ME 6622 Spring 2025

Computer Project: Data Fit

I. Background

The amplitude of the response of an underwater sensor to a time-periodic "ping" of duration T is given by a composite waveform having time-dependent amplitude and frequency:

$$\frac{A}{A_0} = \left[\sin\left(2\pi \cdot f_m \cdot \left(\frac{t}{T}\right)\right)\right]^2 * abs\left(\left[\sin\left(2\pi \cdot f_c(t) \cdot \left(\frac{t}{T}\right)\right)\right]\right)$$

where f_m is the amplitude modulating frequency and $f_c(t)$ is the carrier frequency. In the present implementation $f_m = 0.5$ Hz and $f_c = 4 \cdot (t/T)^2$ Hz, and the corresponding time trace is shown in Figure 1.

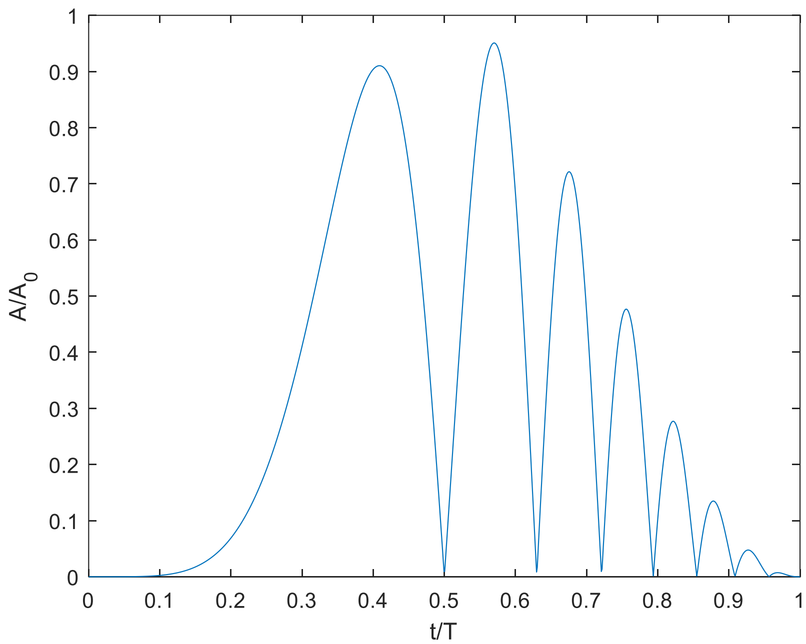

Figure 1 Time trace of one period of the sensor response

II. Project Specifics

The amplitude of the measured signal is affected by noise that modifies the amplitude and results in an offset:

$$\tilde{A} = \frac{A_{\text{actual}}}{A_o} = \left[1 + a_1\varepsilon_1\left(\frac{t}{T}\right)\right]\left(\frac{A}{A_o}\right) + a_2\varepsilon_2\left(\frac{t}{T}\right)$$

where $\varepsilon_i(t)$ are modeled using a noise function $N(t)$ (having a known PDFs) that independently affects the magnitudes of ε_1 and ε_2.

Given a discretized sample of A (in the absence and presence of noise) over $t/T \in [0, 1]$, it is desired to determine an estimate of its functional form using a polynomial fit on a least squares basis. The fit is applied using the three approaches discussed in class:

i. Global fit,
ii. Piecewise fit,

and

iii. "Optimal" fit,

The results of each approach should be analyzed, and the compared to the other approaches.

The project findings should be summarized in a brief, concise report (as outlined in class) that should include the following steps:

1. Select an appropriate discretization level for your data.
2. (5%) Select a (known) noise function $N(t/T)$ and determine its PDF. Apply the noise to the discretized data and check the added noise by plotting the discretized $\tilde{A}$(t/T) for t/T ∈[0, 1]
3. (5%) Select an appropriate criterion for assessing the quality (goodness) of the fit in your work. Please discuss this criterion briefly in the report.
4. Develop algorithms for a polynomial fit to the discretized data for the Global, Piecewise, and "Optimal" fits as described in class. The results obtained with these algorithms form the main body of the project report.
5. (35%) Using your algorithms, perform the polynomial fit for each of Global, Piecewise, and "Optimal" fits *in the absence of noise ($\varepsilon_i = 0$)* using *several* polynomial orders. Discuss the sensitivity of your selected goodness of fit criterion to the degree of the polynomial, and to the window size of the piecewise and optimal fits. *Illustrate this discussion with figures that show the discretized Pand each fit*.
6. (10%) Based on your criterion of goodness of fit adjust the fit parameters to yield your optimized Global, Piecewise, and "Optimal" fits and plot the discretized P and the fit. Include your selected parameters.
7. (45%) Repeat parts (5) and (6) in the presence of noise ($\varepsilon_i > 0$) for several levels of noise amplitude ε_i (include at least one 'low' and one 'high' noise levels). Apply your fit algorithm to the signal in the presence of noise several times (iteratively). Observe the changes in the fit and note any improvements.
8. *Please summarize your conclusions at the end of the report.*

Comments

1. Please *write your own brief code* following the formulation derived in class using a software platform of your choice. ***Please refrain from using built-in functions*** for computing the least square fit. You can use a built-in solver to solve the system of linear equations for the coefficients, and any plotting software.

2. The body of your report should include your entire analysis and the accompanying plots.

3. Each figure should be properly numbered and referenced. All figures should include clear legends, axis labels, and a caption. Please import your figures (MATLAB: edit figure>copy figure> past), *please avoid screen shots*.

4. Please append your code to the report. Make sure that the body of the report and the appendix are clearly separated. *None of the results should be included within the code, nor should parts of the code be included in the body of the report.*