Theorem-6.3 :- [AAA-similarity]

If in two triangles, corresponding angles are equal, then their corresponding sides are in the same ratio (or proportional) and hence the two triangles are similar.

Given that:- In ΔABC and ΔDEF

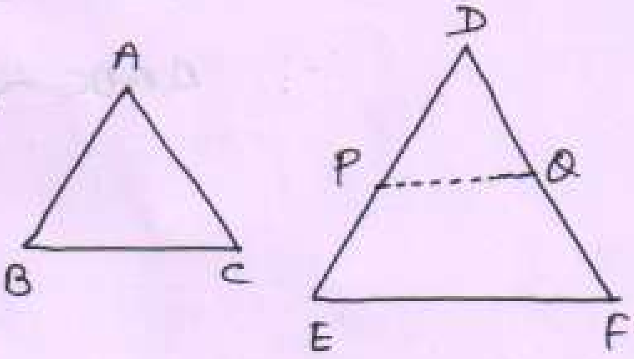
$$\angle A = \angle D$$
$$\angle B = \angle E$$
$$\angle C = \angle F$$

To prove:-

$$\Delta ABC \sim \Delta DEF$$

Construction:- Cut $DP = AB$ and $DQ = AC$. join PQ.

Proof:- In ΔABC and ΔDPQ,



$$AB = DP$$
$$AC = DQ$$
$$\angle A = \angle D$$

$\therefore$ $\Delta ABC \cong \Delta DPQ$ [by SAS-congruence]

$\therefore$ $\angle B = \angle P$ [CPCT]

$\Rightarrow$ $\angle E = \angle P$ [$\because \angle B = \angle E$]

These are corresponding angle.

$\therefore$ $PQ \parallel EF$

Now,

In ΔDEF

$PQ \parallel EF$

$$\therefore \frac{DP}{DE} = \frac{DQ}{DF}$$

$$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF} \quad \text{—(I)} \quad [\because DP = AB \text{ and } DQ = AC]$$

Similarly,

$$\frac{AB}{DE} = \frac{BC}{EF} \quad \text{—(II)}$$

from equ^n (I) and (II),

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

then,

In ΔABC and ΔDEF,

$\angle A = \angle D$

$\angle B = \angle E$

$\angle C = \angle F$

and,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$\therefore \quad \Delta ABC \sim \Delta DEF$

Proved

Theorem - 6.4 → [SSS- similarity]

If in two triangles, sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

Given that:- In ΔABC and ΔDEF,

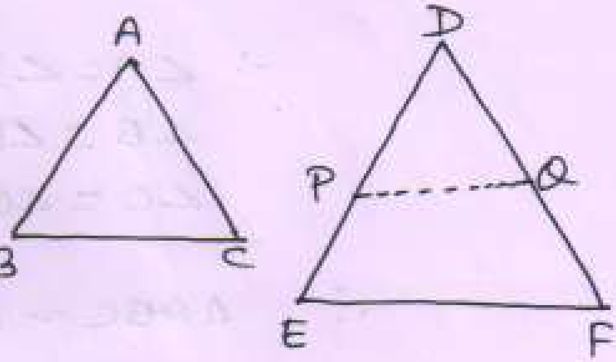
$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

To prove:- $\Delta ABC \sim \Delta DEF$

Construction:- Cut $DP = AB$ and $DQ = AC$.
Join PQ.

Proof:- $\because \frac{AB}{DE} = \frac{AC}{DF}$

$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF}$ $\left[\because \begin{matrix} DP = AB \\ DQ = AC \end{matrix}\right]$



In ΔDEF,

$$\frac{DP}{DE} = \frac{DQ}{DF}$$

$\therefore PQ \parallel EF$ [Inverse of thale's theorem]

$\therefore \left.\begin{matrix} \angle P = \angle E \\ \angle Q = \angle F \end{matrix}\right\}$ - [Corresponding angle]

$\therefore \Delta DPQ \sim \Delta DEF$ [by AA-similarity]

$\therefore \frac{DP}{DE} = \frac{PQ}{EF}$

$\Rightarrow \frac{AB}{DE} = \frac{PQ}{EF}$ — (1) [DP = AB]

but,

$\frac{AB}{DE} = \frac{BC}{EF}$ — (11)

from equ^n (1) and (11), we get

$\frac{PQ}{EF} = \frac{BC}{EF}$

Now, $\therefore$ $BC = PQ$

In ΔABC and ΔDPQ,

$AB = DP$

$AC = DQ$

$BC = PQ$

$\therefore$ $\Delta ABC \cong \Delta DPQ$ [by SSS- Congruence]

$\therefore \angle A = \angle D$

$\angle B = \angle P = \angle E$

$\angle C = \angle Q = \angle F$

$\therefore$ $\Delta ABC \sim \Delta DEF$ [AAA - similarity]

Proved

Theorem - 6.5 → [SAS - similarity]

If one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional, then the two triangles are similar.

Given that :- In ΔABC and ΔDEF

$$\angle A = \angle D$$

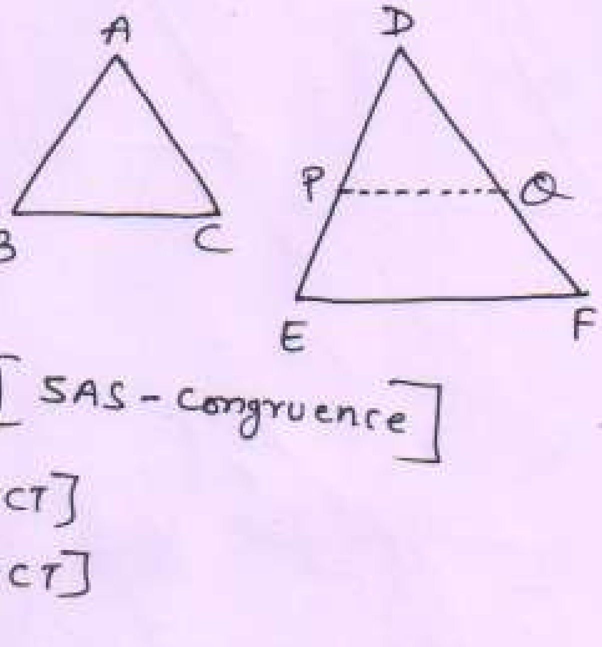
and,

$$\frac{AB}{DE} = \frac{AC}{DF}$$

To prove :- $\Delta ABC \sim \Delta DEF$

Construction :- Cut $DP = AB$ and $DQ = AC$.
join PQ.

Proof :- In ΔABC and ΔDPQ,



$AB = DP$

$AC = DQ$

$\angle A = \angle D$

$\therefore \Delta ABC \cong \Delta DPQ$ [SAS - congruence]

$\therefore \angle B = \angle P$ [CPCT]

$\angle C = \angle Q$ [CPCT]

Again,

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\Rightarrow \frac{DP}{DE} = \frac{DQ}{DF} \quad [\because DP = AB, DQ = AC]$$

$\therefore PQ \parallel EF$ [Inverse of thale's theorem]

$\therefore \left.\begin{array}{l} \angle P = \angle E \\ \angle Q = \angle F \end{array}\right\}$ (Corresponding angle)

$\therefore \angle A = \angle D$

$\angle B = \angle P = \angle E$

$\angle C = \angle Q = \angle F$

In ΔABC and ΔDEF,

$\angle A = \angle D$

$\angle B = \angle E$

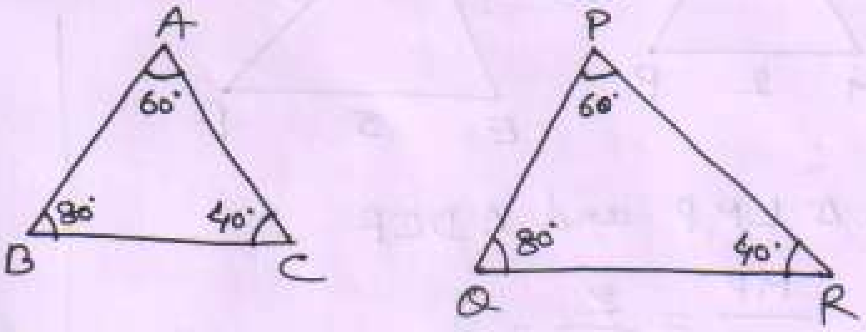
$\angle C = \angle F$

$\therefore \Delta ABC \sim \Delta DEF$ [AAA - similarity]

Proved

Exercise-6.3

1.) (i)



In ΔABC and ΔPQR,

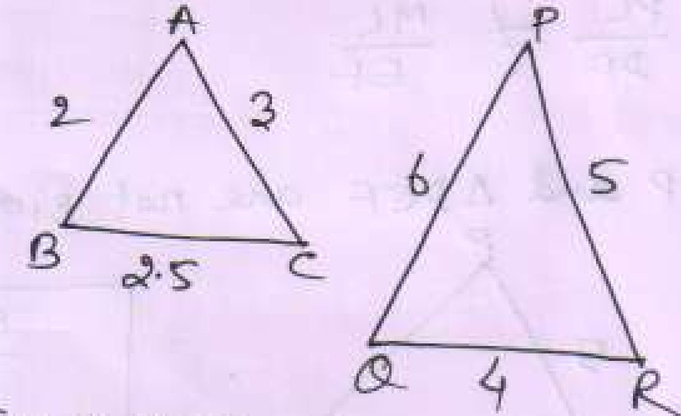
$\angle A = \angle P = 60°$

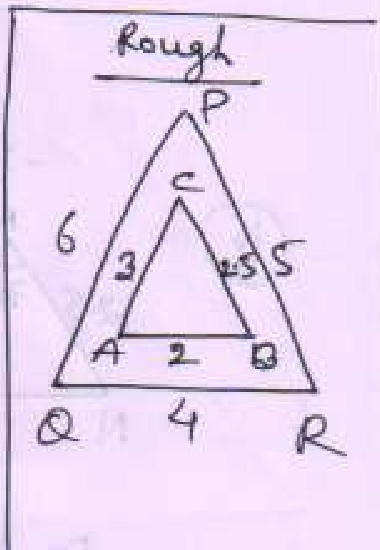
$\angle B = \angle Q = 80°$

$\angle C = \angle R = 40°$

$\therefore$ $\Delta ABC \sim \Delta PQR$ [AAA-similarity]

(ii)





In ΔABC and ΔPQR,

$\therefore$ $\frac{AB}{QR} = \frac{\cancel{2}}{\cancel{4}} = \frac{1}{2}$

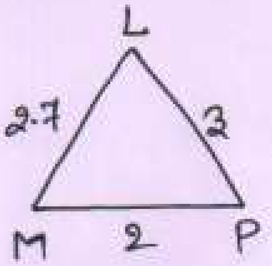
$\frac{BC}{PR} = \frac{\cancel{2.5}}{\cancel{5}} = \frac{1}{2}$

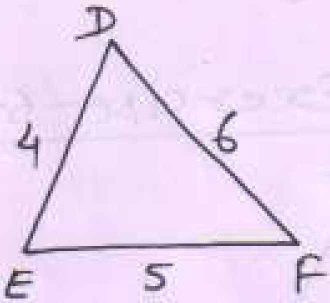
$\frac{AC}{PQ} = \frac{\cancel{3}}{\cancel{6}_2} = \frac{1}{2}$

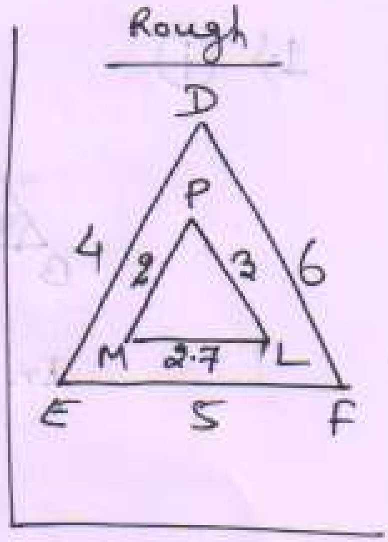
$\therefore$ $\frac{AB}{QR} = \frac{BC}{PR} = \frac{AC}{PQ}$

$\therefore$ $\Delta ABC \sim \Delta PQR$ [S-S-S-similarity]

(iii)







In ΔLMP and ΔDEF

$$\frac{MP}{DE} = \frac{\cancel{2}}{\cancel{4}_2} = \frac{1}{2}$$

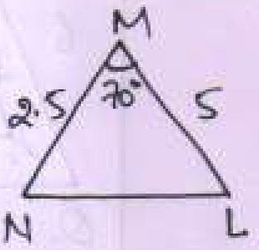
$$\frac{PL}{DF} = \frac{\cancel{3}}{\cancel{6}_2} = \frac{1}{2}$$

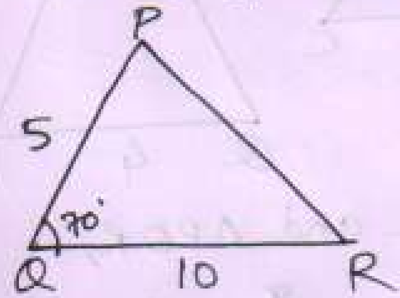
$$\frac{ML}{EF} = \frac{2.7}{5}$$

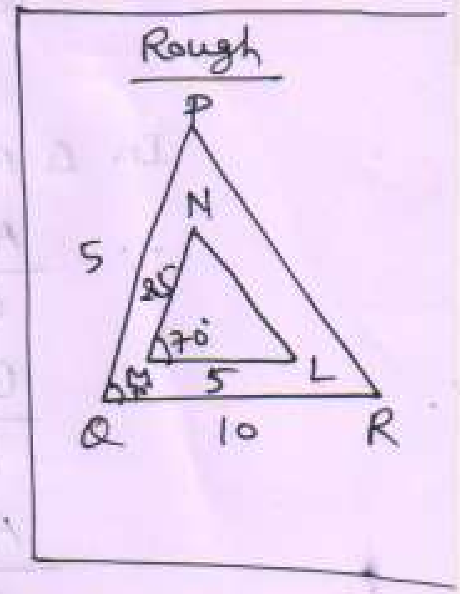
$\therefore$ $$\frac{MP}{DE} = \frac{PL}{DF} \neq \frac{ML}{EF}$$

$\therefore$ ΔLMP and ΔDEF are not similar.

(iv)







In ΔMNL and ΔPQR,

$$\frac{MN}{PQ} = \frac{2.5}{\cancel{5}} = \frac{1}{2}$$

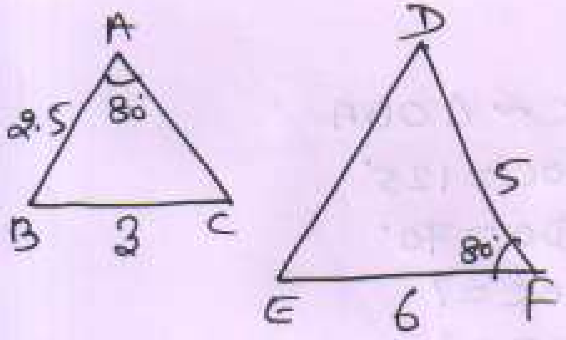
$$\frac{ML}{QR} = \frac{\cancel{5}}{\cancel{10}_2} = \frac{1}{2}$$

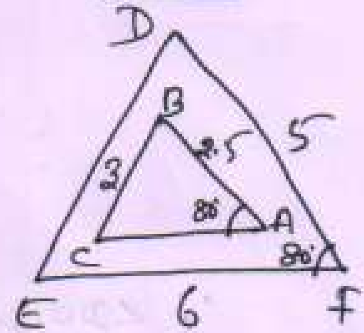
$$\angle M = \angle Q = 70^\circ$$

$\therefore$ $$\frac{MN}{PQ} = \frac{ML}{QR} \text{ and } \angle M = \angle Q$$

$\therefore$ $\Delta MNL \sim \Delta PQR$

(v)





In ΔABC and ΔDEF

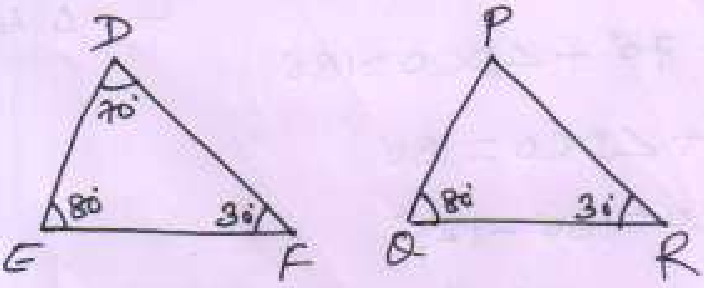
$\frac{AB}{DF} = \frac{\cancel{2.5}}{\cancel{5}_2} = \frac{1}{2}$

$\frac{BC}{DE} = \frac{3}{DE}$

$\frac{AC}{\cancel{E}F} = \frac{AC}{6}$

$\therefore$ ΔABC and ΔDEF are not similar.

(vi)



In ΔDEF and ΔPQR

$\angle E = \angle Q = 80°$

$\angle F = \angle R = 30°$

$\therefore$ $\Delta DEF \sim \Delta PQR$

(2) Given that :- $\Delta ODC \sim \Delta OBA$

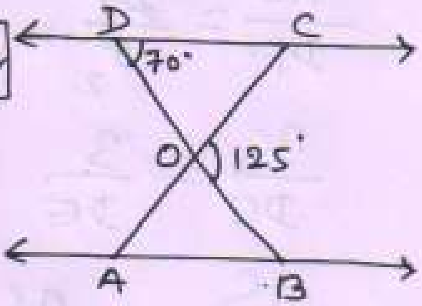
$\angle BOC = 125^\circ$

$\angle CDO = 70^\circ$

$\angle DOC = ?$

$\angle DCO = ?$

$\angle OAB = ?$



$\because \angle DOC + \angle BOC = 180^\circ$ [linear pair]

$\Rightarrow \angle DOC + 125^\circ = 180^\circ$

$\Rightarrow \angle DOC = 180^\circ - 125^\circ$

$\Rightarrow \angle DOC = 55^\circ$ Ans

Again,

In ΔDOC,

$\angle DOC + \angle CDO + \angle DCO = 180^\circ$ [Sum of angles of Δ is 180°]

$\Rightarrow 55^\circ + 70^\circ + \angle DCO = 180^\circ$

$\Rightarrow 125^\circ + \angle DCO = 180^\circ$

$\Rightarrow \angle DCO = 180^\circ - 125^\circ$

$\Rightarrow \angle DCO = 55^\circ$ Ans

Again,

$\Delta ODC \sim \Delta OBA$

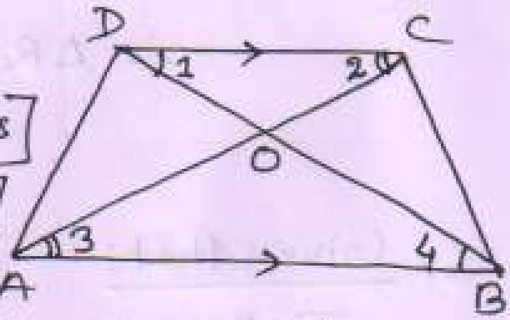
$\therefore \angle DCO = \angle OAB$ [corresponding angles]

$\Rightarrow 55^\circ = \angle OAB$

$\therefore \angle OAB = 55^\circ$ Ans

(3) Given that:- In a trapezium ABCD, $AB \parallel DC$ and diagonals AC and BD intersect each other at the point O.

To prove:- $\frac{OA}{OC} = \frac{OB}{OD}$



Proof:- In $\triangle AOB$ and $\triangle COD$,

$\angle 4 = \angle 1$ [Alt. int. ∠s]

$\angle 3 = \angle 2$ [Alt. int. ∠s]

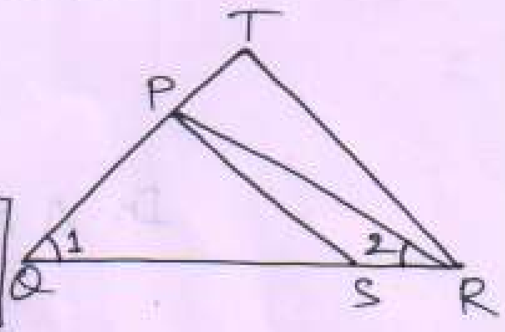
$\therefore \triangle AOB \sim \triangle COD$ [by AA-similarity]

$\therefore \frac{OA}{OC} = \frac{OB}{OD}$

Proved

(4) Given that:- $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$

To Prove:- $\triangle PQS \sim \triangle TQR$



Proof:- In $\triangle PQR$,

$\angle 1 = \angle 2$

$\therefore PR = PQ$ — (1) [opp. sides of equal angles are equal.]

Again,

$\frac{QR}{QS} = \frac{QT}{PR}$

$\Rightarrow \frac{QR}{QS} = \frac{QT}{PQ}$ [from equ^n (1)]

In $\triangle PQS$ and $\triangle TQR$

$\frac{QR}{QS} = \frac{QT}{PQ}$ and $\angle 1 = \angle 1$ [Common]

$\therefore \triangle PQS \sim \triangle TQR$ [by SAS-similarity]

Proved

<5.> Given that:- In ΔPQR,

$\angle P = \angle RTS$

To Prove:- $\Delta RPQ \sim \Delta RTS$

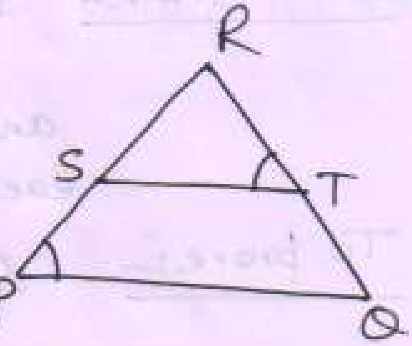
Proof:- In ΔRPQ and ΔRTS,

$\angle P = \angle RTS$

$\angle R = \angle R$ [Common]

$\therefore \Delta RPQ \sim \Delta RTS$ [by AA-Similarity]

Proved



<6.> Given that:- $\Delta ABE \cong \Delta ACD$

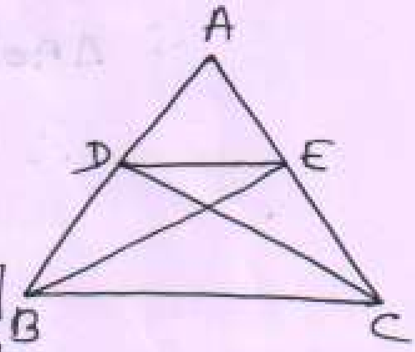
To Prove:- $\Delta ADE \sim \Delta ABC$

Proof:- $\because \Delta ABE \cong \Delta ACD$

$\therefore$ $AB = AC$ — (1) [CPCT]

$AE = AD$ [CPCT]

or, $AD = AE$ — (11)



Dividing equ^n (11) by (1), we get,

$$\frac{AD}{AB} = \frac{AE}{AC}$$

In ΔADE and ΔABC

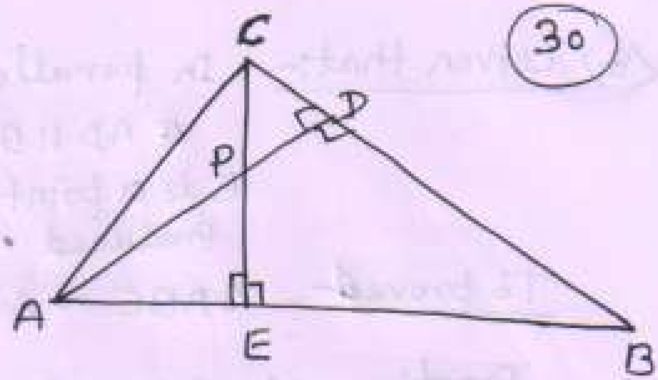
$$\frac{AD}{AB} = \frac{AE}{AC}$$

and,

$\angle A = \angle A$ [Common]

$\therefore \Delta ADE \sim \Delta ABC$ [SAS-similarity]

Proved



⟨7.⟩ Given that:- In ΔABC,
$AD \perp BC$
$CE \perp AB$

To prove:- (i) $\Delta AEP \sim \Delta CDP$
(ii) $\Delta ABD \sim \Delta CBE$
(iii) $\Delta AEP \sim \Delta ADB$
(iv) $\Delta PDC \sim \Delta BEC$

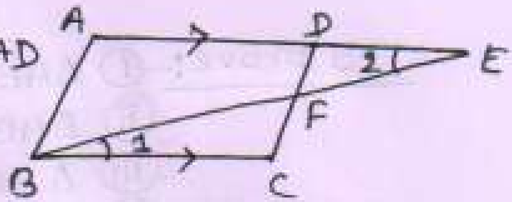
Proof:- (i) In ΔAEP and ΔCDP,
$\angle AEP = \angle CDP$ (90°)
$\angle APE = \angle CPD$ [vertically opp. ∠s]
$\therefore \Delta AEP \sim \Delta CDP$ [AA-similarity]
Proved

(ii) In ΔABD and ΔCBE,
$\angle ADB = \angle CEB$ (90°)
$\angle B = \angle B$ [Common]
$\therefore \Delta ABD \sim \Delta CBE$ [by AA-similarity]
Proved

(iii) In ΔAEP and ΔADB,
$\angle AEP = \angle ADB$ (90°)
$\angle PAE = \angle DAB$ [Common]
$\therefore \Delta AEP \sim \Delta ADB$ [by AA-similarity]
Proved

(iv) In ΔPDC and ΔBEC,
$\angle PDC = \angle BEC$ (90°)
$\angle PCD = \angle BCE$ [Common]
$\therefore \Delta PDC \sim \Delta BEC$ (AA-similarity)
Proved

<8.> **Given that:-** In parallelogram ABCD, AD ∥ BC and E is a point on the side AD Produced.



To prove:- $\Delta ABE \sim \Delta CFB$

Proof:- ∵ AE ∥ BC and BE is a transversal line

∴ $\angle 1 = \angle 2$ —① [Alt. int. ∠s]

In ΔABE and ΔCFB,

$\angle 2 = \angle 1$

$\angle A = \angle C$ [opposite angle of ∥gm]

∴ $\Delta ABE \sim \Delta CFB$ [AA-similarity]

Proved

<9.> **Given that:-** ABC and AMP are two right triangle in which right angled at B and M respectively.

To Prove:- (I) $\Delta ABC \sim \Delta AMP$

(II) $\frac{CA}{PA} = \frac{BC}{MP}$

Proof:- (I) In ΔABC and ΔAMP,

$\angle ABC = \angle AMP$ (90°)

$\angle A = \angle A$ [common]

∴ $\Delta ABC \sim \Delta AMP$ (AA-similarity)

Proved

(II) ∵ $\Delta ABC \sim \Delta AMP$

∴ $\frac{CA}{PA} = \frac{BC}{MP}$

Proved

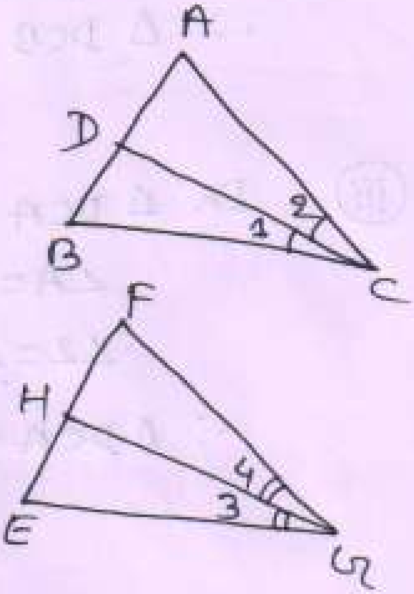
(10) Given that:- $\Delta ABC \sim \Delta FEG$

CD and GH are respectively the bisector of $\angle ACB$ and $\angle EGF$.

To Prove:- (i) $\frac{CD}{GH} = \frac{AC}{FG}$

(ii) $\Delta DCB \sim \Delta HGE$

(iii) $\Delta DCA \sim \Delta HGF$



Proof:- $\because \Delta ABC \sim \Delta FEG$

$\therefore \angle A = \angle F$ —(I)

$\angle B = \angle E$ —(II)

$\angle C = \angle G$ —(III)

Again,

$\because$ CD bisects $\angle ACB$

$\therefore \angle 1 = \angle 2 = \frac{1}{2}\angle C$

and,

GH bisects $\angle EGF$

$\therefore \angle 3 = \angle 4 = \frac{1}{2}\angle G$

from equn (III), we get,

$\angle C = \angle G$

$\Rightarrow \frac{1}{2}\angle C = \frac{1}{2}\angle G$

$\Rightarrow \angle 1 = \angle 2 = \angle 3 = \angle 4$ —(IV)

(I) In ΔACD and ΔFGH,

$\angle A = \angle F$

$\angle 2 = \angle 4$

$\therefore \Delta ACD \sim \Delta FGH$ [AA-similarity]

$\therefore \angle ADC = \angle FHG$

$\therefore \frac{CD}{GH} = \frac{AC}{FG}$

Proved