Techniques of Artificial Intelligence

Exercises – Search Space & Concept Learning

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4. **The Missionaries and Cannibals Puzzle**
   There are three missionaries and three cannibals on the west bank of a river. There is a boat on the west bank that can hold no more than two people. The missionaries wish to cross to the east bank. But they have a problem: If on either bank the cannibals ever outnumber the missionaries, the outnumbered missionaries will be eaten. Is there a way for the missionaries to get to the east bank without losing anyone?

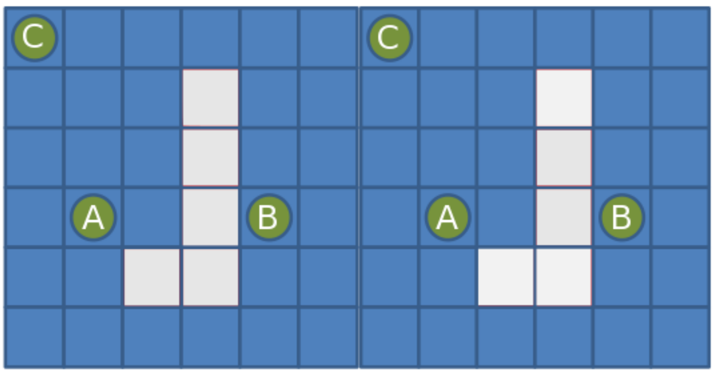
   (a) Represent this puzzle as a search problem by defining the states and actions.

   (b) Investigate your initial representation of the puzzle as you did in part (a): is there any redundant information in your representation? Try to remove any redundant information from it.

   (c) How many different states are there?

   (d) Try to solve the puzzle yourself.

5. **Path planning**
   Imagine a simple blocks world as shown below. A block can be moved a single cell at a time in each of the four directions. We have to find the shortest path from configuration A to configuration B.



   (a) Discuss whether depth first search and breadth first search will always find a path from A to B. Will they find the shortest path?

   (b) Suppose that we want to use informed search in order to guide our search. We will use the Manhattan distance as a search heuristic. Discuss whether best first search will find the shortest path, use the left drawing to add f, g and h costs. Will A* do so? Use the right drawing to add f, g and h costs.

(c) Now suppose that there is a direct underground connection from C to B. Hence, the shortest path from A to B now is over C (it is only five steps). Argue whether A* will find this shortest path.

6. The optimality condition of A* is expressed as follows: If the heuristic h(n) is admissible, then the solution returned by A* is optimal. In order to prove this, we assume that the optimal solution n* has an optimal cost f(n*) = C*.

   (a) Suppose that there is a solution G2 which is not optimal in the agenda. What does this learn about f(G2)?

   (b) Suppose that there is a node n in the agenda which is on the path to the optimal solution G. What does this learn about f(n)?

   (c) Proof that if h(n) is admissible, then the solution returned by A* is optimal.

7. A new game comes with a board which contains 7 cells (as shown below). There are 6 pivots, each containing a number. Each pivot has to be in one cell and each cell can contain at most one pivot. Hence, there is always one empty cell. A pivot can be moved one cell to the left or one cell to the right (if it does not break the rules given above) with a cost of 1. A pivot can also jump over another pivot, with cost 2 (if it does not break the rules given above). The goal of the game is to have the empty cell in the middle, such that the sum of the blocks at the left of the empty cell is equal to the sum of the blocks at the right of the empty cell.

   (a) Model this as a state space search problem.

   (b) Which of the search algorithms studied in class is best suited for solving this puzzle? Motivate (max 5 lines).

   (c) Give a sequence of states leading to the goal state.

8. **Calculating the size of the hypothesis space** (based on exercise 2.1 from the course book)
   Suppose there are m attributes in a learning task and that every attribute i can take k_i possible values. What will be the size of the hypothesis space?

9. **Order of training instances**
   In candidate elimination, suppose you have n training instances $T_1 \ldots T_n$. After the n_{th} training instance, candidate elimination learned the boundaries S and G. Will S and G differ or not when providing the training instances in reverse order: $T_n \ldots T_1$? Explain why (not).

10. What is the version space while tracing the **candidate elimination algorithm** with the following examples?

    $Architecture \in \{Gothic, Romanesque\}$

    $Size \in \{Small, Large\}$

    $Steeples \in \{Zero, One, Two\}$

    Example 1: Arch = G, Sz = S, St = 2 $\rightarrow$ classified building
    Example 2: Arch = R, Sz = S, St = 2 $\rightarrow$ non-classified building
    Example 3: Arch = G, Sz = L, St = 2 $\rightarrow$ classified building
    Example 4: Arch = G, Sz = S, St = 0 $\rightarrow$ non-classified building
    Example 5: Arch = R, Sz = L, St = 2 $\rightarrow$ classified building

11. **Rectangular version spaces and candidate elimination** (exercise 2.4 from the course book)
Consider the instance space consisting of integer points in the x, y plane and the set of hypotheses H consisting of rectangles. More precisely, hypotheses are of the form $a \leqslant x \leqslant b$, $c \leqslant y \leqslant d$, where a, b, c and d can be any integers. Consider the version space with respect to the set of positive (+) and negative (-) training examples:

$$\begin{array}{ll} -(1,3) & -(2,6) \\ +(6,5) & +(5,3) \\ -(9,4) & -(5,1) \\ +(4,4) & -(5,8) \end{array}$$

(a) What is the S boundary of the version space in this case? Write a diagram with the training data and the S boundary.

(b) What is the G boundary of this version space? Draw that in the diagram as well.

(c) Suppose the learner may suggest a new x, y instance and ask the trainer for its classification. Suggest a query guaranteed to reduce the size of the version space, regardless how the trainer classifies it. Suggest one that will not.

(d) Now assume you are the teacher, attempting to reach a particular target concept, $3 \leqslant x \leqslant 5, 2 \leqslant y \leqslant 9$. What is the smallest number of training examples you can provide so that the Candidate- Elimination algorithm will perfectly learn the concept?

12. **Finding a maximally specific consistent hypothesis** (exercise 2.7 from the course book)
Consider a concept learning problem in which each instance is a real number and in which each hypothesis is an interval over the reals. More precisely, each hypothesis in H is of the form: $a < x < b$ as in $4.5 < x < 6.1$, meaning that all real numbers between 4.5 and 6.1 are classified as positive examples and all others are classified as negative examples.

(a) Explain informally why there cannot be a maximally specific consistent hypothesis for any set of positive training examples.

(b) Suggest a modification to the hypothesis representation so this will not happen.