Theorem - 10.1 → The tangent at any point of a circle is perpendicular to the radius through the point of contact.

OR

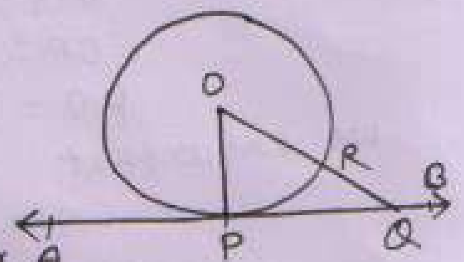
The tangent is the perpendicular to the radius of circle.

OR

The radius of circle is the perpendicular to the tangent.

Given that:- A circle with centre O and a tangent AB at a point P of the circle

To Prove:- $OP \perp AB$



Constr:- Take a point Q, other than P, on AB.
join OQ.

Proof:- ∵ Q lies outside the circle.

$\therefore \quad OQ > OR$

$\Rightarrow \quad OQ > OP$

$\Rightarrow \quad OP < OQ$

Thus,
OP is shorter than any other line segment joining O to any point of AB.

∴ OP is the shortest distance between the point and the line AB.

but, the shortest distance between a point and a line is the perpendicular distance.

$\therefore \; OP \perp AB$ Proved

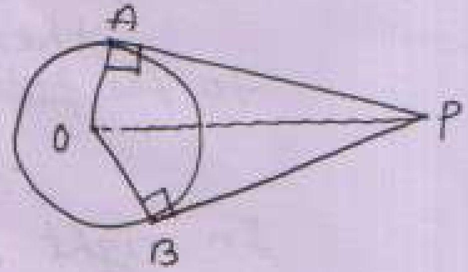
Theorem - 10.2 The length of tangents drawn from an external point to a circle are equal.

Given that:- Two tangents PA and PB are drawn from a point P to a circle with centre O.

To Prove:- (I) $PA = PB$

(II) $\angle POA = \angle POB$

(III) $\angle APO = \angle BPO$



Const:- join OP.

Proof:- Since,

The radius of circle is the perpendicular to the tangent of a circle.

$\therefore OA \perp PA$

$\therefore \angle A = 90^\circ$

and $OB \perp PB$

$\therefore \angle B = 90^\circ$

In right ΔOAP and ΔOBP,

$OA = OB$ (radius)

$\angle A = \angle P$ (90°

$OP = OP$

$\therefore \Delta OAP \cong \Delta OBP$ [R.H.S]

$\therefore PA = PB$ (CPCT)

$\angle POA = \angle POB$ (CPCT)

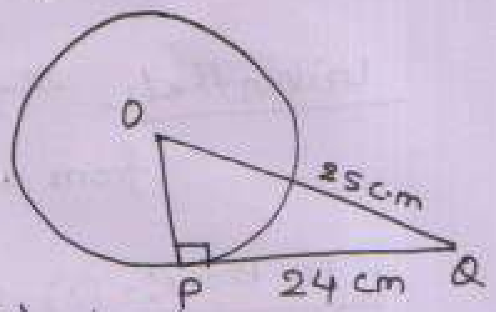
$\angle APO = \angle BPO$ (CPCT)

Ex-10.2

1.) **Given that:-** PQ is a tangent.
OP is radius.

$PQ = 24\,cm$

$OQ = 25\,cm$



Since,
the radius of circle is the perpendicular to the ~~pa~~ tangent of circle.

$\therefore\ OP \perp PQ$

In right $\triangle OPQ$, $\angle P = 90^\circ$

$OP = \sqrt{OQ^2 - PQ^2}$ [Pythagorous theorem]

$= \sqrt{25^2 - 24^2}$

$= \sqrt{625 - 576}$

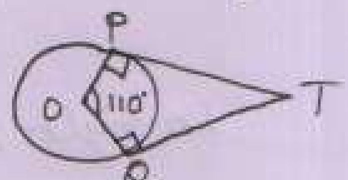
$= \sqrt{49}$

$OP = 7\,cm$ Ans

2. **Given that:-** TP and TQ are the two tangents to a circle with centre O.

$\angle POQ = 110^\circ$

$\angle PTQ = ?$



Since,
the radius of circle is the perpendicular to the tangent of circle.

$OP \perp TP$

$OQ \perp TQ$

In quadrilateral POQT,
the sum of four angles are 360°.

$\therefore\ \angle POQ + \angle P + \angle Q + \angle PTQ = 360^\circ$

$\Rightarrow 110^\circ + 90^\circ + 90^\circ + \angle PTQ = 360^\circ$

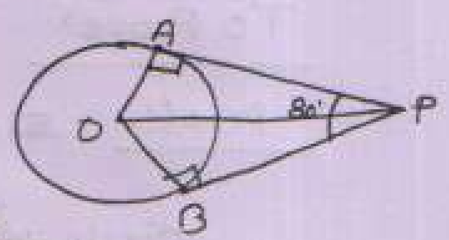
$\Rightarrow 290^\circ + \angle PTQ = 360^\circ$

$\Rightarrow \angle PTQ = 360^\circ - 290^\circ = 70^\circ$ (B) Ans

3.) <u>Given that:-</u> PA and PB are two tangents

$\angle APB = 80^\circ$

$\angle POA = ?$



Since,

the radius of circle is the perpendicular to the tangents.

$\therefore$ $OA \perp PA$

~~Again~~, $OB \perp PB$

two tangents are equally inclined to the line segment joining the centre to that points

$\angle APO = \angle BPO$

$\therefore$ $\angle APO = \angle BPO = \frac{1}{2} \angle APB = \frac{1}{2} \times 80^\circ = 40^\circ$

In right ΔPAO, $\angle A = 90^\circ$

~~$\angle B + \angle BPO + \angle$~~

$\angle A + \angle POA + \angle APO = 180^\circ$ [properties of Δ]

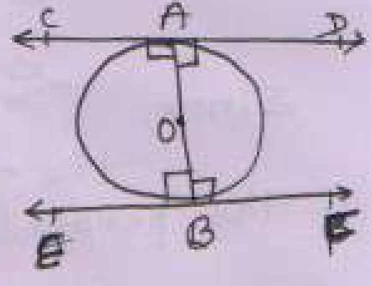
$\Rightarrow$ $90^\circ + \angle POA + 40^\circ = 180^\circ$

$\Rightarrow$ $130^\circ + \angle POA = 180^\circ$

$\Rightarrow$ $\angle POA = 180^\circ - 130^\circ = 50^\circ$. (A) A

4) Given that:- CD and EF are the tangents at the end points A and B of the diameter AB of a circle.

To Prove:- $CD \parallel EF$



Proof:- Since,
the tangent of a circle is the perpendicular to the radius of circle.

$\therefore \quad CD \perp OA$

$\therefore \angle OAD = \angle OAC = 90^\circ$

and,

$EF \perp OB$

$\therefore \angle OBE = \angle OBF = 90^\circ$

then,

~~$\angle BAD$~~ $\angle OAD = \angle OBE = 90^\circ$

$\Rightarrow \angle BAD = \angle ABE \; (90^\circ)$

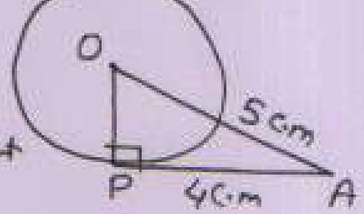
but, they are alternate interior angles.

$CD \parallel EF$

Proved

6) Given that:- A circle with centre O and AP is a tangent.

$OA = 5\,cm$
$AP = 4\,cm$



$\because$ the radius of circle is the perpendicular to the tangent of circle.

$OP \perp AP, \quad \angle P = 90^\circ$

In right $\triangle OAP$, $\angle P = 90^\circ$

$OP = \sqrt{OA^2 - AP^2}$ [Pythagorous theorem]

$= \sqrt{5^2 - 4^2}$

$= \sqrt{25 - 16}$

$= \sqrt{9}$

$= 3\,cm$,

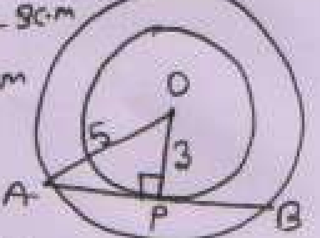
$\therefore$ radius $= OP = 3\,cm$

7) Given that:- Two concentric circles with centre O.

radius of small circle = OP = 3cm

radius of big circle = OA = 5cm

AB = ?



Since,

the radius of circle is the perpendicular to the tangent of circle.

$OP \perp AB$

In right $\triangle OAP$, $\angle OPA = 90°$

$AP = \sqrt{OA^2 - OP^2}$ [Pythagorous theorem]

$= \sqrt{5^2 - 3^2}$

$= \sqrt{25 - 9}$

$= \sqrt{16}$

$\therefore AP = 4$ cm

but,

the perpendicular from the centre to the chord bisects the chord.

$\therefore AP = BP$

$\therefore AP = BP = \frac{1}{2} AB$

$\Rightarrow AB = 2AP$

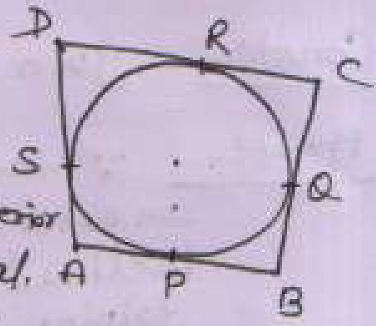
$= 2 \times 4$

$= 8$ cm

8.) Given that:- A quad. ABCD is drawn to circumscribe a circle.

To Prove:- $AB + CD = AD + BC$

Proof:- We know that, the lengths of tangents drawn from an exterior point to a circle are equal.



$\therefore$ from point A,

$AP = AS$ — (I)

from point B,

$BP = BQ$ — (II)

from point C,

$CR = CQ$ — (III)

from point D,

$DR = DS$ — (IV)

adding (I), (II), (III) (IV). We get

$AP + BP + CR + DR = AS + BQ + CQ + DS$

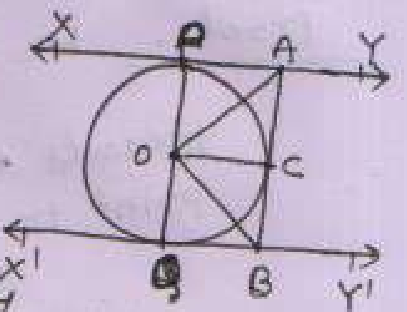
$\Rightarrow AB + CD = AD + BC$

Proved

9.) Given that:- $XY \parallel X'Y'$ are two tangents to a circle with centre O.
AB is another tangent.

To Prove:- $\angle AOB = 90^\circ$

Const:- join OC.



Proof:- ∵ tangent ~~at any~~ from an external point are equally inclined to the line segment joining the centre to that point.

$$\therefore \angle OAP = \angle OAC = \frac{1}{2}\angle A$$

$$\therefore \angle A = 2\angle OAP = 2\angle OAC \quad —(I)$$

and,

$$\angle OBQ = \angle OBC = \frac{1}{2}\angle B$$

$$\Rightarrow \angle B = 2\angle OBQ = 2\angle OBC \quad —(II)$$

Again,

$XY \parallel X'Y'$ and AQ is a transparent line.

$$\therefore \angle A + \angle B = 180^\circ \quad [\text{Co-interior } \angle s]$$

$$\Rightarrow 2\angle OAC + 2\angle OBC = 180^\circ$$

$$\Rightarrow \cancel{2}(\angle OAC + \angle OBC) = \cancel{180^\circ}\ 90^\circ$$

$$\Rightarrow \angle OAC + \angle OBC = 90^\circ$$

$$\Rightarrow \angle OAB + \angle OBA = 90^\circ \quad —(III)$$

In $\triangle AOB$,

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ \quad [\text{Sum of angles of } \triangle \text{ is } 180^\circ]$$

$$\Rightarrow \angle AOB + 90^\circ = 180^\circ$$

$$\Rightarrow \angle AOB = 180^\circ - 90^\circ$$

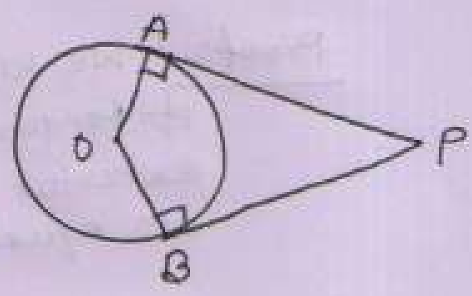
$$\Rightarrow \angle AOB = 90^\circ$$

Proved

10.) Given that :- PA and PB are the tangents drawn from a point P to a circle with centre O.

To Prove :- $\angle APB + \angle AOB = 180^\circ$

Proof:- We know that,



the tangent to a circle is perpendicular to the radius through the point of contact.

$\therefore\ PA \perp OA$

$\Rightarrow \angle OAP = 90^\circ$

and,

$PB \perp OB$

$\Rightarrow \angle OBP = 90^\circ$

But,

We know that the sum of all angles of a quadrilateral is 360°.

$\therefore\ \angle OAP + \angle OBP + \angle APB + \angle AOB = 360^\circ$

$\Rightarrow\ 90^\circ + 90^\circ + \angle APB + \angle AOB = 360^\circ$

$\Rightarrow\ 180^\circ + \angle APB + \angle AOB = 360^\circ$

$\Rightarrow\ \angle APB + \angle AOB = 360^\circ - 180^\circ$

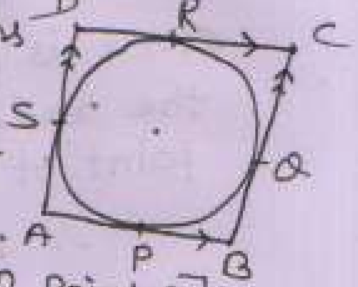
$\Rightarrow\ \angle APB + \angle AOB = 180^\circ$

Proved

11.) Given that :- A parallelogram ABCD circumscribes a circle with centre O.

To. Prove :- $AB = BC = CD = AD$
Such that,
Parallelogram ABCD is a rhombus.

Proof :- We know that the lengths of tangents drawn from an exterior point to a circle are equal.



$\therefore$ $AP = AS$ — (I) [From Point A]
$BP = BQ$ — (II) [" Point B]
$CR = CQ$ — (III) [" Point C]
$DR = DS$ — (IV) [" Point D]

Adding equ^n (I), (II), (II) and (IV), we get
$AP + BP + CR + DR = AS + BQ + CQ + DS$
Now, $\Rightarrow$ $AB + CD = AD + BC$ — (V)

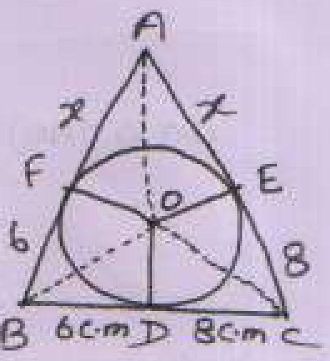
In parallelogram ABCD,
$AB = CD$ — (VI)
$AD = BC$ — (VII)
[opposite sides of a llgm are equal.]

from equ^n (V), we get
$AB + CD = AD + BC$
$\Rightarrow$ $AB + AB = AD + AD$ [from equ^n (VI) and (VI)]
$\Rightarrow$ $\cancel{2}AB = \cancel{2}AD$
$\Rightarrow$ $AB = AD$ — (VIII)

from equ^n (VI), (VII), (VIII), we get
$AB = BC = CD = AD$
$\therefore$ ABCD is a rhombus. Proved

12.) Given that:- A ΔABC is drawn to circumscibe a circle. ~~of radius~~

$OD = OE = OF = 4cm$



$BD = 6cm$

$DC = 8cm$

$AB = ?$

$AC = ?$

join OA, OB, OC

$\because$ We know that the lengths of tangents drawn from an exterior point to a circle are equal.

$AE = AF = x\,cm$ (let)

$BD = BF = 6cm$

$CD = CE = 8cm$

In ΔABC,

$a = BC = 6+8 = 14\,cm$

$b = AC = (x+8)\,cm$

$c = AB = (x+6)\,cm$

$\therefore S = \frac{a+b+c}{2} = \frac{14+x+8+x+6}{2}$

$= \frac{2x+28}{2}$

$= \frac{\cancel{2}(x+14)}{\cancel{2}}$

by heorine formula, $= x+14$

$ar(\Delta ABC) = \sqrt{S(S-a)(S-b)(S-c)}$

$= \sqrt{(x+14)(x+\cancel{14}-\cancel{14})(\cancel{x}+14-\cancel{x}-8)(\cancel{x}+14-\cancel{x}-6)}$

$= \sqrt{(x+14)\cdot x\cdot 6\times 8}$

Again, $= \sqrt{48x(x+14)}$

We know that the radius of circle is the perpendicular to the tangents.

$\therefore OD \perp BC$

$OE \perp AC$

$OF \perp AB$

$\therefore$ $ar(\Delta OBC) = \frac{1}{2} \times BC \times OD$

$= \cancel{\frac{1}{2}} \times 14 \times \cancel{4}^{2}$

$= 28\ cm^2$

$ar(\Delta OAC) = \frac{1}{2} \times AC \times OE$

$= \cancel{\frac{1}{2}} \times (x+8) \times \cancel{4}^{2}$

$= (2x+16)\ cm^2$

$ar(\Delta OAB) = \frac{1}{2} \times (x+6) \times OF$

$= \cancel{\frac{1}{2}} \times (x+6) \times \cancel{4}^{2}$

$= (2x+12)\ cm^2$

~~then~~ Now,

$ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta OAC) + ar(\Delta OAB)$

$\Rightarrow \sqrt{48x(x+14)} = 28 + 2x + 16 + 2x + 12$

$\Rightarrow \sqrt{48x(x+14)} = 4x + 56$

$\Rightarrow \sqrt{48x(x+14)} = 4(x+14)$

squaring both sides,

$\Rightarrow \left(\sqrt{48x(x+14)}\right)^2 = [4(x+14)]^2$

$\Rightarrow {}^{3}\cancel{48}x(x+14) = \cancel{16}(x+14)^2$

$\Rightarrow 3x\cancel{(x+14)} = \cancel{(x+14)}(x+14)$

$\Rightarrow 3x = x + 14$

$\Rightarrow 3x - x = 14$

$\Rightarrow 2x = 14$

$\Rightarrow x = \frac{\cancel{14}}{\cancel{2}} = 7$

$\therefore$ $AB = x + 6 = 7 + 6 = 13\ cm$

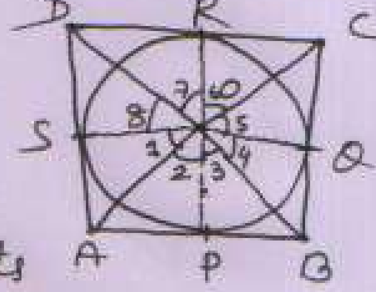
$AC = x + 8 = 7 + 8 = 15\ cm$

13) Given that:- A quad. ABCD circumscribe a circle with centre O.

To Prove:- $\angle AOB + \angle COD = 180°$

$\angle AOD + \angle BOC = 180°$

Const:- join OP, OQ, OR, OS.



Proof:- We know that the tangents drawn from an external point of a circle subtend equal angles at the centre.

$\therefore \angle 1 = \angle 2$ — (i) [from point A]

$\angle 3 = \angle 4$ — (ii) [" " B]

$\angle 5 = \angle 6$ — (iii) [" " C]

$\angle 7 = \angle 8$ — (iv) [" " D]

Adding equ^n (i), (ii), (iii), (iv). we get,

$\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

$\Rightarrow \angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360°$

$\Rightarrow 2\angle 2 + 2\angle 3 + 2\angle 6 + 2\angle 7 = 360°$

$\Rightarrow \cancel{2}(\angle 2 + \angle 3 + \angle 6 + \angle 7) = \cancel{360°}\ 180°$

$\Rightarrow \angle 2 + \angle 3 + \angle 6 + \angle 7 = 180°$

$\Rightarrow \angle AOB + \angle COD = 180°$

Proved

Again,

$\angle AOB + \angle COD + \angle AOD + \angle BOC = 360°$

$\Rightarrow 180° + \angle AOD + \angle BOC = 360°$

$\Rightarrow \angle AOD + \angle BOC = 360° - 180°$

$\Rightarrow \angle AOD + \angle BOC = 180°$

Proved