departamento de eletrónica, telecomunicações e informática

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Report

Multisymbol Systems with Optimum Detection

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1 - Introduction

This assignment aims at studying transmission systems based on optimum detection of multisymbol signals, using for this purpose the QAM signal. The tool used is MATLAB, complemented with the corresponding Communications Toolbox.

2 - QAM Signal

2.1. QAM Transmitter

In this part we implemented a QAM Transmitter, the signals that modulate the quadrature carriers of the QAM signal are of the NRZ type. They can have amplitudes of ±d/2, ±3d/2, …, ±(K)d/2. K is the number of levels that the signal can assume, and d is the difference between consecutive amplitudes that each signal may take.

2.2. Characterization of the transmitted signal

a) In this part of the assignment we designed a 64-QAM signal, indicating the first 4 indicated symbols. The figure obtained was the following:

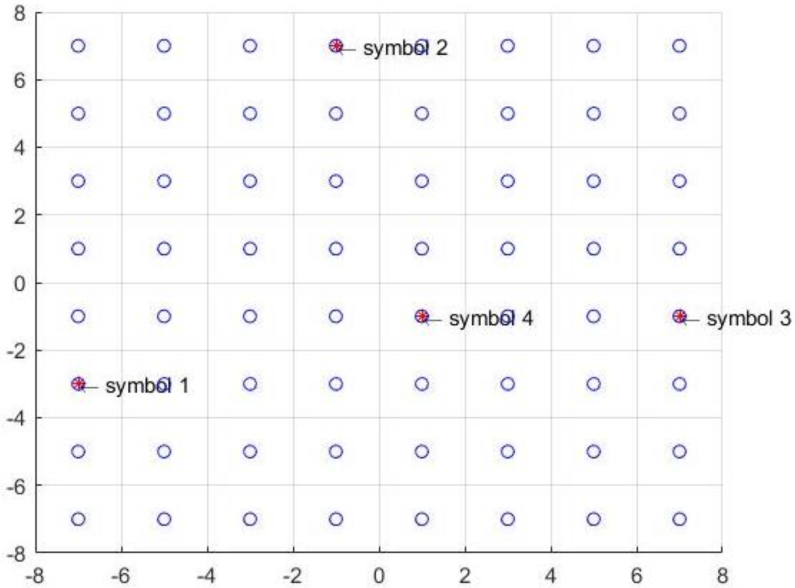

Figure 1 - 64-QAM constellation

b) In this part we must obtain the baseband signals that modulate the in-phase and quadrature carriers, corresponding to the symbols indicated before. The figures obtained were the following:

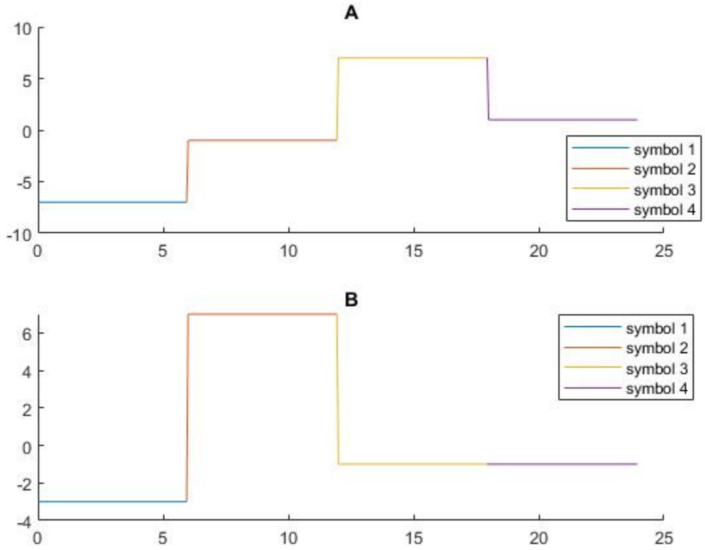

Figure 2 - Baseband signals

c) From Figure 1 we can conclude that the coordinates of symbol 3 and 4 are (7,-1) and (1,-1) respectively. So, it's expected that:

- For the 3rd symbol, the signal A was the value of 7 and the signal B was the value of -1
- For the 4th symbol the signal A was the value of 1 and the signal B was the value of -1

This agrees with Figure 2.

d) The two modulated quadrature carriers (signals C and D in Fig. 1), during the time span corresponding to the 4 symbols mentioned in (a) are the following:

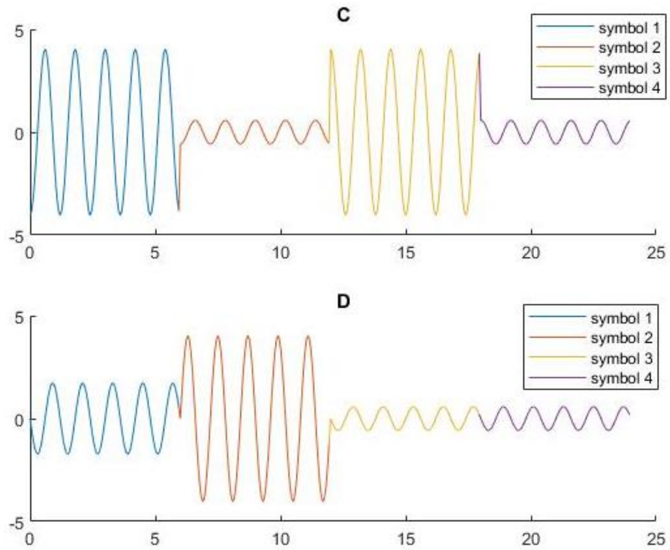

Figure 3 - Modulated Quadrature Carriers

e) In this part the figures obtained were the following:

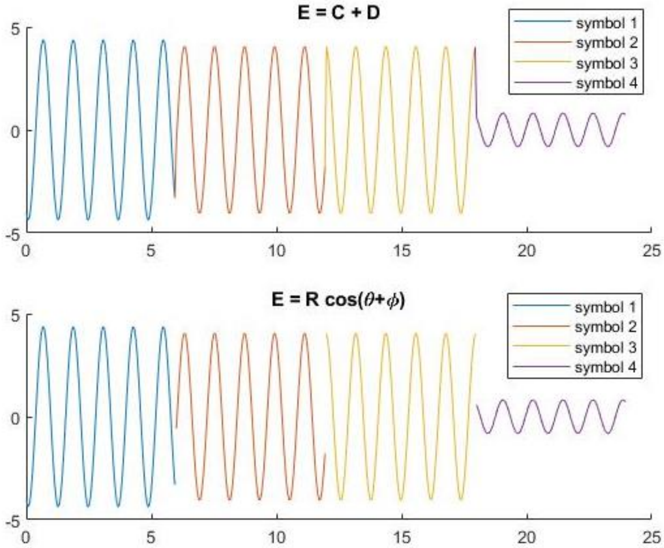

Figure 4 - 64-QAM modulator output

To determinate the amplitude and phase of the 4 symbols we used the following expressions:

$$C\cos(\theta) - S\sin(\theta) = R\cos(\theta + \varphi)\ ;\ R = \sqrt{C^2 + S^2}\ ;\ \varphi = \arctan\left(\frac{S}{C}\right)$$

C was given by $\sqrt{\frac{2}{T}}\ xd$ (xd represents the signal modulating the in-phase carrier) and S was given by $\sqrt{\frac{2}{T}}\ xq$ (represents the signal modulating the quadrature carrier).

It was necessary to consider that if the symbol was in the 3rd or 4th quadrant it was necessary to add π to the value of φ.

f)

Symbol energy is given by:

$$E = \|s\|^2 = (s_{\phi 1})^2 + (s_{\phi 2})^2$$

Average symbol energy is given by

$$E_m = \frac{1}{number\ of\ symbols}\sum E_i$$

By simulation we got, $E_m = 41.98$.

Theoretically

$$E_s = \frac{(M-1)d^2}{6} = 42$$

Was we can see the average symbol energy given by simulation agrees with the theoretical average symbol energy.

g.1 and g.3)

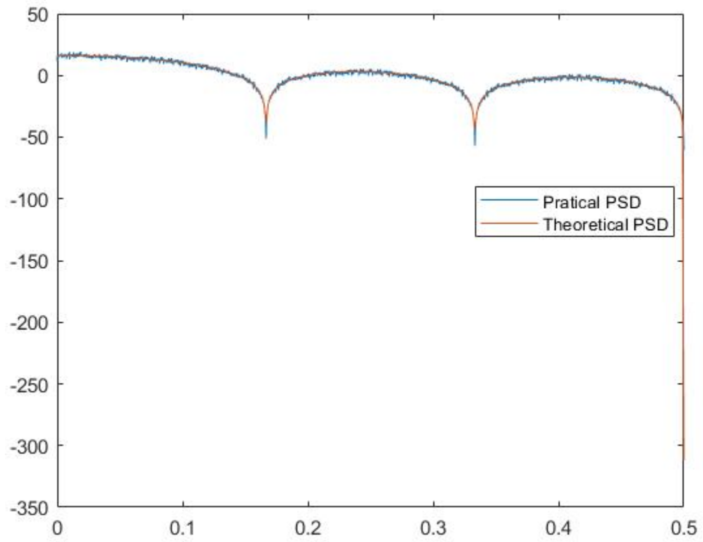

Figure 5 - PSD of signal B

In this part of the assignment we simulated and achieved the unilateral power spectral density (PSD) of the baseband signal, which modulates the quadrature carrier.

We can see that the theorical PSD agrees with the practical one. The theorical was obtained using the following:

$$Em * \left(\frac{\sin(\pi f T)}{\pi f T}\right)^2$$

The transmission rhythm of a symbol is $R_s = \frac{1}{T_s}$. The transmission rhythm of a bit will be $R_b = R_s * \mathrm{k}$. With this, $T_b = \frac{T_s}{k}$, where k is the number of bits of each symbol, k = 6. This way, the frequencies scale will be normalized relative to the transmission rate of the bits. The minimum of the function will be:

$$\min(sinc) := n * T_b = n * \frac{T_s}{6} = \frac{n}{6}, n \,\boldsymbol{\epsilon}\, \boldsymbol{N}$$

The first two minimums will be at $\frac{1}{6}$ and $\frac{2}{6}$, as we can see in the graph obtained.

g.2)

$$Frequency\ Resolution\ \approx \frac{fs}{L} = \frac{n}{\frac{n*m}{10}} = \frac{10}{m}$$

$$Frequency\ Resolution = 10^{-4} \qquad \rightarrow \qquad m = 10^5$$

The practical frequency resolution was calculated:

$$Frequency\ Resolution = f(2) - f(1) = 10^{-4}$$

h) The figure obtained in this part was the following:

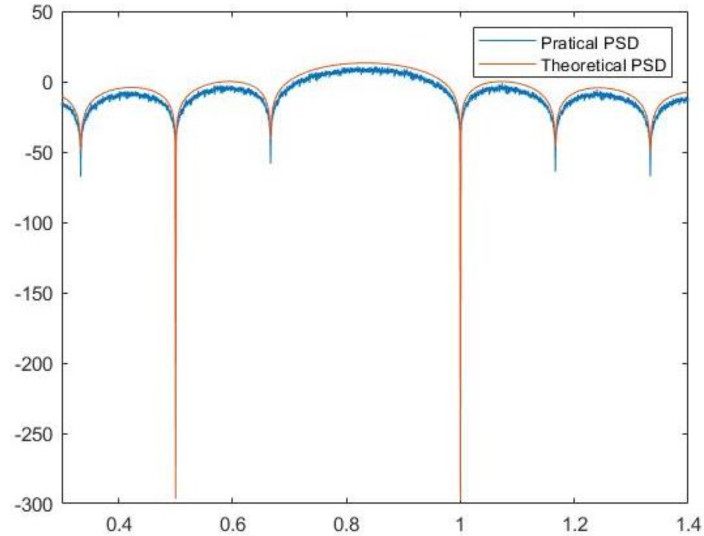

Figure 6 - PSD of signal E

The absolute maximum will be at the frequency of the carrier:

$$fc = Rc * T = \frac{5}{6}$$

The minimums will be at the $fc + \frac{n}{6}$, that means it will be at $\frac{5}{6} + \frac{1}{6} = 1$ and at $\frac{5}{6} + \frac{2}{6} = 1{,}1667$.

i) We repeated point (h) for a 4-QAM signal, with the same average symbol energy of the 64-QAM signal studied above (the average symbol energy was 42). For this to be true we had to change the d (distance between symbols):

$$\frac{(M-1) * d^2}{6} = 42 \quad (=) \quad d = \sqrt{\frac{252}{M-1}} \quad (=) \quad d = 9{,}1652$$

The absolute maximum will be at the frequency of the carrier:

$$fc = Rc * T = \frac{5}{2} = 2{,}5$$

The minimums will be at the $fc + \frac{n}{2}$, that means it will be at $\frac{5}{2} + \frac{1}{2} = 3$ and at $\frac{5}{2} + \frac{2}{2} = 3{,}5$.

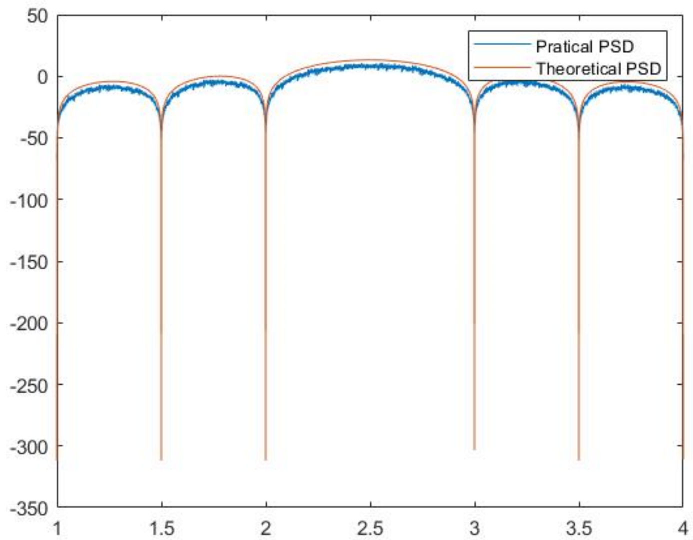

Figure 7 - PSD of 4-QAM signal

The value of L to show the 4 secondary lobes will be from the 3 left minimums to the 3 right minimums, using the same logic as above, this means that $L = \frac{5}{2} - \frac{3}{2} = 1$ and $U = \frac{5}{2} + \frac{3}{2} = 4$.

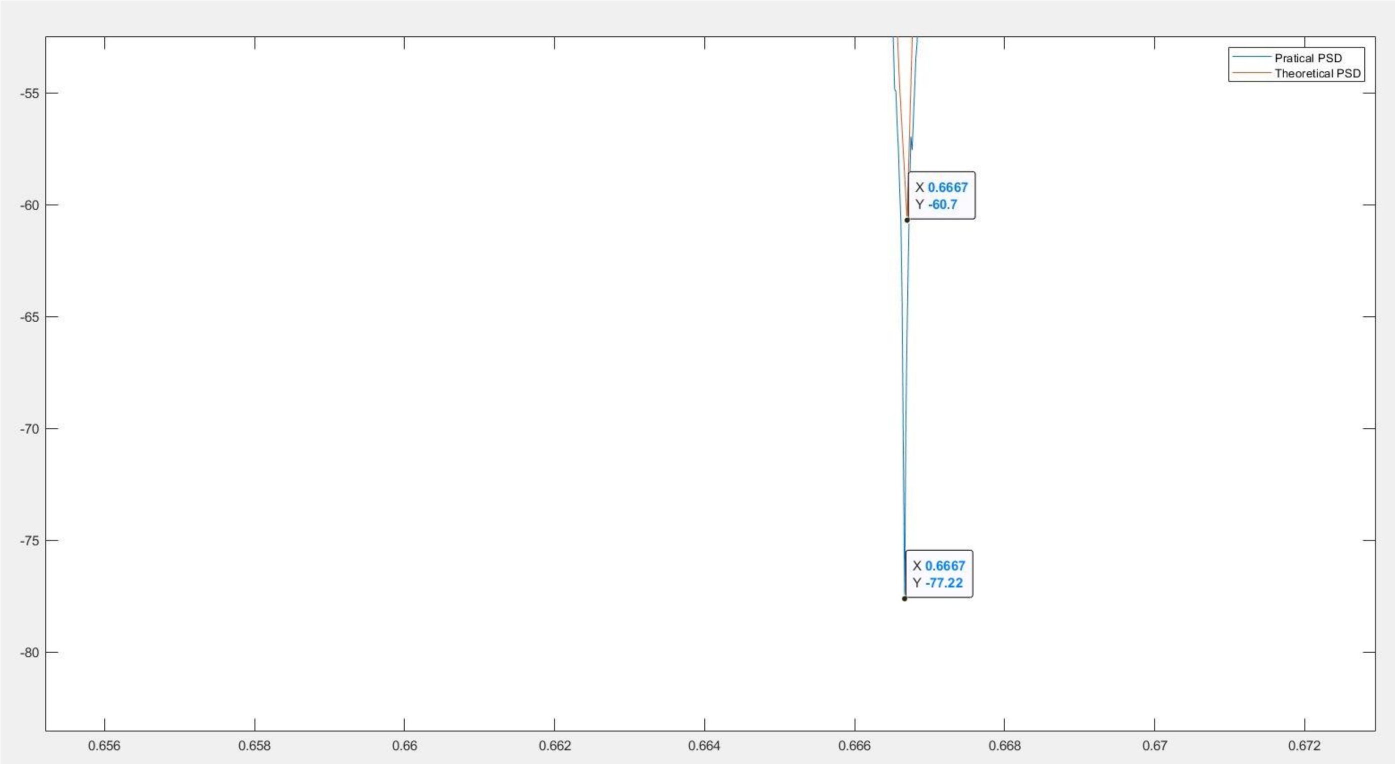

Figure 8 - Approximation left minimum 64-QAM

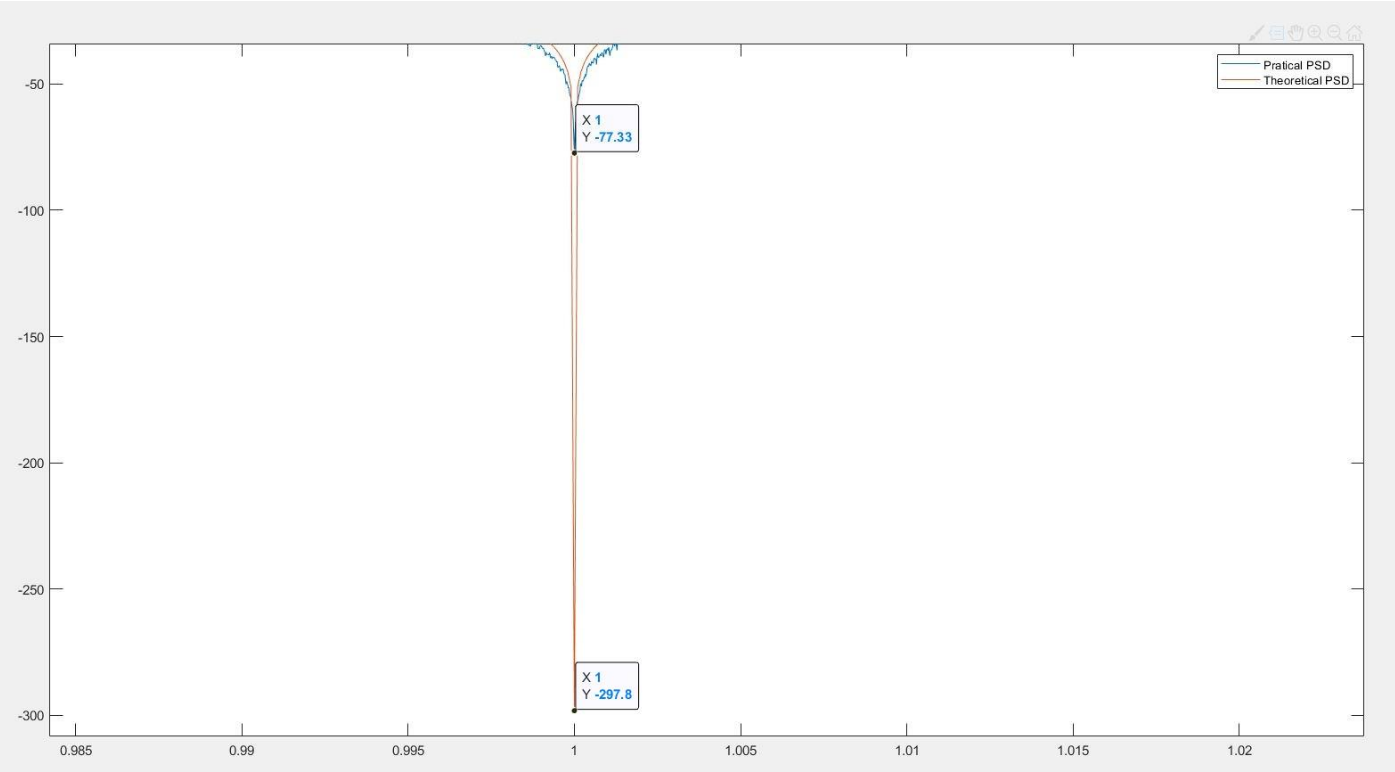

Figure 9 - Approximation right minimum 64-QAM

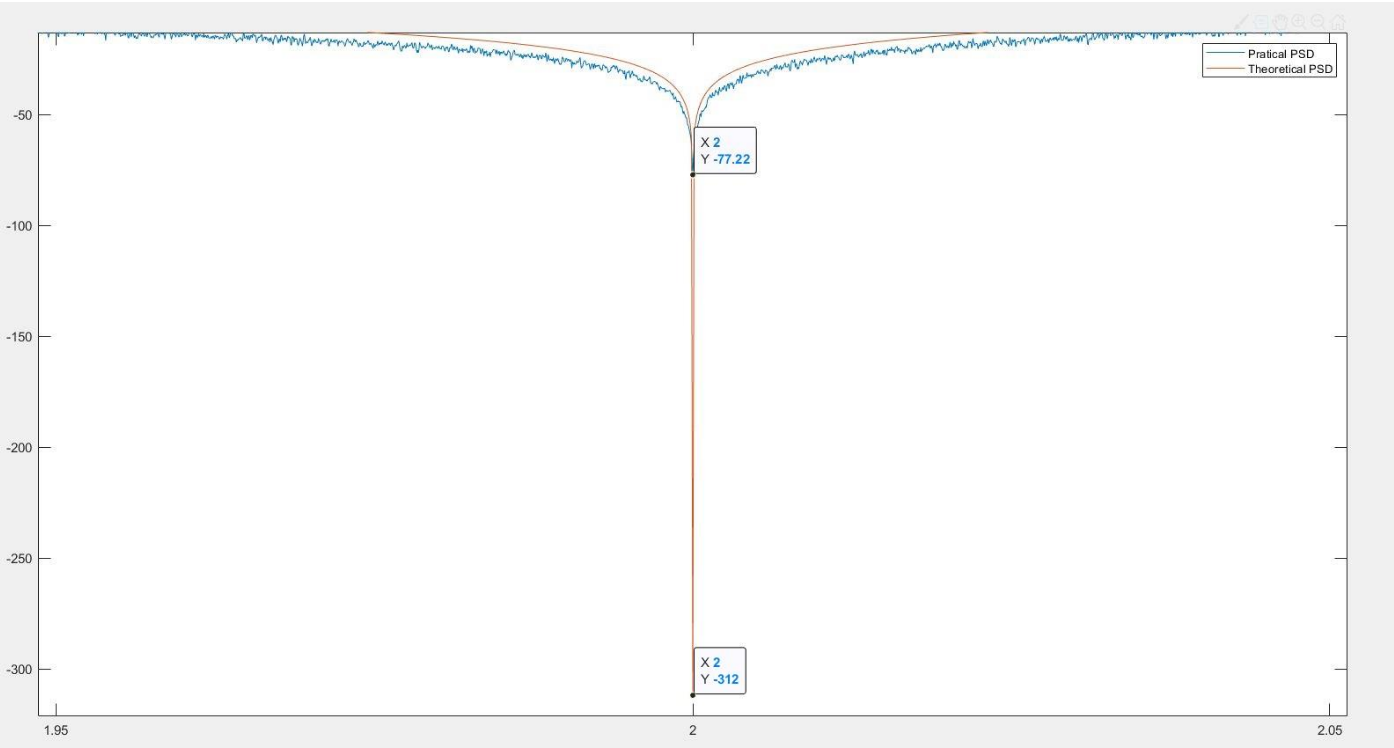

Figure 10 -Approximation left minimum 4-QAM

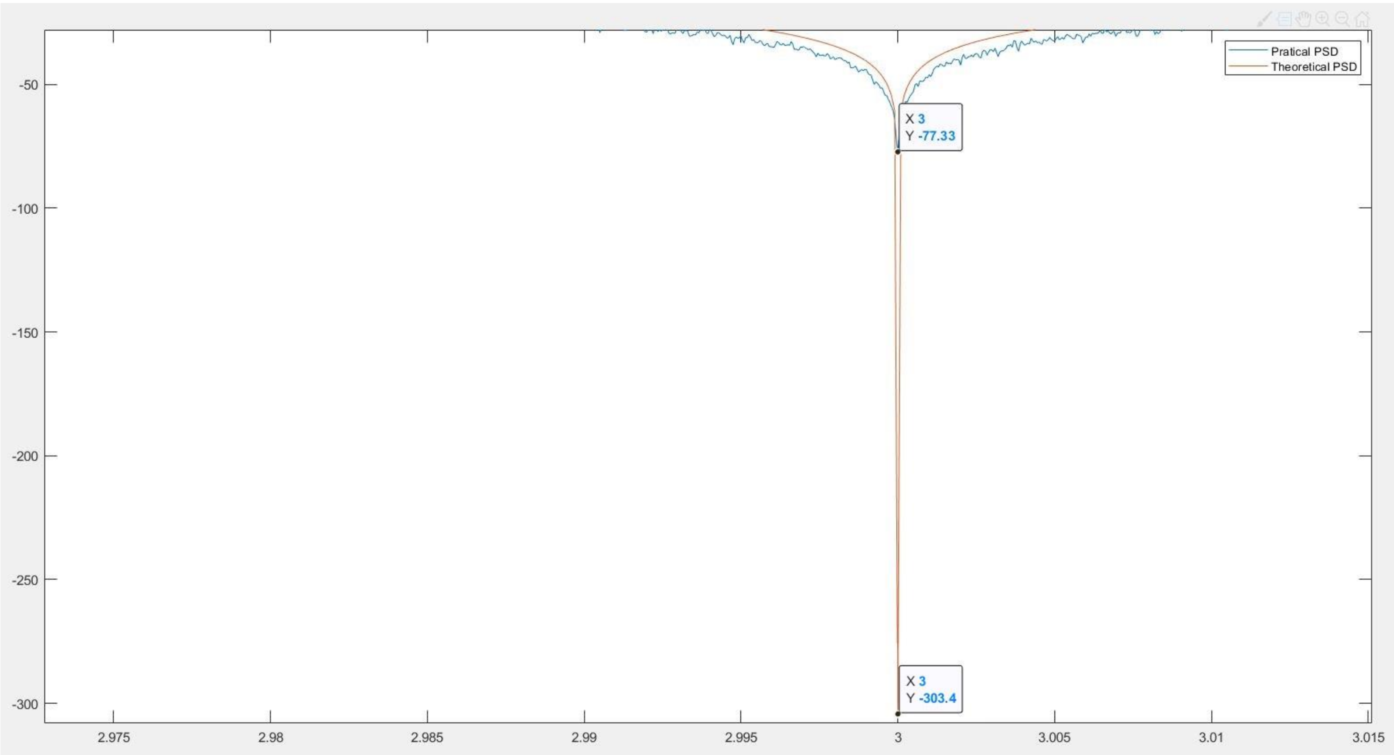

Figure 11 - Approximation right minimum 4-QAM

The practical ratio between the width of the biggest lobes of the 64-QAM signal and the 4-QAM signal is the following (Figure 8, Figure 9, Figure 10, Figure 11):

$$ratio = \frac{\Delta 64 - QAM}{\Delta 4 - QAM} = \frac{1 - 0,6667}{3 - 2} = 0,3333$$

The theorical ratio is given by the following (Figure 8, Figure 9, Figure 10, Figure 11):

$$ratio = \frac{\Delta 64 - QAM}{\Delta 4 - QAM} = \frac{1 - 0{,}6667}{3 - 2} = 0{,}3333$$

3 – System Performance

3.1. Implementation

In this part of the work, we loaded the Constellation_16_QAM.m to MATLAB, added white noise and developed an Optimum QAM Receiver.

3.2. Verification

a) To find the correct d for the given PES we had to do the following:

$$P_{es} = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3log_2(M)E_b}{2(M-1)n_0}}\right)\right]^2$$

$$P_{es} = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3\,(M-1)d^2}{6 \times 2(M-1)n_0}}\right)\right]^2 = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{d^2}{4\,n_0}}\right)\right]^2$$

$$\sqrt{1 - P_{es}} = 1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{d^2}{4\,n_0}}\right) (=) \frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}} = erfc\left(\sqrt{\frac{d^2}{4\,n_0}}\right)$$

$$erfcinv\left(\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}}\right) = \frac{d}{2\sqrt{n_0}} \rightarrow d = erfcinv\left(\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}}\right) 2\sqrt{n_0}$$

The d obtained was 5,1233.

(b) The figure obtained was the following:

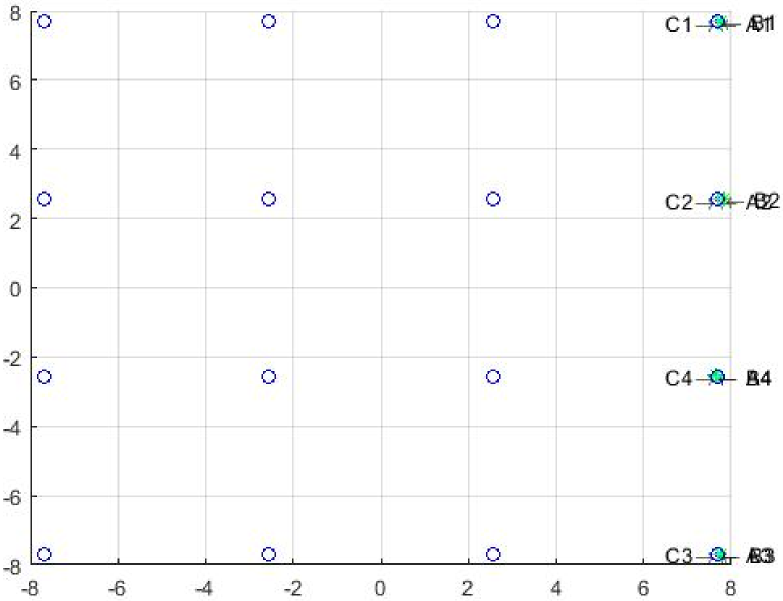

Figure 12 - Point A, B and C

To understand better the differences between the 3 points, we zoomed the points of interest and we obtained:

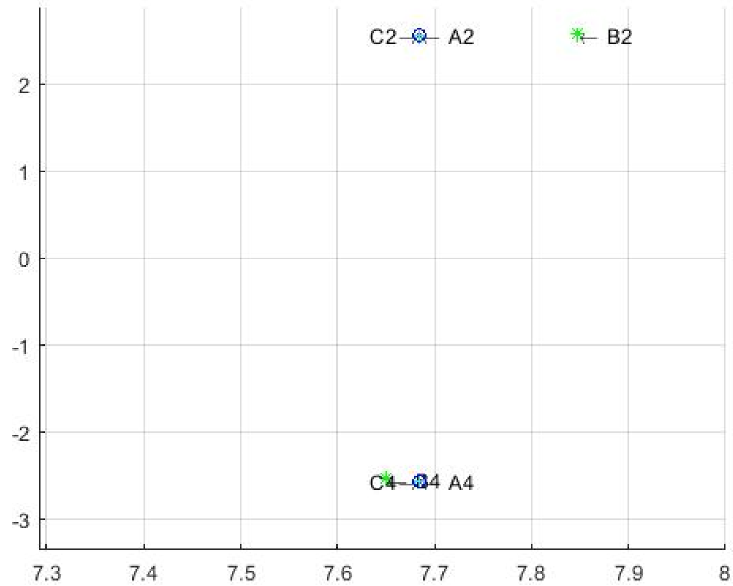

Figure 13 - Points of Interest

(c) We can conclude that for each one the four illustrated cases the receiver made a maximum likelihood detection because in each case the receiver based on the signal received choose the closest symbol.

3.3. Symbol Error Rate

a)

$$P_{es} = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)\right]^2$$

$$\sqrt{1 - P_{es}} = 1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)$$

$$\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}} = erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)$$

$$erfcinv\left(\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}}\right) = \sqrt{\frac{3E_s}{2(M-1)n_0}}$$

$$erfcinv\left(\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}}\right)^2 = \frac{3E_s}{2(M-1)n_0}$$

$$\frac{3E_s}{erfcinv\left(\frac{1 - \sqrt{1 - P_{es}}}{1 - \frac{1}{\sqrt{M}}}\right)^2 2(M-1)} = n_0$$

To obtain the value of INI and FIN, we used the same formula and we changed the value of PES to 10^{-3} and 0.5, respectively. The values obtained were 0.0691 for INI and 1.0850 for FIN.

b)

$$P_{cI} = P\left(-\frac{d}{2} < n1 < \frac{d}{2}\right) * P\left(-\frac{d}{2} < n2 < \frac{d}{2}\right)$$

$$P_{cI} = \left(1 - erfc\left(\frac{d}{2\sqrt{n_0}}\right)\right) * \left(1 - erfc\left(\frac{d}{2\sqrt{n_0}}\right)\right)$$

As,

$$E_s = \frac{(M-1)\,d^2}{6} \quad ; \quad 4 * E_b = E_s \qquad d = \sqrt{\frac{6E_s}{M-1}} = \sqrt{\frac{6 * \ 4E_b}{M-1}}$$

$$P_{cI} = \left(1 - erfc\left(\frac{\sqrt{\frac{6 * 4E_b}{M-1}}}{2\sqrt{n_0}}\right)\right)^2$$

$$P_{cI} = \left(1 - erfc\left(\sqrt{\frac{6E_b}{n_0\,(M-1)}}\right)\right)^2$$

$$P_{eI} = 1 - P_{cI} = 1 - \left(1 - erfc\left(\sqrt{\frac{6E_b}{n_0\,(M-1)}}\right)\right)^2$$

And this was the formula we used to fill the tables requested.

Table 1 - 16-QAM signal

Noise PSD	**18.3940**	**25.0930**	**27.6122**	**29.1962**	**30.3544**
$\boldsymbol{P_{eI}}$	0.0013	0.2178	0.4209	0.5464	0.6285
$\boldsymbol{P_{es}}$	1.0000e-03	0.1659	0.3264	0.4298	0.5000
$\boldsymbol{\frac{P_{eI}}{P_{es}}}$	1.3332	1.3132	1.2896	1.2713	1.2571

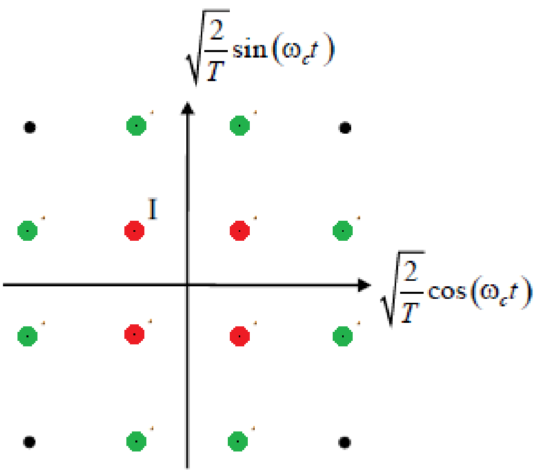

Figure 14 - 16-QAM

The $P_{error\ black\ sysmbol} < P_{error\ green\ sysmbol} < P_{error\ red\ sysmbol}$ (Figure 14 - 16-QAM)

So is to expect that $\frac{P_{error\ red\ sysmbol}}{0{,}25\ P_{error\ black\ sysmbol} + 0{,}5\ P_{error\ green\ sysmbol} +\ 0{,}25\ P_{error\ red\ sysmbol}} > 1.$

That happen as shown in Table 1.

c)

$$E_s = \frac{(M-1)\,d^2}{6} \quad \rightarrow \quad d = \sqrt{\frac{6E_s}{M-1}}$$

$$P_{es} = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)\right]^2$$

Table 2 - 16-QAM signal

Noise PSD (dBm/Hz)	**18.3940**	**25.0930**	**27.6122**	**29.1962**	**30.3544**
Number of symbols used in each simulation	10000	10000	10000	10000	10000
Symbol error rate (SER)	0.0010	0.1662	0.3254	0.4305	0.5038
Probability of symbol error (exact value Pes)	0.0010	0.1659	0.3264	0.4298	0.5000
|SER-Pes|/Pes (%)	2.0000	0.2201	0.3120	0.1559	0.7560

To notice that the last row of the table was calculated taken in account all bits, that's why in the first column it isn't 0.

d) There's a small difference between the SER and the exact value of Pes, this probably happened because the noise was introduced randomly in the system.

When the noise increases the percentage of the difference between SER and the exact value of Pes tends to rise. To decrease this, we could increase the number of symbols simulated.

e)

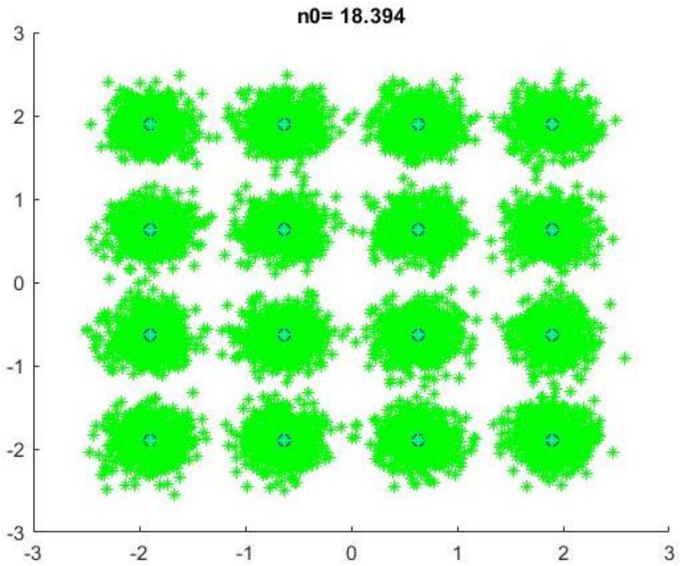

Figure 15 - 16-QAM with n0 = 18.394

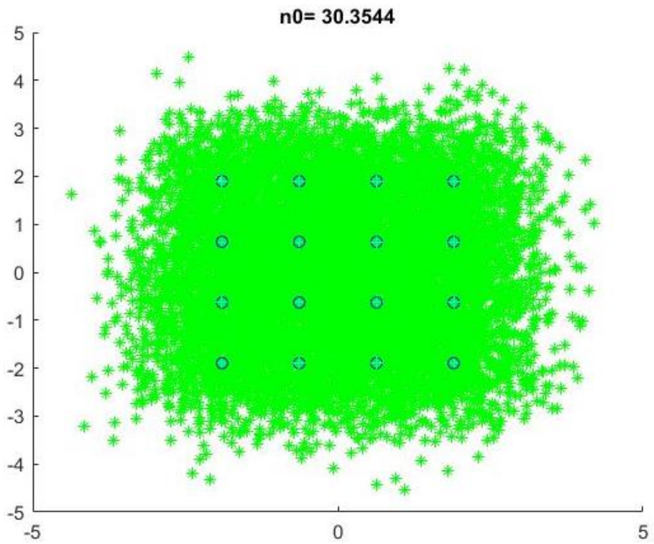

Figure 16 - 16-QAM with n0 = 30.3544

As we can see in Figure 15 the signals are very close to the symbols. That happens because we have a small noise that don't spread the signals far away from the original symbol. The probability of error is less was shows Table 2 - 16-QAM signal

In Figure 16 we can't identify clearly every symbol. That happens because we have more noise and it will spread some of the signals further from the original symbol. The probability of error is bigger was shows Table 2 - 16-QAM signal

f)

Table 3 - 64-QAM signal

Noise PSD	18.3940	25.0930	27.6122	29.1962	30.3544
Number of symbols used in each simulation	62	62	62	62	62
Symbol error rate (SER)	0.1581	0.6484	0.7333	0.7677	0.8065
Probability of symbol error (exactvalue Pes)	0.1623	0.6245	0.7449	0.8003	0.8326
|SER-Pes|/Pes (%)	2.5925	3.8168	1.7695	4.0661	3.1375

g) The value of SER increases from Table 2 to Table 3

$$P_{es} = 1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)\right]^2$$

From 16-QAM to 64-QAM the only value that changes is M.
16-QAM -> M=16
64-QAM -> M=64

If M rises (considering that $\sqrt{\frac{3E_s}{2(M-1)n_0}} < 1$)

- $\sqrt{\frac{3E_s}{2(M-1)n_0}}$ falls -> $erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)$ falls
- $\left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)\right]^2$ rises
- $1 - \left[1 - \left(1 - \frac{1}{\sqrt{M}}\right) erfc\left(\sqrt{\frac{3E_s}{2(M-1)n_0}}\right)\right]^2$ falls

So, the probability of error increases from 16-QAM to 64-QAM

Note: the variation of the term $1 - \frac{1}{\sqrt{M}}$ when M increases is insignificant compared to the variation of the term $\sqrt{\frac{3E_s}{2(M-1)n_0}}$

3.4. Bit Error Rate

a)

16-QAM

Table 4 - 16-QAM

Noise PSD	18.3940	25.0930	27.6122	29.1962	30.3544
Number of symbols used in each simulation	10000	10000	10000	10000	10000
Bit error rate (BER)	0.0002	0.0434	0.0892	0.1234	0.1494
BER/SER	0.2500	0.2612	0.2743	0.2866	0.2966

64-QAM

Table 5 - 64-QAM

Noise PSD	18.3940	25.0930	27.6122	29.1962	30.3544
Number of symbols used in each simulation	62	62	62	62	62
Bit error rate (BER)	0.0151	0.0796	0.1108	0.1194	0.1366
BER/SER	0.0952	0.1227	0.1461	0.1555	0.1693

b)

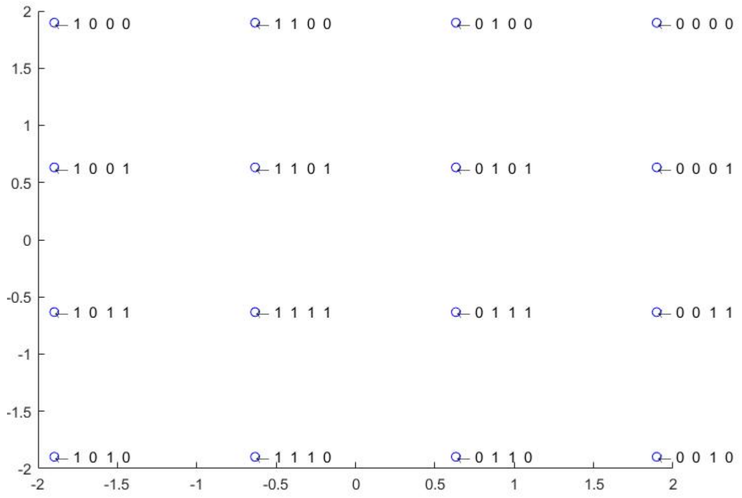

Figure 17 - 16-QAM constellation

$$\boldsymbol{BER} = \frac{\boldsymbol{SER}}{\log_2 \boldsymbol{M}} \rightarrow \frac{\boldsymbol{BER}}{\boldsymbol{SER}} = \frac{\mathbf{1}}{\log_2 \boldsymbol{M}} = \mathbf{0.1667}$$

The relation between BER and SER will be approximately 0.1667, that justifies the values obtained. When we increase the noise power, the probability of error increases and that provokes a higher BER/SER.

c) We know that in this constellation (Figure 17) 2 adjacent symbols have a unitary distance 'd'. When we increase the noise, there’s a higher probability of the symbol be incorrectly evaluated at the receiver and evaluated as a non-adjacent symbol to the correct symbol.

In this case, the distance ‘d’ will be greater than one, so there will be a symbol error. In this way, the BER will increase as well as the BER/SER ratio.