
Global Convergence Newton

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Abstract

1 The abstract paragraph should be indented 1/2 inch (3 picas) on both the left- and
2 right-hand margins. Use 10 point type, with a vertical spacing (leading) of 11 points.
3 The word **Abstract** must be centered, bold, and in point size 12. Two line spaces
4 precede the abstract. The abstract must be limited to one paragraph.

5 1 Introduction

6 Newton's method is a popular optimization algorithm that is commonly used to solve optimization
7 problems. It is a second-order optimization algorithm since it uses second-order information of the
8 objective function. Newton's method is known to have fast local convergence guarantees for convex
9 functions. However, the global convergence properties of Newton's method are still an active area of
10 research. The purpose of this project is to survey and analyze various strategies to achieve global
11 convergence.

12 This sentence will be cited by the sources so i can test if bibtex is working properly [1]

13 2 Background

14 In the following we will expore the theoretical foundations of different variants of Newton's method
15 including line search strategies, regularization techniques, and the trust region method. We

16 2.1 Classic Newton's Method

17 Newton's method is a second-order method. This means it uses information from the second derivative
18 of the function that we are trying to minimize. It is used to find a local minimum in a function. It
19 uses the update rule:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - \frac{\nabla f(\mathbf{x}_k)}{\nabla^2 f(\mathbf{x}_k)} \quad (1)$$

20 2.2 Regularized Newton

21 In their 2023 article Michenko presents a variation of Newton's method that uses the update rule [2]:

$$\mathbf{x}_{k+1} = \mathbf{x}_k - (\nabla^2 f(\mathbf{x}_k) + \sqrt{H||\nabla f(\mathbf{x}_k)||\mathbf{I}})^0 \nabla f(\mathbf{x}_k) \quad (2)$$

22 where $H > 0$ is a constant. The convergence rate of this algorithm is $\mathcal{O}(\frac{1}{k^2})$.

23 References

- 24 [1] Slavomír Hanzely, Dmitry Kamzolov, Dmitry Pasechnyuk, Alexander Gasnikov, Peter Richtárik,
25 and Martin Takáč. A damped newton method achieves global $(o)(\frac{1}{k^2})$ and local quadratic
26 convergence rate. *Advances in Neural Information Processing Systems*, 35:25320–25334, 2022.
- 27 [2] Konstantin Mishchenko. Regularized newton method with global convergence. *SIAM Journal on*
28 *Optimization*, 33(3):1440–1462, 2023.

29 Checklist

30 The checklist follows the references. Please read the checklist guidelines carefully for information on
31 how to answer these questions. For each question, change the default **[TODO]** to **[Yes]**, **[No]**, or
32 **[N/A]**. You are strongly encouraged to include a **justification to your answer**, either by referencing
33 the appropriate section of your paper or providing a brief inline description. For example:

- 34 • Did you include the license to the code and datasets? **[Yes]** See Section
- 35 • Did you include the license to the code and datasets? **[No]** The code and the data are
36 proprietary.
- 37 • Did you include the license to the code and datasets? **[N/A]**

38 Please do not modify the questions and only use the provided macros for your answers. Note that the
39 Checklist section does not count towards the page limit. In your paper, please delete this instructions
40 block and only keep the Checklist section heading above along with the questions/answers below.

- 41 1. For all authors...
- 42 (a) Do the main claims made in the abstract and introduction accurately reflect the paper’s
43 contributions and scope? **[TODO]**
- 44 (b) Did you describe the limitations of your work? **[TODO]**
- 45 (c) Did you discuss any potential negative societal impacts of your work? **[TODO]**
- 46 (d) Have you read the ethics review guidelines and ensured that your paper conforms to
47 them? **[TODO]**
- 48 2. If you are including theoretical results...
- 49 (a) Did you state the full set of assumptions of all theoretical results? **[TODO]**
- 50 (b) Did you include complete proofs of all theoretical results? **[TODO]**
- 51 3. If you ran experiments...
- 52 (a) Did you include the code, data, and instructions needed to reproduce the main experi-
53 mental results (either in the supplemental material or as a URL)? **[TODO]**
- 54 (b) Did you specify all the training details (e.g., data splits, hyperparameters, how they
55 were chosen)? **[TODO]**
- 56 (c) Did you report error bars (e.g., with respect to the random seed after running experi-
57 ments multiple times)? **[TODO]**
- 58 (d) Did you include the total amount of compute and the type of resources used (e.g., type
59 of GPUs, internal cluster, or cloud provider)? **[TODO]**
- 60 4. If you are using existing assets (e.g., code, data, models) or curating/releasing new assets...
- 61 (a) If your work uses existing assets, did you cite the creators? **[TODO]**
- 62 (b) Did you mention the license of the assets? **[TODO]**
- 63 (c) Did you include any new assets either in the supplemental material or as a URL?
64 **[TODO]**
- 65 (d) Did you discuss whether and how consent was obtained from people whose data you’re
66 using/curating? **[TODO]**

- 67 (e) Did you discuss whether the data you are using/curating contains personally identifiable
68 information or offensive content? **[TODO]**
- 69 5. If you used crowdsourcing or conducted research with human subjects...
- 70 (a) Did you include the full text of instructions given to participants and screenshots, if
71 applicable? **[TODO]**
- 72 (b) Did you describe any potential participant risks, with links to Institutional Review
73 Board (IRB) approvals, if applicable? **[TODO]**
- 74 (c) Did you include the estimated hourly wage paid to participants and the total amount
75 spent on participant compensation? **[TODO]**

76 **A Appendix**

77 Optionally include extra information (complete proofs, additional experiments and plots) in the
78 appendix. This section will often be part of the supplemental material.