AM5630 Foundations of Computational Fluid Dynamics

Computer Assignment 3

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Contents

1 Lid-driven cavity

1.1 Problem Statement

Consider a square cavity of side, a=1 unit, as shown in Figure 1. The cavity is filled with an incompressible fluid. Assume the flow to be a steady, 2-dimensional flow. Determine the fluid flow pattern and pressure distribution inside the cavity at Reynolds number, Re = 100, using any finite difference method. Use the non-dimensional form of the governing equation with appropriate boundary conditions. This setup is called the Lid-driven cavity, a benchmark problem in the field of Computational Fluid Dynamics.

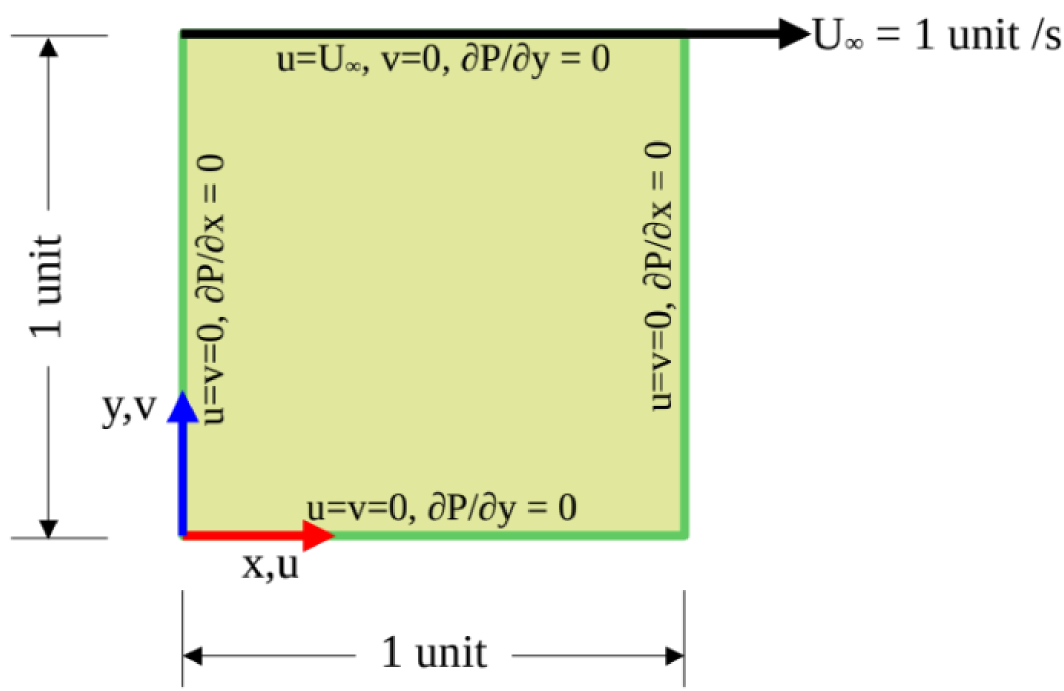

Figure 1: Computational domain of Lid-driven square cavity

1.2 Governing Equations

We will be using the stream function-vorticity approach to solve this problem. To obtain the governing equations, we use the Navier-Stokes equation, the vorticity equation, the relation between vorticity and stream function, and the continuity equation, which are as given below:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -(1/\rho)\frac{\partial p}{\partial x} + (1/Re)[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}] \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -(1/\rho)\frac{\partial p}{\partial y} + (1/Re)[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}] \tag{2}$$

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \tag{3}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{4}$$

$$\nabla^2\psi = -\omega \tag{5}$$

partially differentiating eq (2) with x, and eq (1) with y and subtracting both gives us:

$$\frac{\partial \omega}{\partial t} + u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = (1/Re)[\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}] \tag{6}$$

partially differentiating eq (1) with x and eq (2) with y and adding both gives us:

$$\frac{\partial^2 p}{\partial^2 x} + \frac{\partial^2 p}{\partial^2 y} = 2\rho[\frac{\partial^2 \psi}{\partial x^2}\frac{\partial^2 \psi}{\partial y^2} - (\frac{\partial^2 \psi}{\partial y \partial x})^2] \tag{7}$$

1.3 Boundary and Initial Conditions

The following boundary conditions are imposed:

Top wall velocity $U_\infty = 1$ m/s

remaining walls u , $v = 0$

$\frac{\partial p}{\partial n} = 0$ (Neumann boundary condition)

1.4 Numerical Formulation (Stream function Vorticity approach)

Equation (6) is called the vorticity transport equation (parabolic in nature), and equation (5) is Poisson's equation. We will use these equations to find the steady-state vorticity and stream function contours. The algorithm to compute these values is as follows:

Algorithm:

1. Initialize ω and ψ at t = 0, and velocities as per boundary condition.
2. Solve vorticity transport equation (6) at interior grid points to find ω.
3. Solve the Poisson's equation (5) at all interior points to find ψ.
4. Stream function is a constant on the walls and can be set to zero.
5. Find u and v using the relation $u = \frac{\partial \psi}{\partial y}$ and $v =$ -$\frac{\partial \psi}{\partial y}$.
6. Determine ω on boundaries using ψ and ω at interior points.
7. Check for convergence. If not, return to step 2.

1.5 Pseudo Code

1. Define all variables as shown below:

```
tolerance = 1e-4;
Nx = 51;
Ny = 51;
Re = 100;
gamma = 1/Re;
delta = 1/(Nx-1);
u_inf = 1;
rho = 1;

psi_k = zeros(Ny,Nx);
psi_k_1 = zeros(Ny,Nx);

vor_k = zeros(Ny,Nx);
vor_k_1 = zeros(Ny,Nx);

P_k = zeros(Ny,Nx);
P_k_1 = zeros(Ny,Nx);

u = zeros(Ny,Nx);
u(Ny,:) = u_inf;
v= zeros(Ny,Nx);

condition = true;
iter = 0;
beta = delta/(2*gamma);
```

2. Set up a while loop with the condition over ω. If abs$(\omega_{k+1} - \omega_k)$ is $<$ threshold value, break the loop, or else continue.

3. Update the interior points of ω using the discretized vorticity transport equation. (temporal term can be dropped as we analyze only the steady-state flow properties)

```
for i = 2:Ny-1
    for j = 2:Nx-1
        vor_k_1(i,j) = 0.25*(vor_k_1(i,j-1)*(1 + beta*u(i,j)) + vor_k_1(i-1,j)*(1 + beta*v(i,j))
                       + (1 - beta*u(i,j))*vor_k(i,j+1) + (1 - beta*v(i,j))*vor_k(i+1,j));
    end
end
```

4. Update the interior points of ψ using the discretized Poisson's equation and the ω values obtained in the previous step. I have used **point Gauss-Seidel** for discretization.

```
for i = 2:Ny-1
    for j = 2:Nx-1
        psi_k_1(i,j) = 0.25*(psi_k(i,j+1) + psi_k(i+1,j) + psi_k_1(i-1,j) + psi_k_1(i,j-1) + (delta^2)*vor_k_1(i,j));
    end
end
```

5. u and v can be computed after obtaining the ψ values in the previous step.

```
for i = 2:Ny-1
    for j = 2:Nx-1
        u(i,j) = (psi_k_1(i+1,j) - psi_k_1(i-1,j))/(2*delta);
        v(i,j) = -(psi_k_1(i,j+1) - psi_k_1(i,j-1))/(2*delta);
    end
end
```

6. ω on the boundaries can be computed using the ψ values calculated above.

```
vor_k_1(1,:) = -2*psi_k_1(2,:)/(delta^2);
vor_k_1(Ny,:) = -2*(u_inf*delta + psi_k_1(Ny-1,:))/(delta^2);
vor_k_1(:,1) = -2*psi_k_1(:,2)/(delta^2);
vor_k_1(:,Nx) = -2*psi_k_1(:,Nx-1)/(delta^2);
```

7. Repeat the above steps as long as the while loop isn't exited.

1.6 Plots

Below are the plots that have been obtained after implementing the above scheme.

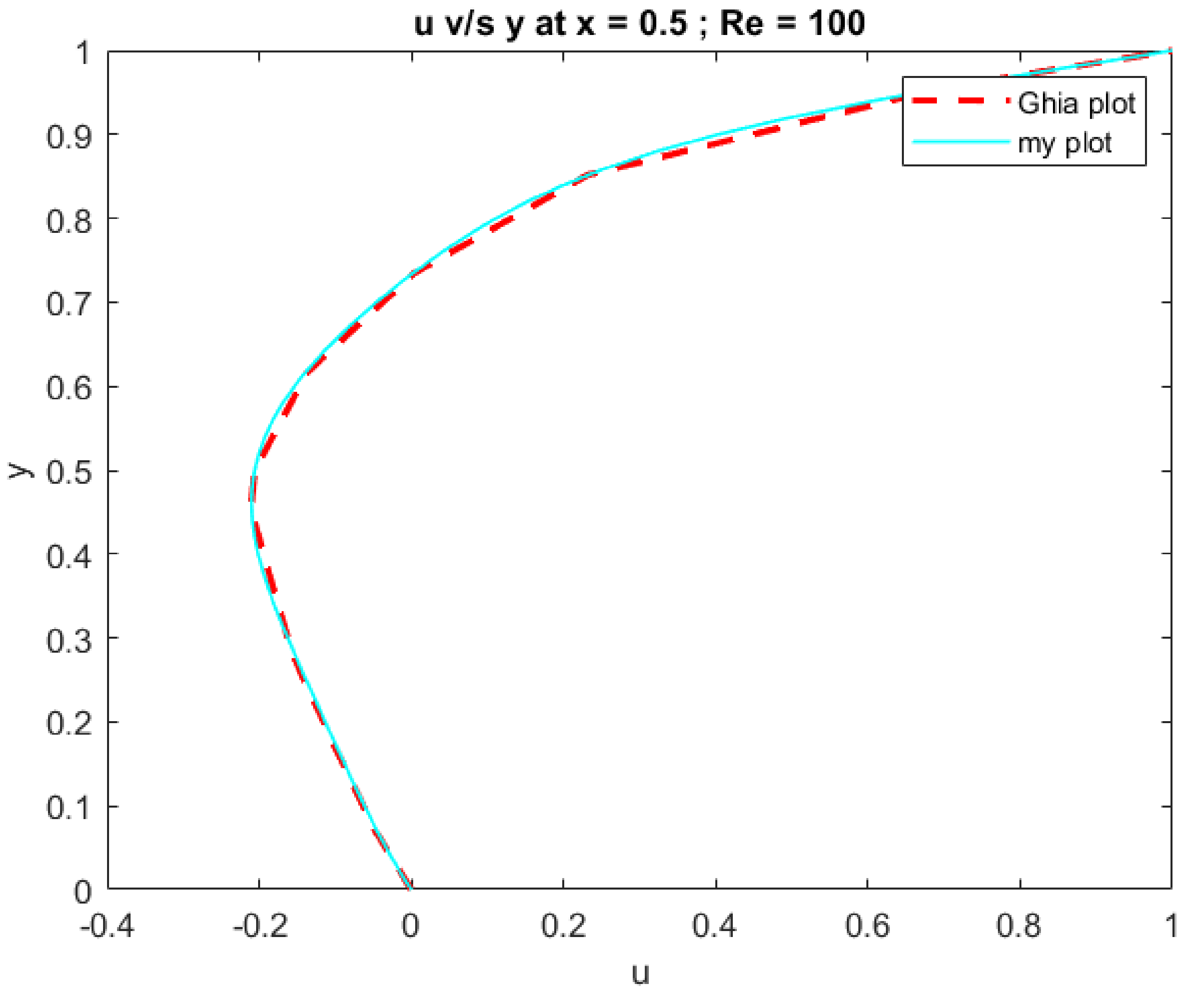

Figure 1: u vs y at x = 0.5 plot comparison with the actual paper

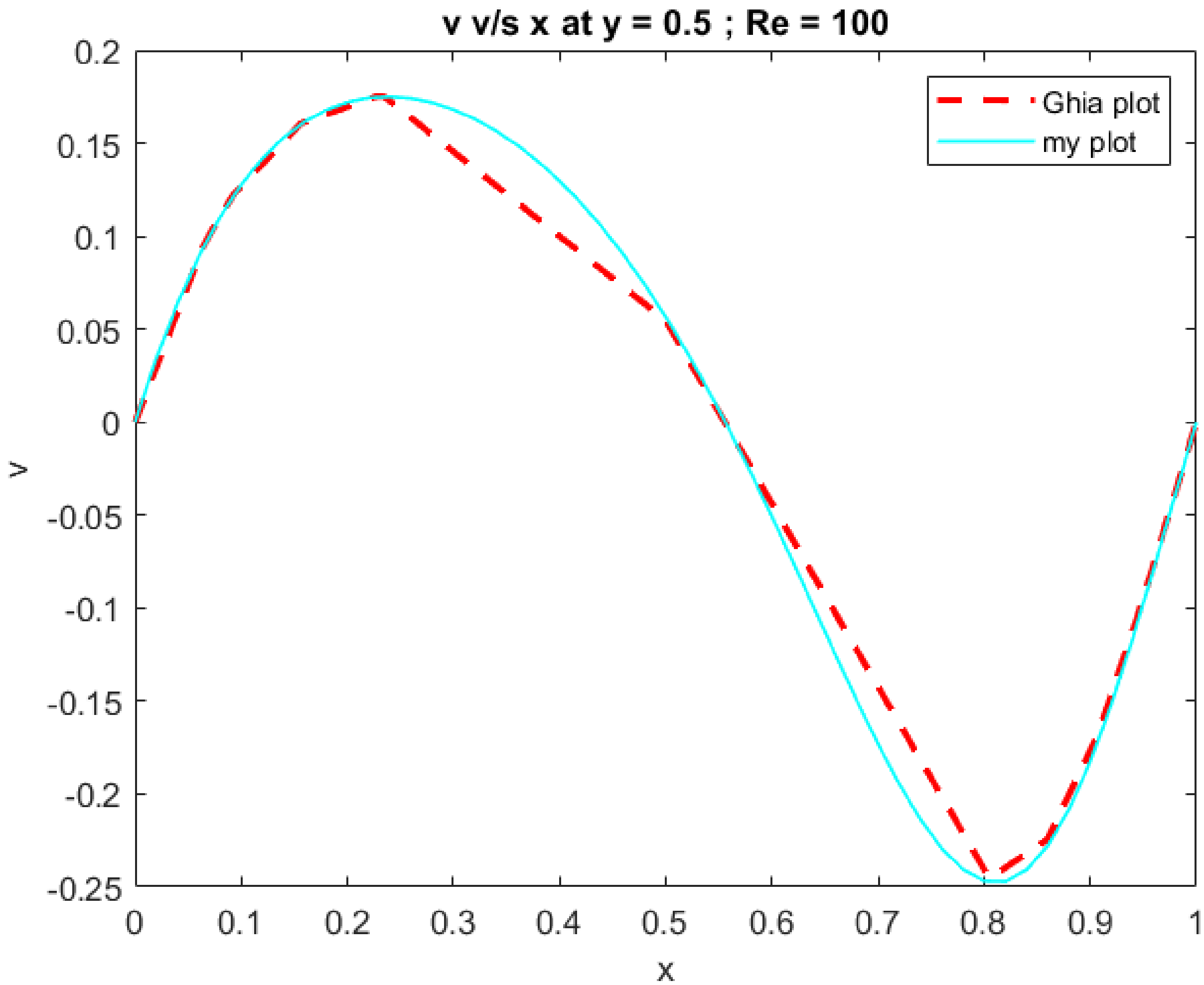

Figure 2: v vs x at y = 0.5 plot comparison with the actual paper

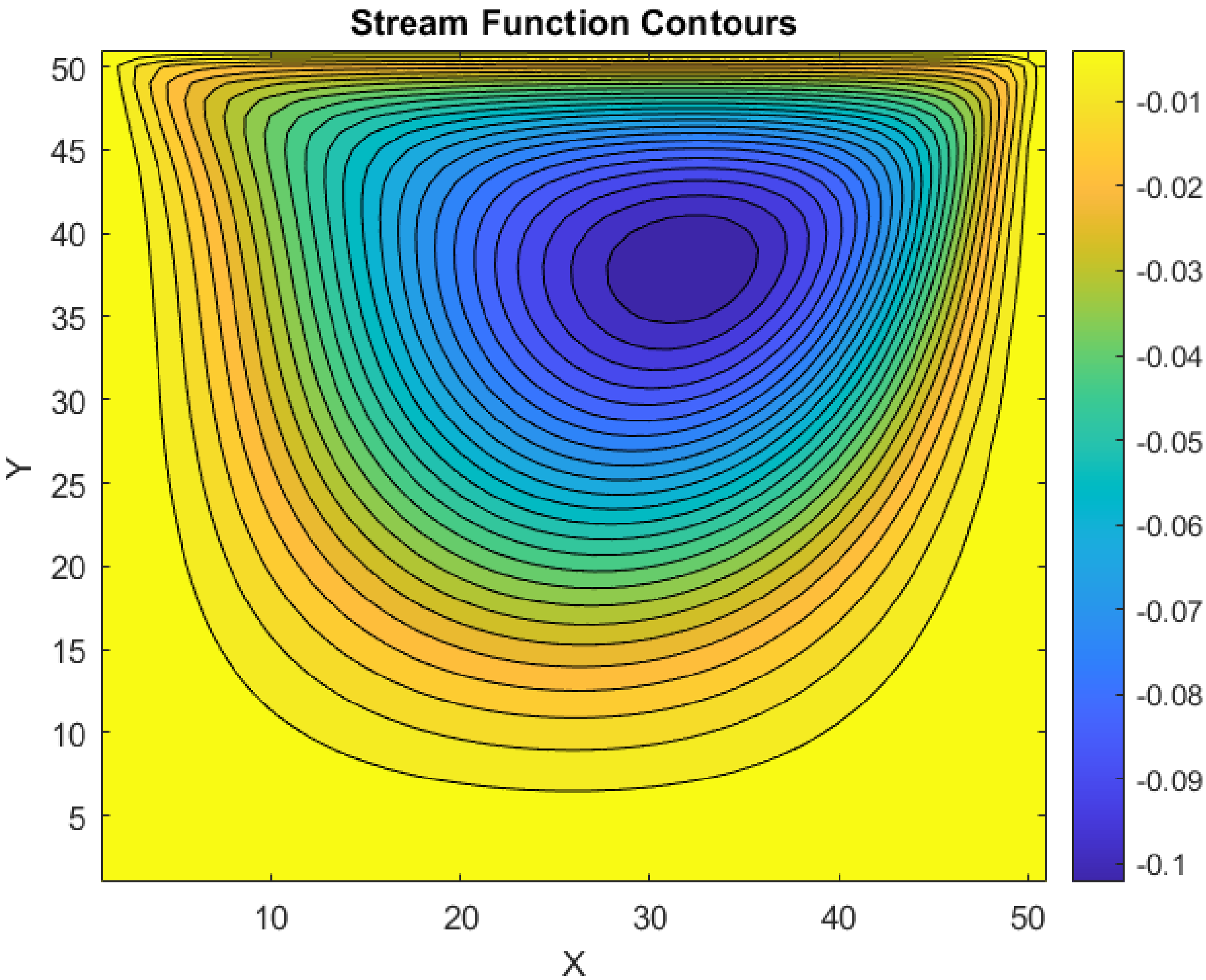

Figure 3: stream function contours plot

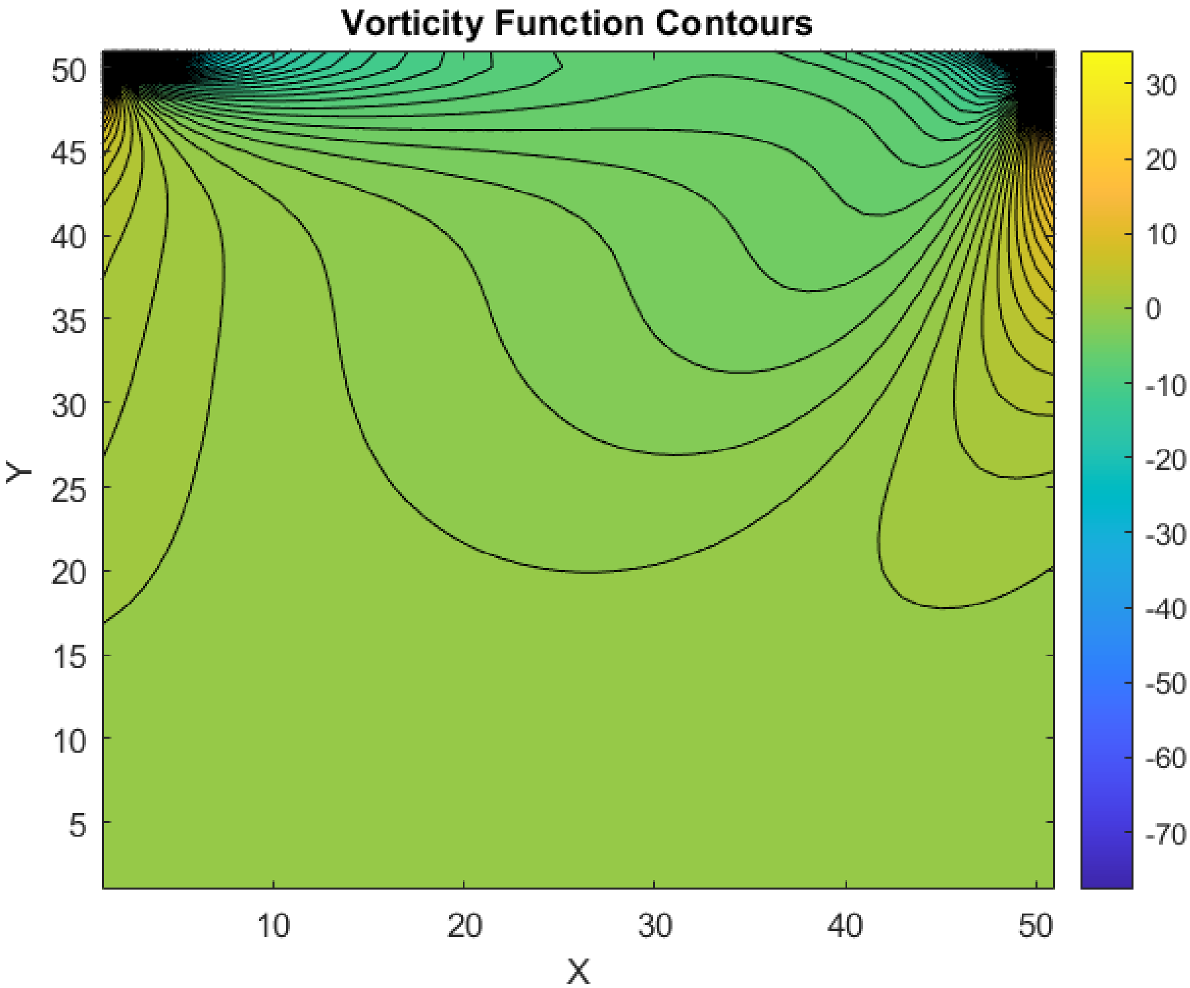

Figure 4: vorticity contours plot

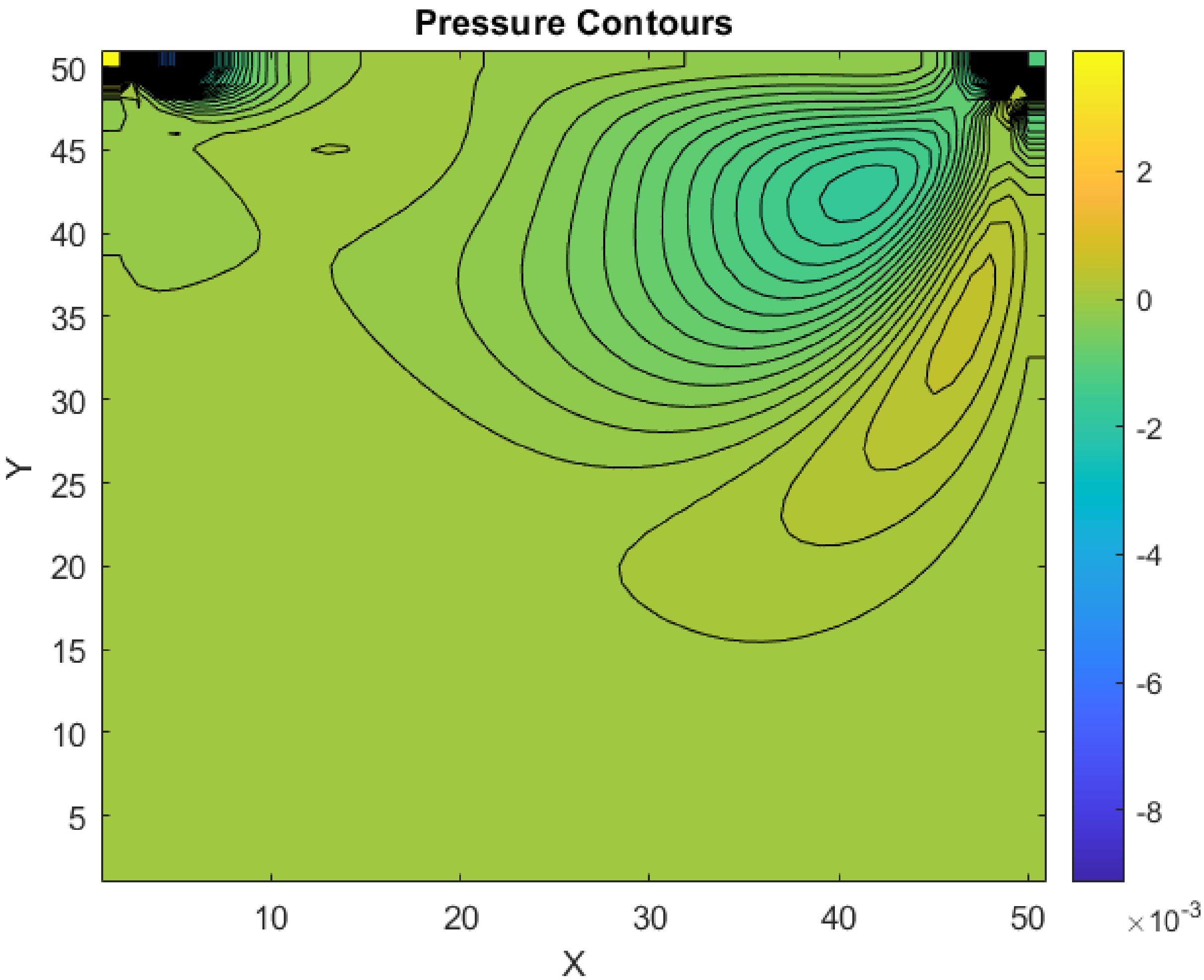

Figure 5: pressure contours produced using the stream function formulation

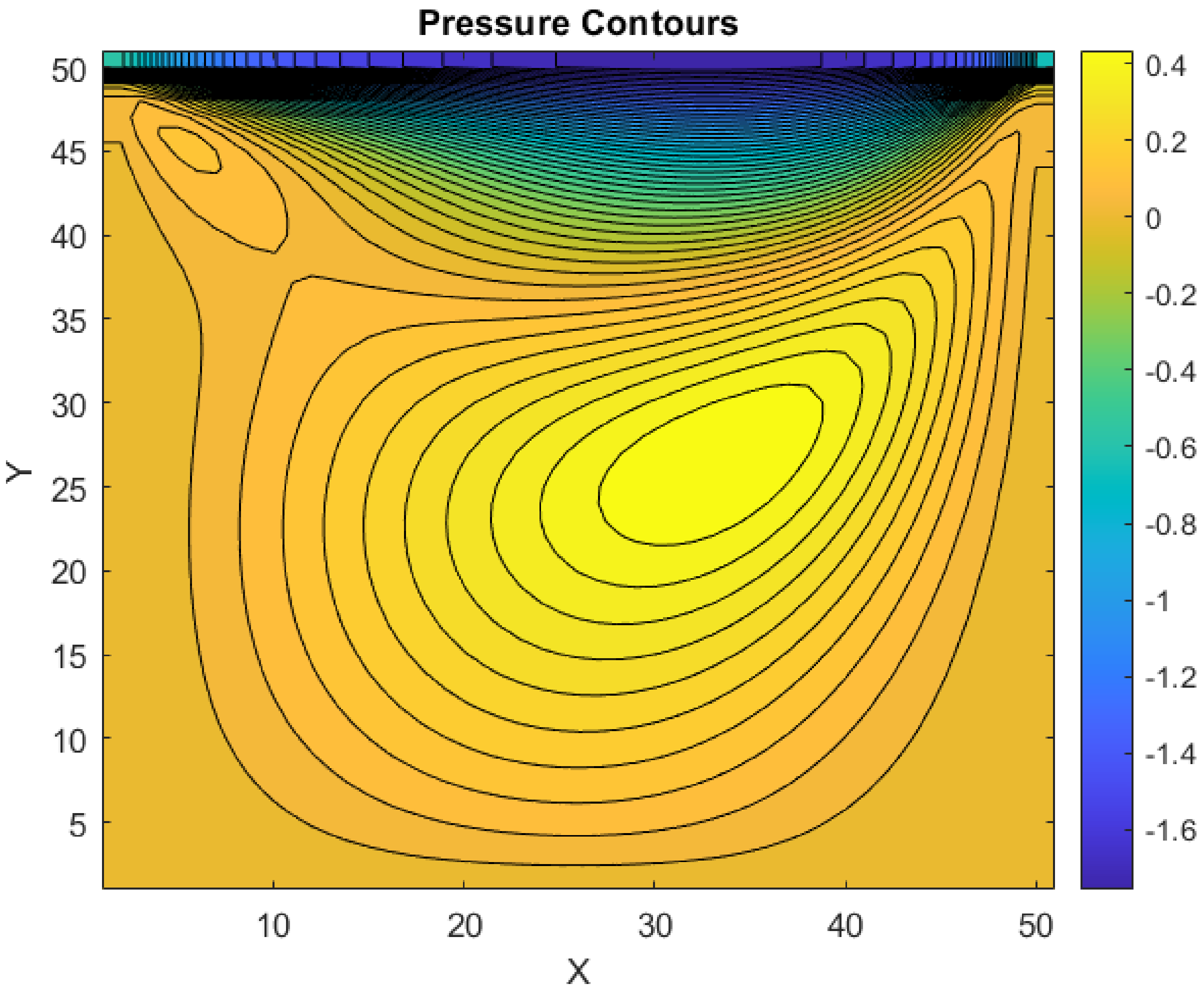

Figure 6: pressure contours produced using NS equation

1.7 Results

The plots obtained match the results mentioned in the paper. The pressure plots are inconsistent, as seen above. (different results for different formulations) This may be because the lid-driven cavity problem is not a pressure-driven problem; hence, our primary emphasis is only on stream function, vorticity, and velocity plots.

1.8 References

Ghia, U., Ghia, K. N. and Shin, C. T. (1982) High Re Solutions for Incompressible Flow Using the Navier-Stokes Equations and a Multigrid Method. Journal of Computational Physics, 48, 387-411.