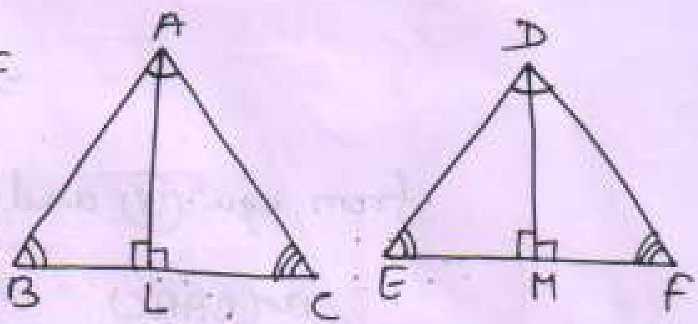
Theorem - 6.6 → The ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Given that :- $\Delta ABC \sim \Delta DEF$

To Prove :- $\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$

Construction :- Draw $AL \perp BC$ and $DM \perp EF$.



Proof :- ∵ $\Delta ABC \sim \Delta DEF$

$\angle A = \angle D$ — (i)

$\angle B = \angle E$ — (ii)

$\angle C = \angle F$ — (iii)

and,

$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$ — (iv)

Now,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{\frac{1}{2} \times BC \times AL}{\frac{1}{2} \times EF \times DM} = \frac{BC}{EF} \times \frac{AL}{DM} \quad \text{— (v)}$$

∵ In ΔALB and ΔDME

$\angle B = \angle E$ [from equⁿ (ii)]

$\angle ALB = \angle DME$ (90°)

∴ $\Delta ALB \sim \Delta DME$ [AA-criteria]

∴ $\frac{AB}{DE} = \frac{BL}{EM} = \frac{AL}{DM}$

$$\because \quad \frac{AB}{DE} = \frac{AL}{DM}$$

$$\Rightarrow \quad \frac{BC}{EF} = \frac{AL}{DM} \quad \text{— (VI)} \quad \left[\text{from equ}^n \text{ (IV)}\right]$$

from equ^n (V),

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$= \frac{BC}{EF} \times \frac{BC}{EF}$$

$$= \frac{BC^2}{EF^2} \quad \text{— (VII)}$$

from equ^n (IV) and (VII), we get

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Proved

Exercise - 6.4

(1) Given that:- $\Delta ABC \sim \Delta DEF$

$ar(\Delta ABC) = 64\ cm^2$

$ar(\Delta DEF) = 121\ cm^2$

$EF = 15.4\ cm$

$BC = ?$

$\because \Delta ABC \sim \Delta DEF$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \frac{64}{121} = \left(\frac{BC}{EF}\right)^2$$

$$\Rightarrow \sqrt{\frac{64}{121}} = \frac{BC}{EF}$$

$$\Rightarrow \frac{8}{11} = \frac{BC}{15.4}$$

$$\Rightarrow BC = \frac{8}{11} \times 15.4$$

$$= \frac{8}{\cancel{11}} \times \frac{\overset{14}{\cancel{154}}}{10}$$

$$= \frac{112}{10}$$

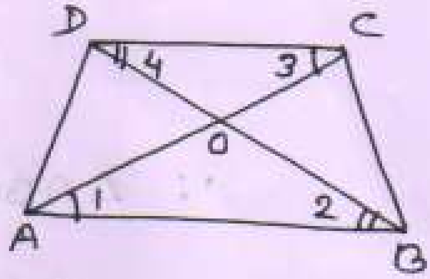
$$= 11.2\ cm$$

⟨2⟩ Given that:- In traperium ABCD,

$AB \parallel DC$

$AB = 2DC$

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = ?$$



∵ In ΔAOB and ΔCOD,

$\angle 1 = \angle 3$ [Alt. int. ∠s]

$\angle 2 = \angle 4$ [Alt. int. ∠s]

∴ $\Delta AOB \sim \Delta COD$ [AA-criteria]

$$\frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{DC^2}$$

$$= \frac{(2DC)^2}{DC^2}$$

$$= \frac{4\cancel{DC^2}}{\cancel{DC^2}}$$

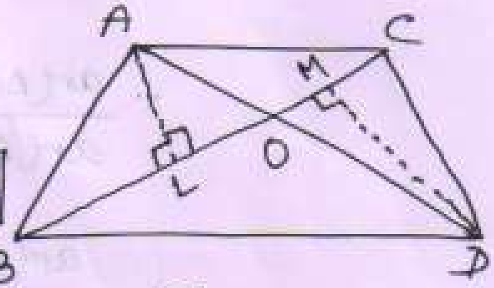
$$= \frac{4}{1}$$

$= 4:1$ Ans

⟨3.⟩ **Given that:-** ΔABC and ΔDBC are two triangles on the same base BC.

To Prove:- $\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$

Const:- Draw $AL \perp BC$ and $DM \perp BC$.



Proof:- In ΔALO and ΔDMO,

$\angle ALO = \angle DMO$ (90°)

$\angle AOL = \angle DOM$ [vert. opp. ∠s]

$\therefore \Delta ALO \sim \Delta DMO$ [AA-criteria]

$\therefore \frac{AL}{DM} = \frac{AO}{DO}$ —— (1)

Now,

$$\frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{\cancel{\frac{1}{2}} \times \cancel{BC} \times AL}{\cancel{\frac{1}{2}} \times \cancel{BC} \times DM} = \frac{AL}{DM}$$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DBC)} = \frac{AO}{DO} \quad \text{[from equ}^n \text{ (1)]}$$

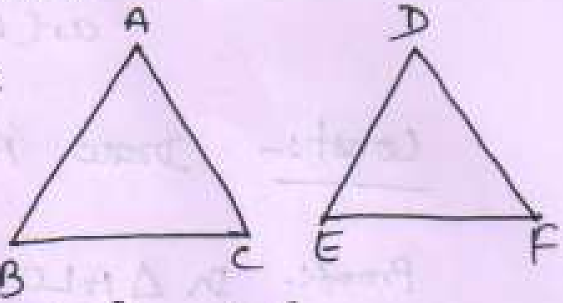
Proved

⟨4⟩ Given that:- $\Delta ABC \sim \Delta DEF$

$ar(\Delta ABC) = ar(\Delta DEF)$ —— (1)

To prove :- $\Delta ABC \cong \Delta DEF$

Proof:- $\because \Delta ABC \sim \Delta DEF$



$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\Rightarrow \frac{\cancel{ar(\Delta ABC)}}{\cancel{ar(\Delta ABC)}} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\Rightarrow 1 = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

$$\because \frac{AB^2}{DE^2} = 1$$

$$\Rightarrow \left(\frac{AB}{DE}\right)^2 = 1$$

$$\Rightarrow \frac{AB}{DE} = \sqrt{1}$$

$$\Rightarrow \frac{AB}{DE} = 1$$

$\Rightarrow AB = DE$

Similarly,

$BC = EF$

and $AC = DF$

In ΔABC and ΔDEF

$AB = DE$, $BC = EF$, $AC = DF$

$\therefore \Delta ABC \cong \Delta DEF$ [by S-S-S congruence]

Proved

⟨5⟩ Given that:- D, E and F are respectively the mid-points of sides AB, BC and CA of ΔABC.

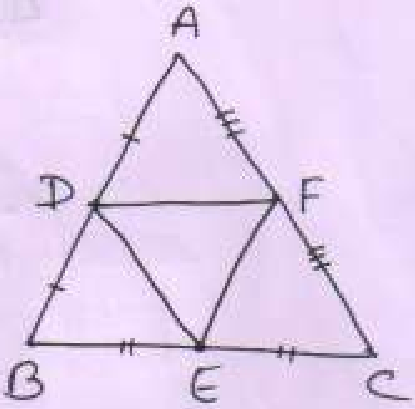
$$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = ?$$

$\because$ D and F are the mid point of sides AB and AC respectively.

$\therefore$ by mid point theorem,

DF ∥ BC and

$DF = \frac{1}{2} BC$



$\therefore \frac{DF}{BC} = \frac{1}{2}$ — (I)

Again,

$\because$ D and E are the mid-point of AB and BC respectively,

$\therefore$ by mid point theorem,

DE ∥ AC and

$\Rightarrow DE = \frac{1}{2} AC$

$\Rightarrow \frac{DE}{AC} = \frac{1}{2}$ — (II)

Again,

$\because$ E and F are the mid-point of BC and AC respectively.

$\therefore$ EF ∥ AB and

$EF = \frac{1}{2} AB$

$\Rightarrow \frac{EF}{AB} = \frac{1}{2}$ — (III) from equ^n (I), (II) and (III),

$$\frac{DE}{AC} = \frac{DF}{BC} = \frac{EF}{AB} = \frac{1}{2}$$

∴ Corresponding sides are proportion.

∴ $\Delta DEF \sim \Delta ABC$ [SSS-similarity]

∴ $$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{DE^2}{AC^2}$$

$$= \left(\frac{DE}{AC}\right)^2$$

$$= \left(\frac{1}{2}\right)^2$$

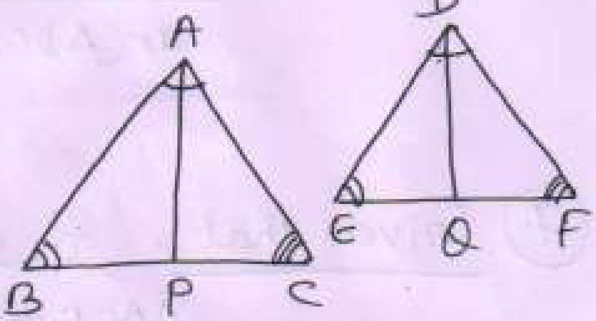
$$= \frac{1}{4}$$

$$= 1:4$$

<6> <u>Given that</u>:- $\Delta ABC \sim \Delta DEF$

AP and DQ are the medium of ΔABC and ΔDEF respectively.

<u>To prove</u>:- $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AP^2}{DQ^2}$



<u>Proof</u>:- $\because \Delta ABC \sim \Delta DEF$

$$\therefore \left.\begin{aligned} \angle A &= \angle D \\ \angle B &= \angle E \\ \angle C &= \angle F \end{aligned}\right\} \text{—(1)}$$

and

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \text{—(11)}$$

$\because \Delta ABC \sim \Delta DEF$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2} \quad \text{—(111)}$$

from equ^n (11), then

$$\frac{AB}{DE} = \frac{BC}{EF}$$

$$\Rightarrow \frac{AB}{DE} = \frac{\cancel{2}BP}{\cancel{2}EQ} \quad \left[\begin{aligned} &\text{P is mid-point of BC} \\ &\text{Q is the mid-point of EF} \end{aligned}\right]$$

$$\Rightarrow \frac{AB}{DE} = \frac{BP}{EQ}$$

In ΔABP and ΔDEQ,

$$\frac{AB}{DE} = \frac{BP}{EQ} \text{ and } \angle B = \angle E$$

$\therefore \Delta ABP \sim \Delta DEQ$ [S-A-S criteria]

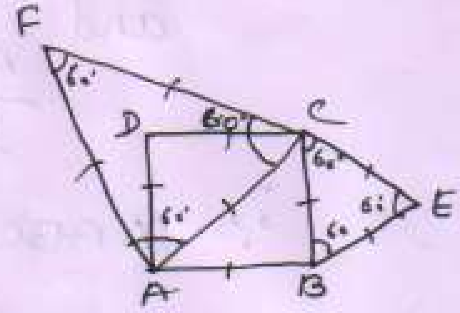
$$\therefore \frac{AB}{DE} = \frac{BP}{EQ} = \frac{AP}{DQ}$$

from equⁿ (III),

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \left(\frac{AB}{DE}\right)^2 = \left(\frac{AP}{DQ}\right)^2 = \frac{AP^2}{DQ^2}$$

Proy

(7.) Given that:- In Square ABCD,
ΔACF and ΔBCE are two equilateral triangle described on the diagonals AC and side BC of the square ABCD.



To prove:- $ar(\Delta BCE) = \frac{1}{2} ar(\Delta ACF)$

Proof:- Diagonal of square $= AC = \sqrt{2}\, BC$

In ΔBCE and ΔACF

$\angle E = \angle F$ (60°)

$\angle EBC = \angle ACF$ (60°)

$\angle ECB = \angle CAF$ (60°)

$\therefore \Delta BCE \sim \Delta ACF$ [A-A Criteria]

$$\therefore \frac{ar(\Delta BCE)}{ar(\Delta ACF)} = \frac{BC^2}{AC^2} = \frac{BC^2}{(\sqrt{2}BC)^2}$$

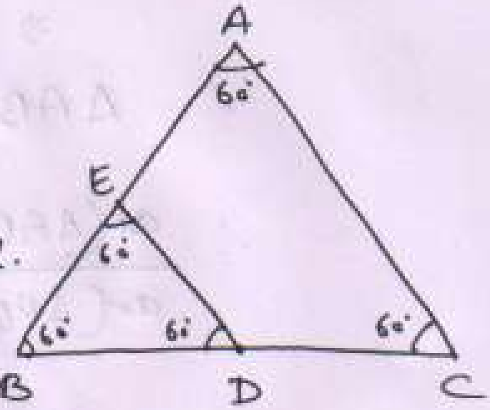
$$= \frac{\cancel{BC^2}}{2\cancel{BC^2}}$$

$$= \frac{1}{2}$$

$\therefore ar(\Delta BCE) = \frac{1}{2} ar(\Delta ACF)$

Proy

<8> Given that: ΔABC and ΔBDE are two equilateral triangle and D is the mid point of BC.



$\therefore \quad BD = DC = \frac{1}{2} BC$

let each side of triangle is a.

Now,

$\Delta ABC \sim \Delta BDE$

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{BC^2}{BD^2}$$

$$= \frac{a^2}{\left(\frac{1}{2}BC\right)^2}$$

$$= \frac{a^2}{\left(\frac{1}{2}a\right)^2}$$

$$= \frac{a^2}{\frac{1}{4}a^2}$$

$$= \frac{1}{\frac{1}{4}}$$

$$= \frac{4}{1}$$

$$= 4:1$$

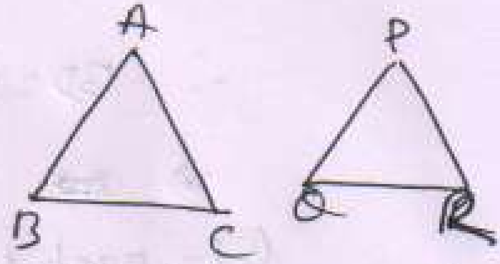
C

9. let ΔABC and ΔPQR in which

$$BC : QR = 4 : 9$$

$$\Rightarrow \frac{BC}{QR} = \frac{4}{9}$$

$$\Delta ABC \sim \Delta PQR$$



$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

the ratio of the area of triangles will be equal to the square of the ratio of the corresponding sides.

$$= \left(\frac{BC}{QR}\right)^2$$

$$= \left(\frac{4}{9}\right)^2$$

$$= \frac{16}{81}$$

$$= 16 : 81$$