The Sleeping Beauty Problem: a Monte Carlo Simulation

R. C. Jamen[1, a*]
[1]Prk 2, San Vicente St., Brgy. Maon, Butuan City, Agusan del Norte, Philippines
[a]ramelcary.jamen@g.msuiit.edu.ph

Abstract— The Sleeping Beauty problem is a puzzle that has required philosophers to refine their concepts and principles of rationality in an attempt to better understand how and when our credences ought to change. The aim of the paper is to integrate Monte Carlo Simulation to the problem, in order to assess the ideal approach of *halfers* and *thirders* through gambling. A based-case modeling was utilized and showed a promising result for choosing tails on any given scenario from the Sleeping Beauty Problem.

Keywords—Puzzle, decision theory, monte carlo, simulation

I. INTRODUCTION

Decision theory is a theory of the reasoning behind an agent's choices, such as taking the bus or getting a taxi, pursuing college or accepting a job offer. Standard thinking suggests that what an agent chooses to do is determined by their beliefs and desires or values. As much as it is a theory of choice, it is also a theory of beliefs, desires, and other important attitudes (call them "preference attitudes") cohere together [1]. One of its puzzles is the Sleeping Beauty Problem, which was proposed by Arnold Zuboff in the 1980's. Later on, people such as Robert Stalnaker and Adam Elga popularized the problem [2]. The Sleeping Beauty problem presents a scenario where Sleeping Beauty is put to sleep, and a fair coin is tossed. If the coin lands heads, Beauty is only interviewed on Monday. However, if the coin lands tails, Beauty is interviewed on both Monday and Tuesday, with an amnesia-inducing drug administered between the two interviews to erase her memory of the first one. During each interview, Beauty is briefly awakened and asked to assess the probability of the coin showing heads, without any access to additional information such as the coin toss outcome or knowledge of previous interviews or the day of the week [3]. The inclusion of Self-locating beliefs that define one's position or situation in the world, as opposed to beliefs about how the world is in itself [4], produces the following approach to the Sleeping Beauty Problem:

A. *Intuitive Approach (Halfers)*

A halfer's position expresses that Beauty was informed about the experiment in advance, she doesn't receive any extra information about her situation throughout the experiment. Before the experiment begins, she believes that the probability of getting heads is 1/2. Since she doesn't gain any new relevant information when she wakes up during the experiment, it logically follows that her belief in the probability of heads, P(Heads) = 1/2, should remain unchanged [5].

B. *Counterintuitive Approach (Thirders)*

A thirder is someone who believes that the probability of heads is 1/3. This is because the thirder position holds that when Sleeping Beauty wakes up, she should reason three possible scenarios as follows: heads on Monday, tails on Monday, and tails on Tuesday. Thirders argue that heads on Monday and tails on Tuesday are equally likely, each with a probability of 1/2. Therefore, since there are two possibilities for it being Tuesday (tails on Monday and tails on Tuesday), each with a probability of 1/2, the probability of it being Monday must be one-third (1/3) [1].

C. *Monte Carlo Simulation*

The Monte Carlo simulation is a simulation technique that utilizes repetitive random sampling and statistical analysis to calculate outcomes. This simulation method is closely associated with random experiments, where the exact outcome is unknown beforehand.

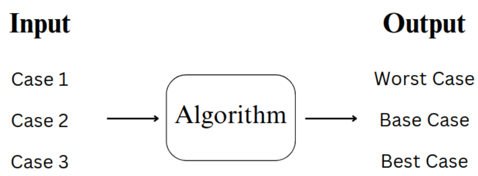

Figure 1. Base-Cased Modeling [6].

In our context, we will utilize Monte Carlo with Base-Cased Modeling to estimate the probability of a certain income from the Sleeping Beauty Problem when integrated with Gambling.

II. METHODOLOGY

To satisfy the objectives of this study, the following will be the required specification:

A. *Algorithm*

Using Python and its dependencies, an algorithm that is modified and developed for implementation to illustrate the Sleeping Beauty problem [8].

Algorithm 1. Sleeping Beauty Pseudocode

```
Toss a Coin on Sunday with Bet
        If Head, SB wakes up Monday
        If Tails, SB wakes up Monday to Friday
For each wake up
        Guess Coin, if True, add Gains
```

B. Test Cases

In the Monte Carlo Simulation, given the mechanics of the Sleeping Beauty and to assess the money gained by SB, we will differentiate between 3 cases of the probabilistic approaches with their corresponding assumptions:

- Halfers - SB bets full heads every awakening
- Random - SB bets randomly between heads and tails every awakening
- Thirders - SB bets full tails every awakening

C. Setup of the Simulation

SB owns an initial ₱1,000,000 on the bank, for every Sunday she will bet ₱2 to play on the succeeding days or to have a chance to guess on Monday to Friday (5 Days), where she can guess (according to test cases) to win ₱3 for each correct answer on every wake up. A total experiment of 3 reruns with n-number of attempts (300,000/600,000/1,000,000) sought to answer the difference of gains between the 3 above-stated approaches.

III. RESULTS AND DISCUSSION

Table 1. Simulation Setup

N - Attempts	Toss Results (H/T) %			
	On Sunday		On Wakeup	
300,000	49.82	50.18	16.70	83.30
600,000	50.02	49.98	16.68	83.32
1,000,000	49,94	50.06	16.63	83.37

In table 1, Toss results from the N - Attempts on Sundays shows an approximation of ½ and an indication of a fair coin that are equally likely to show when the coin is flipper [9]. However, changes upon wakeup shows that tails are more likely to come up as the credence of the tails adds up as SB waking up from Monday to Friday [10].

Table 2. Money Gains at 300,000 attempts

Approach (H/T)%	Number Count in Thousands(K) and Million (M)			
	Bet Win	Bet Lose	Wakeups	Gains
Halfers (16/83)	≈150K	≈748K	≈898K	≈ -148K
Random(49/50)	≈449K	≈448K	≈898K	≈748K
Thirders(83/16)	≈749K	≈150K	≈899K	≈1.6M

Table 3. Money Gains at 600,000 attempts

Approach (H/T)%	Number Count in Thousands(K) and Million (M)			
	Bet Win	Bet Lose	Wakeups	Gains
Halfers (16/83)	≈300K	≈1.5M	≈1.8M	≈ -297K
Random(49/50)	≈896K	≈902K	≈1.7M	≈1.4M
Thirders(83/16)	≈1.5M	≈300K	≈1.8M	≈3.3M

Table 4. Money Gains at 1,000,000 attempts

Approach (H/T)%	Number Count in Thousands(K) and Million (M)			
	Bet Win	Bet Lose	Wakeups	Gains
Halfers (16/83)	≈500K	≈2.5M	≈2.9M	≈ -499K
Random(49/50)	≈1.5M	≈1.4M	≈3M	≈2.5M
Thirders(83/16)	≈2.4M	≈500K	≈2.9M	≈5.4M

The three tables (2,3,4), shows an approximation of the values resulted from the simulation, and with these given following insights:

- The halfers gain results show an inverse relationship with the number of wakeups, as the number of attempts increases shows undesirable decrease in gains for all simulation.
- A negative gain shows a loss from the simulation from the halfers
- Random choice of guessing from SB upon waking up shows promising results with gains.
- The thirders money gains peaked at ≈5.4M from 1M initial money, from the 1,000,000 reruns of the simulation.

The counterintuitive approach (thirders) or betting tails upon waking up was the best choice for the gambling setup on this Monte Carlo Simulation, because of the tail's additional number of days and exploitation of the game's rule (SB doesn't remember which day she woke up), which gives SB the chances of winning consecutively guessing that she is on the day where tails landed.

IV. CONCLUSION

In conclusion, this paper has addressed the intriguing Sleeping Beauty problem, which has challenged philosophers in refining their notions of rationality and decision-making. By integrating Monte Carlo Simulation into the analysis, our study aimed to assess the optimal approach of both halfers and thirders through the lens of gambling. Through our carefully designed base-case modeling, we have obtained promising results that favor the selection of tails in any given scenario from the Sleeping Beauty Problem.

By providing empirical evidence, we contribute to the ongoing discourse surrounding rationality and decision theory. Further research could explore alternative modeling techniques, consider additional variables, or examine different perspectives on the problem. Nonetheless, our investigation serves as a valuable contribution to the understanding of the Sleeping Beauty problem and its implications for rational belief updating.

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Appendix

Fig 1. Halfer at 300,000

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Total experiment attempts: 300000
**Halfer Position**

Toss result was HEADS: 150634 [50.21%]
Toss result was TAILS: 149366 [49.79%]

Toss result was HEADS on wakeup: 150634 [16.78%]
Toss result was TAILS on wakeup: 746830 [83.22%]

Bet was HEADS on wakeup: [16.78%]
Bet was TAILS on wakeup: [83.22%]
Total wake ups: 897464
Bet Wins: 150634
Bet Lose: 746830

Total Money: 851902
Gains: -148098
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Fig 2. Random at 300,000

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Total experiment attempts: 300000
**Random Position**

Toss result was HEADS: 149913 [49.97%]
Toss result was TAILS: 150087 [50.03%]

Toss result was HEADS on wakeup: 149913 [16.65%]
Toss result was TAILS on wakeup: 750435 [83.35%]

Bet was HEADS on wakeup: [49.94%]
Bet was TAILS on wakeup: [50.06%]
Total wake ups: 900348
Bet Wins: 449602
Bet Lose: 450746

Total Money: 1748806
Gains: 748806
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Fig 3. Thirder at 300,000

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Total experiment attempts: 300000
**Thirder Position**

Toss result was HEADS: 150150 [50.05%]
Toss result was TAILS: 149850 [49.95%]

Toss result was HEADS on wakeup: 150150 [16.69%]
Toss result was TAILS on wakeup: 749250 [83.31%]

Bet was HEADS on wakeup: [83.31%]
Bet was TAILS on wakeup: [16.69%]
Total wake ups: 899400
Bet Wins: 749250
Bet Lose: 150150

Total Money: 2647750
Gains: 1647750
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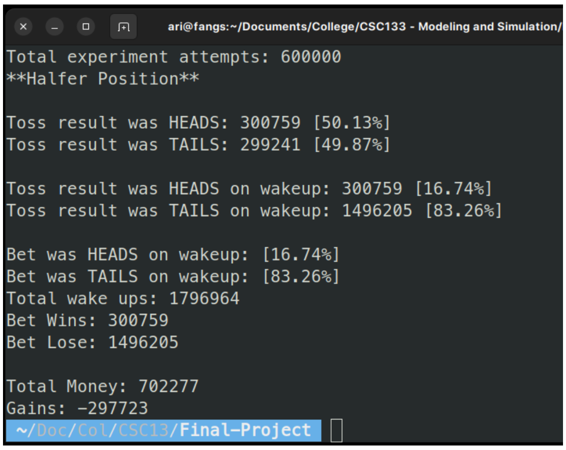
Fig 4. Halfer at 600,000



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Total experiment attempts: 600000
**Halfer Position**

Toss result was HEADS: 300759 [50.13%]
Toss result was TAILS: 299241 [49.87%]

Toss result was HEADS on wakeup: 300759 [16.74%]
Toss result was TAILS on wakeup: 1496205 [83.26%]

Bet was HEADS on wakeup: [16.74%]
Bet was TAILS on wakeup: [83.26%]
Total wake ups: 1796964
Bet Wins: 300759
Bet Lose: 1496205

Total Money: 702277
Gains: -297723
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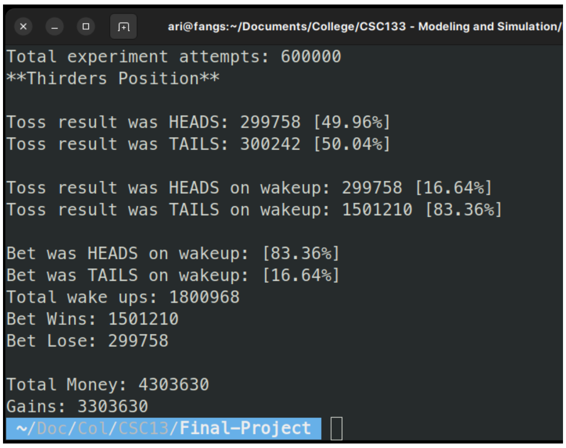
Fig 5. Random at 600,000



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Total experiment attempts: 600000
**Thirders Position**

Toss result was HEADS: 299758 [49.96%]
Toss result was TAILS: 300242 [50.04%]

Toss result was HEADS on wakeup: 299758 [16.64%]
Toss result was TAILS on wakeup: 1501210 [83.36%]

Bet was HEADS on wakeup: [83.36%]
Bet was TAILS on wakeup: [16.64%]
Total wake ups: 1800968
Bet Wins: 1501210
Bet Lose: 299758

Total Money: 4303630
Gains: 3303630
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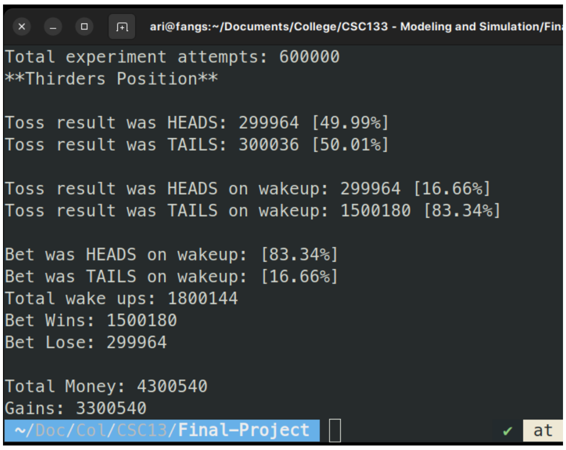
Fig 6. Thirder at 600,000



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Total experiment attempts: 600000
**Thirders Position**

Toss result was HEADS: 299964 [49.99%]
Toss result was TAILS: 300036 [50.01%]

Toss result was HEADS on wakeup: 299964 [16.66%]
Toss result was TAILS on wakeup: 1500180 [83.34%]

Bet was HEADS on wakeup: [83.34%]
Bet was TAILS on wakeup: [16.66%]
Total wake ups: 1800144
Bet Wins: 1500180
Bet Lose: 299964

Total Money: 4300540
Gains: 3300540
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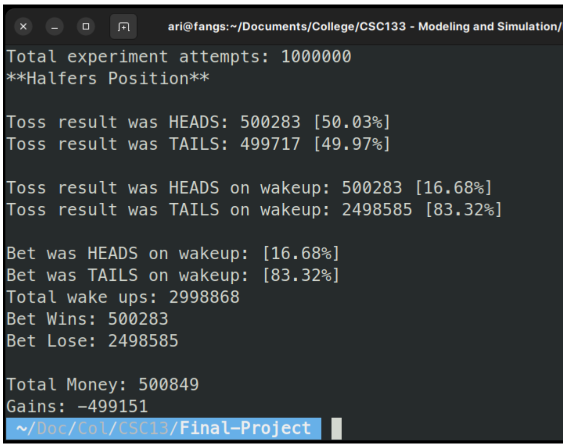
Fig 7. Halfer at 1,000,000



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Total experiment attempts: 1000000
**Halfers Position**

Toss result was HEADS: 500283 [50.03%]
Toss result was TAILS: 499717 [49.97%]

Toss result was HEADS on wakeup: 500283 [16.68%]
Toss result was TAILS on wakeup: 2498585 [83.32%]

Bet was HEADS on wakeup: [16.68%]
Bet was TAILS on wakeup: [83.32%]
Total wake ups: 2998868
Bet Wins: 500283
Bet Lose: 2498585

Total Money: 500849
Gains: -499151
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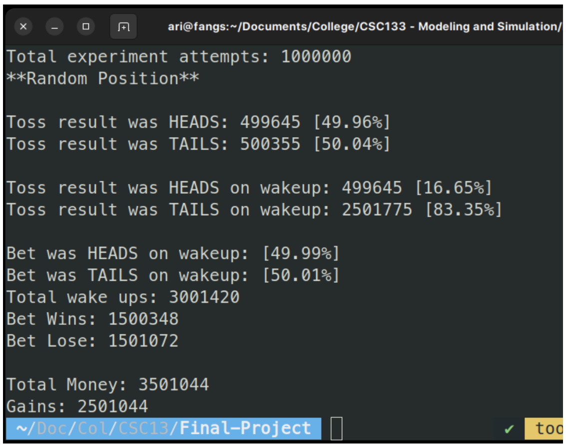
Fig 8. Random at 1,000,000



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Total experiment attempts: 1000000
**Random Position**

Toss result was HEADS: 499645 [49.96%]
Toss result was TAILS: 500355 [50.04%]

Toss result was HEADS on wakeup: 499645 [16.65%]
Toss result was TAILS on wakeup: 2501775 [83.35%]

Bet was HEADS on wakeup: [49.99%]
Bet was TAILS on wakeup: [50.01%]
Total wake ups: 3001420
Bet Wins: 1500348
Bet Lose: 1501072

Total Money: 3501044
Gains: 2501044
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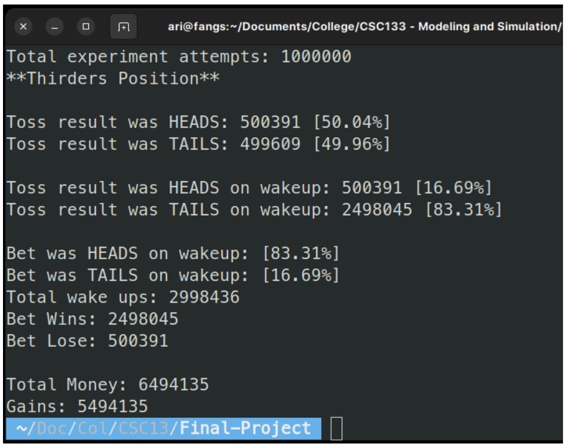
Fig 9. Thirder at 1,000,000



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Total experiment attempts: 1000000
**Thirders Position**

Toss result was HEADS: 500391 [50.04%]
Toss result was TAILS: 499609 [49.96%]

Toss result was HEADS on wakeup: 500391 [16.69%]
Toss result was TAILS on wakeup: 2498045 [83.31%]

Bet was HEADS on wakeup: [83.31%]
Bet was TAILS on wakeup: [16.69%]
Total wake ups: 2998436
Bet Wins: 2498045
Bet Lose: 500391

Total Money: 6494135
Gains: 5494135
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