ANA670 Applied Optimization
Assignment Module 1

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P1 – Rectangle with Greatest Area [10 points]

In about 300BC, Euclid proved that a square encloses the greatest area among all rectangles, assuming the total length of the four edges is fixed. Provide your own version of such proof.

Solution

A. Define Variables and Constraints

L = length of rectangle
W = width of rectangle
P = fixed perimeter of rectangle

$P = 2L + 2W$

$L + W = P/2$
$W = P/2 – L$

B. Define Area in Terms of One Variable

A = area of rectangle

$A = L \times W$

Substitute $W = P/2 - L$ into the area formula:

$A = L \times (P/2 – L)$
$A = L \times P/2 – L^2$
$A = (P/2)L – L^2$

Area A is now a function of L:

$A(L) = -L^2 + (P/2)L$

This is a quadratic function in L. Since the coefficient of L^2 is negative (-1), the parabola opens downward, meaning it has a maximum point.

C. Determine the Maximum Area Using Calculus

Take the derivative of A(L) with respect to L and set it to zero.

$A(L) = -L^2 + (P/2)L$

Differentiate:

$dA/dL = d/dL(-L^2 + (P/2)L)$
$dA/dL = -2L + P/2$

Set the derivative equal to zero to find the critical point:

$-2L + P/2 = 0$
$2L = P/2$
$L = P/4$

Substitute $L = P/4$ back into the equation for W:

$W = P/2 – L$
$W = P/2 – P/4$
$W = P/4$

$L = W$, and the rectangle is a square with side length $P/4$.

D. Compute the Area of the Square

$A = L \times W$
$A = P/4 \times P/4$
$A = P^2/16$

E. Proof Summary

Let the perimeter be $P = 2L + 2W$, so $L + W = P/2$. The area is $A = L \times W$. Substitute $W = P/2 – L$ into the area: $A = L(P/2 – L) = (P/2)L - L^2$. This is a quadratic in L. The maximum occurs at the vertex: $dA/dL = P/2 – 2L = 0$ → $L = P/4$, so $W = P/4$. Thus, the rectangle is a square with side $P/4$, and its area is $P^2/16$, the largest possible.

P2 - Cylindrical Water Tank using the Minimal Materials [20 points]

Design a cylindrical water tank which uses the minimal materials and holds the largest volume of water. What is the relationship between the radius r of the base and the height h?

Solution

A. Define the Problem

A cylindrical tank has:

- **A circular base and top (if it has a lid), each with area πr^2.**

- **A curved side surface, which is a rectangle when unrolled, with area $2\pi rh$ (circumference times height).**

The total surface area S (material used) depends on whether the tank has a top. Since the problem requires minimal materials and the largest volume, two cases must be considered:

- **Case 1: The tank has no top (open at the top).**

- **Case 2: The tank has a top and bottom (closed cylinder).**

The volume V of the cylinder is:

$V = \pi r^2 h$

Need to minimize the surface area S subject to maximizing V, finding the relationship between r and h that optimizes this.

B. Surface Area for an Open Cylinder (no lid)

If the tank is open at the top (no lid), the surface area S is the area of the base plus the lateral surface:

$S = \pi r^2 + 2\pi rh$

C. Optimize for an Open Cylinder (no lid)

To find the relationship between r and h, treat S as a constant (minimal material) and express h in terms of r and S:

Solve for h:

$S = \pi r^2 + 2\pi rh$
$2\pi rh = S - \pi r^2$
$h = (S - \pi r^2)/(2\pi r)$

Substitute h into the volume formula:

$V = \pi r^2 h$
$V = \pi r^2((S - \pi r^2)/(2\pi r))$
$V = r(S - \pi r^2)/2$
$V = (Sr/2) - (\pi r^3/2)$

This is a function of r. To maximize V, take the derivative with respect to r and set it to zero:

$V(r) = -(\pi r^3/2) + (Sr/2)$

Differentiate:

$dV/dr = d/dr(-(\pi r^3/2) + (Sr/2))$
$dV/dr = -(3\pi r^2/2) + (S/2)$

Set the derivative equal to zero to find the critical point related to S:

$-(3\pi r^2/2) + (S/2) = 0$
$S/2 = 3\pi r^2/2$
$S = 3\pi r^2$

Set the original surface area S equation equal to the critical point equation for S to determine the relationship between r and h:

$dV/dr = 0 \rightarrow S = 3\pi r^2$
$S = \pi r^2 + 2\pi rh$

$3\pi r^2 = \pi r^2 + 2\pi rh$
$2\pi r^2 = 2\pi rh$
$r = h$

The radius r of the base and the height h of the cylindrical open tank are equal.

D. Surface Area for a Closed Cylinder

If the tank is closed at the top, the surface area S is 2 times the area of the base plus the lateral surface:

$S = 2\pi r^2 + 2\pi rh$

E. Optimize for a Closed Cylinder

To find the relationship between r and h, treat S as a constant (minimal material) and express h in terms of r and S:

Solve for h:

$S = 2\pi r^2 + 2\pi rh$
$2\pi rh = S - 2\pi r^2$
$h = (S - 2\pi r^2)/(2\pi r)$

Substitute h into the volume formula:

$V = \pi r^2 h$
$V = \pi r^2((S - 2\pi r^2)/(2\pi r))$
$V = r(S - 2\pi r^2)/2$
$V = (Sr/2) – (2\pi r^3/2)$

This is a function of r. To maximize V, take the derivative with respect to r and set it to zero:

$V(r) = -(2\pi r^3/2) + (Sr/2)$

Differentiate:

$dV/dr = d/dr(-(2\pi r^3/2) + (Sr/2))$
$dV/dr = -(6\pi r^2/2) + (S/2)$

Set the derivative equal to zero to find the critical point related to S:

$-(6\pi r^2/2) + (S/2) = 0$
$S/2 = 6\pi r^2/2$
$S = 6\pi r^2$

Set the original surface area S equation equal to the critical point equation for S to determine the relationship between r and h:

$dV/dr = 0 \rightarrow S = 6\pi r^2$
$S = 2\pi r^2 + 2\pi rh$

$6\pi r^2 = 2\pi r^2 + 2\pi rh$
$4\pi r^2 = 2\pi rh$
$2r = h$

The height h is twice the size of the radius r of the cylindrical closed tank.

P3 - Type of the Optimization Problem [10 points]

State the type of the following optimization problems:

a) max *exp(-x2)* subject to *-2 < x < 5*;

b) min $x^2 + y^2$;

c) max 2x + 3y subject to *x > 0, 0 < y < 10, y > x*;

d) min x^2y^6 subject to $x^2 + y^2 / 4 = 1$;

e) Design a quieter, more efficient, low cost car engine;

f) Design a software package to predict the stock market.

Solution

a) Single Objective, Constrained (Inequality), Unimodal, Non-Linear, Continuous, Deterministic

b) Single Objective, Unconstrained, Unimodal, Non-Linear, Continuous, Deterministic

c) Single Objective, Constrained (Inequality), Unimodal, Linear, Continuous, Deterministic

d) Single Objective, Constrained (Equality), Multimodal, Non-Linear, Continuous, Deterministic

e) Multi-Objective, Unconstrained, Multimodal, Non-Linear, Mixed, Deterministic

f) Single Objective, Unconstrained, Multimodal, Non-Linear, Continuous, Stochastic

P4 – Python coding: Cluster center [10 points]

Compute the following using Python with the attached input file: guassian_blob.txt.

Solution

1. Maximum in the x direction: 7.335
2. Minimum in the x direction: -0.968

3. Maximum in the y direction: 16.816
4. Minimum in the y direction: 8.013

5. Sum of the Euclidean distances from each point to the point (x =4 , y= 15)

Euclidean Distance Formula: $\sqrt{((x - 4)^2 + (y - 15)^2)}$

Sum of Euclidean Distances: 128,385.833

6. **Include your code below**

```
import numpy as np

# Step 1: Load the data from gaussian_blob.txt
data = np.genfromtxt('guassian_blob.txt', delimiter=',', skip_header=1)  # Load
the file as a 2D array.

# Extract x and y columns.
x = data[:, 0]  # First column is x.
y = data[:, 1]  # Second column is y.

# Step 2: Compute the required statistics.
# 1. Maximum in the x direction
max_x = np.max(x)

# 2. Minimum in the x direction
min_x = np.min(x)

# 3. Maximum in the y direction
max_y = np.max(y)

# 4. Minimum in the y direction
min_y = np.min(y)

# 5. Sum of the Euclidean distances from each point to (4, 15).
# Euclidean distance formula: sqrt((x_i - 4)^2 + (y_i - 15)^2)
distances = np.sqrt((x - 4) ** 2 + (y - 15) ** 2)
sum_distances = np.sum(distances)

# Step 3: Print the results.
print(f"1. Maximum in the x direction: {max_x:,.3f}")
print(f"2. Minimum in the x direction: {min_x:,.3f}")
print(f"3. Maximum in the y direction: {max_y:,.3f}")
print(f"4. Minimum in the y direction: {min_y:,.3f}")
print(f"5. Sum of the Euclidean distances to (4, 15): {sum_distances:,.3f}")
```

P4 - Find the Global Maximum for Linear Optimization [15 points]

Find the global maximum for linear optimization $f(x, y) = x + y$ subject to $0 < x < 5$, $0 < y < 2$ and $2x + y = 8$.

Solution

A. Determine the Feasible Region

Constraints:

$0 < x < 5$
$0 < y < 2$
$2x + y = 8$

Solve for the Feasible Points:

From $2x + y = 8$, solve for y :

Apply the bounds:

$y > 0$: $8 - 2x > 0 \rightarrow 2x < 8 \rightarrow x < 4$
$y < 2$: $8 - 2x < 2 \rightarrow -2x < -6 \rightarrow x > 3$.

Combine the constraints on x from $0 < x < 5$:

$x > 0$ and $x < 4$ (from $y > 0$)
$x > 3$ and $x < 5$ (from $y < 2$)

The intersection is $3 < x < 4$ (since $x > 3$ and $x < 4$ are the tightest bounds).

As x ranges from just above 3 to just below 4, y ranges from just below 2 to just above 0.
The feasible region is the open line segment where $2x + y = 8$ intersects $0 < y < 2$ and $0 < x < 5$, specifically the open interval $3 < x < 4$, with $y = -2x + 8$ satisfying $0 < y < 2$.

B. Evaluate the Objective Function

$f(x, y) = x + y$

Substitute $y = -2x + 8$:

This is a linear function of x, decreasing as x increases.

Evaluate the behavior in the feasible interval $3 < x < 4$:

As x approaches 3 from above, y approaches 2 from below, so f(x) approaches $3 + 2 = 5$.
As x approaches 4 from below, y approaches 0 from above, so f(x) approaches $4 + 0 = 4$.

C. **Address the Open Interval**

Because the constraints are strict ($0 < x < 5$, $0 < y < 2$), the feasible region is an open set. The points where $x = 3$ and $y = 2$ (3, 2) and where $x = 4$ and $y = 0$ (4, 0) are on the boundary and excluded due to strict inequalities.

The maximum value of $f(x, y)$ approaches 5 as (x, y) approaches (3, 2) from within the feasible region, but since the region is open, 5 is a supremum.

D. **Conclusion**

The global maximum of 5 is approached as $(x, y) \rightarrow (3, 2)$ within the feasible region $3 < x < 4$, $0 < y < 2$.
Solution: The global maximum (supremum) is 5 at the limit point $(x, y) \rightarrow (3, 2)$.

P5 - Find the Stationary Points and Inflection Points [20 points]

Find the stationary points and inflection points of the following functions.

a) $f(x) = \cos(x)$
b) $f(x) = exp(-x^2)$

Solution

Stationary Points: These occur where the first derivative is zero.

Inflection Points: These occur where the second derivative is zero and changes the sign, or where the third derivative exists and the second derivative's concavity changes.

a) **$f(x) = \cos(x)$**

First Derivative:

$f'(x) = -\sin(x)$

Set $f'(x) = 0$:

$-\sin(x) = 0 \rightarrow \sin(x) = 0$

Solutions are $x = n\pi$, where n is an integer (e.g., $x = 0, +/-\pi, +/-2\pi, +/-3\pi, \ldots$).

Second Derivative:

$f''(x) = -\cos(x)$

Evaluate the change:

At $x = 0$: $f''(0) = -\cos(0) = -1 < 0$ (local maximum).
At $x = \pi$: $f''(\pi) = -\cos(\pi) = -(-1) = 1 > 0$ (local minimum).

The points alternate between maxima and minima.

Set $f''(x) = 0$:

$-\cos(x) = 0 \rightarrow \cos(x) = 0$

Solutions are $x = \pi/2 + n\pi$, where n is an integer (e.g., $x = \pi/2, 3\pi/2, 5\pi/2, \ldots$).

Third Derivative:

$f'''(x) = \sin(x)$

Evaluate the change:

At $x = \pi/2$: $f'''(\pi/2) = \sin(\pi/2) = 1 \neq 0$, and $f''(x)$ changes from negative to positive, indicating an inflection.
At $x = 3\pi/2$: $f'''(3\pi/2) = \sin(3\pi/2) = -1 \neq 0$, and $f''(x)$ changes from positive to negative, indicating an inflection.

Solution Summary:

Stationary Points: $x = n\pi$ (maxima at $x = 2n\pi$, minima at $x = (2n +1)\pi$).

Inflection Points: $x = \pi/2 + n\pi$ (from negative to positive at odd multiples, positive to negative at even multiples after π).

b) $f(x) = \exp(-x^2)$

First Derivative:

$f'(x) = -2x \exp(-x^2)$

Set $f'(x) = 0$:

$-2x \exp(-x^2) = 0$

Since $\exp(-x^2) > 0$ for all x, $-2x = 0 \rightarrow x = 0$

Second Derivative:

$f''(x) = d/dx[-2x \exp(-x^2)] = -2\exp(-x^2) + (-2x)(-2x \exp(-x^2) = -2\exp(-x^2) + 4x^2 \exp(-x^2) = \exp(-x^2)(4x^2 – 2)$

Evaluate the change:

At $x = 0$: $f''(0) = \exp(0)(0 - 2) = -2 < 0$ (local maximum).

Set $f''(x) = 0$:

$\exp(-x^2)(4x^2 – 2) = 0$

$\exp(-x^2) \neq 0$, so $4x^2 - 2 = 0 \rightarrow x^2 = 2/4 = 0.5 \rightarrow x = \pm\sqrt{0.5} \rightarrow |x| = \sqrt{0.5}$

$\sqrt{0.5} = \sqrt{(1/2)} = \sqrt{1}/\sqrt{2} = 1/\sqrt{2} = \sqrt{2}/2$

Sign analysis for $f''(x)$:

$\exp(-x^2)$ is always positive for all real x, and the sign of $f''(x)$ depends on $4x^2 – 2$.

For $x < -\sqrt{2}/2$: $x^2 > 0.5$, $4x^2 – 2 > 0$, $f''(x) > 0$ (concave up).
For $-\sqrt{2}/2 < x < \sqrt{2}/2$: $x^2 < 0.5$, $4x^2 – 2 < 0$, $f''(x) < 0$ (concave down).
For $x > \sqrt{2}/2$: $x^2 > 0.5$, $4x^2 – 2 > 0$, $f''(x) > 0$ (concave up).

At $x = -\sqrt{2}/2$: $f''(x)$ changes from positive (concave up) to negative (concave down).
At $x = \sqrt{2}/2$: $f''(x)$ changes from negative (concave down) to positive (concave up).

Third Derivative:

$f'''(x) = d/dx[\exp(-x^2) (4x^2 – 2)]$

Use the product rule:

$f'''(x) = [\exp(-x^2)(8x)] + [(4x^2 – 2)(-2x \exp(-x^2))]$

Evaluate the change:

At $x = \sqrt{2}/2$: $f'''(\sqrt{2}/2) \neq 0$ (non-zero, supports a change in concavity).

Solution Summary:

Stationary Points: $x = 0$ (local maximum, since $f''(0) = -2 < 0$).

Inflection Points: $x = \pm\sqrt{2}/2$ (for $x = -\sqrt{2}/2$, $f''(x)$ changes from positive to negative, and for $x = \sqrt{2}/2$, $f''(x)$ changes from negative).

P6 – Convex Functions [20 points]

Find if the following functions are convex or concave or neither?

a) $x^2+\exp(y)$;
b) $1-x^2 -y^2$;
c) xy;
d) x^2 /y for $y > 0$.

Solution

For functions of two variables $f(x, y)$, a Hessian matrix needs to be computed:

First partial derivatives: f_x, f_y
Second partial derivatives: f_{xx}, f_{xy}, f_{yy}

Hessian matrix: $H = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{vmatrix}$

A function is convex if H is positive semi-definite, meaning for all (x, y), the eigenvalues of H are non-negative. Checked by:

$f_{xx} \geq 0$
Determinant of H, $\det(H) = f_{xx} f_{yy} - (f_{xy})^2 \geq 0$

Concave if eigenvalues are non-positive semi-definite. Checked by:

$f_{xx} \leq 0$
$\det(H) \geq 0$

a) **$f(x, y) = x^2 + \exp(y)$**

First Partial Derivatives:

$f_x = \partial/\partial x(x^2 + \exp(y)) = 2x$
$f_y = \partial/\partial y(x^2 + \exp(y)) = \exp(y)$

Second Partial Derivatives:

$f_{xx} = \partial/\partial x(2x) = 2$
$f_{xy} = \partial/\partial y(2x) = 0$
$f_{yy} = \partial/\partial y(\exp(y)) = \exp(y)$

Hessian Matrix:

$$H = \begin{vmatrix} 2 & 0 \\ 0 & \exp(y) \end{vmatrix}$$

Convexity Check:

Since H is diagonal, the eigenvalues are the diagonal entries: 2 and exp(y)

$f_{xx} = 2 > 0$
$f_{yy} = \exp(y) > 0$ for all y
$\det(H) = (2)(\exp(y)) - (0)^2 = 2\exp(y) > 0$

Since H is positive definite (all eigenvalues are positive), f(x, y) is convex.

b) **$f(x, y) = 1 - x^2 - y^2$**

First Partial Derivatives:

$f_x = \partial/\partial x(1 - x^2 - y^2) = -2x$
$f_y = \partial/\partial y(1 - x^2 - y^2) = -2y$

Second Partial Derivatives:

$f_{xx} = \partial/\partial x(-2x) = -2$
$f_{xy} = \partial/\partial y(-2x) = 0$
$f_{yy} = \partial/\partial y(-2y) = -2$

Hessian Matrix:

$$H = \begin{vmatrix} -2 & 0 \\ 0 & -2 \end{vmatrix}$$

Concavity Check:

Since H is diagonal, the eigenvalues are the diagonal entries: -2 and -2

$f_{xx} = -2 < 0$
$f_{yy} = -2 < 0$
$\det(H) = (-2)(-2) - (0)^2 = 4 > 0$

Since H is negative definite (all eigenvalues are negative), f(x, y) is concave.

c) **$f(x, y) = xy$**

First Partial Derivatives:

$f_x = \partial/\partial x(xy) = y$
$f_y = \partial/\partial y(xy) = x$

Second Partial Derivatives:

$f_{xx} = \partial/\partial x(y) = 0$
$f_{xy} = \partial/\partial y(y) = 1$
$f_{yy} = \partial/\partial y(x) = 0$

Hessian Matrix:

$$H = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix}$$

Convexity/Concavity Check:

Eigenvalues: Solve $\det(H - \lambda I) = 0$

$$H - \lambda I = \begin{vmatrix} -\lambda & 0 \\ 0 & -\lambda \end{vmatrix}$$

$\det(H - \lambda I) = (-\lambda)(-\lambda) - (1)^2 = \lambda^2 - 1 = 0 \rightarrow \lambda = \pm 1$

Eigenvalues are 1 and -1.

Since there are both positive and negative eigenvalues, H is indefinite, so f(x, y) is neither convex nor concave.

d) **$f(x, y) = x^2/y$ for $y > 0$**

First Partial Derivatives:

$f_x = \partial/\partial x(x^2/y) = 2x/y$
$f_y = \partial/\partial y(x^2/y) = x^2(-1/y^2) = -x^2/y^2$

Second Partial Derivatives:

$f_{xx} = \partial/\partial x(2x/y) = 2/y$
$f_{xy} = \partial/\partial y(2x/y) = 2x(-1/y^2) = -2x/y^2$
$f_{yy} = \partial/\partial y(-x^2/y^2) = -x^2(-2y^{-3}) = 2x^2/y^3$

Hessian Matrix:

$$H = \begin{vmatrix} 2/y & -2x/y^2 \\ -2x/y^2 & 2x^2/y^3 \end{vmatrix}$$

Convexity Check:

Since $y > 0$:

$f_{xx} = 2/y > 0$
$f_{yy} = 2x^2/y^3 > 0$
$\det(H) = (2/y)(2x^2/y^3) - (-2x/y^2)^2 = 4x^2/y^4 - 4x^2/y^4 = 0$

Since H is positive semi-definite (eigenvalues are a positive value and zero), f(x, y) is convex.

P7 – Python Coding [20 points]

Find the minima of *$f(x) = x^2 + k\cos^2(x)$* with *k* > 0. How many minima? Will the number of minima depend on *k*?

Python Programming: Plot the function for -30 < x < 30 using for k=1,2,3. Show the minima for each case on the plot.

Your Code

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function using the identity cos^2(x) = (1 + cos(2x))/2.
# ***f(x) = x^2 + (k/2) * (1 + cos(2x))***

def f(x, k):
    return x**2 + (k/2) * (1 + np.cos(2*x))

# Create x ranges
x_full = np.linspace(-30, 30, 1000)  # Full range
x_zoom = np.linspace(-5, 5, 200)     # Zoomed range

# Generate plots for each k value
k_values = [1, 2, 3]
```

```
for k in k_values:
    # Compute function values
    y_full = f(x_full, k)
    y_zoom = f(x_zoom, k)

    # Approximate minima visually (simple peak detection)
    minima_indices_full = []
    for i in range(1, len(x_full) - 1):
        if y_full[i] < y_full[i-1] and y_full[i] < y_full[i+1]:
            minima_indices_full.append(i)

    minima_indices_zoom = []
    for i in range(1, len(x_zoom) - 1):
        if y_zoom[i] < y_zoom[i-1] and y_zoom[i] < y_zoom[i+1]:
            minima_indices_zoom.append(i)

    # Full-scale plot
    plt.figure(figsize=(8, 5))
    plt.plot(x_full, y_full, label=f'f(x) with k = {k}', color='blue')
    for idx in minima_indices_full:
        plt.plot(x_full[idx], y_full[idx], 'ro') # Red dot at each minimum
    plt.title(f'f(x) = x^2 + (k/2)(1 + cos(2x)) for k = {k}, -30 < x < 30')
    plt.xlabel('x')
    plt.xticks(np.arange(-30, 31, 5))
    plt.ylabel('f(x)')
    plt.yticks(np.arange(0, 1000, 50))
    plt.legend()
    plt.grid(True)
    plt.show()

    # Zoomed-in plot with coordinates
    plt.figure(figsize=(8, 5))
    plt.plot(x_zoom, y_zoom, label=f'f(x) with k = {k}', color='blue')
    for idx in minima_indices_zoom:
        x_min = x_zoom[idx]
        y_min = y_zoom[idx]
        plt.plot(x_min, y_min, 'ro') # Red dot at each minimum
        plt.text(x_min, y_min - 0.2, f'({x_min:.2f}, {y_min:.2f})',
                 ha='center', va='top', fontsize=8) # Annotate coordinates
    plt.title(f'Zoomed: f(x) = x^2 + (k/2)(1 + cos(2x)) for k = {k}, -5 < x <
5')
    plt.xlabel('x')
    plt.xticks(np.arange(-5, 6, 1))
```

```
    plt.ylabel('f(x)')
    plt.yticks(np.arange(0, 10, 0.5))
    plt.ylim(0, 10)
    plt.legend()
    plt.grid(True)
    plt.show()

    # Print a note for visual inspection
    print(f"For k = {k}, visually inspect the zoomed plot (-5 < x < 5, 0 < f(x)
< 10) for the number of minima (red dots are approximate).")
```

Analysis:

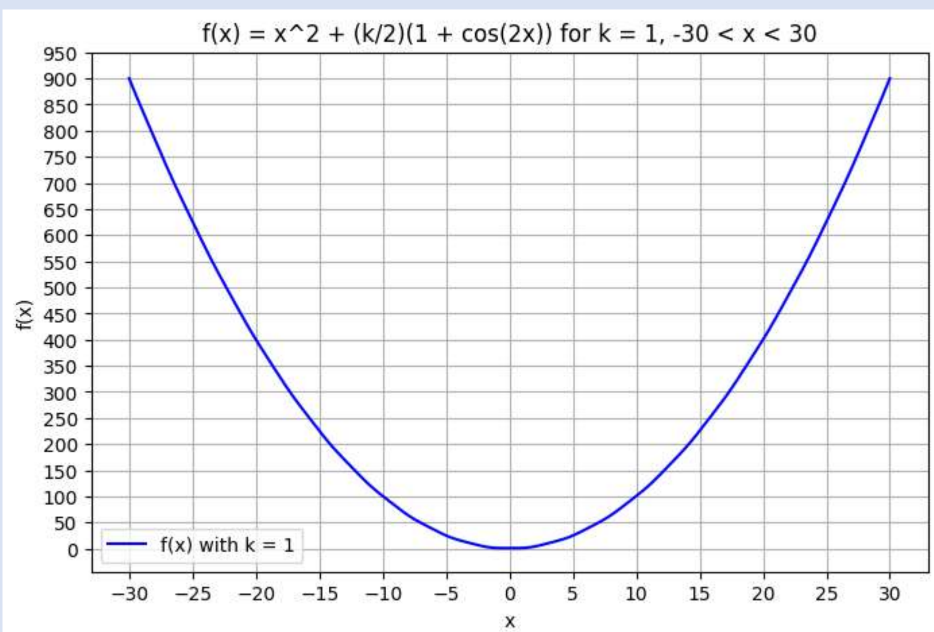
Based on visual inspection of the zoomed plots with coordinates below:

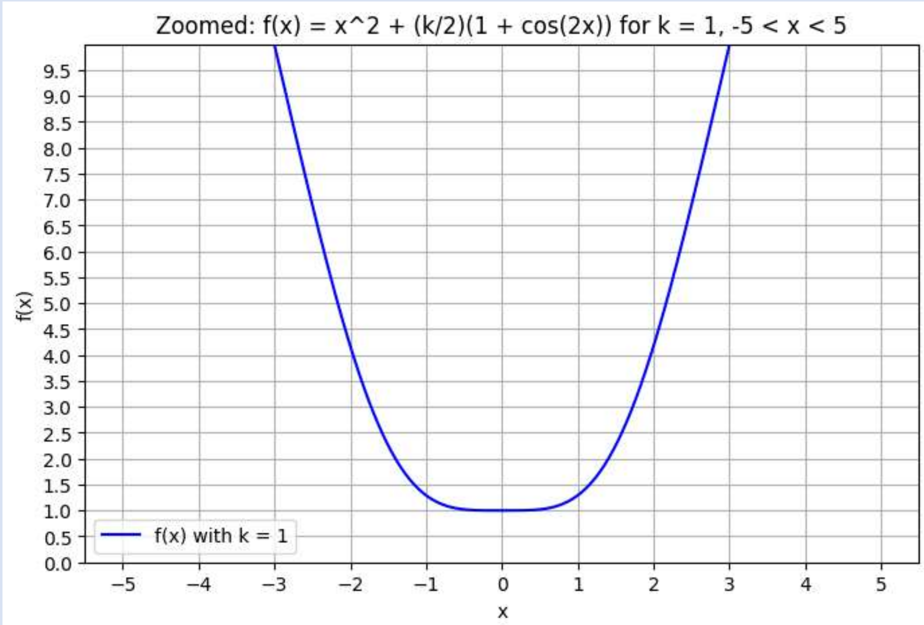
For k = 1: Appears to have 1 minimum at (0, 1)
For k = 2: Appears to have 2 minima at (-0.95, 1.58) and (0.95, 1.58) approximately.
For k = 3: Appears to have 2 minima at (-1.13, 1.82) and (1.13, 1.82) approximately.

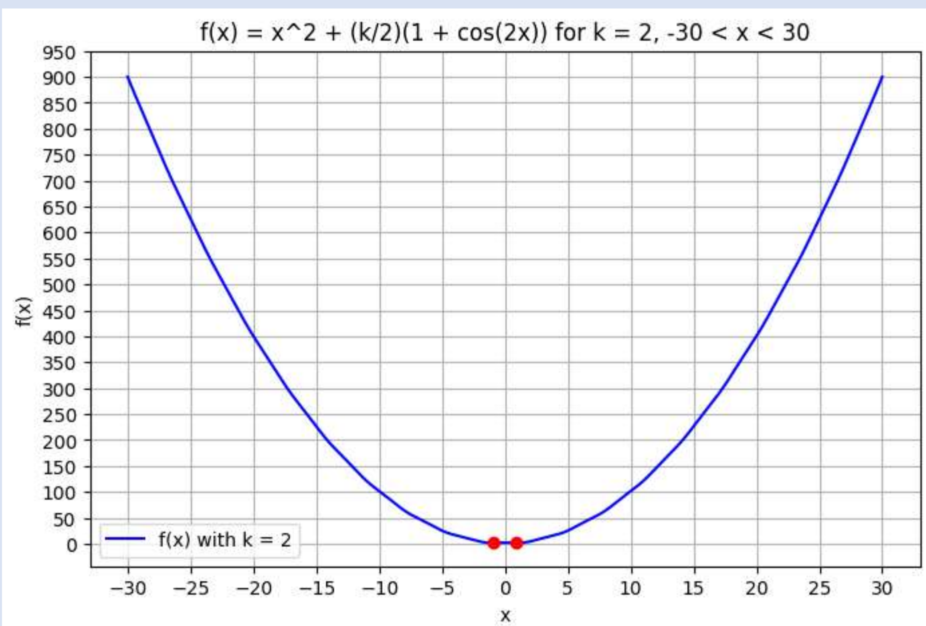
At k > 1, the number of strong global minima looks to be 2, and their respective coordinates (±x, f(x)) increase as k increases. Weak local minima are created as k increases (i.e. at k=8), due to the amplified oscillatory effect of (k/2)(cos(2x)). A weak local maxima is created at x=0, and its f(x) increases as k increases.

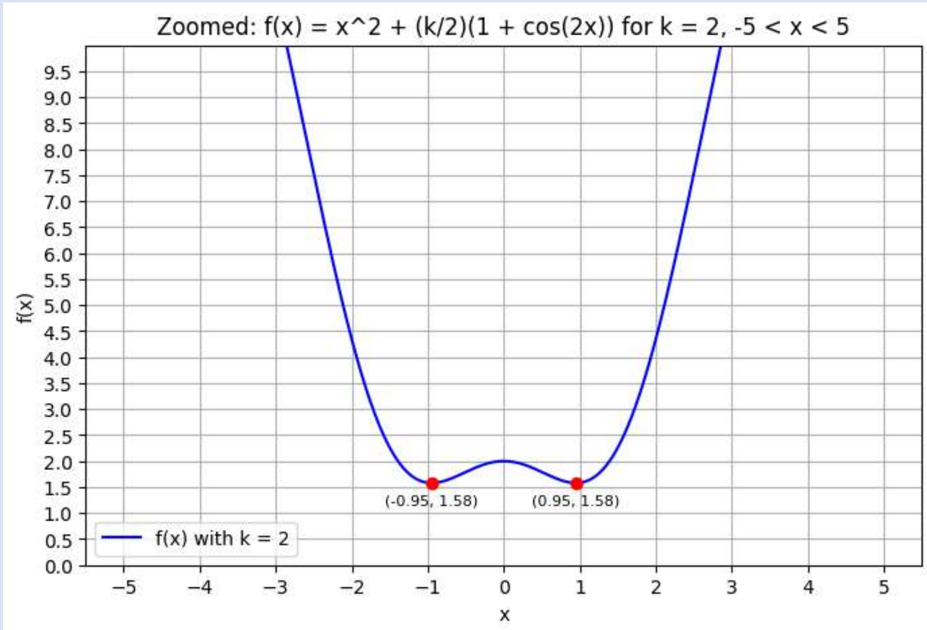
Run the code and insert the result in the following box.

Plots

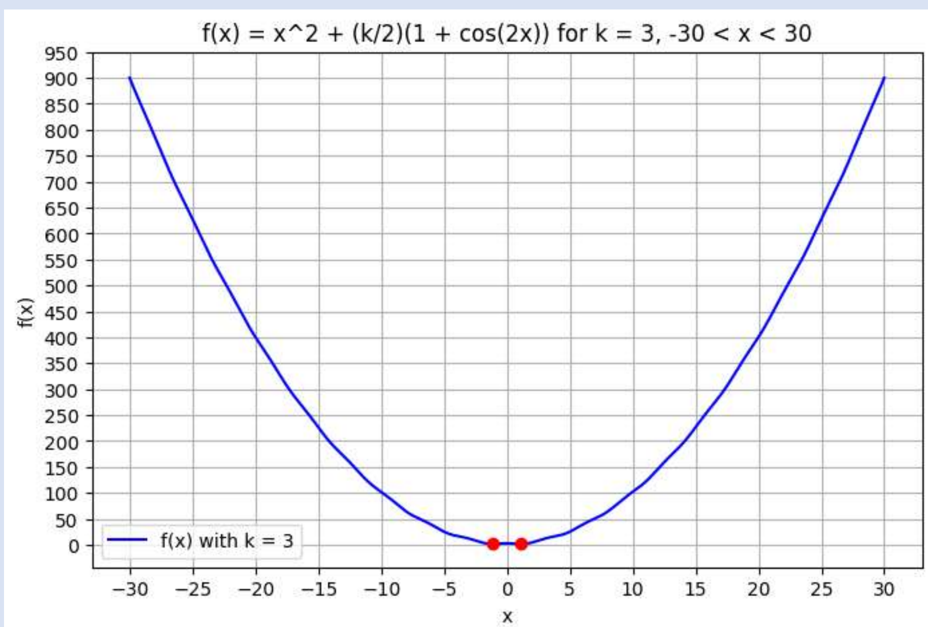



Zoomed: f(x) = x^2 + (k/2)(1 + cos(2x)) for k = 1, -5 < x < 5
f(x) with k = 1
f(x)
x

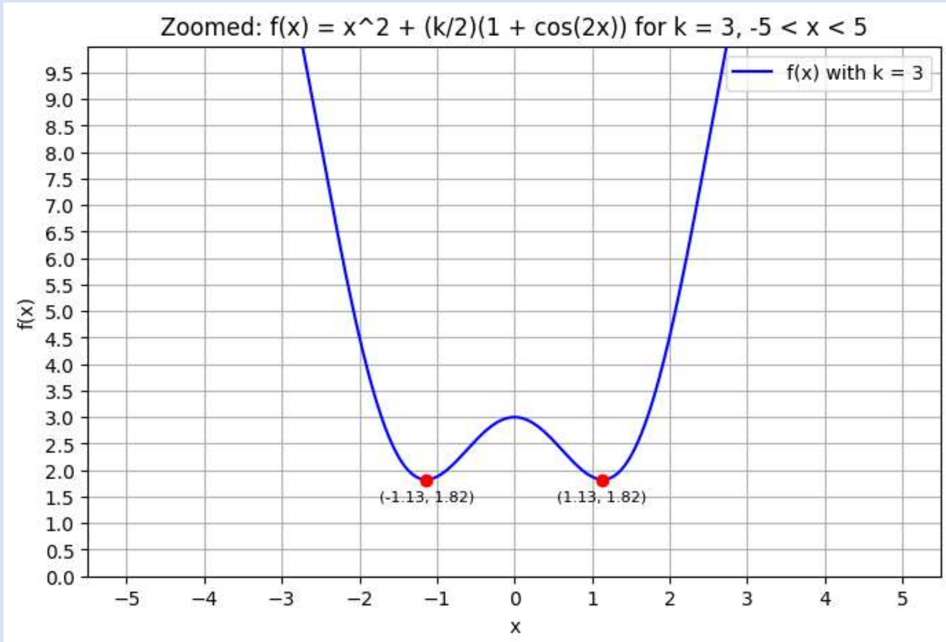

f(x) = x^2 + (k/2)(1 + cos(2x)) for k = 2, -30 < x < 30
f(x) with k = 2
f(x)
x



Zoomed: f(x) = x^2 + (k/2)(1 + cos(2x)) for k = 2, -5 < x < 5
f(x)
x
(-0.95, 1.58)
(0.95, 1.58)
f(x) with k = 2



f(x) = x^2 + (k/2)(1 + cos(2x)) for k = 3, -30 < x < 30
f(x)
x
f(x) with k = 3



The End