Techniques of Artificial Intelligence

Exercises – Neural Networks & Evaluating Hypothesis

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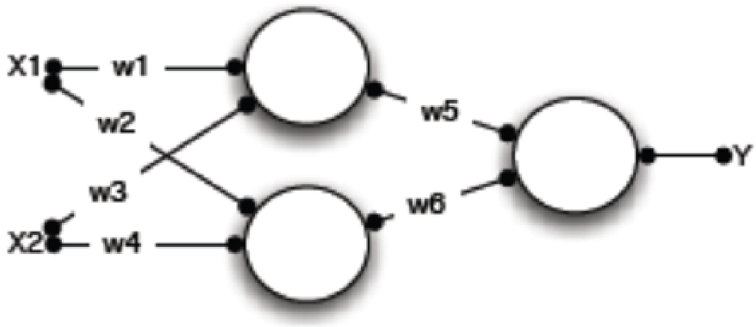
18. **Neural networks**
 Consider the neural network which is shown above. Suppose it is a linear neural network, such



that the output value of each node is the weighted some of the inputs.

Give an equation for the values of the three output nodes.

Design a perceptron which will provide exactly the same output as the linear neural network above. Show that the output equation of the perceptron is the same as the one of the neural net shown above.

Answer:

$N_1 = w_1x_1 + w_3x_2$
$N_2 = w_2x_2 + w_4x_2$
$Y = w_5N_1 + w_6N_2$
We can rewrite this as follows:
$Y = w_5w_1x_1 + w_5w_3x_2 + w_6w_2x_1 + w_6w_4x_2 = (w_5w_1 + w_6w_2)x_1 + (w_5w_3 + w_6w_4)x_2$
This shows that a perceptron with two inputs x_1 an x_2 and weights $(w_5w_1 + w_6w_2)$ and $(w_5w_3 + w_6w_4)$ has the same output function.

19. **Perceptrons** (exercise 4.3 in the course book)
 Consider two perceptrons defined by the threshold expression $w_0 + w_1x_1 + w_2x_2 > 0$.
 Perceptron A has weight values $w_0 = 1$, $w_1 = 2$, $w_2 = 1$
 Perceptron B has weight values $w_0 = 0$, $w_1 = -2$, $w_2 = -1$
 Is perceptron A more general than perceptron B? Motivate.

 Answer:

 We need to investigate if when an input (x_1, x_2) is classified as positive by perceptron B, it is also always classified as positive by perceptron A.

 This is not the case, take for instance $(-1, -1)$ which is classified as positive by perceptron B $(3 > 0)$, but is classified as negative by perceptron A $(-2 < 0)$.

20. **Perceptrons** (exercise 4.5 in the course book)
 Derive a gradient descent training rule for a single unit with output o, where:
 $O = w_0 + w_1x_1 + w_1x_1^2 + ... + w_nx_n + w_nx_n^2$

Answer:

The gradient descent training rule is:
$w_i = w_i + \Delta w_i$, where $\Delta w_i = -\eta \frac{\partial E}{\partial w_i}$ and $E = \frac{1}{2} \sum\limits_{d \in D} (t_d - o_d)^2$

We need to compute $\frac{\partial E}{\partial w_i}$:

$$\begin{aligned}
\frac{\partial E}{\partial w_i} &= \frac{\partial}{\partial w_i} \left(\frac{1}{2} \sum_{d \in D} (t_d - o_d)^2 \right) = \frac{1}{2} \sum_{d \in D} 2 (t_d - o_d) \frac{\partial (t_d - o_d)}{\partial w_i} \\
&= \sum_{d \in D} (t_d - o_d) \frac{\partial}{\partial w_i} \left[t_d - (w_0 + w_1 x_{1d} + w_1 x_{1d}^2 + ... + w_n x_{nd} + w_n x_{nd}^2) \right] \\
&= \sum_{d \in D} (t_d - o_d) \left(-x_{id} - x_{id}^2 \right)
\end{aligned} \tag{1}$$

21. **Evaluating Hypothesis**
When testing a hypothesis h, we used a sample set containing 30 samples. We learnt that 3 samples were misclassified. Calculate the 95% and the 50% confidence interval. You observe that the 95% confidence interval is almost 4 times bigger than the 50% confidence interval. What is the meaning of this interval? Using 95% confidence intervals, is it possible that the true error rate is:

    (a) actually 1% instead of 10%?
    (b) actually 0% instead of 10%?

    Answer:

    The following equation is used to estimate the true error given the sample error:

$$error_D(h) = error_s(h) \pm z_n \sqrt{\frac{error_s(h)(1 - error_s(h))}{n}} \tag{2}$$

For 95% confidence intervals, z_n equals 1.96, for 50% confidence intervals z_n equals 0.67.
In this case, we get: $error_s(h) = 3/30 = 0.1$
95%CI: $error_D(h) = 0.1 \pm 1.96\sqrt{\frac{0.1(1-0.1}{30}} = 0.1 \pm 0.10735 = [-0.00735, 0.20735]$
50%CI: $error_D(h) = 0.1 \pm 0.67\sqrt{\frac{0.1(1-0.1}{30}} = 0.1 \pm 0.0366 = [0.0634, 0.1366]$
So, although we experienced a sample error of 0.1, the true error might seem to be with 95% chance in between [-0.00735 , 0.20735]. A 0.01 is indeed in the interval, it is possible that the true error is actually 1% instead of the 10% we experienced.

However, the true error cannot be 0%, even if 0 is in the interval: As we have experienced that 3 samples were classified wrong, the error is above 0.

Obviously, the true error, cannot be in the range [-0.00735 , 0.20735] as an error rate cannot be negative. Moreover, it cannot be 0 as we have evidence that there are at least 3 misclassifications. Hence (0,0.20735] is a better estimation of the true error.

22. **Evaluating Hypothesis**
When testing a hypothesis h, we used a sample set containing n samples. We learnt that 10% of the samples were misclassified. We want to be 95% sure (this means "with 95% confidence") that the true error rate is between 5% and 15%. How many samples do we need in order to be able to assure this?

    Answer:

    $error_D(h) = 0.1 \pm 1.96\sqrt{\frac{0.1(1-0.1)}{n}} = [0.05, 0.15] = 0.1 \pm 0.05$
    $0.05 = 1.96\sqrt{\frac{0.1(1-0.1)}{n}}$
    $n = 138.2$
    As the number of samples is a discrete number, we need 139 samples to be 95% confident that the true error is in the given interval.