF$

$= 2.56 - 0.36$

$= 2.20$ c.m

Here,

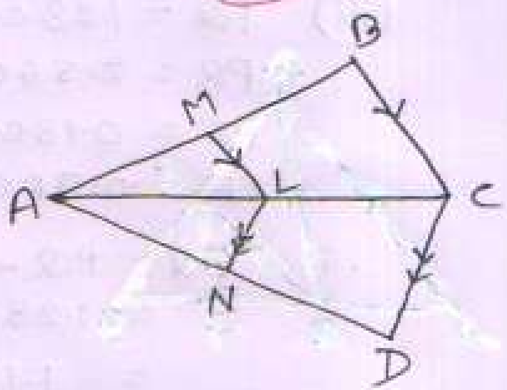
$$\frac{PE}{EQ} = \frac{0.18}{1.10} = \frac{\overset{9}{\cancel{18}} \times \cancel{100}}{\cancel{100} \times \underset{55}{\cancel{110}}} = \frac{9}{55}$$

$$\frac{PF}{FR} = \frac{0.36}{2.20} = \frac{\overset{9}{\cancel{36}} \times \cancel{100}}{\cancel{100} \times \underset{55}{\cancel{220}}} = \frac{9}{55}$$

$\because$ $\frac{PE}{EQ} = \frac{PF}{FR}$

So, $EF \parallel QR$ Yes

3 <u>Given that:-</u> LM ∥ CB
LN ∥ CD



To Prove:- $\frac{AM}{AB} = \frac{AN}{AD}$

<u>Proof:-</u> In ΔABC,

LM ∥ CB

$\therefore \frac{AM}{AB} = \frac{AL}{AC}$ — (I) [Thales' Theorem]

and,

In ΔADC,

LN ∥ CD

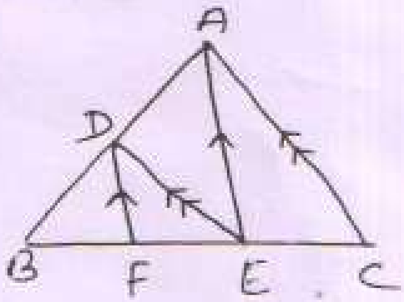
$\therefore \frac{AN}{AD} = \frac{AL}{AC}$ — (II) [Thales Theorem]

from equation (I) and (II), we get,

$$\frac{AM}{AB} = \frac{AN}{AD}$$

<u>Proved</u>

4 <u>Given that:-</u> DE ∥ AC
DF ∥ AE



To Prove:- $\frac{BF}{FE} = \frac{BE}{EC}$

<u>Proof:-</u> In ΔABC,

DE ∥ AC

$\therefore \frac{BD}{DA} = \frac{BE}{EC}$ — (I) [Thales' Theorem]

and,

In ΔABE,

DF ∥ AE

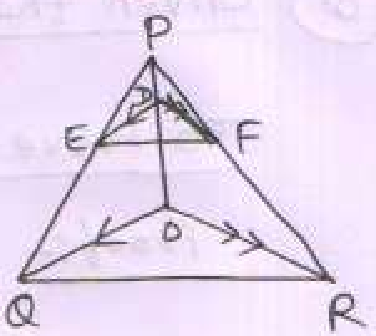
$\therefore \frac{BD}{DA} = \frac{BF}{FE}$ — (II) [Thales Theorem]

From equ^n (I) and (II), we get, $\frac{BF}{FE} = \frac{BE}{EC}$ Proved

5 Given that:- DE || OQ

DF || OR

To Prove:- EF || QR



Proof:- In ΔPOQ,

DE || OQ

$\therefore \frac{PE}{EQ} = \frac{PD}{DO}$ — (I) [Thales' Theorem]

and,

In ΔPOR,

DF || OR

$\therefore \frac{PF}{FR} = \frac{PD}{DO}$ — (II) [Thales' Theorem]

From eqn (I) and (II)

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Now,

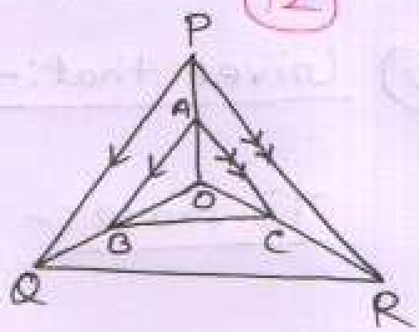
In ΔPQR,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

$\therefore$ EF || QR [Converse of Thales' Theorem]

Proved

6. <u>Given that :-</u> $AB \parallel PQ$
$AC \parallel PR$



<u>To Prove :-</u> $BC \parallel QR$

<u>Proof :-</u> In ΔPOQ,

$AB \parallel PQ$

$\therefore \dfrac{OB}{BQ} = \dfrac{OA}{AP}$ —— (I) [Thales' Theorem]

Again,

In ΔPOR,

$AC \parallel PR$

$\therefore \dfrac{OC}{CR} = \dfrac{OA}{AP}$ —— (II) [Thales' Theorem]

From equ^n (I) and (II), we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

Now,

In ΔOQR,

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

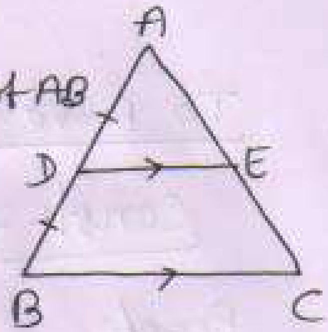
$\therefore BC \parallel QR$ [Converse of Thales' Theorem]

<u>Proved</u>

<7.>

Given that:- In ΔABC
D is the mid-point of AB
$DE \parallel BC$

To Prove :- E is the mid-point of AC.



Proof:- $\because$ D is the mid-point of AB

$\therefore$ $AD = DB$

$\Rightarrow \frac{AD}{DB} = 1$ — (1)

Again,

In ΔABC,

$DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{AE}{EC}$ [Thale's theorem]

$\Rightarrow 1 = \frac{AE}{EC}$

$\Rightarrow AE = EC$

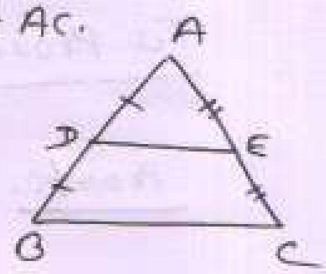
$\therefore$ E is the mid-point of AC

proved

8 **Given That:-** In $\triangle ABC$,

D is the mid point of AB

E is the mid point of AC.



To Prove:- $DE \parallel BC$

Proof:- Since,

D is the mid point of AB

$\therefore AD = DB$

$\Rightarrow \dfrac{AD}{DB} = 1$ — (1)

Again,

E is the mid point of AC

$\therefore AE = EC$

$\Rightarrow \dfrac{AE}{EC} = 1$ — (11)

From equ^n (1) and (11), We get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

Now,

In $\triangle ABC$,

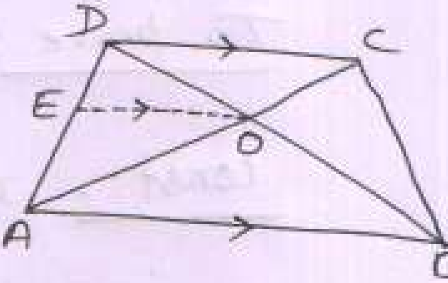
$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\therefore DE \parallel BC$ [Converse of Thales' Theorem]

Proved

9. <u>Given That:-</u> ABCD is a trapezium in which
AB ∥ DC
diagonals AC and BD intersect each other at the point O.

<u>To Prove:-</u> $\frac{AO}{BO} = \frac{CO}{DO}$



<u>Const:-</u> Through O, draw
EO ∥ AB ∥ DC

<u>Proof:-</u> In ΔADC,
EO ∥ DC

$\therefore \frac{AE}{ED} = \frac{AO}{CO}$ — (I) [Thales' Theorem]

Again,

In ΔDAB,
EO ∥ AB

$\therefore \frac{AE}{ED} = \frac{BO}{DO}$ — (II) [Thales' Theorem]

from equ^n (I) and (II), We have

$$\frac{AO}{CO} = \frac{BO}{DO}$$

$$\Rightarrow \frac{AO}{BO} = \frac{CO}{DO}$$ [replace BO and CO each other]

Proved

Given that:- ABCD is a quadrilateral in which Diagonal AC and BD intersect each other at the point O.

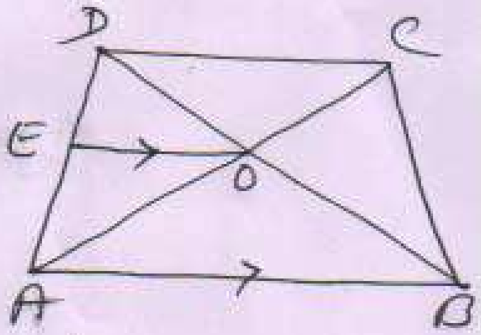
and,

$$\frac{AO}{BO} = \frac{CO}{DO}$$

To Prove :- ABCD is a traperium.

Const:- → Draw OE ∥ AB.

Proof:-



In ΔABD,

$OE \parallel AB$

$$\therefore \frac{AE}{ED} = \frac{BO}{DO} \quad \text{— (I)} \quad [\text{Thale's therrem}]$$

but,

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\Rightarrow \frac{AO}{CO} = \frac{BO}{DO} \quad \text{— (II)} \quad \left[\begin{array}{l}\text{replace BO and CO} \\ \text{each other.}\end{array}\right.$$

from equn (I) and (II) we get

$$\frac{AE}{ED} = \frac{AO}{CO}$$

In ΔADC,

$$\frac{AE}{ED} = \frac{AO}{CO}$$

$\therefore OE \parallel CD$ [Inverse of thale's theorem]

but, $OE \parallel AB$

$\therefore AB \parallel CD$

$\therefore ABCD$ is a traperium.

Proved