New way of creating and standardizing phenotype data with variance partitioning

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$$V_P = V_G + V_E + V_{GE} + Cov_{GE}$$

In a reciprocal transplant experiment, there are g genotypes transplanted into e environmental patches, for a total of $g*e=n_{GE}$ genotype-environment combinations.

g is levels of genotypes for i = 1, 2...g e is levels of environments for j = 1, 2...e

Assuming the equal sample sizes r(1, 2, ...k) within each genotype-environment combination, could partitioning the variance be as simple as:

$$V_G = re \sum_{i=1}^g (\bar{y}_i - \bar{y})^2$$

$$V_E = rg \sum_{j=1}^{e} (\bar{y}_j - \bar{y})^2$$

$$V_{GE} = r \sum_{i=1}^{g} \sum_{j=1}^{e} (\bar{y}_{ij} - \bar{y}_{i} - \bar{y}_{j} + \bar{y})^{2}$$

$$V_{Cov_{GE}} = (g-1)er \sum_{i=1}^{g} \sum_{j=1}^{e} (\bar{y}_i - \bar{y})(\bar{y}_j - \bar{y})I$$

- I is an indicator variable that is 1 when the genotype i originated from environment j and 0 otherwise
- basically, each summation needs to count the same number of times to be comparable as a SS on the same scale. If we have a 2x2 reciprocal transplant, we have 2 of 4 treatment combinations that count toward the sum (and 2 that should but do not). If we have a 4x4 reciprocal transplant, we have 4 of 16 reciprocal transplants that count toward the sum (and 12 that should but do not). We can get the SS to the right amount by multiplying (g-1)*e to get the same number of observations as the other SS calculations. At this point I'm only confident this will work in a reciprocal transplant case, would have to test g=4 and e=2 level cases etc.

$$V_{error} = \sum_{i=1}^{g} \sum_{j=1}^{e} \sum_{k=1}^{r} (y_{ijk} - \bar{y}_{ij})^2$$

Some evidence that this is behaving as expected:

if you set beta 1 = 1 and beta 2 = 1 and int = 0, you get equal amounts of variance explained for G, E, and CovGE for 2 pops, but CovGE increases with more pops

if you set beta 1 = 1 and beta 2 = 0.1 and int = 0, you get more variance explained for V_E, a little bit for V_G, and a decent amount for cov_GE

if you set beta 1 = 0.1 and beta 2 = 1 and int = 0, you get the same pattern as above but V_E and V_G switched

if you set beta1 or beta2 negative and the other positive, you get negative covGE

try adding noise and see if outputs make sense * Adding noise can sometimes "create" some CovGE when there are enough pops and beta1=0 or beta2=0 * But for most part seems to work (?)

try increasing n_pop and see if outputs make sense * increasing n_pop can lead to a greater amount of variance explained by CovGE and less by VG or VE

Step 1: create true phenotype data without error

```
beta1 = 10 # the amount the phenotype changes across 1 value of the environment (i.e., the slope). This
n_genotypes = 10 # the number of genotypes collected from different locations
n = 10 \# sample size
beta2 = 10 # the amount the phenotype changes from one genotype to the next. This is essitially the inc
scale = 0.01 # the scale for sd_noise within pops compared to sd_means among pops
# I'll let you explore the space, but I think the upper would be scale = 1
# I'd start with scale = 1 and scale = 1/2 and see what that looks like
sd_int = 0#n_genotypes # sd of the interaction terms that will be drawn
    # beta1 <- 1
    # beta2 <- 1
    # Generate data
   n_environments = n_genotypes # the number of common garden environments that the genotypes are grow
   ## Create levels of genotypes and environments ####
   G = rep(1:n_genotypes, each=n*n_genotypes) # '0' n times, '1' n times
   E = rep(1:n\_environments, times=n*n\_genotypes) # '0'x50, '1'x50, '0'x50, '1'x50
    #set.seed(97)
    ## Create interaction effect for each level of both factors ####
    # In this case we assume the GxE interactions are a
    # normally distributed random variable with a mean of O
    # As the sd increases, so does the GxE among treatments
   if(sd int == 0){
      int = 0
    int <- rnorm(n_genotypes * n_environments, 0, sd=sd_int)</pre>
    # no GxE
     # this sd determines the amount of GxE
    int_df <- data.frame(expand.grid(G=1:n_genotypes, E=1:n_environments),</pre>
                         int)
    ### Create the model dataframe ####
   model_df <- data.frame(G, E)</pre>
   model_df$GE_factor <- paste0("G",model_df$G, "E", model_df$E)</pre>
   model_df <- merge(model_df, int_df)</pre>
   model_df <- model_df[order(model_df$G, model_df$E),]</pre>
   dim(model_df)
## [1] 1000
   head(model_df, 30)
     G E GE_factor int
## 1 1 1
               G1E1
## 2 1 1
               G1E1
## 3 1 1
               G1E1
```

```
## 4 1 1
               G1E1
## 5 1 1
               G1E1
## 6 1 1
               G1E1
## 7 1 1
               G1E1
                       0
## 8 1 1
               G1E1
                       0
## 9 1 1
               G1E1
                       0
## 10 1 1
               G1E1
## 21 1 2
               G1E2
                       0
## 22 1 2
               G1E2
                       0
## 23 1 2
               G1E2
## 24 1 2
               G1E2
## 25 1 2
               G1E2
                       0
## 26 1 2
               G1E2
                       0
## 27 1 2
               G1E2
## 28 1 2
               G1E2
                       0
## 29 1 2
               G1E2
## 30 1 2
               G1E2
## 31 1 3
               G1E3
## 32 1 3
               G1E3
                       0
## 33 1 3
               G1E3
## 34 1 3
               G1E3
                       0
## 35 1 3
               G1E3
## 36 1 3
               G1E3
                       0
## 37 1 3
               G1E3
                       0
## 38 1 3
               G1E3
                       0
## 39 1 3
               G1E3
                       0
## 40 1 3
               G1E3
                       0
    # Generate phenotype data using the regression equation ####
    model_df$GE_true = beta1*model_df$E + beta2*model_df$G + model_df$int
    G_true <- data.frame(G=1:n_genotypes, G_true=tapply(model_df$GE_true, model_df$G, mean))
    E_true <- data.frame(E=1:n_genotypes , E_true=tapply(model_df$GE_true, model_df$E, mean))
    model_df1 <- merge(model_df,G_true)</pre>
    model_df2 <- merge(model_df1,E_true)</pre>
    model_df <- model_df2
    model_df$mean_true <- mean(model_df$GE_true)</pre>
    model_df$int_true <- model_df$mean_true + model_df$GE_true -</pre>
                         model_df$G_true -
                                              model_df$E_true
    head(model_df)
     E G GE_factor int GE_true G_true E_true mean_true int_true
## 1 1 1
              G1E1
                             20
                                     65
                                            65
                                                      110
## 2 1 1
              G1E1
                      0
                             20
                                            65
                                                      110
                                                                 0
                                     65
                             20
                                            65
                                                                 0
## 3 1 1
              G1E1
                      0
                                     65
                                                      110
## 4 1 1
              G1E1
                      0
                             20
                                     65
                                            65
                                                      110
                                                                 0
## 5 1 1
                                                                 0
              G1E1
                      0
                             20
                                     65
                                            65
                                                      110
## 6 1 1
                             20
                                                                 0
              G1E1
                      0
                                     65
                                            65
                                                      110
    tail(model_df)
         E G GE_factor int GE_true G_true E_true mean_true int_true
```

155

110

155

200

995 10 10

G10E10

```
## 996
         10 10
                    G10E10
                                     200
                                             155
                                                      155
                                                                  110
   997
         10 10
                   G10E10
                              0
                                     200
                                             155
                                                      155
                                                                  110
                                                                              0
         10 10
   998
                    G10E10
                                     200
                                             155
                                                      155
                                                                  110
                                                                              0
                                                      155
                                                                              0
## 999
         10 10
                    G10E10
                              0
                                     200
                                             155
                                                                  110
   1000 10 10
                    G10E10
                                     200
                                             155
                                                      155
                                                                  110
    par(mfrow=c(1,1))
    plot(model_df$int, model_df$int_true)
    abline(0,1)
      S
model_df$int_true
      0.0
      5
```

Note that the way we add "int" to create phenotypic variation is not the same way that the actual interaction is calculated from the data. Our simulations create an interaction, but as we create an interaction we also change the G-means and E-means, and so the calculated interaction is a bit different.

0.0

model df\$int

0.5

1.0

Step 2: standardize true phenotype data

-0.5

-1.0

```
GE_true_means <- tapply(model_df$GE_true, model_df$GE_factor, mean)

model_df$GE_stn_true <- (model_df$GE_true - mean(GE_true_means))/sd(GE_true_means)

GE_stn_true_means <- tapply(model_df$GE_stn_true, model_df$GE_factor, mean)

G_stn_true <- data.frame(G=1:n_genotypes, G_stn_true=tapply(model_df$GE_stn_true, model_df$G, mean)

E_stn_true <- data.frame(E=1:n_genotypes , E_stn_true=tapply(model_df$GE_stn_true, model_df$E, mean)

model_df1 <- merge(model_df,G_stn_true)

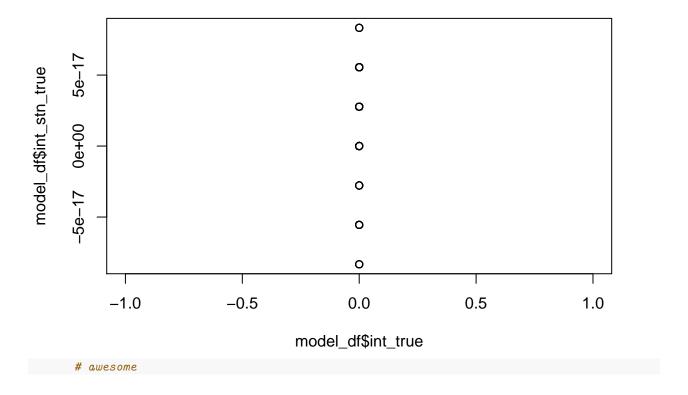
model_df2 <- merge(model_df1,E_stn_true)

model_df3 <- model_df2

model_df3 mean_stn_true <- mean(model_df$GE_stn_true)

model_df$int_stn_true <- model_df$mean_stn_true + model_df$GE_stn_true -
```

```
model_df$G_stn_true - model_df$E_stn_true
    head(model_df)
##
     E G GE_factor int GE_true G_true E_true mean_true int_true GE_stn_true
## 1 1 1
              G1E1
                      0
                             20
                                    65
                                            65
                                                     110
                                                                     -2.204541
## 2 1 1
                                            65
                                                                     -2.204541
              G1E1
                      0
                             20
                                    65
                                                     110
                                                                 0
## 3 1 1
              G1E1
                      0
                             20
                                    65
                                            65
                                                     110
                                                                 0
                                                                     -2.204541
## 4 1 1
              G1E1
                      0
                             20
                                    65
                                            65
                                                     110
                                                                 0
                                                                    -2.204541
## 5 1 1
              G1E1
                      0
                             20
                                    65
                                            65
                                                     110
                                                                    -2.204541
## 6 1 1
              G1E1
                      0
                             20
                                    65
                                            65
                                                                 0 -2.204541
                                                     110
    G_stn_true E_stn_true mean_stn_true int_stn_true
## 1
      -1.10227
                  -1.10227
                                        0
                                                      0
## 2
      -1.10227
                  -1.10227
                                        0
                                                      0
## 3
       -1.10227
                  -1.10227
                                        0
                                                      0
                                        0
                                                      0
## 4
      -1.10227
                  -1.10227
                                        0
## 5
      -1.10227
                  -1.10227
                                                      0
      -1.10227
                  -1.10227
                                         0
                                                      0
## 6
    tail(model df)
         E G GE_factor int GE_true G_true E_true mean_true int_true GE_stn_true
##
## 995
        10 9
                 G9E10
                          0
                                190
                                        145
                                               155
                                                         110
                                                                     0
                                                                         1.9595918
## 996
        10 7
                 G7E10
                          0
                                170
                                        125
                                               155
                                                         110
                                                                         1.4696938
        10 6
                 G6E10
                                160
                                               155
                                                                         1.2247449
## 997
                          0
                                        115
                                                         110
                                                                     0
## 998
        10 3
                 G3E10
                          0
                                130
                                        85
                                               155
                                                         110
                                                                     0
                                                                         0.4898979
## 999
        10 8
                 G8E10
                          0
                                180
                                        135
                                               155
                                                         110
                                                                     0
                                                                         1.7146428
## 1000 10 6
                 G6E10
                          0
                                160
                                        115
                                               155
                                                         110
                                                                         1.2247449
        G_stn_true E_stn_true mean_stn_true int_stn_true
##
## 995
         0.8573214
                      1.10227
                                            0
                                                         0
## 996
         0.3674235
                       1.10227
                                            0
## 997
         0.1224745
                      1.10227
                                            0
                                                         0
                                            0
                                                         0
## 998
        -0.6123724
                       1.10227
## 999
         0.6123724
                       1.10227
                                            0
                                                         0
## 1000 0.1224745
                       1.10227
                                            0
                                                         0
    par(mfrow=c(1,1))
    plot(model_df$int_true, model_df$int_stn_true)
```

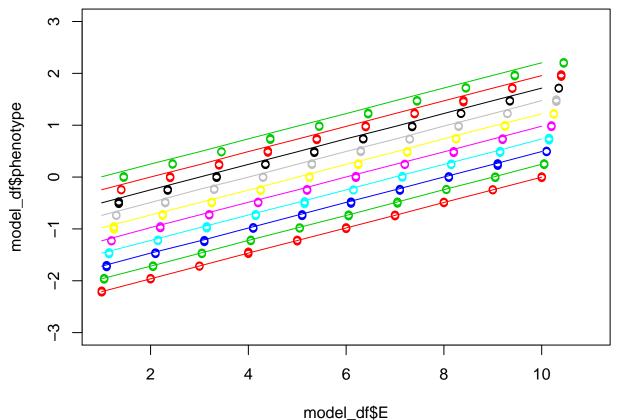


Step 3: add error to stnd phenotype data

```
scaled to the variation in the phenotype means
```

```
sd(GE_stn_true_means)
## [1] 1
# I guess this will always be 1 because we standardize.
sd_noise <- sd(as.numeric(GE_stn_true_means))*scale</pre>
model_df$e = rnorm(n*n_genotypes*n_environments, 0, sd=sd_noise) # Random noise
model_df$phenotype <- model_df$GE_stn_true + model_df$e</pre>
head(model_df)
     E G GE_factor int GE_true G_true E_true mean_true int_true GE_stn_true
## 1 1 1
              G1E1
                      0
                             20
                                     65
                                            65
                                                      110
                                                                  0
                                                                      -2.204541
## 2 1 1
                             20
               G1E1
                      0
                                     65
                                            65
                                                      110
                                                                      -2.204541
## 3 1 1
              G1E1
                      0
                             20
                                     65
                                            65
                                                                  0
                                                                      -2.204541
                                                      110
## 4 1 1
              G1E1
                      0
                              20
                                     65
                                            65
                                                      110
                                                                      -2.204541
## 5 1 1
              G1E1
                      0
                              20
                                     65
                                            65
                                                      110
                                                                      -2.204541
## 6 1 1
              G1E1
                      0
                              20
                                            65
                                                      110
                                                                      -2.204541
##
     G_stn_true E_stn_true mean_stn_true int_stn_true
                                                                     e phenotype
                   -1.10227
       -1.10227
                                                       0 -0.020174463 -2.224715
## 1
                                         0
                                         0
## 2
       -1.10227
                  -1.10227
                                                       0 -0.010773675 -2.215314
## 3
       -1.10227
                   -1.10227
                                         0
                                                       0 0.006140117 -2.198401
       -1.10227
                                         0
                                                       0 -0.011312425 -2.215853
## 4
                   -1.10227
## 5
       -1.10227
                   -1.10227
                                         0
                                                       0 -0.013353474 -2.217894
                                         0
## 6
       -1.10227
                   -1.10227
                                                       0 -0.011807912 -2.216349
```

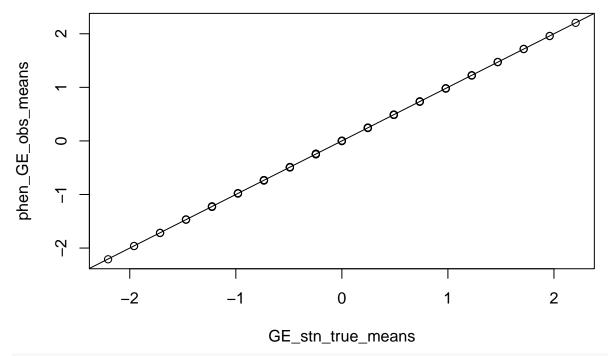
Step 4: plot the pattern so we can see what it looks like



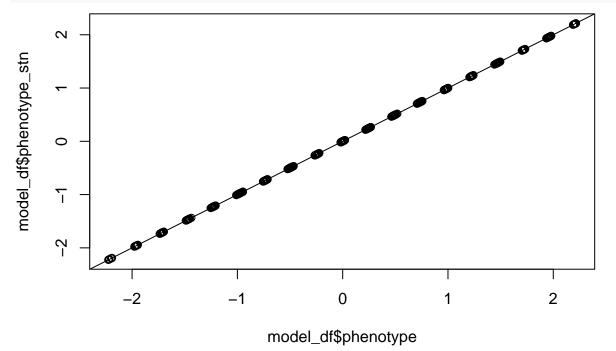
Step 5: pretend we start with noisy simulated data

Go through the steps we will go through in the study

```
# Step A: standardize by mean(GE_means) and sd(GE_means)
phen_GE_obs_means <- tapply(model_df$phenotype, model_df$GE_factor, mean)
plot(GE_stn_true_means, phen_GE_obs_means) #sanity check
abline(0,1)</pre>
```



model_df\$phenotype_stn <- (model_df\$phenotype_mean(phen_GE_obs_means))/sd(phen_GE_obs_means)
plot(model_df\$phenotype, model_df\$phenotype_stn)
abline(0,1)</pre>

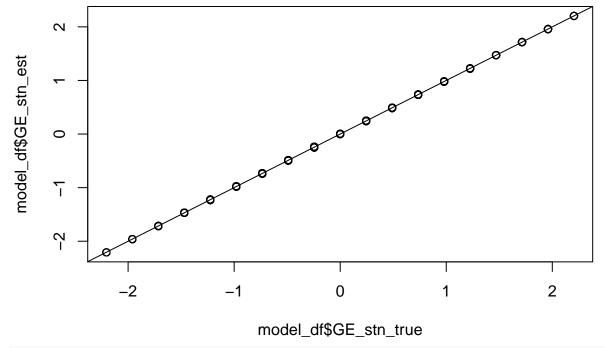


```
# Step B: calculate observed G_means and E_means and interaction
   G_stn_est <- data.frame(G=1:n_genotypes, G_stn_est=tapply(model_df$phenotype_stn, model_df$G, mean)
   E_stn_est <- data.frame(E=1:n_genotypes , E_stn_est=tapply(model_df$phenotype_stn, model_df$E, mean

   GE_stn_est_means <- tapply(model_df$phenotype_stn, model_df$GE_factor, mean)
   GE_stn_est_means_df <- data.frame(GE_factor=names(GE_stn_est_means) , GE_stn_est=GE_stn_est_means)</pre>
```

```
model_df1 <- merge(model_df,G_stn_est)
model_df2 <- merge(model_df1,E_stn_est)
model_df3 <- merge(model_df2, GE_stn_est_means_df)
model_df <- model_df3

plot(model_df$GE_stn_true, model_df$GE_stn_est)
abline(0,1) # looks good</pre>
```



Step 6: Compare true values to estimated values

```
head(model_df)
##
     GE_factor E G int GE_true G_true E_true mean_true int_true GE_stn_true
## 1
         G10E1 1 10
                             110
                                     155
                                             65
                                                       110
## 2
         G10E1 1 10
                       0
                             110
                                     155
                                             65
                                                       110
                                                                  0
                                                                               0
## 3
         G10E1 1 10
                       0
                             110
                                     155
                                             65
                                                                               0
                                                       110
                                                                  0
## 4
         G10E1 1 10
                             110
                                     155
                                             65
                                                                  0
                                                                               0
                       0
                                                       110
## 5
         G10E1 1 10
                             110
                                     155
                                             65
                                                       110
                                                                  0
                                                                               0
         G10E1 1 10
                                     155
                                             65
                                                                               0
## 6
                       0
                             110
                                                       110
                                                                  0
                                                                            phenotype
##
     G_stn_true E_stn_true mean_stn_true int_stn_true
## 1
        1.10227
                                                       0 -6.073255e-04 -6.073255e-04
                  -1.10227
                                         0
## 2
        1.10227
                  -1.10227
                                         0
                                                        1.423871e-02 1.423871e-02
## 3
        1.10227
                   -1.10227
                                         0
                                                          1.364169e-02
                                                                        1.364169e-02
## 4
        1.10227
                  -1.10227
                                         0
                                                       0 1.226333e-02 1.226333e-02
## 5
        1.10227
                  -1.10227
                                                       0 5.721586e-03 5.721586e-03
```

```
## 6
        1.10227
                  -1.10227
                                                     0 -4.673862e-05 -4.673862e-05
##
     phenotype_stn G_stn_est E_stn_est GE_stn_est mean_stn_est int_stn_est
## 1 -0.0001974582
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
     0.0146421836
     0.0140454142
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
     0.0126676471
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
     0.0061287256
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
## 6 0.0003628870
(true_int <- mean(abs(model_df$int_stn_true)))</pre>
## [1] 1.221245e-17
(obs_int <- mean(abs(model_df$int_stn_est)))</pre>
```

[1] 0.002389785

New step 6.5: make ANOVA table with CovGE

Here, I didn't write loops to calculate the equations, so the code will look different from the equations.

TO DO: check to code the loops as written in the equations and make sure it gives the same answer as what I calculated here

```
head(model_df)
     GE_factor E G int GE_true G_true E_true mean_true int_true GE_stn_true
##
## 1
         G10E1 1 10
                      0
                             110
                                    155
                                            65
                                                      110
## 2
         G10E1 1 10
                       0
                             110
                                    155
                                            65
                                                      110
                                                                 0
                                                                              0
## 3
                       0
                                    155
                                            65
                                                                              0
         G10E1 1 10
                             110
                                                      110
                                                                 0
## 4
         G10E1 1 10
                       0
                             110
                                    155
                                            65
                                                                 0
                                                                              0
                                                      110
## 5
         G10E1 1 10
                       0
                             110
                                    155
                                            65
                                                      110
                                                                 0
                                                                              0
## 6
         G10E1 1 10
                      0
                                                                              0
                             110
                                    155
                                            65
                                                      110
                                                                 0
     G_stn_true E_stn_true mean_stn_true int_stn_true
                                                                           phenotype
## 1
        1.10227
                                        0
                  -1.10227
                                                      0 -6.073255e-04 -6.073255e-04
## 2
        1.10227
                   -1.10227
                                        0
                                                         1.423871e-02
                                                                       1.423871e-02
## 3
                                        0
        1.10227
                  -1.10227
                                                         1.364169e-02
                                                                       1.364169e-02
                                                      0
## 4
        1.10227
                  -1.10227
                                        0
                                                         1.226333e-02
                                                                       1.226333e-02
## 5
        1.10227
                  -1.10227
                                        0
                                                        5.721586e-03 5.721586e-03
        1.10227
                  -1.10227
                                                      0 -4.673862e-05 -4.673862e-05
## 6
     phenotype stn G stn est E stn est GE stn est mean stn est int stn est
                     1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
## 1 -0.0001974582
     0.0146421836
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
## 3
     0.0140454142
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
     0.0126676471
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
## 5
     0.0061287256
     0.0003628870
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823
V_G_SS = sum((model_df$G_stn_est-model_df$mean_stn_est)^2)
V_E_SS = sum((model_df$E_stn_est-model_df$mean_stn_est)^2)
V_GE_SS = sum(model_df\sint_stn_est^2)
V_error = sum((model_df$phenotype_stn - model_df$GE_stn_est)^2)
model_df$I = model_df$E==model_df$G
# Covariance pattern (ignore the 0,0 points)
par(mfrow=c(2,1))
```

```
plot(model_df$G_stn_est*model_df$I, model_df$E_stn_est*model_df$I)
plot(model_df$G_stn_est*model_df$I, model_df$E_stn_est*model_df$I, xlim=c(-1,1), ylim=c(-1,1))
nodel_df$E_stn_est * model_model_df$E_stn_est * model_c
                                                                                O
                                                                        0
                                                                0
                                                       0
                                                   0
                                               O
                                       0
                               0
                -1.0
                                 -0.5
                                                                                    1.0
                                                  0.0
                                                                   0.5
                                 model_df$G_stn_est * model_df$I
                                                                                   σ
                                                                          O
                                                                 0
                                                        0
                                                   0
                                               O
                                      0
             -1.0
                               -0.5
                                                  0.0
                                                                     0.5
                                                                                        1.0
                                 model_df$G_stn_est * model_df$I
V_Cov_GE <- (nlevels(factor(model_df$G))-1)*</pre>
  nlevels(factor(model df$E))*
  sum((model_df$G_stn_est-model_df$mean_stn_est)*(model_df$E_stn_est-model_df$mean_stn_est)*model_df$I)
SS <- round(rbind(V_G_SS, V_E_SS, V_GE_SS, V_Cov_GE, V_error),2)
omega2 <- round(abs(SS)/sum(abs(SS)),2)</pre>
data.frame(SS, abs(SS), omega2)
##
                  SS abs.SS. omega2
## V_G_SS
              495.30
                       495.30
                                 0.09
## V_E_SS
              494.69
                       494.69
                                 0.09
                                 0.00
## V_GE_SS
                0.01
                         0.01
## V_Cov_GE 4454.95 4454.95
                                 0.82
## V error
                0.09
                         0.09
                                 0.00
```

Step 7: pretend we start with noisy empirical data

Let's back transform the noisy data, then go through the steps and see what happens

```
# what we did to standardize: model_df\$GE_stn_true \leftarrow (model_df\$GE_true - mean(GE_true_means))/sd(GE_true_model_df2 \leftarrow model_df[,1:3] model_df2\$phenotype2 \leftarrow model_df\$phenotype*sd(GE_true_means)+mean(GE_true_means) #backtransform
```

```
# Step A: standardize by mean(GE_means) and sd(GE_means)
phen_GE_obs_means <- tapply(model_df2$phenotype2, model_df2$GE_factor, mean)

model_df2$phenotype2_stn <- (model_df2$phenotype2-mean(phen_GE_obs_means))/sd(phen_GE_obs_means)

plot(model_df2$phenotype2, model_df2$phenotype2_stn) # straight line check

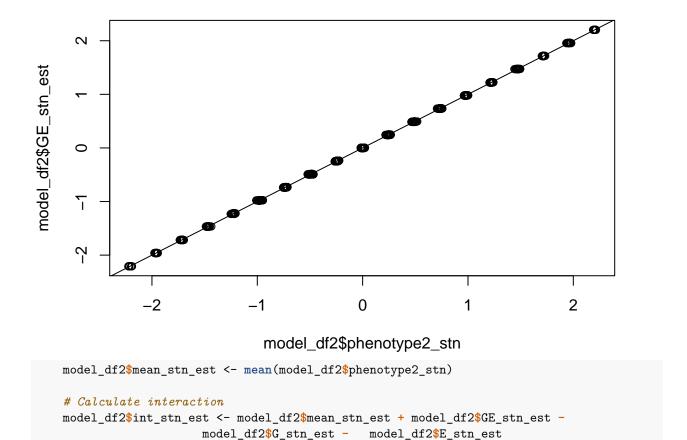
US_OBORDONE

OR OBORDONE

OR OBORD
```

model_df2\$phenotype2

```
\# Step B: calculate observed G_means and E_means and interaction
    G_stn_est <- data.frame(G=1:n_genotypes, G_stn_est=tapply(model_df2$phenotype2_stn, model_df2$G, me
   E_stn_est <- data.frame(E=1:n_genotypes , E_stn_est=tapply(model_df2$phenotype2_stn, model_df2$E, m
   GE_stn_est_means <- tapply(model_df2$phenotype2_stn, model_df2$GE_factor, mean)
   GE_stn_est_means_df <- data.frame(GE_factor=names(GE_stn_est_means) , GE_stn_est=GE_stn_est_means)
   model_dfa <- merge(model_df2,G_stn_est)</pre>
   model_dfb <- merge(model_dfa,E_stn_est)</pre>
   model_dfc <- merge(model_dfb, GE_stn_est_means_df)</pre>
   head(model_dfc)
##
     GE_factor E G phenotype2 phenotype2_stn G_stn_est E_stn_est GE_stn_est
## 1
         G10E1 1 10
                      110.5569
                                 0.0140454142
                                                 1.10397 -1.103204 0.005632742
## 2
         G10E1 1 10
                      109.9752 -0.0001974582
                                                 1.10397 -1.103204 0.005632742
## 3
                      110.5813
         G10E1 1 10
                                 0.0146421836
                                                 1.10397 -1.103204 0.005632742
## 4
         G10E1 1 10
                      110.6155
                                 0.0154790149
                                                 1.10397 -1.103204 0.005632742
## 5
         G10E1 1 10
                      110.0082
                                 0.0006110880
                                                 1.10397 -1.103204 0.005632742
## 6
         G10E1 1 10
                      109.3813 -0.0147391211
                                                 1.10397 -1.103204 0.005632742
   model_df2 <- model_dfc</pre>
   plot(model_df2$phenotype2_stn, model_df2$GE_stn_est)
    abline(0,1) # looks good
```



Step 8: Compare true values to estimated values

```
head(model_df)
     GE_factor E G int GE_true G_true E_true mean_true int_true GE_stn_true
##
## 1
         G10E1 1 10
                                     155
                                              65
                              110
                                                        110
## 2
         G10E1 1 10
                              110
                                      155
                                              65
                                                        110
                                                                    0
                                                                                 0
                       0
## 3
         G10E1 1 10
                       0
                              110
                                      155
                                              65
                                                        110
                                                                    0
                                                                                 0
                                              65
                                                                                 0
## 4
         G10E1 1 10
                       0
                              110
                                      155
                                                        110
                                                                    0
## 5
         G10E1 1 10
                       0
                              110
                                      155
                                              65
                                                        110
                                                                    0
                                                                                 0
                                                                                 0
## 6
         G10E1 1 10
                       0
                              110
                                     155
                                              65
                                                        110
     {\tt G\_stn\_true} \ {\tt E\_stn\_true} \ {\tt mean\_stn\_true} \ {\tt int\_stn\_true}
                                                                              phenotype
## 1
        1.10227
                   -1.10227
                                          0
                                                        0 -6.073255e-04 -6.073255e-04
## 2
        1.10227
                   -1.10227
                                          0
                                                           1.423871e-02 1.423871e-02
## 3
        1.10227
                   -1.10227
                                          0
                                                           1.364169e-02
                                                                          1.364169e-02
## 4
        1.10227
                   -1.10227
                                          0
                                                           1.226333e-02
                                                                          1.226333e-02
                                          0
## 5
        1.10227
                   -1.10227
                                                           5.721586e-03 5.721586e-03
## 6
        1.10227
                   -1.10227
                                          0
                                                        0 -4.673862e-05 -4.673862e-05
     phenotype_stn G_stn_est E_stn_est GE_stn_est mean_stn_est int_stn_est
## 1 -0.0001974582
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
      0.0146421836
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
## 3
      0.0140454142
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
      0.0126676471
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
      0.0061287256
      0.0003628870
                      1.10397 -1.103204 0.005632742 2.716078e-17 0.004866823 FALSE
```

```
(true_int <- mean(abs(model_df$int_stn_true)))

## [1] 1.221245e-17

# Although we call this "true_int", when int=0 this should be 0
# note this is a shortcut that only works with equal sample sizes

(obs_int_sim <- mean(abs(model_df$int_stn_est)))

## [1] 0.002389785

# when int=0, this will increase as the within-population mean gets less accurate

(obs_int_emp <- mean(abs(model_df2$int_stn_est)))

## [1] 0.002389785</pre>
```

Drop the mic.