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Homework Complexity 1.

对数 Logarithms are the inverse of exponentiation.

$\log_2 8) = 3.$ $\log_b(x) = y \Rightarrow b^y = x.$

In the lecture and homework, following convention:

1. $\log(x)$ means $\log_2(x)$.

tined. 2. $\ln(x)$ means $\log_e(x)$.

exercise one: 按增长顺序排序.

1. n. constant.   2. $n - n^3 + 7n^5$ polynomial.

③ 2^n exponential   4. n^2 quadratic

⑤ $n\ln(n)$ linearithmic   6. $\sqrt{n}$. fractional

7. $n!$ factorial.   ⑧ e^n exponential function

⑨ $\ln(\ln(n))$.

$\log_e(x)$ $\log_e(\log_e(n)) = y$

$e^y = \log_e(n)$

start with the slowest growing.

$9 - 1 - 6 - 5 - 4 - 2 - 8 - 3 - 7.$

exercise two:

$T(n) = C_{op} C(n).$

running time ↓ → Number of times
执行时间 basic operation is execute.
for basic operation   基本运算执行的次数

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1 million ($=10^6$) operations per second.

how many elements can be processed in 1 minute.

$60 \cdot 10^6 = 6 \cdot 10^7$

denote. 表示   number of operations $6 \cdot 10^7$.

running time of some algorithms : mine formulas.
(base operation)

running time : 1 minute.

Q: how many elements?

1. the elements for n. $= 6 \cdot 10^7 = 60000000$

2. $n^2 = 6 \cdot 10^7 = 60 \cdot 10^6 \Rightarrow n = \sqrt{60} \cdot 10^3 = \cancel{}$
$= 2\sqrt{15} \cdot 10^3$
≈ 7745.97

3. $n^3 = 6 \cdot 10^7 = 60 \cdot 10^6 \Rightarrow n = \sqrt[3]{60} \cdot 10^3 \approx 891.49$

4. $n! = 60 \cdot 10^6 \Rightarrow n = 11.$
$11! = 39,916,800$ $12! = 4.79006 \times 10^8.$
$= 3.99,1680 \times 10^7$

5. $n\ln(n) = 6 \cdot 10^7 \Rightarrow n = 3.9504 \times 10^6.$

6. $n\log(n) = 6 \cdot 10^7 \Rightarrow n = 2.801420 \times 10^6.$

7. $2^n = 6 \cdot 10^7$
$\Rightarrow \log(2^n) = \log(6 \cdot 10^7)$
$\cancel{n \log 2n} = \log_2(6 \cdot 10^7)$
$n = 25.838459.$

4^3 $(4 \times 4 \times 4.$
$2^{2 \cdot 3}$ $2 \times 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$
$\therefore 1961386$ ⑨ $n^{100} = 6 \cdot 10^7 \Rightarrow n = \sqrt[100]{6 \cdot 10^7}$ $n = \frac{\log_2(6 \cdot 10^7)}{2} \Rightarrow 2.919$

8. $n\sqrt{n} = 6 \cdot 10^7 \Rightarrow n = 1.532616$
1.532616×10^5 $\frac{\log_2(6 \cdot 10^8)}{2}$

$2n = \log_2(6 \cdot 10^7)$

10. $4^n = 6 \cdot 10^7 \Rightarrow \log(2^{2n}) = 6 \cdot 10^7 \log_2(6 \cdot 10^7)$

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Exercise Three:

3.1 A:

1. $g(n) = O(f(n)) \Leftarrow \ln(n^2) = O(\sqrt{n}).$

2. Both.

3. $n = O(\log(n)) \therefore f(n) = O(g(n)).$

4. $n\ln(n) + n = O(\ln(n)) \therefore f(n) = O(g(n))$

5. $10 = O(\ln(\ln(n))) \therefore f(n) = O(g(n))$

6. $10n^2 = O(2^n) \therefore g(n) = O(f(n))$

7. $3^n = O(2^n) \therefore g(n) = O(f(n)).$

3.2 B:

$c > 0, N > 0$ Prove: $f(n) = \log(n^2), g(n) = \log(n).$

$f(n) \leq c \cdot g(n).$ ① assum $c = 2.$ $N = 1.$

for every $n > N.$ $f(n) = \log(n^2) = 2\log(n) = 2 \cdot g(n)$

for every $n > N,$ $f(n) = \log(n^2) \leq 2 \cdot g(n) = 2\log(n)$
$\Rightarrow f(n) = O(g(n)).$

② assum $c = 3,$ $N = 1.$

$g(n) = \log(n) \leq 3 \cdot \log(n^2) = 3 \cdot f(n).$

$\Rightarrow g(n) = O(f(n)).$

Exercise 4.

```
def harmonic-number (n):
    if n <= 0:
        return " n is a positive integer."
    else:
        Hn = 0
        for i in range (1, n+1):
            Hn = 1/i   < return Hn.
```

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Exercise 5:

```
def factorial (n):
    if n = 0:
        return "1"
    else:
        return n * factorial (n).
```