

1.7

$$\because x+1=0$$

$$\Rightarrow x = -1$$

$$\textcircled{i} \quad P(x) = x^3 + x^2 + x + 1$$

$$P(-1) = (-1)^3 + (-1)^2 + (-1) + 1$$

$$= -1 + 1 - 1 + 1$$

$$= -2 + 2$$

$$= 0$$

$\therefore x+1$, बहुपद $P(x)$ का गुणखंड है।

$$\textcircled{ii} \quad P(x) = x^4 + x^3 + x^2 + x + 1$$

$$P(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$$

$$= 1 + (-1) + 1 - 1 + 1$$

$$= 1 - 1 + 1 - 1 + 1$$

$$= 3 - 2$$

$$= 1$$

$\therefore x+1$, बहुपद $P(x)$ का गुणखंड नहीं है।

iii) $P(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$$P(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$$

$$= 1 + 3(-1) + 3(1) - 1 + 1$$

$$= 1 - 3 + 3 - 1 + 1$$

$$= -4 + 5$$

$$= 1$$

$\therefore x+1$, बहुपद $P(x)$ का गुणखंड नहीं है।

iv) $P(x) = x^3 - x^2 - (2+\sqrt{2})x + \sqrt{2}$

$$P(-1) = (-1)^3 - (-1)^2 - (2+\sqrt{2})(-1) + \sqrt{2}$$

$$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$$

$$= -\cancel{1} + \cancel{1} + \sqrt{2} + \sqrt{2}$$

$$= 2\sqrt{2}$$

$\therefore x+1$, बहुपद $P(x)$ का गुणखंड नहीं है।

2.) (i) $P(x) = 2x^3 + x^2 - 2x - 1$
 $g(x) = x + 1$

$$\therefore x + 1 = 0$$

$$\Rightarrow x = -1$$

$$\therefore P(x) = 2x^3 + x^2 - 2x - 1$$

$$P(-1) = 2x(-1)^3 + (-1)^2 - 2(-1) - 1$$

$$= 2x(-1) + 1 + 2 - 1$$

$$= -2 + 1 + 2 - 1$$

$$= -3 + 3$$

$$= 0$$

$\therefore g(x), P(x)$ का गुणनखंड है।

(ii) $P(x) = x^3 + 3x^2 + 3x + 1$
 $g(x) = x + 2$

$$\therefore x + 2 = 0$$

$$\Rightarrow x = -2$$

$$\therefore P(x) = x^3 + 3x^2 + 3x + 1$$

$$= (-2)^3 + 3x(-2)^2 + 3x(-2) + 1$$

$$= -8 + 3 \times 4 - 6 + 1$$

$$= -8 + 12 - 6 + 1$$

$$= -14 + 13$$

$$= -1$$

$\therefore g(x), P(x)$ का गुणनखंड नहीं है।

iii)

$$P(x) = x^3 - 4x^2 + x + 6$$

$$g(x) = x - 3$$

$$\therefore x - 3 = 0$$

$$\Rightarrow x = 3$$

$$P(x) = x^3 - 4x^2 + x + 6$$

$$= (3)^3 - 4 \times 3^2 + 3 + 6$$

$$= 27 - 36 + 9$$

$$= 36 - 36$$

$$= 0$$

$\therefore g(x), P(x)$ का गुणनखंड है।

3)

$$\textcircled{1} P(x) = x^2 + x + K$$

$$\therefore x + 1 = 0$$

$$\Rightarrow x = -1$$

गुणनखण्ड प्रमेय से,

$x + 1$, बहुपद $P(x)$ का गुणनखण्ड है

$$\therefore P(-1) = 0$$

$$\Rightarrow 1^2 + 1 + K = 0$$

$$\Rightarrow 1 + 1 + K = 0$$

$$\Rightarrow 2 + K = 0$$

$$\Rightarrow K = -2$$

$$(ii) \quad P(x) = 2x^2 + Kx + \sqrt{2}$$

$$\therefore x-1=0$$

$$\Rightarrow x=1$$

$$P(1) = 2 \times 1^2 + K \times 1 + \sqrt{2}$$

$$= 2 \times 1 + K + \sqrt{2}$$

$$= 2 + K + \sqrt{2}$$

गुणनखंड प्रमेय से,

$x-1$, बहुपद $P(x)$ का गुणनखंड है।

$$\therefore P(1) = 0$$

$$\Rightarrow 2 + K + \sqrt{2} = 0$$

$$\Rightarrow K = -2 - \sqrt{2}$$

$$= -(2 + \sqrt{2})$$



$$(iii) \quad P(x) = Kx^2 - \sqrt{2}x + 1$$

$$\therefore x-1=0$$

$$\Rightarrow x=1$$

$$P(1) = K \times 1^2 - \sqrt{2} \times 1 + 1$$

$$= K - \sqrt{2} + 1$$

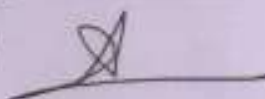
गुणनखंड प्रमेय से,

$x-1$, बहुपद $P(x)$ का गुणनखंड है।

$$\therefore P(1) = 0$$

$$\Rightarrow K - \sqrt{2} + 1 = 0$$

$$\Rightarrow K = \sqrt{2} - 1$$



$$(iv) \quad P(x) = Kx^2 - 3x + K$$

$$\therefore x-1=0$$

$$\Rightarrow x=1$$

$$P(1) = K(1)^2 - 3(1) + K$$

$$= K - 3 + K$$

$$= 2K - 3$$

गुणनखंड प्रमेय से,

$x-1$, बहुपद $P(x)$ का गुणनखंड है।

$$\therefore P(1) = 0$$

$$\Rightarrow 2K - 3 = 0$$

$$\Rightarrow 2K = 3$$

$$\Rightarrow K = \frac{3}{2}$$



$$4. \textcircled{i} \quad 12x^2 - 7x + 1$$

$$= 12x^2 - 4x - 3x + 1$$

$$= 4x(3x-1) - 1(3x-1)$$

$$= (4x-1)(3x-1)$$



$$\textcircled{ii} \quad 2x^2 + 7x + 3$$

$$= 2x^2 + 6x + x + 3$$

$$= 2x(x+3) + 1(x+3)$$

$$= (2x+1)(x+3)$$



$$\textcircled{iii} \quad 6x^2 + 5x - 6$$

$$= 6x^2 + 9x - 4x - 6$$

$$= 3x(2x+3) - 2(2x+3)$$

$$= (3x-2)(2x+3)$$



$$\textcircled{iv} \quad 3x^2 - x - 4$$

$$= 3x^2 - 4x + 3x - 4$$

$$= x(3x-4) + 1(3x-4)$$

$$= (x+1)(3x-4)$$



$$12x = \textcircled{27}$$

$$\textcircled{-4} \quad \textcircled{-3}$$

$$\textcircled{+2} \times \textcircled{+3} = \textcircled{+6}$$

$$\textcircled{+6} \cdot \textcircled{+1}$$

$$\textcircled{+6} \times \textcircled{-6} = \textcircled{-36}$$

$$\textcircled{+9} \quad \textcircled{-4}$$

$$\textcircled{+3} \times \textcircled{-4} = \textcircled{-12}$$

$$\textcircled{-4} \quad \textcircled{+3}$$

5. > (i) $P(x) = x^3 - 2x^2 - x + 2$

यहाँ, अचर पद 2 का गुणनखंड $= \pm 1, \pm 2$

$$\therefore x = 1$$

$$P(1) = 1^3 - 2 \times 1^2 - 1 + 2$$

$$= 1 - 2 - 1 + 2$$

$$= 3 - 3$$

$$= 0$$

अतः $(x-1)$, $P(x)$ का एक गुणनखंड होगा।

$$P(x) = x^3 - 2x^2 - x + 2$$

$$= x^2(x-1) - x(x-1) - 2(x-1)$$

$$= (x-1)(x^2 - x - 2)$$

$$= (x-1)(x^2 - 2x + x - 2)$$

$$= (x-1)[x(x-2) + 1(x-2)]$$

$$= (x-1)(x+1)(x-2)$$



Rough (28)

$$\begin{aligned} P(1) &= 1^3 - 2 \times 1^2 - 1 + 2 \\ &= 1 - 2 - 1 + 2 \\ &= 3 - 3 \\ &= 0 \end{aligned}$$

$$\begin{array}{r} -x^2 \\ -2x^2 \\ + \\ \hline -x^2 \end{array}$$

$$\begin{array}{r} -x \\ +x \\ \hline -2x \\ -2(x) \end{array}$$

(ii) $P(x) = x^3 - 3x^2 - 9x - 5$

यहाँ,

5 का गुणनखंड = $\pm 1, \pm 5$

$\therefore x = -1$

$P(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 \times 1 + 9 - 5$

$= -1 - 3 + 9 - 5$

$= -9 + 9$

$= 0$

अतः $x+1$, $P(x)$ का गुणनखण्ड होगा /

$P(x) = x^3 - 3x^2 - 9x - 5$

$= x^2(x+1) - 4x(x+1) - 5(x+1)$

$= (x+1)(x^2 - 4x - 5)$

$= (x+1)(x^2 - 5x + x - 5)$

$= (x+1)[x(x-5) + 1(x-5)]$

$= (x+1)(x+1)(x-5)$



Rough

29

$P(1) = 1^3 - 3 \times 1^2 - 9 \times 1 + 5$

$= 1 - 3 - 9 + 5$

$= 6 - 12 = -6$

~~$P(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$~~

~~$= -1 - 3 + 9 - 5$~~

$P(-1) = (-1)^3 - 3 \times (-1)^2 - 9 \times (-1) - 5$

$= -1 - 3 + 9 - 5$

$= -9 + 9$

$= 0$

$-3x^2$
 $+ x^2$

$\hline -4x^2$

$-4x(x)$

$-9x$

$-4x$

$\hline -9x$

$-5(x)$

(iii) $P(x) = x^3 + 13x^2 + 32x + 20$

यहाँ,

$\therefore 20$ का गुणनखंड $= \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$

$\therefore x = -1$

$P(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= -1 + 13 - 32 + 20$

$= -33 + 33$

$= 0$

अतः $x+1$, $P(x)$ का गुणनखंड होगा।

$P(x) = x^3 + 13x^2 + 32x + 20$

$\therefore = x^2(x+1) + 12x(x+1) + 20(x+1)$

$= (x+1)(x^2 + 12x + 20)$

$= (x+1)(x^2 + 10x + 2x + 20)$

$= (x+1)[x(x+10) + 2(x+10)]$

$= (x+1)(x+2)(x+10)$



Rough

(30)

$P(1) = 1^3 + 13 \times 1^2 + 32 \times 1 + 20$

$P(-1) = (-1)^3 + 13(-1)^2 + 32(-1) + 20$

$= -1 + 13 - 32 + 20$

$= -33 + 33$

$= 0$

$+13x^2$
 $+x^2$

$+12x^2$

$+12x(x)$

$+32x$

$+12x$

$+20x$

(iv)

$$P(y) = 2y^3 + y^2 - 2y + 1$$

यहाँ,

$$1 \text{ का गुणनखंड } = \pm 1$$

$$\therefore y = 1$$

$$P(1) = 2 \times 1^3 + 1^2 - 2 \times 1 - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

अतः $y = 1$, $P(y)$ का गुणनखंड होगा।

$$P(y) = 2y^3 + y^2 - 2y - 1$$

$$= 2y^2(y-1) + 3y(y-1) + 1(y-1)$$

$$= (y-1)(2y^2 + 3y + 1)$$

$$= (y-1)(2y^2 + 2y + y + 1)$$

$$= (y-1)[2y(y+1) + 1(y+1)]$$

$$= (y-1)(y+1)(2y+1)$$



(3)

$$P(1) = 2 \times 1^3 + 1^2 - 2 \times 1 - 1$$

$$= 2 + 1 - 2 - 1$$

$$= 3 - 3$$

$$= 0$$

$$\begin{array}{r} +y^2 \\ -2y^2 \\ \hline +3y^2 \\ +3y(y) \end{array}$$

$$\begin{array}{r} -2y \\ -3y \\ \hline +y \\ +1(y) \end{array}$$