

Ex-2.5

1) उपयुक्त सर्वसमिकाओं का प्रयोग करके निम्नलिखित गुणनफल ज्ञात कीजिए :-

(i) $(x+4)(x+10)$

सर्वसमिकाएँ ले,

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

माना कि,

$$a = 4$$

$$b = 10$$

$$\begin{aligned} \therefore (x+4)(x+10) &= x^2 + (4+10)x + 4 \times 10 \\ &= x^2 + 14x + 40 \end{aligned}$$

(ii) $(x+8)(x-10)$

सर्वसमिकाओं ले,

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

इसमें

$$a = 8, b = -10 \text{ रखने पर}$$

$$\begin{aligned} \Rightarrow (x+8)(x-10) &= x^2 + [8+(-10)]x + 8 \times (-10) \\ &= x^2 + (8-10)x - 80 \\ &= x^2 - 2x - 80 \end{aligned}$$

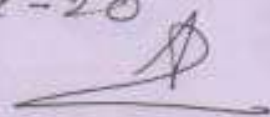
iii) $(3x+4)(3x-5)$

सर्वसमिकाएँ ले

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

इसमें $x = 3x$, $a = 4$, $b = -5$ रखने पर

$$\begin{aligned} \Rightarrow (3x+4)(3x-5) &= (3x)^2 + [4+(-5)] \times 3x + 4 \times (-5) \\ &= 9x^2 + (4-5) \times 3x - 20 \\ &= 9x^2 + (-1) \times 3x - 20 \\ &= 9x^2 - 3x - 20 \end{aligned}$$



iv) $(y^2 + \frac{3}{2})(y^2 - \frac{3}{2})$

सर्वसमिकाएँ ले,

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

इसमें $x = y^2$, $a = \frac{3}{2}$, $b = -\frac{3}{2}$ रखने पर

$$\Rightarrow (y^2 + \frac{3}{2})(y^2 - \frac{3}{2}) = (y^2)^2 + [\frac{3}{2} + (-\frac{3}{2})] \times y^2 + \frac{3}{2} \times (-\frac{3}{2})$$

सर्वसमिकाएँ ले,

$$(a+b)(a-b) = a^2 - b^2$$

इसमें $a = y^2$, $b = \frac{3}{2}$ रखने पर

$$(y^2 + \frac{3}{2})(y^2 - \frac{3}{2}) = (y^2)^2 - (\frac{3}{2})^2$$

$$= y^4 - \frac{9}{4}$$



$$= y^4 + (\frac{3}{2} - \frac{3}{2}) y^2 - \frac{3}{2} \times \frac{3}{2}$$

$$= y^4 + 0 \cdot y^2 - \frac{9}{4}$$

$$= y^4 - \frac{9}{4}$$



$$\textcircled{v} (3-2x)(3+2x)$$

सर्वसमिका ले,

$$(a-b)(a+b) = a^2 - b^2$$

इसमें,

$$a = 3, b = 2x \text{ रखने पर}$$

$$\Rightarrow (3-2x)(3+2x) = 3^2 - (2x)^2$$

$$= 9 - 4x^2$$

2.) सीधे गुणा किए बिना निम्नलिखित गुणनफल के मान ज्ञात कीजिए :-

$$\textcircled{i} 103 \times 107 = (105-2)(105+2)$$

$$\text{माना कि } a = 105$$

$$b = 2$$

सर्वसमिका ले,

$$\textcircled{ii} (a-b)(a+b) = a^2 - b^2$$

$$\Rightarrow (105-2)(105+2) = (105)^2 - 2^2$$

$$\Rightarrow 103 \times 107 = 11025 - 4$$

$$= 11021$$

दूसरी विधि:

$$103 \times 107 = (105-2)(105+2)$$

$$= (105)^2 - 2^2 \quad [(a-b)(a+b) = a^2 - b^2]$$

$$= 11025 - 4 = 11021$$

$$(ii) \quad 95 \times 96 = (100-5)(100-4)$$

यहाँ,

$$x = 100, a = -5, b = -4$$

लक्समिका ले,

$$(x+a)(x+b) = x^2 + (a+b)x + ab$$

$$\Rightarrow (100-5)(100-4) = (100)^2 + (-5-4) \times 100 + (-5) \times (-4)$$

$$\Rightarrow 95 \times 96 = 10000 + (-9) \times 100 + 20$$

$$= 10000 - 900 + 20$$

$$= 9100 + 20$$

$$= 9120$$

(iii)

$$104 \times 96 = (100+4)(100-4)$$

$$= (100)^2 - 4^2 \quad [(a+b)(a-b) = a^2 - b^2]$$

$$= 10000 - 16$$

$$= 9984$$

3) उपयुक्त सर्वसमिकाएँ प्रयोग करके निम्नलिखित का गुणनखण्डन कीजिए :-

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(i) $9x^2 + 6xy + y^2$

$$= (3x)^2 + 2 \times 3x \times y + y^2$$

$$= (3x + y)^2 \left[a^2 + 2ab + b^2 = (a + b)^2 \right]$$

$$= (3x + y)(3x + y)$$

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(ii) $4y^2 - 4y + 1$

$$= (2y)^2 - 2 \times 2y \times 1 + 1^2$$

$$= (2y - 1)^2 \left[a^2 - 2ab + b^2 = (a - b)^2 \right]$$

$$= (2y - 1)(2y - 1)$$

A

(iii) $x^2 - \frac{y^2}{100}$

$$= x^2 - \left(\frac{y}{10} \right)^2$$

$$= \left(x + \frac{y}{10} \right) \left(x - \frac{y}{10} \right) \left[a^2 - b^2 = (a + b)(a - b) \right]$$

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4) उपयुक्त सर्वसमिकाओं का प्रयोग करके निम्नालिखित में से प्रत्येक का प्रसार कीजिए :-

(i) $(x+2y+4z)^2$

माना कि, $a = x$

$$b = 2y$$

$$c = 4z$$

अब,

$$(x+2y+4z)^2 = (a+b+c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= x^2 + (2y)^2 + (4z)^2 + 2 \times x \times 2y + 2 \times 2y \times 4z + 2 \times 4z \times x$$

$$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8zx$$

(ii) $(2x - y + z)^2 = (2x + (-y) + z)^2$ ~~$(2x - y + z)^2$~~

माना कि $a = 2x$

$$b = -y$$

$$c = z$$

$$\therefore (2x - y + z)^2 = (a+b+c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (2x)^2 + (-y)^2 + z^2 + 2 \times 2x \times (-y) + 2 \times (-y) \times z + 2 \times z \times 2x$$

$$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4zx$$

$$(iii) (-2x+3y+2z)^2 = [(-2x)+3y+2z]^2$$

माना कि

$$a = -2x$$

$$b = 3y$$

$$c = 2z$$

$$\therefore (-2x+3y+2z)^2 = (a+b+c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (-2x)^2 + (3y)^2 + (2z)^2 + 2x(-2x) \times 3y + 2 \times 3y \times 2z + 2 \times 2z \times (-2x)$$

$$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8zx$$

$$(iv) (3a-7b-c)^2 = [3a+(-7b)+(-c)]^2$$

माना कि,

$$a = 3a$$

$$b = -7b$$

$$c = -c$$

$$\therefore (3a-7b-c)^2 = (a+b+c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (3a)^2 + (-7b)^2 + (-c)^2 + 2 \times 3a \times (-7b) + 2 \times (-7b) \times (-c) + 2 \times (-c) \times 3a$$

$$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ca$$

$$(v) \quad (-2x + 5y - 3z)^2 = [(-2x) + 5y + (-3z)]^2$$

माना कि,

$$a = -2x$$

$$b = 5y$$

$$c = -3z$$

$$\therefore (-2x + 5y - 3z)^2 = (a + b + c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= (-2x)^2 + (5y)^2 + (-3z)^2 + 2x(-2x) \times 5y + 2x5y \times (-3z) + 2x(-3z) \times (-2x)$$

$$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12zx$$

$$(vi) \quad \left[\frac{1}{4}a - \frac{1}{2}b + 1 \right]^2 = \left[\frac{1}{4}a + \left(-\frac{1}{2}b\right) + 1 \right]^2$$

माना कि,

$$a = \frac{1}{4}a$$

$$b = -\frac{1}{2}b$$

$$c = 1$$

$$\therefore \left(\frac{1}{4}a - \frac{1}{2}b + 1 \right)^2 = (a + b + c)^2$$

$$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$$

$$= \left(\frac{1}{4}a\right)^2 + \left(-\frac{1}{2}b\right)^2 + 1^2 + 2 \times \frac{1}{4}a \times \left(-\frac{1}{2}b\right) + 2 \times \left(-\frac{1}{2}b\right) \times 1 + 2 \times 1 \times \frac{1}{4}a$$

$$= \frac{1}{16}a^2 + \frac{1}{4}b^2 + 1 - \frac{1}{4}ab - b + \frac{1}{2}a$$

5.) गुणनखंड कीजिए:-

(46)

$$\begin{aligned} \textcircled{i} \quad & 4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz \\ &= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \times 2x \times 3y + 2 \times 3y \times (-4z) + 2 \times (-4z) \times 2x \\ &= [2x + 3y + (-4z)]^2 \cdot [a^2 + b^2 + c^2 + 2ab + 2bc + 2ca = (a+b+c)^2] \\ &= (2x + 3y - 4z)^2 \\ &= (2x + 3y - 4z)(2x + 3y - 4z) \end{aligned}$$

$$\begin{aligned} \textcircled{ii} \quad & 2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz \\ &= (\sqrt{2}x)^2 + (-y)^2 + (-2\sqrt{2}z)^2 + 2 \times \sqrt{2}x \times (-y) + 2 \times (-y) \times (-2\sqrt{2}z) \\ &\quad + 2 \times (-2\sqrt{2}z) \times \sqrt{2}x \\ &= [\sqrt{2}x + (-y) + (-2\sqrt{2}z)]^2 \\ &= (\sqrt{2}x - y - 2\sqrt{2}z)^2 \\ &= (\sqrt{2}x - y - 2\sqrt{2}z)(\sqrt{2}x - y - 2\sqrt{2}z) \end{aligned}$$

6. निम्नलिखित घनों को प्रसारित रूप में लिखिए -

$$(i) (2x+1)^3 = (2x)^3 + 1^3 + 3 \times 2x \times 1 (2x+1)$$

$$= 8x^3 + 1 + 6x(2x+1)$$

$$= 8x^3 + 1 + 12x^2 + 6x$$

$$= 8x^3 + 12x^2 + 6x + 1$$

$$(ii) (2a-3b)^3 = (2a)^3 - (3b)^3 - 3 \times 2a \times 3b (2a-3b)$$

$$= 8a^3 - 27b^3 - 18ab(2a-3b)$$

$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

$$(iii) \left(\frac{3}{2}x+1\right)^3 = \left(\frac{3}{2}x\right)^3 + 1^3 + 3 \times \frac{3}{2}x \times 1 \left(\frac{3}{2}x+1\right)$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \left(\frac{3}{2}x+1\right)$$

$$= \frac{27}{8}x^3 + 1 + \frac{9}{2}x \times \frac{3}{2}x + \frac{9}{2}x$$

$$= \frac{27}{8}x^3 + 1 + \frac{27}{4}x^2 + \frac{9}{2}x$$

$$= \frac{27}{8}x^3 + \frac{27}{4}x^2 + \frac{9}{2}x + 1$$

$$\textcircled{\text{iv}} \quad \left(x - \frac{2}{3}y\right)^3 = x^3 - \left(\frac{2}{3}y\right)^3 - 3 \times x \times \frac{2}{3}y \left(x - \frac{2}{3}y\right)$$

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$$= x^3 - \frac{8}{27}y^3 - 2xy \left(x - \frac{2}{3}y\right)$$

$$= x^3 - \frac{8}{27}y^3 - 2x^2y + \frac{4}{3}xy^2$$

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7.) उपयुक्त सर्वसमिकाएँ प्रयोग करके निम्नलिखित के मान ज्ञात कीजिए -

$$\textcircled{\text{i}} \quad (99)^3 = (100 - 1)^3$$

$$= (100)^3 - 1^3 - 3 \times 100 \times 1 (100 - 1)$$

$$= 1000000 - 1 - 300 \times 99$$

$$= 1000000 - 1 - 29700$$

$$= 1000000 - 29701$$

$$= 970299$$

$$\textcircled{\text{ii}} \quad (102)^3 = (100 + 2)^3$$

$$= (100)^3 + 2^3 + 3 \times 100 \times 2 (100 + 2)$$

$$= 1000000 + 8 + 600 \times 102$$

$$= 1000000 + 8 + 61200$$

$$= 1061208$$

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$$\begin{aligned}
 \textcircled{\text{iii}} \quad (998)^3 &= (1000-2)^3 \\
 &= (1000)^3 - 2^3 - 3 \times 1000 \times 2 (1000-2) \\
 &= 1000000000 - 8 - 6000 \times 998 \\
 &= 1000000000 - 8 - 5988000 \\
 &= 1000000000 - 5988008 \\
 &= 994011992
 \end{aligned}$$

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8.) निम्नलिखित में से प्रत्येक का गुणनखंड कीजिए -

① $8a^3 + b^3 + 12a^2b + 6ab^2$

$$= (2a)^3 + b^3 + 6ab(2a+b)$$

$$= (2a)^3 + b^3 + 3 \times 2a \times b (2a+b)$$

$$= (2a+b)^3 \quad [\because a^3 + b^3 + 3ab(a+b) = (a+b)^3]$$

$$= (2a+b)(2a+b)(2a+b)$$

② $8a^3 - b^3 - 12a^2b + 6ab^2$

$$= (2a)^3 - b^3 - 6ab(2a-b)$$

$$= (2a)^3 - b^3 - 3 \times 2a \times b (2a-b)$$

$$= (2a-b)^3 \quad [\because a^3 - b^3 - 3ab(a-b) = (a-b)^3]$$

$$= (2a-b)(2a-b)(2a-b)$$

$$\textcircled{\text{iii}} \quad 27 - 125a^3 - 135a + 225a^2$$

$$= 3^3 - (5a)^3 - 45a(3 - 5a)$$

$$= 3^3 - (5a)^3 - 3 \times 3 \times 5a(3 - 5a)$$

$$= (3 - 5a)^3 \quad \left[\because a^3 - b^3 - 3ab(a - b) = (a - b)^3 \right]$$

$$= (3 - 5a)(3 - 5a)(3 - 5a) \quad \underline{\Delta}$$

$$\textcircled{\text{iv}} \quad 64a^3 - 27b^3 - 144a^2b + 108ab^2$$

$$= (4a)^3 - (3b)^3 - 36ab(4a - 3b)$$

$$= (4a)^3 - (3b)^3 - 3 \times 4a \times 3b(4a - 3b)$$

$$= (4a - 3b)^3 \cdot \left[\because a^3 - b^3 - 3ab(a - b) = (a - b)^3 \right]$$

$$= (4a - 3b)(4a - 3b)(4a - 3b) \quad \underline{\Delta}$$

$$\textcircled{\text{v}} \quad 27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - \frac{3}{2}p\left(3p - \frac{1}{6}\right)$$

$$= (3p)^3 - \left(\frac{1}{6}\right)^3 - 3 \times 3p \times \frac{1}{6}\left(3p - \frac{1}{6}\right)$$

$$= \left(3p - \frac{1}{6}\right)^3 \quad \left[\because a^3 - b^3 - 3ab(a - b) = (a - b)^3 \right]$$

$$= \left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right)\left(3p - \frac{1}{6}\right) \quad \underline{\Delta}$$