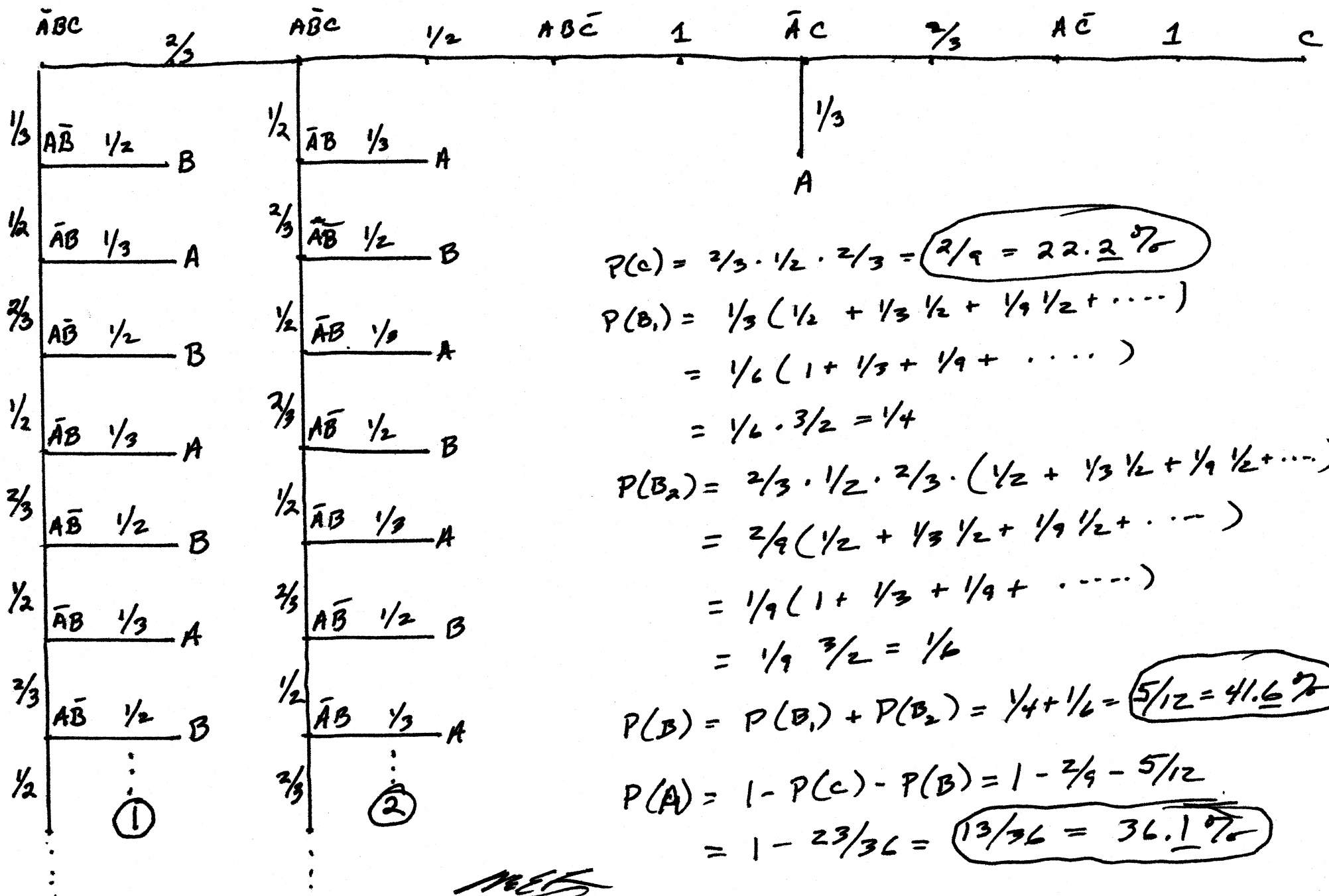


Compare Calculations to Simulation



$$P(C) = \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} = \frac{2}{9} = 22.\underline{2}\%.$$

$$\begin{aligned} P(B_1) &= \frac{1}{3} \left(\frac{1}{2} + \frac{1}{3} \frac{1}{2} + \frac{1}{9} \frac{1}{2} + \dots \right) \\ &= \frac{1}{6} \left(1 + \frac{1}{3} + \frac{1}{9} + \dots \right) \\ &= \frac{1}{6} \cdot \frac{3}{2} = \frac{1}{4} \end{aligned}$$

$$\begin{aligned} P(B_2) &= \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \left(\frac{1}{2} + \frac{1}{3} \frac{1}{2} + \frac{1}{9} \frac{1}{2} + \dots \right) \\ &= \frac{2}{9} \left(\frac{1}{2} + \frac{1}{3} \frac{1}{2} + \frac{1}{9} \frac{1}{2} + \dots \right) \\ &= \frac{1}{9} \left(1 + \frac{1}{3} + \frac{1}{9} + \dots \right) \\ &= \frac{1}{9} \cdot \frac{3}{2} = \frac{1}{6} \end{aligned}$$

$$P(B) = P(B_1) + P(B_2) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12} = 41.6\%$$

$$P(A) = 1 - P(C) - P(B) = 1 - \frac{2}{9} - \frac{5}{12}$$
$$= 1 - \frac{23}{36} = \frac{13}{36} = 36.1\%$$

or calculate $P(A)$ directly

$$P(A_3) = \frac{2}{3} \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{3} = \underline{\underline{\frac{1}{9}}}$$

$$\begin{aligned} P(A_2) &= \frac{2}{3} \cdot \frac{1}{2} \cdot \left(\frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \left(\frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \left(\frac{1}{3} \dots \right) \dots \right) \dots \right) \\ &= \frac{1}{9} (1 + \frac{1}{3} + \frac{1}{9} \dots) \end{aligned}$$

$$= \frac{1}{9} \cdot \frac{3}{2} = \underline{\underline{\frac{1}{6}}}$$

$$P(A_1) = \frac{1}{3} \cdot \frac{1}{2} \left(\frac{1}{3} + \frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \dots \right)$$

$$= \frac{1}{18} (1 + \frac{1}{3} + \frac{1}{9} \dots)$$

$$= \frac{1}{18} \cdot \frac{3}{2} = \underline{\underline{\frac{1}{12}}}$$

$$P(A) = P(A_1) + P(A_2) + P(A_3)$$

$$= \frac{1}{9} + \frac{1}{6} + \frac{1}{12} = \frac{4+6+3}{36}$$

$$= \underline{\underline{\frac{13}{36}}} = 36.1\bar{1}\%$$