Model-Checking Library Support for Dense-time Systems with Decision Diagrams

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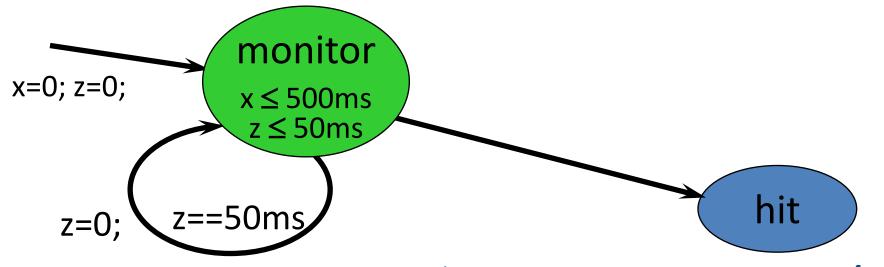


Plan of Presentation

- Theoretical background
- Clock-restriction diagrams (CRD) for timed systems
- Preprocessing of events for efficient model-checking
- Other features of RED technology
- REDLIB
 - APIs
 - examples
- Future research issues

$A=\langle Q, X, I, \mu, E, \tau, \pi \rangle$

```
Q=\{\text{monitor}, \text{ hit}\} \qquad E=\{(\text{monitor}, \text{monitor}), (\text{monitor}, \text{hit})\} \\ \chi=\{x,z\} \qquad \qquad \tau \text{ (monitor}, \text{monitor}): z=50 \\ I=\text{monitor} \qquad \qquad \tau \text{ (monitor, hit): } true \\ \mu(\text{monitor}): x \leq 500 \land z \leq 50 \qquad \pi \text{ (monitor, monitor): } \{z\} \\ \mu(\text{hit}): true \qquad \qquad \pi \text{ (monitor, hit): } \{\}
```



TCTL (Timed Computation-Tree Logic)

$$\phi := q \mid x \le c \mid \neg \phi \mid \phi_1 \lor \phi_2 \mid x.\phi \mid \exists \phi_1 \cup \phi_2 \mid \exists \Box \phi$$

Example: It is possible that I will get my salary at the 7th day.

$$day.\exists \diamondsuit (day=7 \land salary)$$

Example: No matter what, I will be married in 10 years.

$$year$$
. \forall \diamondsuit ($year$ < 10∧married)

TCTL

Example: After you are married, you can remain happy in five days.

 $\forall \Box (married \rightarrow day. \exists \Box (day < 5 \rightarrow happy))$

Example: After you are married, you will remain happy in five days.

 $\forall \Box (married \rightarrow day. \forall \Box (day < 5 \rightarrow happy))$

TCTLF (TCTL with fairness)

Example: If I will be married for infinitely many times, I will always be happy.

 \forall married \Box happy

Example: If process 0 and 2 each infinitely many opportunities to execute, then boot completes someday.

∀^{turn0, turn1} **boot_complete**

TCTL verification complexities

- TCTLF model-checking problem against timed automata is PSPACE-complete.
- TCTL satisfiability problem is undecidable.
 Alur, Cocoubetis, Dill [IEEE LICS 1990]

Zones

basic objects for manipulation and representation

A zone is a state-space bounded by clock difference constraints.

- x-x' < d, $x,x' \in X \cup \{0\}$; $d \in Z \cup \{\infty\}$
- $x-x' \le d$, $x,x' \in X \cup \{0\}$; $d \in Z$

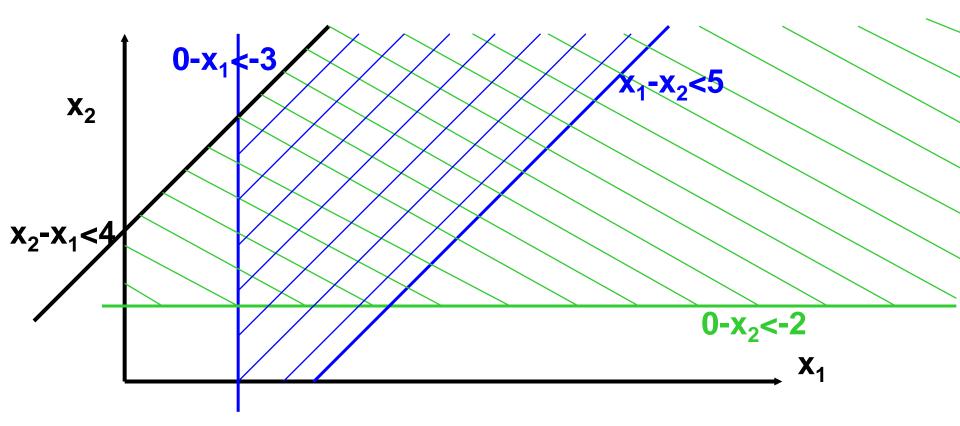
Features:

- convex
- the reachable state-space is a union of zones
- can intersect with one another
- can contain one another → redundancy
- non-canonical needs normal (or canonical) forms

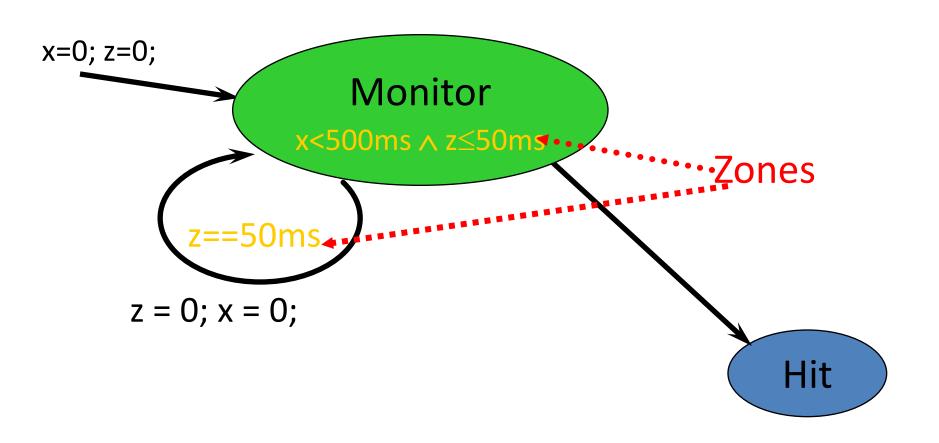
Zones

Two zones

$$(0-x_1 \le -3 \land x_1-x_2 < 5 \land x_2-x_1 < 4) \lor (0-x_2 < -2 \land x_2-x_1 < 4)$$



Zones in a timed automata



Zones & normalization

Two zones:

$$(0-x_1 \le -3 \land x_1-x_2 < 5 \land x_2-x_1 < 4) \lor (0-x_2 < -2 \land x_2-x_1 < 4)$$

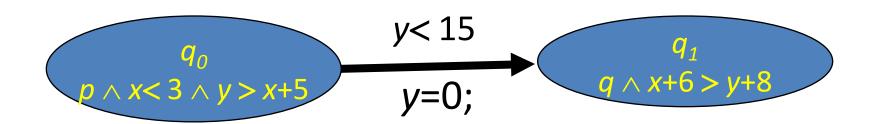
Normal forms

•closure form: all-pair shortest-path form $(0-x_1 \le -3 \land 0-x_2 < 2 \land x_1-x_2 < 5 \land x_2-x_1 < 4) \lor (0-x_2 < -2 \land 0-x_1 < 2 \land x_2-x_1 < 4)$ always the most number of constraints

•reduced form: minimum number of constraints $(0-x_1 \le -3 \land x_1-x_2 < 5 \land x_2-x_1 < 4) \lor (0-x_2 < -2 \land x_2-x_1 < 4)$

Symbolic precondition of dense-time state-space

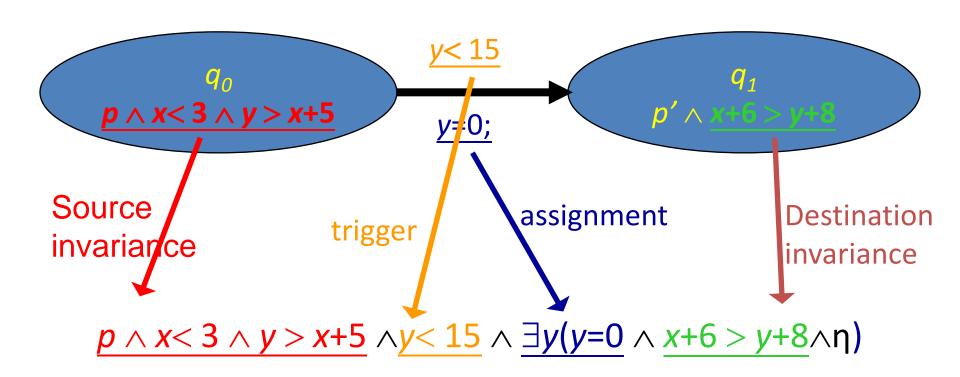
Henzinger, Nicollins, Sifakis, Yovine [IEEE LICS 1992]



What is the weakest precondition in q_0 that

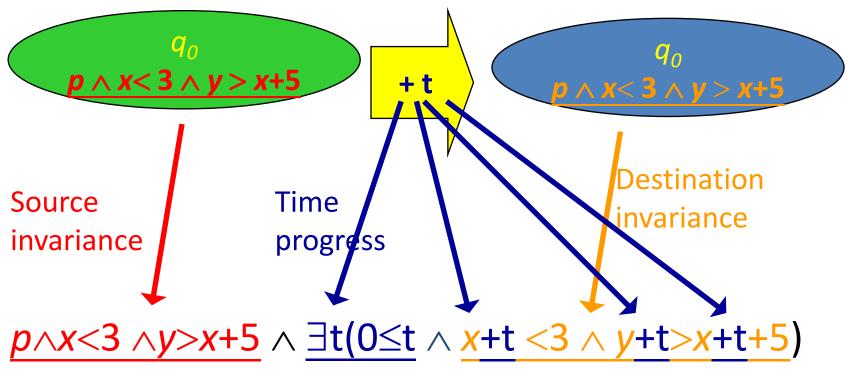
- can transit from q_0 to q_1 ? **xbck(e,\eta): the weakest precondito in \eta after transition e?
- after time $t \ge 0$, remains in s_0 ? $T(\varphi, \eta)$: the weakest precondition to η through time-progress in φ ?

xbck(e,η) weakest preconditoin to e?



How to get rid of the $\exists y$?

T(φ,η): the weakest precondition to η through time-progress t?



How to get rid of the ∃t?

Evaluation of existential quantification

$$\exists y (x \le y \land y < z+5) \equiv x < z+5$$



Joseph Fourier, 1824! (1768-1830)

TCTL Model checking procedures

Basic procedures

$$xbck(e, \eta)$$

weakest precondition of discrete transitions

$$T(\phi,\eta)$$

- backward time-progression
- Backward reachability:

$$rbck(\phi,\eta) \equiv IfpY.(\eta \lor T(\phi \land \lor xbck(e,Y)))$$

least fixpoint operator

TCTLF Model checking

Lemma: given d≥1,

 $A,v \models \exists \Box \eta$ iff there is a finite run ρ

- from v
- of duration ≥d
- along ρ every state satisfies η and
- ρ ends at a state satisfying $\exists \Box \eta$

$$\exists \Box \eta \equiv gfp \ Y. \ \exists z(rbck(\eta, Y \land z \ge d))$$

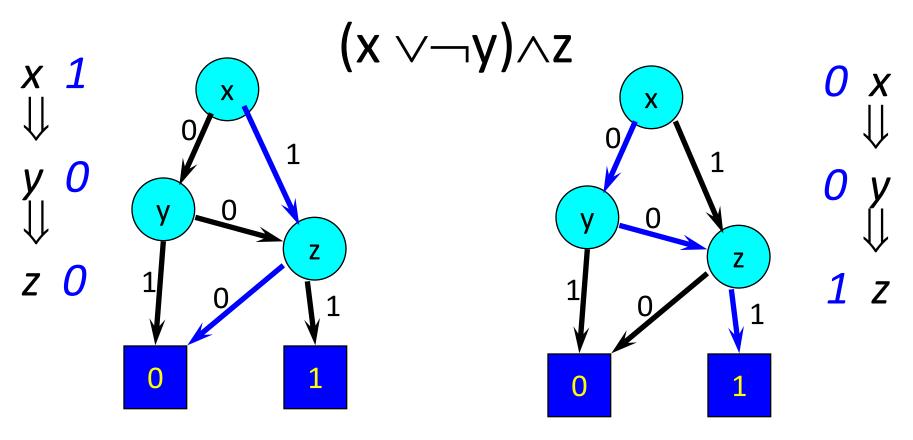
greatest fixpoint operator

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BDD in evaluating function values

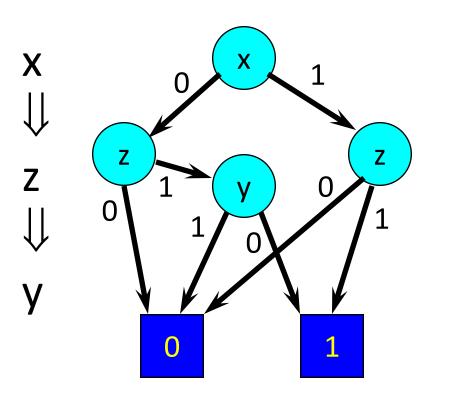
Minimal & Canonicity

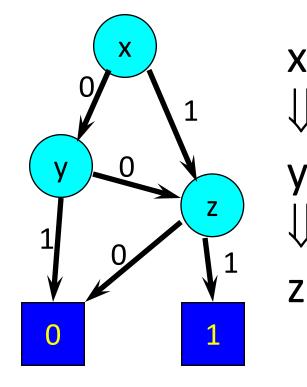


BDD

Effects of variable ordering

$$(x \vee \neg y) \wedge z$$





BDD operations

Given 2 BDDs : $B_1 \setminus B_2$, efficient algorithms for the following:

- $B_1 \land B_2$
- $B_1 \vee B_2$
- ¬B₁
- ∃z B₁

R. Bryant [IEEE Trans. Computers, 1986]

Clock-Restriction Diagram (CRD) Background

Intuition: To repeat the success of BDD in circuit verificaito.

Up to 2000, many proposals for BDD variations for dense-time state-spaces.

- NDD, DDD, CDD, RED
- None of them shows advantage over DBM.

Motivation

Find out the reasons and ways for improvements.

CRD

Related work (I)

- BDD: for untimed systems [Bryant 86; Burch, Clarke, et al, 90]
- DBM: 2-dimensional matrix for a region. [Dill 89]
- NDD: BDD to encode discrete time state-spaces. [ABKMPR97]
- CDD: a decision diagram for dense-time state-space membership [BLPWW99]. Like CRD, except:
 - default value is $(-\infty, \infty)$.
 - a value [c,d] of variable x-x' means c≤x-x'≤d.
- DDD: variable like x-y~ c [WME92,Balarin96,MLAH99]
 - Exponential number of BDD atoms
- RED: encodes the ordering of the fractional parts of clock readings with a single-bit varible [Wang 00]
 - for symmetric systems

CRD

Related work (II)

Difference-Bound Matrix (DBM) [Dill 89]

- 2-dimensional matrix for a convex clock value space.
- A zone is represented as a pair (b,z)
 - b is a BDD for the discrete part of the states
 - z is a DBM for clock values
- Can only represent a convex state space.
- To represent a non-convex state space, a linked list of such pairs are used.
- Inefficient in conjunction and complementation.
- Adopted by UPPAAL, Kronos, Prism, ...

Clock-Restriction Diagram (CRD)

- A BDD variation
- Recording device for (zone) DBM sets
- Variables of the form x-x'
- Arc values of the form:

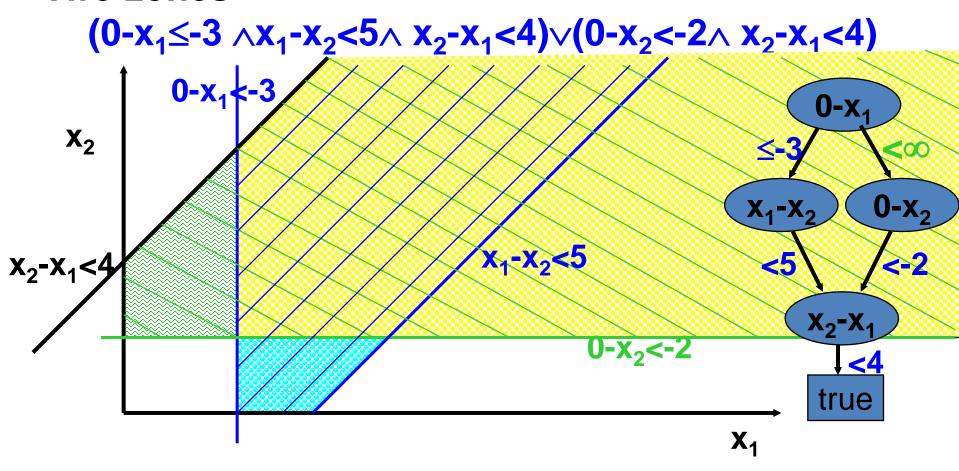
$$(<, d), d \in [-c_A, c_A] \cup \{\infty\}; or$$

 $(\le, d), d \in [-c_A, c_A]$

- Default value on arcs: $(<, \infty)$
 - representing no constraint!

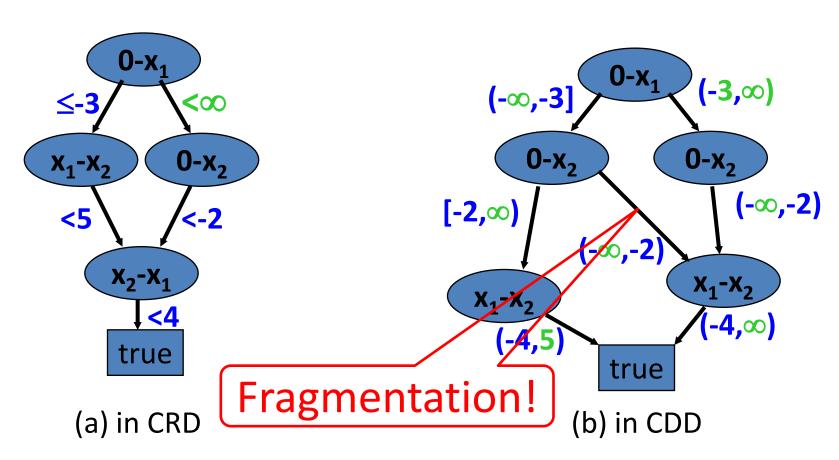
Representation in CRD

Two zones



CRD Example

$$(0-x_1 \le -3 \land x_1-x_2 < 5 \land x_2-x_1 < 4) \lor (0-x_2 < -2 \land x_2-x_1 < 4)$$



CRD

Characteristics

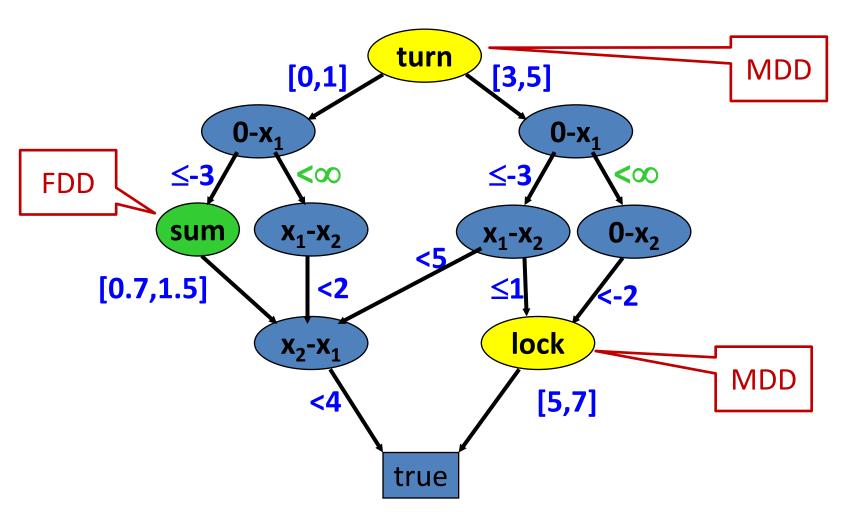
- Integrated representation for convex and nonconvex state spaces
- Efficient operation for union and intersection
- Complementation is a little bit complex.
- No natural canonicity
 - needs normalization

CRD+MDD+FDD

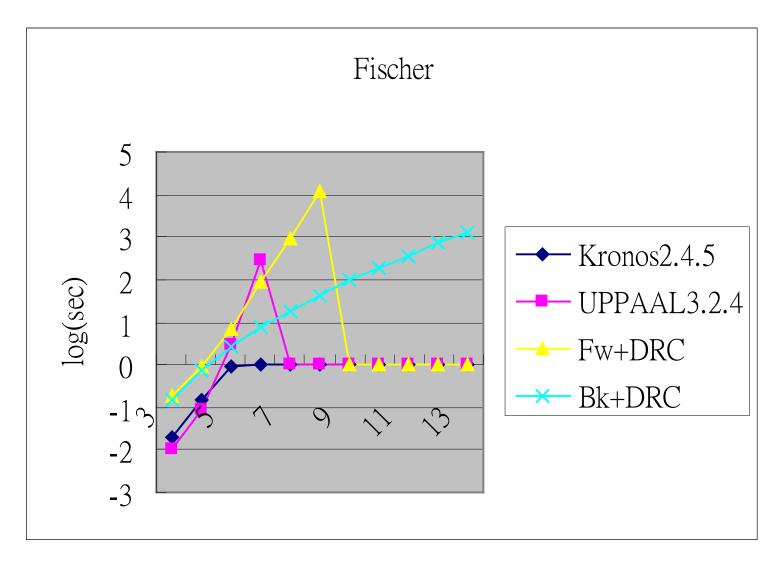
A winner for **RED** technology.

- Integrated representation for convex and nonconvex state spaces
- Efficient operation for union and intersection
- Complementation is a little bit complex.
- No natural canonicity
 - needs normalization

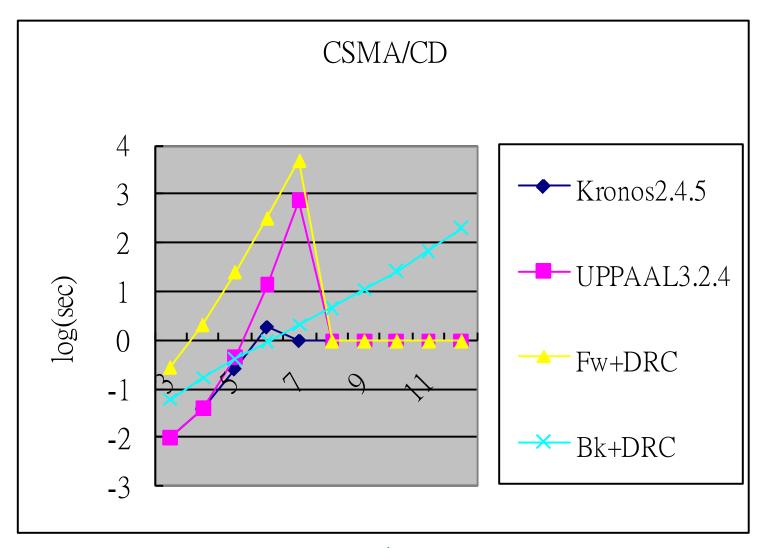
CRD+MDD+FDD Example



Fischer's mutual exclusion

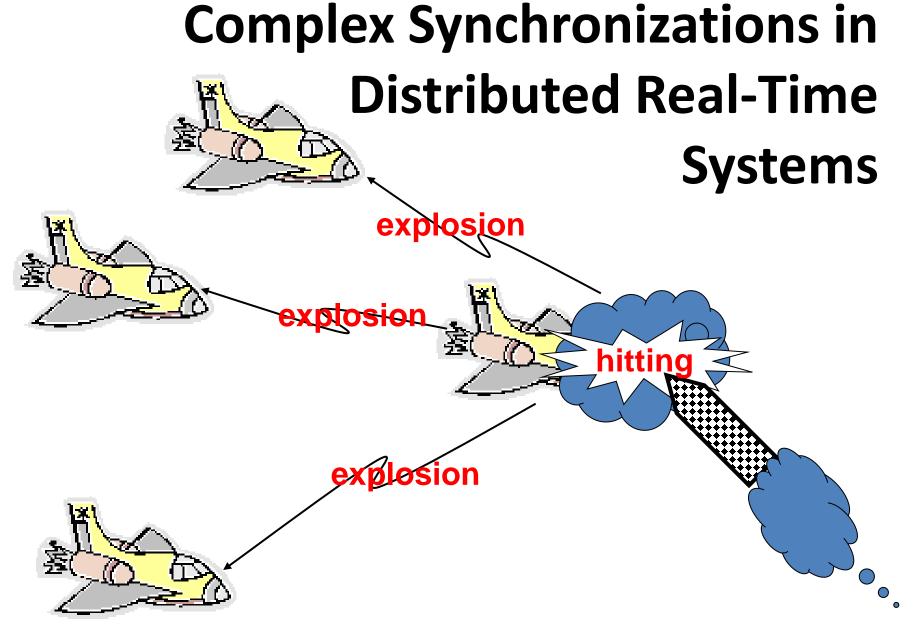


CSMA/CD



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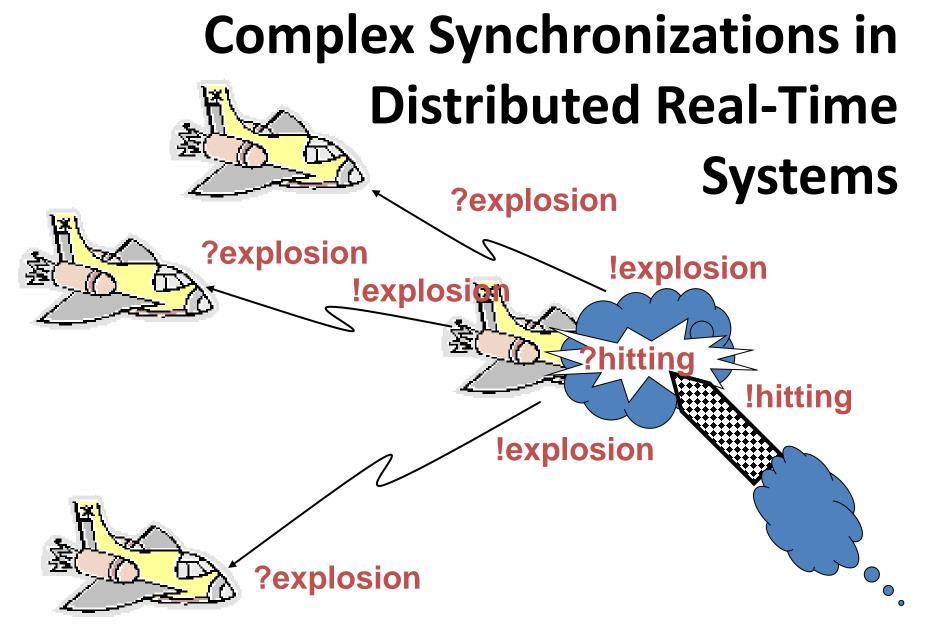


Modeling granularity?

Did the hit and explosion happen at the same time?

- No, at the fine level of quantum mechanics
 - Extremely fine models
 - Extreme confidence in the verification result

- Yes, for the complexity of verification
 - Intuitive, natural, and acceptable
 - Usable analysis result



Complex Synchronizations in the CSP style

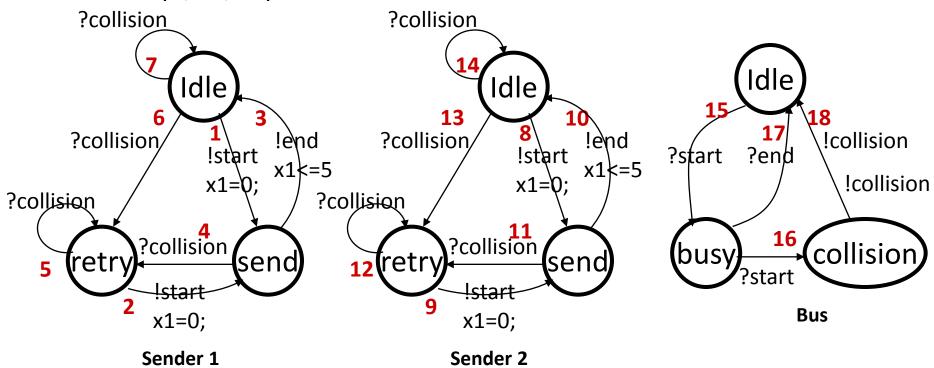
- Multiple-party synchronizations constructed through binary synchronizations
 - Global transitions constructed through process transitions
 - For each channel,
 - # input event = # output event
 - For interleaving semantics
 - Minimality of global transitions
- Modular descriptions and specifications
 - Flexibility in the descriptions of process response variations

Network of TAs Communicating timed automaton (CTA)

Legitimate global transitions

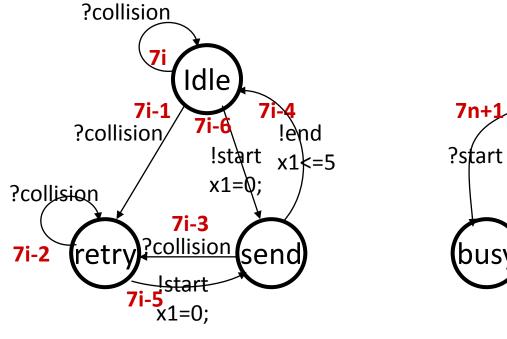
start: (1,15), (1,18), (2,15), (2,18), (8,15), (8,18), (9,15), (9,18)

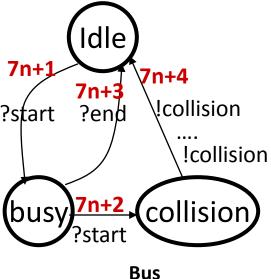
collision: (4,11,18)



Legitimate global transitions with *n* senders

When n = 3, collision: (4,11,19,25) (4,11,20,25) (4,11,21,25) (4,12,18,25) (4,13,18,25) (4,14,18,25) (5,11,18,25) (6,11,18,25) (7,11,18,25)



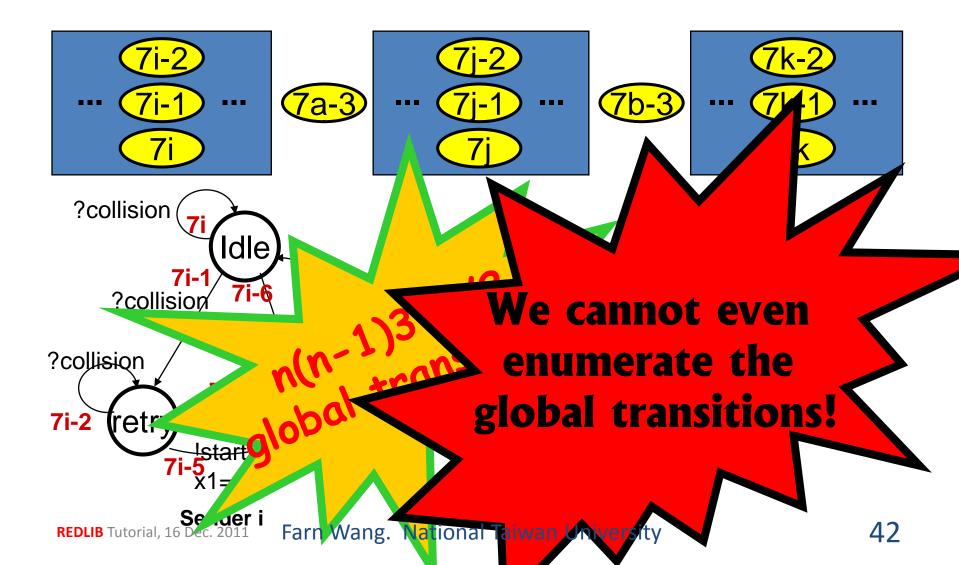


Sender i

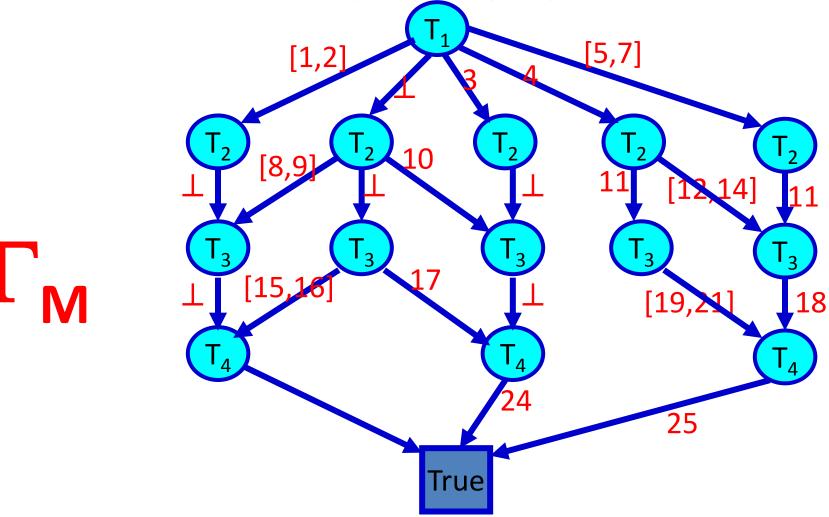
Pre/post condition calculation in the traditional style

```
\psi:= false;
for each global transition T {
 for each 1 \le p \le m with T(p) \ne \bot and
  \phi := \eta \land (\bigwedge_{x \in \pi p(e)} x=0) \land mode_p = q
                                                  An
  \phi := FM_{elim}(\phi, \pi_{p}(e) \cup \{ mode_{p} \})
                                                  enumeration
 Add in the triggering conditions
                                                  of global
   φ.
                                                  transitions
 \psi := \psi \vee \phi;
Return \psi;
```

Legitimate global transitions with *n* senders in the traditional style



Efficient representation for global transitions



Xplans_bck(η)

```
Let \eta := \eta \wedge \Gamma_{M};
for p:=1 to m {
 \psi := \eta \wedge T_p = \bot;
 for each e \in E_p with \gamma(e) = (q,q') {
   \phi:=\eta \wedge T_p=e \wedge (\bigwedge_{x \in \pi p(e)} x=0) \wedge mode_p=q';
   \psi := \psi \vee FM_{elim}(\phi, \pi_{p}(e) \cup \{ mode_{p} \});
 \eta := \psi;
Add in all the triggering conditions of the participating
    process transitions to \eta.
return FM_elim(\eta, {T<sub>1</sub>,...,T<sub>m</sub>});
```

An observation in the experiments

- Simple synchronizations
 - 2 or 3 processes involved
 - Perform well with the traditional style
- Complex synchronizations
 - Perform well with the new style
- Strategy, β : a parameter Synchronization
 - $-<\beta \rightarrow$ traditional style
 - $\ge \beta \rightarrow$ new style

Performance data

| spec. | m | $\beta = 0$ | medium β values | $\beta = \# \text{ procs}$ |
|----------------|-----|--|-----------------------|----------------------------|
| CSMA/CD | 2 | 0.01 s/47 k | 0.00s/28k | 0.00 s/21 k |
| | 3 | 0.05 s / 86 k | 0.03 s/54 k | 0.06 s/43 k |
| medium β | 4 | 0.20 s / 134 k | 0.11s/89k | 0.26 s/93 k |
| values = 2 | 5 | 0.42s/192k | 0.40 s/138 k | 1.73 s/208 k |
| | 6 | 0.84 s/304 k | 0.78s/299k | 12.9s/499k |
| | 7 | 1.70 s/681 k | 1.76s/674k | 132s/1262k |
| | 8 | 3.37 s / 1551 k | 3.79 s / 1546 k | 2572s/3316k |
| | 9 | 7.56 s/3548 k | 7.76s/3541k | *** |
| | 10 | 18.4s/8075k | 18.5 s / 8072 k | |
| | 11 | 54.2s/18272k | 51.5s/18274k | N/A |
| | 12 | 144s/41105k | 146s/41083k | |
| | 13 | 385s/91826k | 397s/91873k | |
| SAM | 2 | 0.18s/92k | 0.17 s/45 k | 0.10 s/45 k |
| | 3 | 1.29s/188k | 0.75 s / 132 k | 0.80 s / 130 k |
| medium β | 4 | 6.01s/387k | 4.03 s/352 k | 4.60 s / 400 k |
| values = 2 | 5 | 26.5s/899k | 17.3s/752k | 23.6s/931k |
| | 6 | 99.4s/1763k | 61.5s/1344k | 144s/1959k |
| | 7 | 284s/3195k | 185s/2205k | 750s/5703k |
| | 8 | 734s/5204k | 537s/3384k | N/A |
| FIFO | 2 | 0.04 s / 72 k | 0.02 s/51 k | 0.02 s/51 k |
| 3001 | 3 | 0.22 s / 172 k | 0.15s/103k | 0.17 s / 152 k |
| medium β | 4 | 0.59 s/346 k | 0.31s/231k | 0.79 s / 428 k |
| values = 4 | 5 | 1.46 s/620 k | 1.27s/471k | 4.97 s / 1230 k |
| | 6 | 5.07 s / 1023 k | 4.32s/872k | 84.9s/3728k |
| | 7 | 24.0s/1665k | 18.2s/1540k | 938s/11734k |
| | 8 | 90.0s/3853k | 76.7s/3372k | N/A |
| | 9 | 304s/9035k | 290s/9894k | |
| ARP | 2 | 0.55 s / 186 k | 0.40s/149k | 0.40 s / 141 k |
| | 3 | 5.59 s/526 k | 3.21 s / 460 k | 3.92 s/615 k |
| medium β | 4 | 72.0s/1578k | 33.0s/1105k | 58.7s/2709k |
| values = 2 | 5 | 872s/7318k | 435s/5837k | 703s/13494k |
| | 6 | 8575s/84086k | 4194s/84087k | N/A |
| ed on a Pent | inn | The state of the s | GHz with 256MB | memory runn |

data collected on a Pentium 4 Mobile 1.6GHz with 256MB memory running LINUX s: seconds; k: kilobytes of memory in data-structure; N/A: not available;

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Other features of RED technology

- Full TCTLF model-checking
 - also with event extensions in TCTLF
 - early termination techniques in greatest fixpoint evaluation.
- Fair simulation for communicating timed automatas
 - model vs. spec TAs in an environment
- Time progress evaluation speed-up
 - [ATVA 2008, RTSS 2008, RTSJ 2011]
 - TCXTL for fast time progress evaluation
 - fast approximation in timed inevitabilities
- Abstraction based on game concepts

REDLIB

library for shared CRD+MDD+FDD

- Support modeling C data-structrures
 - multi-dimensional arrays.
 - structures
 - dynamic memory allocation
 - must first declare the size of memory
 - address arithmetics and indirection
- For model/simulation-checking,
 - a forward untimed reachability analysis is performed.
 - Then a backward timed model/simulation checking is performed
 - to avoid blow-up of the backward state-space.

REDLIB API

- Input
 - parsing, table construction
- Low-level
 - true, false, \vee , \wedge , \neg , ...
- Mid-level
 - discrete pre/post-condition,
 - time pre/post-condition,
 - normalization, abstraction
 - garbage-collection (GC), protection from GC
- High-level
 - fwd/bck reachability,
 - TCTLF model-checking,
 - simulation-checking

Model construction A while loop_

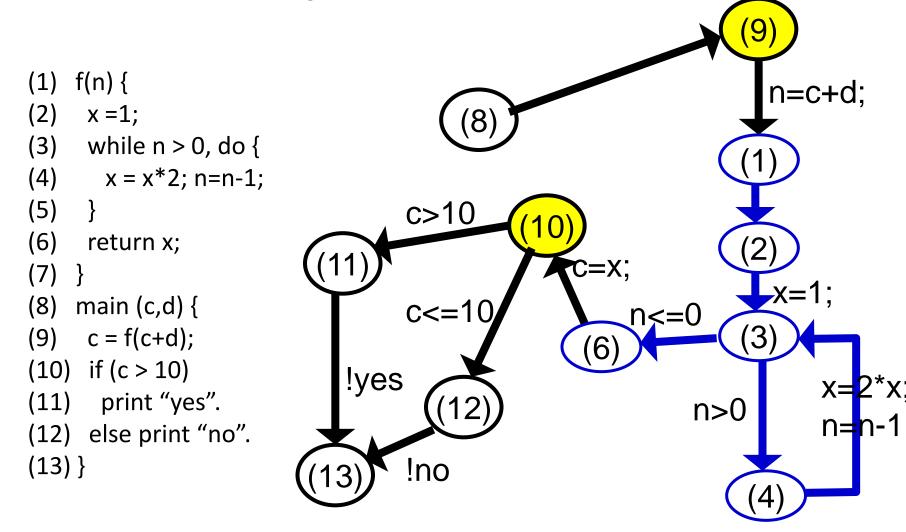
- (1) f(n) {
 (2) x = 1;
 (3) while n > 0, do {
 (4) x = x*2; n=n-1;
 (5) }
 (6) return x;
- x=1;n<=0 (3)(6) x=2*x;n=n-1;n>0 (4)

(7) }

Model construction A while loop

```
process count=1;
global discrete n:0..5;
global discrete x:0..32;
mode one (true) {
 when (true) may goto two; }
mode two (true) {
 when (true) may x=1; goto three; }
                                                             x=1;
                                               n<=0
mode three (true) {
 when (n>0) may goto four;
 when (n<=0) may goto six; }
mode four (true) {
                                                     n>0
 when (true) may x=2*x; n--1; goto three; }
mode six (true) { }
initially one@(1);
```

Model construction
A procedure call

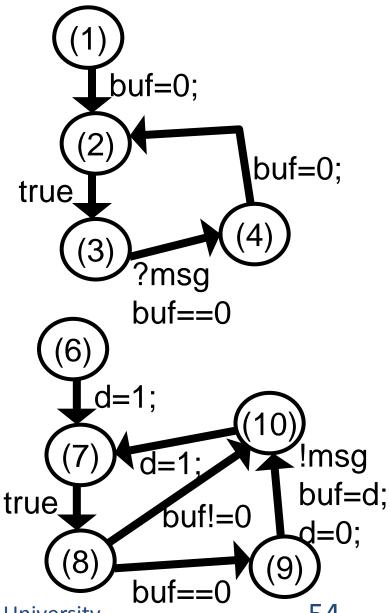


Model construction Synchronization

```
The reader process
(1) buf = 0;
(2) while true, do {
(3) if (buf == 0), read;
(4) buf = 0;
(5) }
```

The writer process

```
(6) d = 1;
(7) while true, do {
(8) if (buf == 0),
(9) write d to buf;d=0;
(10) d = 1;
(11) }
```



CSMA/CD protocol (1/2)

```
#define A
#define B
                                                            constants
#define LAMBDA808
process count = 3; /* 1 is for bus, the others for senders. *,
                                                           thread count
local clock x;
global synchronizer begin, end, cd, busy;
/* The following 3 modes are for the bus. */
                                                               variables
mode idle (true) {
   when ?begin (true) may x= 0; goto busy;
                                                              oval,
mode busy (true) {
                                                              mode,
   when ?end (true) may x=0; goto idle;
                                                              location
   when ?begin (x < A) may x = 0; goto collision;
mode collision (x < A) {
                                                               arrow,
   when !(\#PS-1)cd(x < A) may x=0; goto idle;
                                                               transition
```

CSMA/CD protocol (2/2)

```
/* The following 3 modes are for the senders. */
mode wait (true) {
   when !begin (true) may x= 0; goto send;
   when ?cd (true) may x= 0;
   when ?cd (true) may x = 0; goto retry;
mode send (x \le LAMBDA) {
   when !end (x==LAMBDA) may x= 0; goto wait;
   when ?cd(x < B) may x = 0; goto retry;
mode retry (x < B) {
   when !begin (x < B) may x = 0; goto send;
   when ?cd(x < B) may x = 0;
                                                          parameterized
         idle@(1)&& x@(1) =<del>= 0</del>
initially
       && forall i:2..#PS, (wait@(i)&& x@(i) == 0);
                                                            declaration
risk send@(2) && send@(3) && (x@(2) >= B | |x@(3) >= B);
```

REDLIB application examples Sudoku (1/2)

```
#include <stdlib.h>
#include "redlib.h"
#include "redlib.e"
char *s[9][9]; int c[9][9]; int count print = 0;
redgram slot_constraint(d, i, j) redgram d; int i, j; {
 int value, h, k, nc, ac, gs; redgram conj;
 for(value =1; value <= 9; value++) /* To enumerate the value of s[i][j]. */ {
  for (h = 0; h < 9; h++)
   if (h != i) {
   // regulation for s[i][j] != s[h][j] in the same column.
   conj = red diagram("!(%s==%d && %s==%d)", s[i][j], value, s[h][j], value);
    if ((++count print) < 100 \&\& (gs = red diagram size(d, &nc, &ac)) < 30) {
     fprintf(RED_OUT, "\nMutual exclusion to value:%1d at %s and %s:\nconstraint:\n",
      value, s[i][j], s[h][j]
```

REDLIB application examples Sudoku (2/2)

```
red_print_line(conj); fprintf(RED_OUT, "\n"); red_print_diagram(conj);
    fprintf(RED_OUT, "input diagram:\n"); red_print_line(d);
    fprintf(RED OUT, "\n"); red print diagram(d);
   d = red and(d, conj);
   if ((h/3) == (i/3)) {
    for (k = 0; k < 9; k++) {
     if ((k/3)==(i/3) \&\& (k!=i)) {
       d = red and(d, red_diagram("!(%s==%d && %s==%d)", s[i][j], value, s[h][k], value) );
  }}}
  if (h != j) {
   d = red_and(d, red_diagram("!(%s==%d && %s==%d)", s[i][j], value, s[i][h], value));
}}}
h = red_push(d); red_garbage_collect(RED_GARBAGE_SILENT); red_pop(h); return(d);
/* slot constraint() */
```

REDLIB application examples red (1/4)

```
main(argc, argv) int argc; char **argv; {
redgram sub, conj; char *spec;
 if (!my_process_command_line(argc, argv) /* the number of files */)
 fprintf(RED_OUT, "Use a line beginning with \"%%end\" to end the formula input.\n\n");
 red begin session(flag my system type, model fname, my proc count);
switch (task type) {
case SIM CHECK: case BISIM CHECK:
  red input model(model fname, RED NO REFINED GLOBAL INVARIANCE); break;
default:
  if (flag analysis direction == FLAG ANALYSIS FORWARD)
   red input model(model fname, RED NO REFINED GLOBAL INVARIANCE);
 else
   red input model(model fname, RED REFINE GLOBAL INVARIANCE);
```

REDLIB application examples red (2/4)

```
spec = red_file_to_string(spec_fname);
my_verifier(task_type, spec);
red_end_session(model_fname);
} /* main() */
```

REDLIB application examples red (3/4)

REDLIB application examples red (4/4)

```
switch (tt) {
case SIM CHECK:
 sr = red sim check(
  red query diagram initial(), red query diagram global invariance(),
  RED FULL REACHABILITY, RED NO REACHABILITY DEPTH BOUND,
  flag_counter_example, RED_TIME_PROGRESS,
  flag normality, flag action approx,
  flag_reduction, flag_approx, flag_symmetry, flag_zeno,
  flag tconvexity shared partitions | flag time progress options
  | flag gfp on the fly | flag fairness assumptions eval
  | flag gfp path,
  flag print, s
 );
 red print sim check return(sr); return (sr->iteration count);
 break;
```

REDLIB application examples some API (1/2)

A Procedure for calculating the precondition synchronous transitions with fineresolution control.

```
redgram red sync xtion bck(
 redgram
                   ddst,
 redgram
                   dpath,
 int
                   flag sync xtion table choice,
 int
                   SXİ,
 int
                   flag game roles,
 int
                   flag time progress,
                   flag normality,
 int
 int
                   flag action approx,
 int
                   flag reduction,
 int
                   flag state approx,
                   flag symmetry,
 int
                   flag experiment
 int
);
```

REDLIB application examples some API (2/2)

```
redgram red_norm(
redgram d,
int op
```

A procedure for normalizing decision diagrams.

op: option for normal forms.

A procedure for abstracting decision diagrams.

op: abstraction resolution based on roles.

three roles: model, spec, envr.

Potential research issues

- PAT input → REDLIB input
 - What is the best translation for verification performance? Broadcasting semantics?
 - Pre-analysis of event models ?
- REDLIB output → PAT output
- Garbage-collection for CRD+MDD ?
- Probabilistic model-checking with REDLIB
 - how to outperform prism (DBM+BDD) ?
- Bound analysis ?