Homework 2: problems about exponential families

your name

Due: February 9th at 11:59 PM

This homework set will cover problems concerning exponential family theory. All derivations must be typed. Screenshots of work done with pen and paper will not be accepted.

Problem 1: Verify that displayed equation 7 in the exponential family notes holds for the binomial distribution, the Poisson distribution, and the normal distribution with both μ and σ^2 unknown.

Problem 2: This problem concerns the proof of Theorem 3 in the exponential family notes. Do the following:

- **part a**: Show that the second derivative of the map h is equal to $-\nabla^2 c(\theta)$ and justify that this matrix is negative definite when the exponential family model is identifiable.
- **part a**: Finish the proof of Theorem 3.

Note that part a will be referenced later in this course. Hence, it is treated as its own sub-problem.

Problem 3: Let y be a regular full exponential family with canonical parameter θ. Verify that y is sub-exponential.

Problem 4: In the notes it was claimed that the scalar products of $\sum_{i=1}^{n} \{y_i - \nabla c(\theta)\}$ are also sub-exponential (see the "Finite sample concentration of MLE" section in the exponential family notes). Show that this is in fact true when the observations y_i are iid realizations from a regular full exponential family.

Problem 5: Derive the MLEs of the canonical parameters of the binomial distribution and the normal distribution with both μ and σ^2 unknown.

Problem 6: Derive the asymptotic distribution for $\hat{\tau}$, the MLE of the submodel mean value parameter vector. Hint: use the Delta method.

Problem 7: Prove Lemma 1 in the exponential family notes.