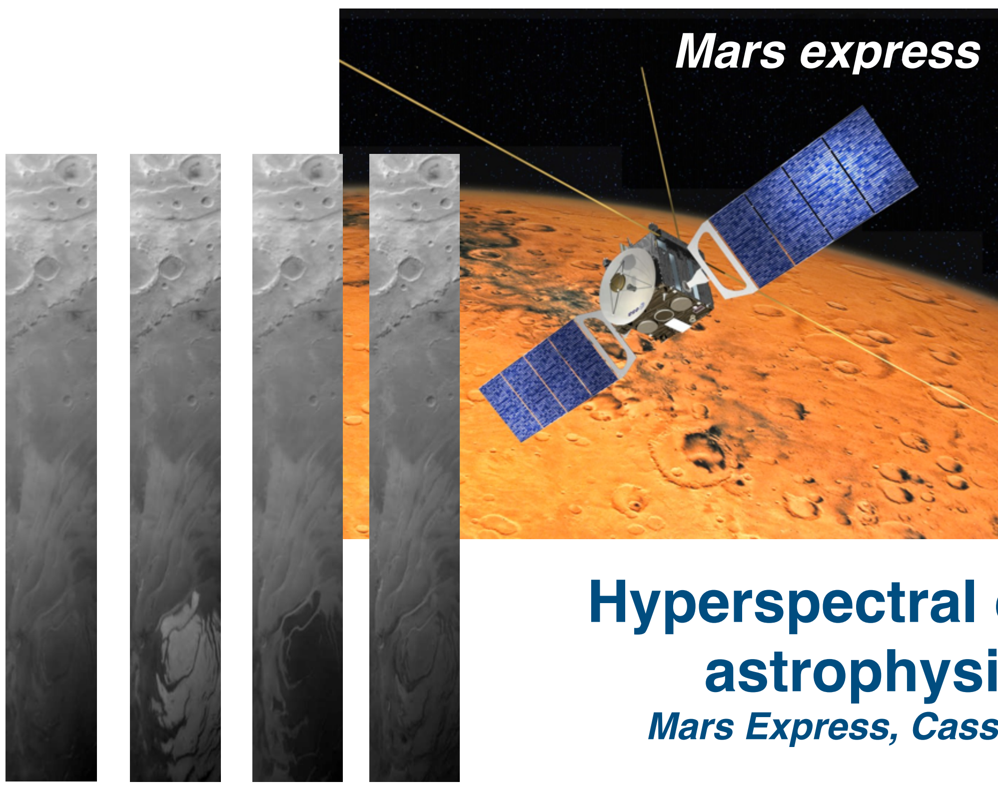
Joint Deconvolution and Blind Source Separation on the Sphere with an Application in Radio-Astronomy

R. Carloni Gertosio, J. Bobin
IRFU/CEA Saclay, Université Paris-Saclay

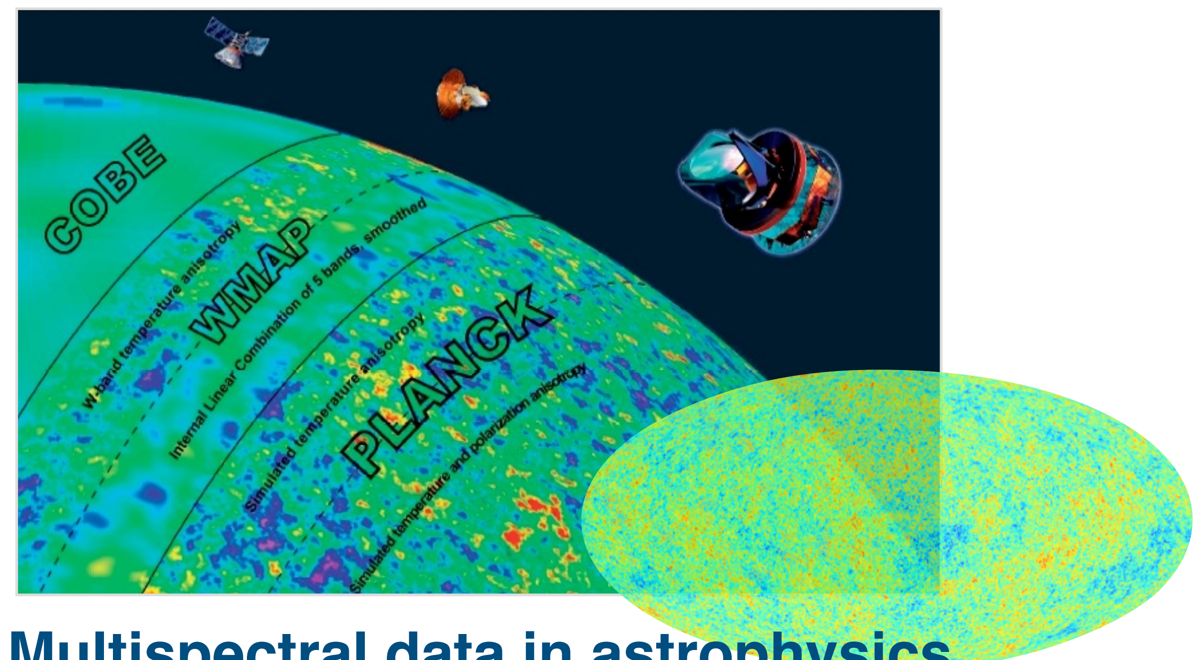
iTwist 2020

Blind source separation

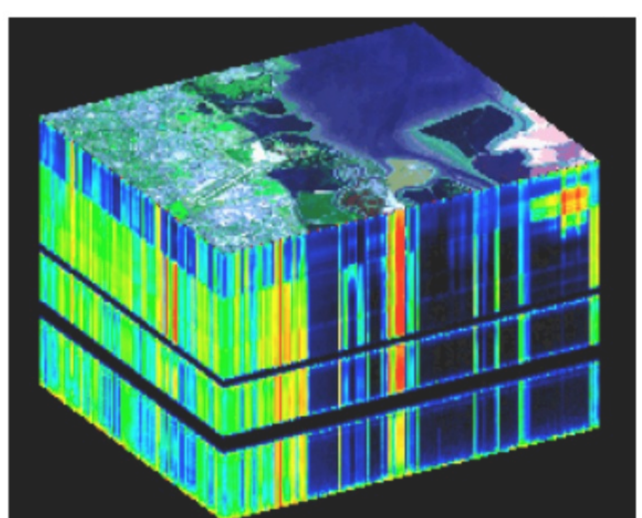
- Blind source separation (BSS) methods are employed to **decompose multi-spectral data** in **elementary components**



Hyperspectral data in astrophysics
Mars Express, Cassini, etc.



Multispectral data in astrophysics
Planck, Fermi, radio-interferometry (Lofar/SKA/…), etc.



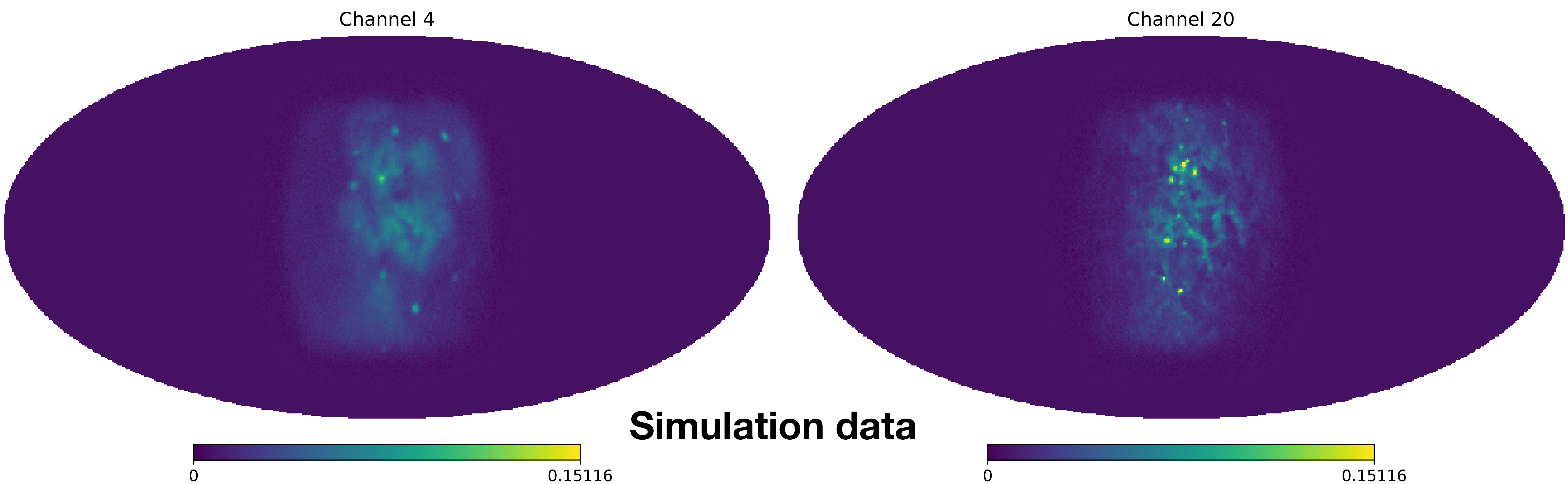
Courtesy of M. Lennon

Hyperspectral data
remote sensing, aerial data, etc.

Context

Forthcoming large-scale radio-telescopes (e.g. SKA) will produce:

1. **Spherical** data
2. Multi-wavelength data with **diverse resolutions**

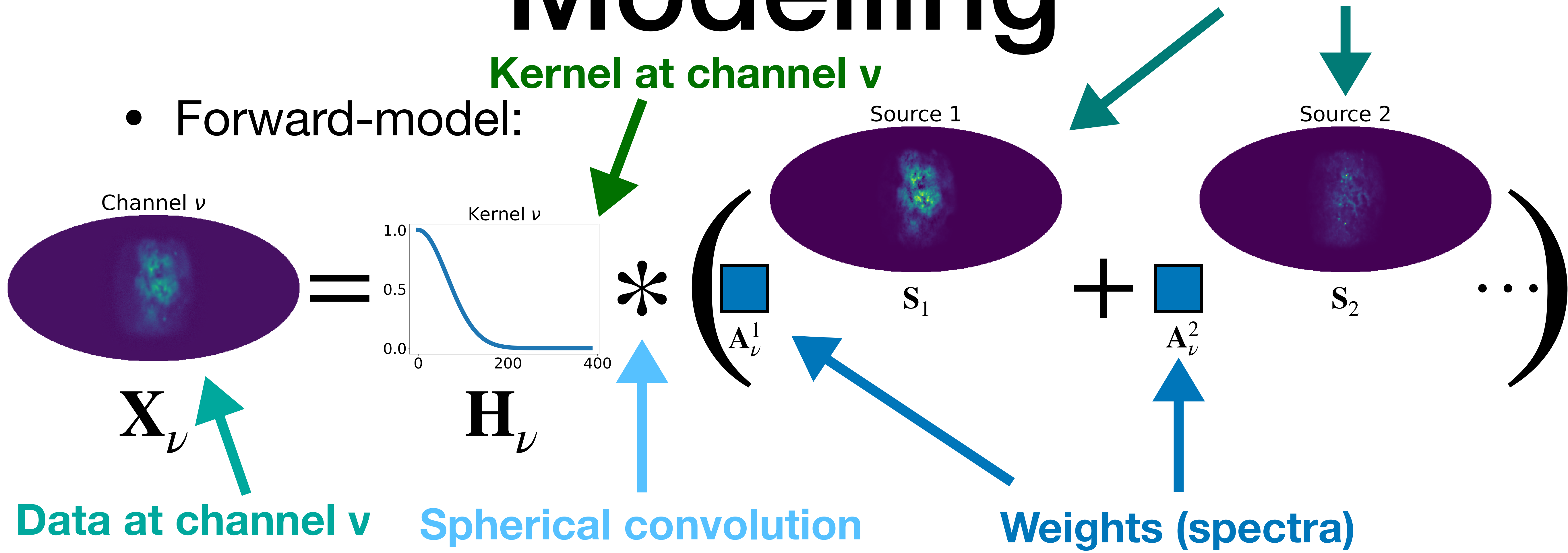


Simulation data

Need to develop joint **spherical deconvolution** and **blind source separation** methods

Modelling

- Forward-model:



$$\mathbf{X}_\nu = \mathbf{H}_\nu * (\mathbf{A}_\nu \mathbf{S}) + \mathbf{N}_\nu$$

Simplified in the spherical harmonic domain:

Harmonic multipole (frequency)

$$\hat{\mathbf{X}}^{l,m} = \mathrm{diag}(\hat{\mathbf{H}}^{l})\mathbf{A}\hat{\mathbf{S}}^{l,m} + \hat{\mathbf{N}}^{l,m}$$

Toward SDecGMCA

- Cost function: Data-fidelity term Sparse penalization

$$\mathrm{argmin}_{\mathbf{A},\mathbf{S}} \quad \frac{1}{2}\sum_{l,m} || \hat{\mathbf{X}}^{l,m} - \mathrm{diag}\left(\hat{\mathbf{H}}^{l}\right)\mathbf{A}\hat{\mathbf{S}}^{l,m}||_2^2 + ||\mathbf{\Lambda} \odot \left(\mathbf{S}\mathbf{\Phi}^{T}\right)||_1$$

- Based on a **Projected Alternate Least-Square** (fast, robust and automatic choice of $\mathbf{\Lambda}$, GMCA *Bobin et al. 2007*)
  - Update $\mathbf{S}$ with $\mathbf{A}$ fixed:
    - Least-square: $\hat{\mathbf{S}}^{l,m} \leftarrow (\mathbf{A}^{T}\mathrm{diag}(\hat{\mathbf{H}}^{l})^{2}\mathbf{A})^{-1}\mathbf{A}^{T}\mathrm{diag}(\hat{\mathbf{H}}^{l})\hat{\mathbf{X}}^{l,m}$
    - Sparsity constraint

      Ill-conditioned!
      To be regularized
  - Update $\mathbf{A}$ with $\mathbf{S}$ fixed:
    - Least-square: $\mathbf{A}_{\nu} \leftarrow (\Sigma_{l,m}\hat{\mathbf{X}}_{\nu}^{l,m}\hat{\mathbf{H}}_{\nu}^{l}\hat{\mathbf{S}}^{l,m\,\dagger})(\Sigma_{l,m}\hat{\mathbf{H}}_{\nu}^{l\,2}\hat{\mathbf{S}}^{l,m}\hat{\mathbf{S}}^{l,m\,\dagger})^{-1}$

Source regularization

- Extra **Tikhonov regularization** (*Jiang et al. 2017*):

$$\hat{\mathbf{S}}^{l,m} \leftarrow (\mathbf{A}^T \mathrm{diag}(\hat{\mathbf{H}}^l)^2 \mathbf{A} + \underline{\mathrm{diag}_n\left(\varepsilon_{n,l}\right)})^{-1} \mathbf{A}^T \mathrm{diag}(\hat{\mathbf{H}}^l) \hat{\mathbf{Y}}^{l,m}$$

- Choice of $\{\varepsilon_{n,l}\}$? We propose **2 strategies**:

<u>Mixing-matrix based:</u>

Smallest eigenvalue

$$\varepsilon_{n,l} = \max\left(0,\ c - \frac{\lambda_{\min}(\mathbf{A}^T \mathrm{diag}(\hat{\mathbf{H}}^l)^2 \mathbf{A})}{\lambda_{\min}(\mathbf{A}^T \mathbf{A})}\right)$$

Regularization hyperparameter

<u>Gist:</u> limit noise amplification

Robust

<u>Source based:</u>

Angular power spectra

$$\varepsilon_{n,l} = c \frac{\mathbf{c}_{\mathbf{N}}[l]}{\mathbf{c}_{\mathbf{S}_n}[l]}$$

Regularization hyperparameter

<u>Gist:</u> ~ Wiener filter

Precise, not robust

SDecGMCA

- Two-step procedure, starting from a PCA-based estimation:

1. Warm-up: while has not converged
    - Update $\mathbf{S}$ with $\mathbf{A}$ fixed (**mixing-matrix-based** regularization strategy)
    - Update $\mathbf{A}$ with $\mathbf{S}$ fixed

First guess, robustness to initial point

2. Refinement: while has not converged
    - Update $\mathbf{S}$ with $\mathbf{A}$ fixed (**source-based** regularization strategy)
    - Update $\mathbf{A}$ with $\mathbf{S}$ fixed

Refines the results

Use the angular power spectra of the sources estimated at last iteration

Numerical experiments

- Objectives:

  1. **Characterize** SDecGMCA under various observation scenarios (SNR, nb. channels, resolution range, mix. mat. condition nb.)
  2. **Compare** SDecGMCA with other BSS methods

- Separation performance metrics:

Asterisk denotes ground truth

$$\mathrm{NMSE} = -10\log_{10}\left(\frac{||\mathbf{S}^* - \mathbf{S}||_{\ell_2}^2}{||\mathbf{S}^*||_{\ell_2}^2}\right)$$

$$\mathrm{C_A} = -10\log_{10}\left(\mathrm{mean}(|\mathbf{A}^+\mathbf{A}^* - \mathbf{I}|)\right)$$

The larger, the better

- Oracle: estimate using ground truth mixing-matrix $\mathbf{A}^*$

  ➡ Provides an upper-bound for the NMSE

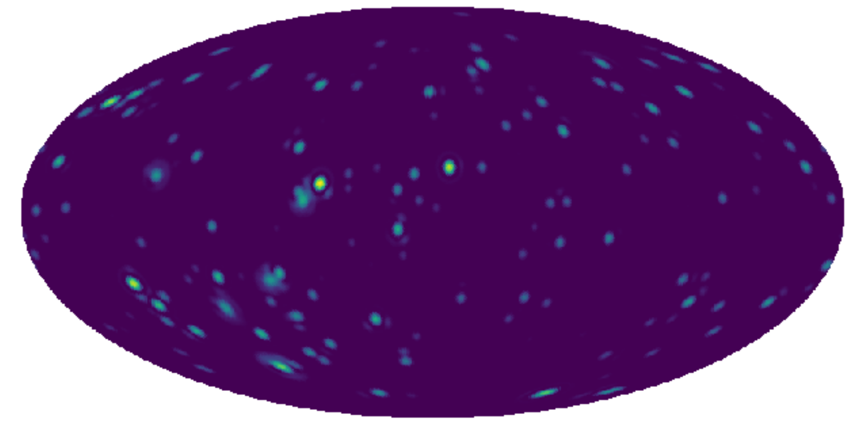
Synthetic data

<u>Objective 1:</u> characterize SDecGMCA

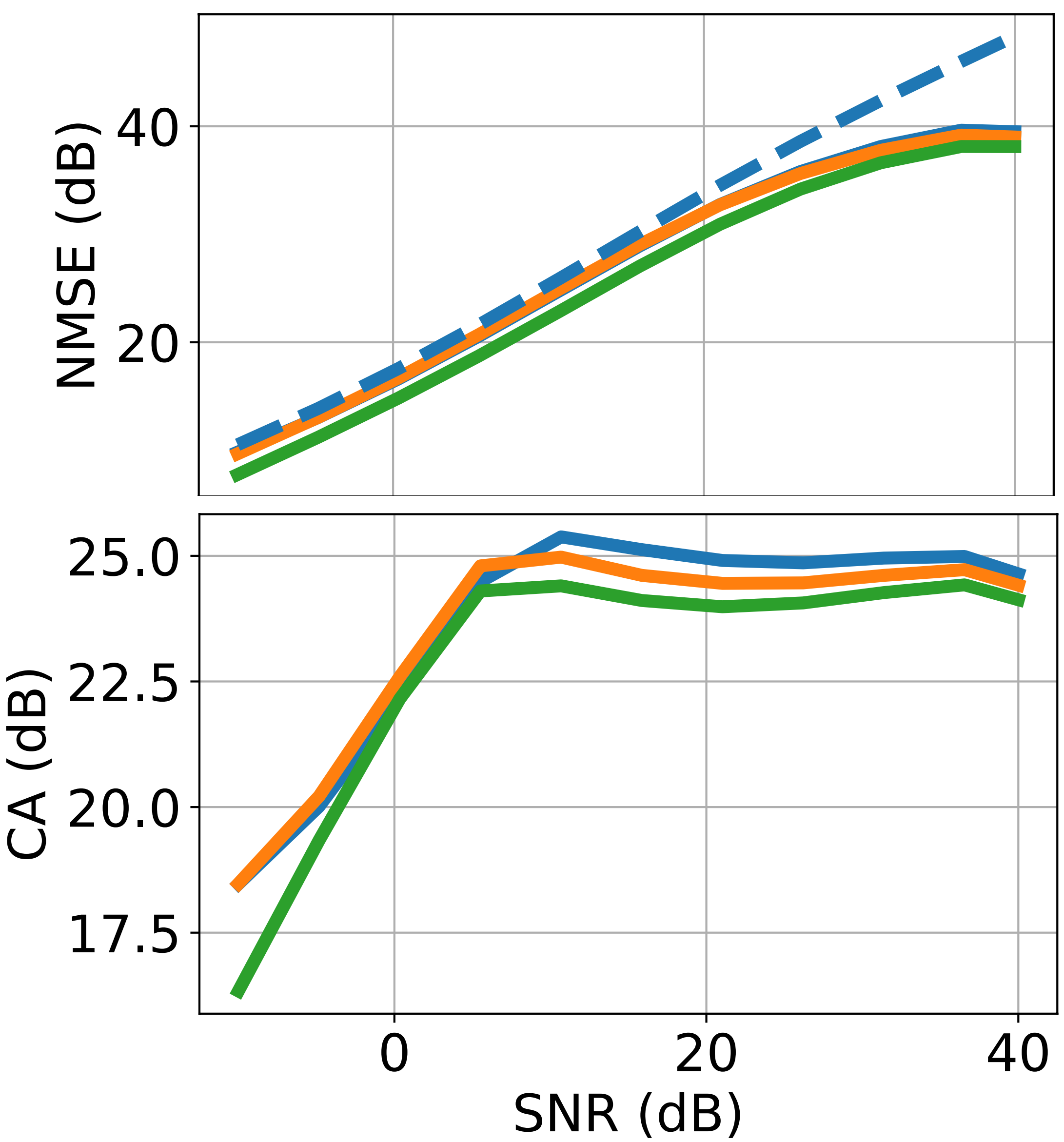
i. Regularization hyperparameter **c** ?

$c_{opt} \approx 0.5$

ii. Performances ? Sensibility of **c** ?



Source example (among 4)



--- Oracle

— $c_{opt} \approx 0.5$

— $10^{0.5}\,c_{opt}$

— $10^{-0.5}\,c_{opt}$

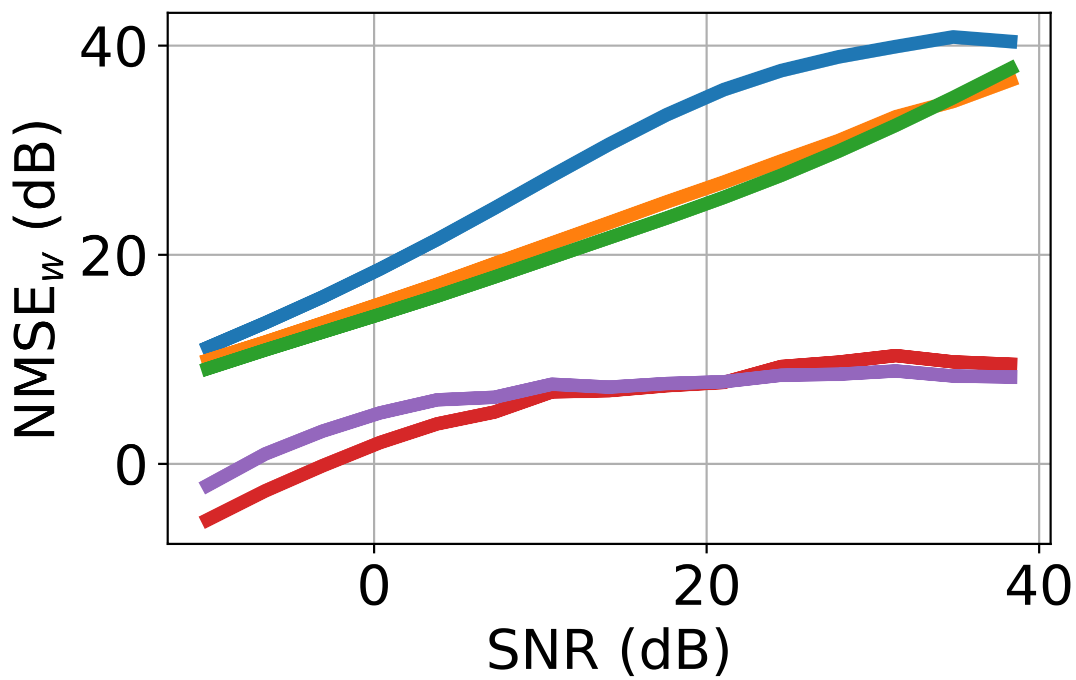
➔ Sources close to oracle

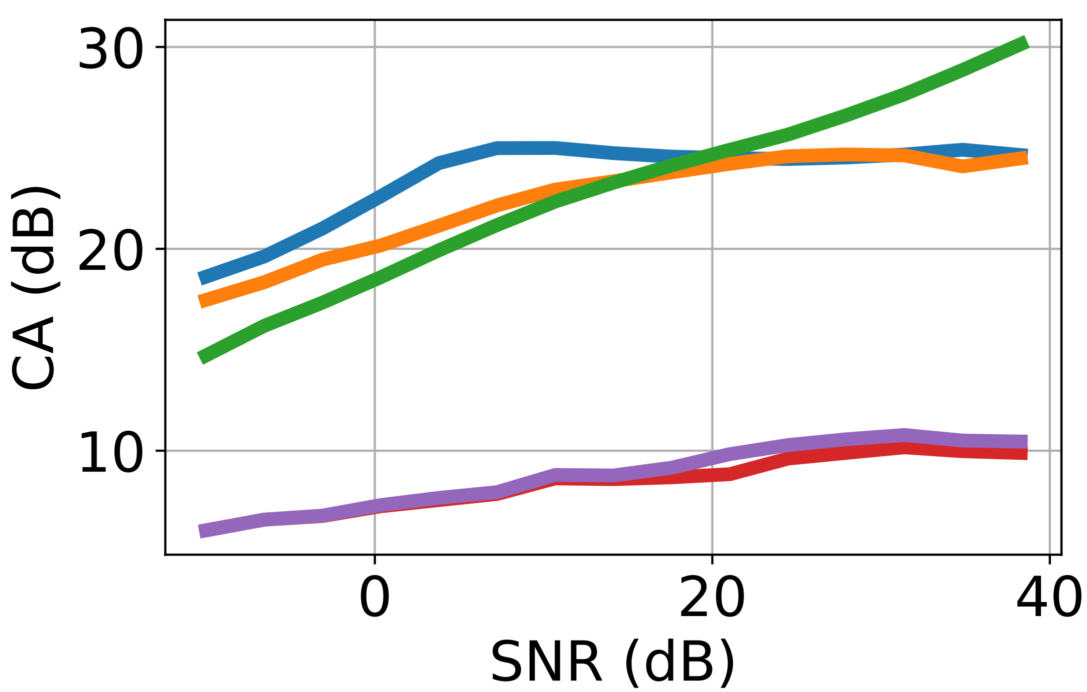
➔ Quite insensitive to c

Similar results when varying the **mix. mat. condition nb.**, the **resolution range** & the **nb. of channels**

Synthetic data

<u>Objective 2:</u> compare SDecGMCA





DBSS methods:

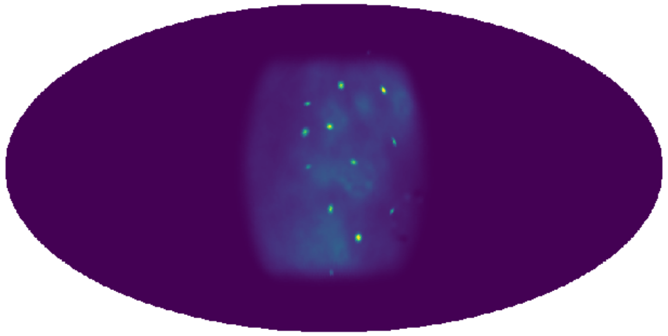
- **SDecGMCA**
- ***Jiang et al. 2017***

BSS/NMF methods:

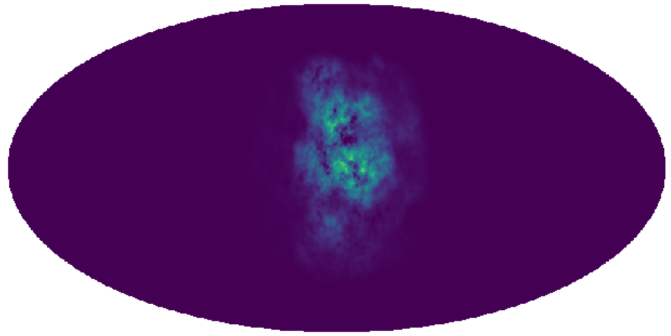
- **GMCA, *Bobin et al. 2007***
- **HALS, *Cichocki et al. 2007***
- **β-NMF S, *Cherni et al. 2020***

Similar results when varying the **mix. mat. condition nb.**, the **resolution range** & the **nb. of channels**

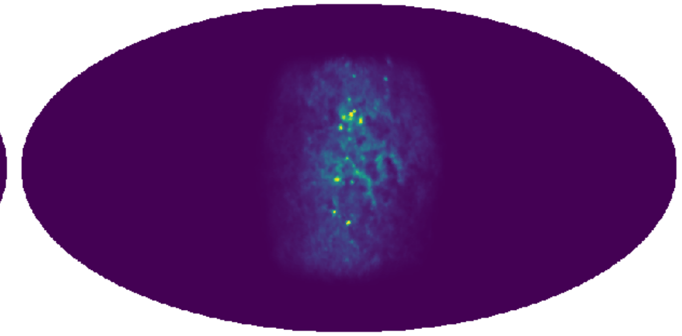
Realistic data



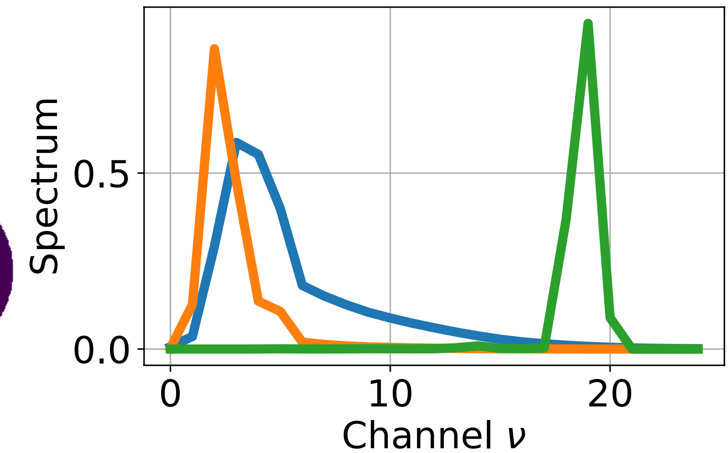
Synchrotron source



Thermal source



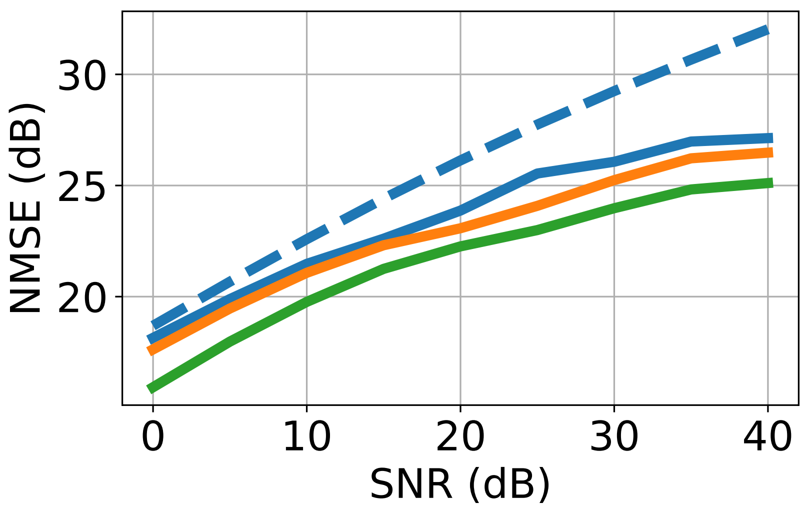
Emission line source

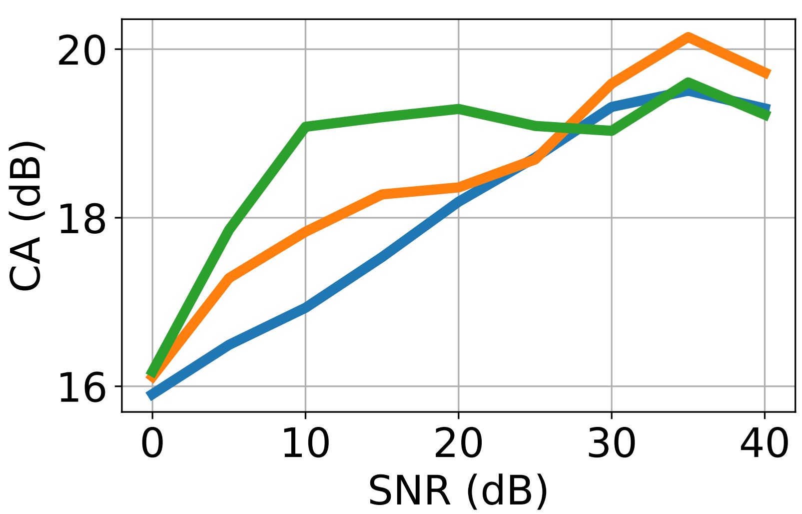


Mixing matrix

<u>Objective 1:</u>

characterize SDecGMCA





--- Oracle

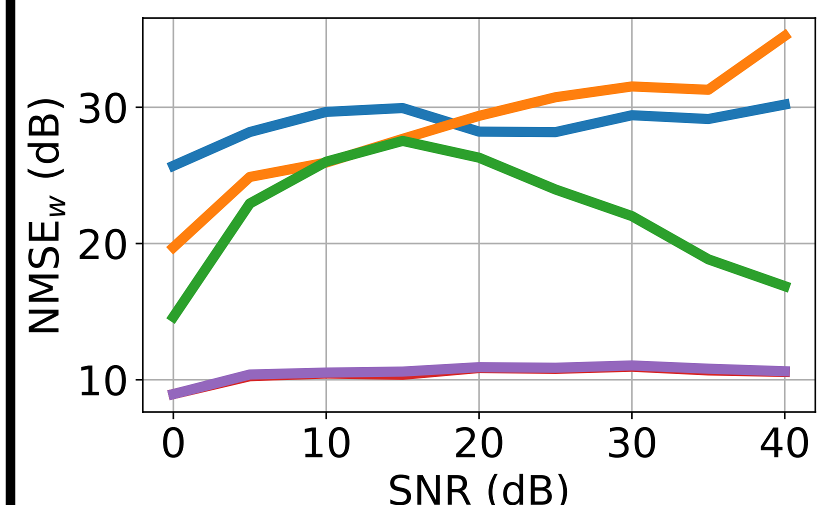
— $c_{opt} \approx \underline{0.5}$

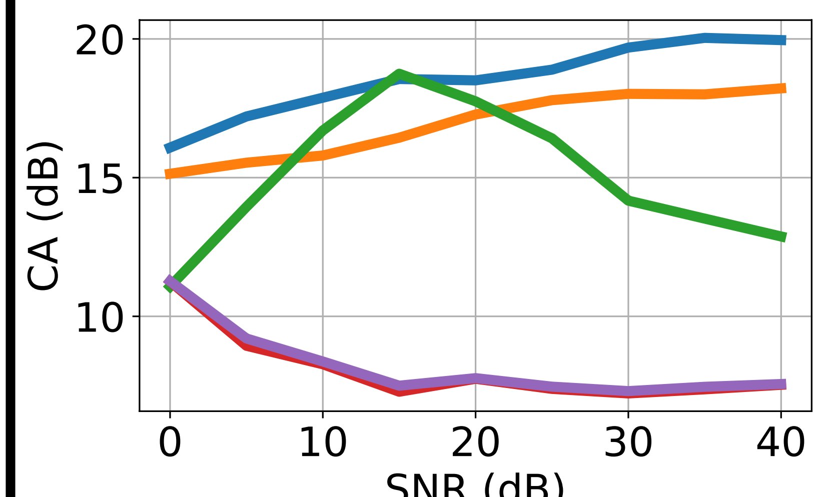
— $10^{0.5}$ c_{opt}

— $10^{-0.5}$ c_{opt}

<u>Objective 2:</u>

compare SDecGMCA





— SDecGMCA

— *Jiang et al.*

— GMCA

— HALS

— β-NMF S

Conclusion

- New method to perform joint **spherical deconvolution** and **blind source separation** problems
- **Robust** and **effective** minimization algorithm, evaluated on both **synthetic** and **realistic** data
- Open-source Python code available online
  github.com/RCarloniGertosio/SDecGMCA
- **Perspectives:**
  - Journal paper under revision
  - Method to be tested on MeerKAT data (SKA precursor)