Global Convergence Newton

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1 Introduction

Newton's method is a popular optimization algorithm that is commonly used to solve optimization problems. It is a second-order optimization algorithm since it uses second-order information of the objective function. Newton's method is known to have fast local convergence guarantees for convex functions. However, the global convergence properties of Newton's method are still an active area of research. The purpose of this project is to survey and analyze various strategies to achieve global convergence.

This sentence will be cited by the sources so i can test if bibtex is working properly [1]

2 Background

2.1 Classic Newton

2.2 Regularized Newton

In their 2023 article Michenko presents a variation of Newton's method that uses the update rule [2]:

$$x^{k+1} = x^k - (\nabla^2 f(x^k) + \sqrt{H||\nabla f(x^k)||}I)^{-1}\nabla f(x^k) \tag{1}$$

where $H > 0$ is a constant. The convergence rate of this algorithm is $\mathcal{O}(\frac{1}{k^2})$.

References

[1] Slavomír Hanzely, Dmitry Kamzolov, Dmitry Pasechnyuk, Alexander Gasnikov, Peter Richtárik, and Martin Takác. A damped newton method achieves global $(o)(\frac{1}{k^2})$ and local quadratic convergence rate. *Advances in Neural Information Processing Systems*, 35:25320–25334, 2022.

[2] Konstantin Mishchenko. Regularized newton method with global convergence. *SIAM Journal on Optimization*, 33(3):1440–1462, 2023.

Figure 1: