Contents

INTERMEDIATE FAQ

November 8, 2013

1 Output

Pre-climate change output $Y(t)$ depends on a (constant-returns-to-scale) Cobb-Douglas production function in technology, capital and labor, denoted by $A(t), K(t)$ and $L(t)$, respectively.

$$Y(t) = A(t)K(t)^{\gamma}L(t)^{1-\gamma} \tag{1}$$

The capital share, γ, is set to 30%.

Final usuable output, $Q(t)$, is pre-damages output reduced by the fraction of harms from climate change, $\Omega(t) < 1$, and by resources spent on abatement, $\Lambda(t) < 1$.

$$Q(t) = \Omega(t)[1 - \Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}. \tag{2}$$

Each of the components of output is discussed below.

1.1 Population growth

In DICE2010, the population in period t, $L(t)$ evolves according to the weighted average formula between the initial population in year 2005, $L(0)$, and the asymptotic population in the last year we consider, $L(Tmax)$:

$$L(t) = [L(t-1) \times L(Tmax)]^{0.5}. \tag{3}$$

The model sets the initial population in 2005, $L(0)$, to 6.411 billion and assumes a population adjustment rate of 0.5 per decade. The default asymptotic population, $L(Tmax)$, is set to 8.6 billion; users can choose to adjust this parameter between 8.0 billion and 12.0 billion. Figure 1 illustrates the population trajectory for different user choices of $L(Tmax)$:

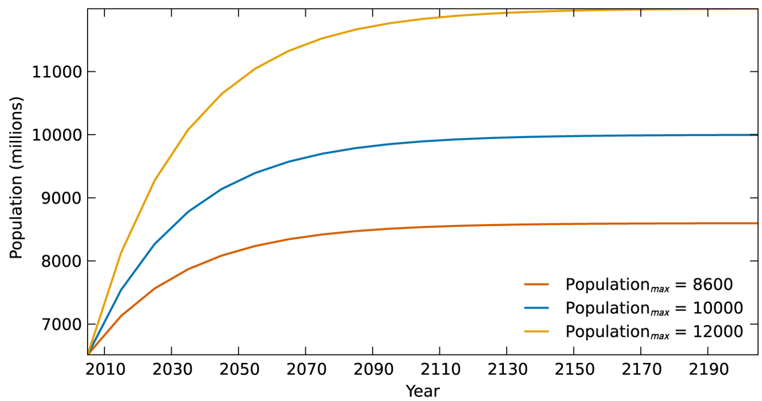

Figure 1: Population pathway for choice of $L(Tmax)$

In DICE2007, population evolves according to:

$$L(t) = L(0) \times \left(1 - \frac{e^{L_g \times (t-1)} - 1}{e^{L_g \times (t-1)}}\right) + L(Tmax) \times \left(\frac{e^{L_g \times (t-1)} - 1}{e^{L_g \times (t-1)}}\right) \tag{4}$$

where $L(0) = 6514$, $L(Tmax) = 8600$ and $L_g = 0.35$.

1.2 Capital and Savings

The amount of capital available for use in a given time period, $K(t)$, is the sum of depreciated capital from the prior period, $(1 - \delta_K)K(t-1)$, and the amount saved, $I(t)$:

$$K(t) = I(t) + (1 - \delta_K)K(t-1), \tag{5}$$

The model sets the default depreciation rate of capital, δ_K, to 10%; users can choose to adjust this parameter between 8% and 20%.

Total savings, $I(t)$, is a fixed fraction of output, $Q(t)$, determined by the savings rate, s.

$$I(t) = s \times Q(t) \tag{6}$$

Output, $Q(t)$, is split between consumption, $C(t)$, and savings, $I(t)$.

$$Q(t) = C(t) + I(t). \tag{7}$$

The model sets the default savings rate, s, to 22% of output. Some versions of DICE solve for the optimal savings rate. webDICE does not support optimization of the savings rate because Nordhaus reports that the optimal savings rate is relatively insensitive to assumptions. Instead of optimization, webDICE allows users to choose a savings rate between 15% and 25%.

1.3 Total Factor Productivity

Total factor productivity (TFP), $A(t)$, describes how efficiently capital and labor produce output. The initial value, $A(0)$, is set to produce 2005 output given the observed 2005 capital and labor inputs. TPF is assumed to evolve for $t > 1$, given the initial technology level, $A(0) = 0.03$, according the equation:

$$A(t) = \frac{A(t-1)}{1 - A_g(t-1)}, \tag{8}$$

This model assumes that the rate of productivity growth, $A_g(t)$, will decrease in the coming decades. The following equation determines the growth rate of TFP:

$$A_g(t) = A_g(0) \times exp(-\Delta_a \times (t-1)), \tag{9}$$

The model sets the rate at which TFP growth declines, Δ_a, to 0.1; users can choose to adjust this parameter between 0.05 and 0.15.

More formally, Δ_a is percentage growth rate of $A_g(t)$, the growth rate of TFP. To see this note that

$$\frac{d \ln A_g(t)}{dt} = \underbrace{\frac{A_g(t+1) - A_g(t)}{A_g(t)}}_{\text{growth rate in \%}} = -\Delta_a \tag{10}$$

Figure 2 shows how the evoluation of TFP changes for a choice of Δ_a :

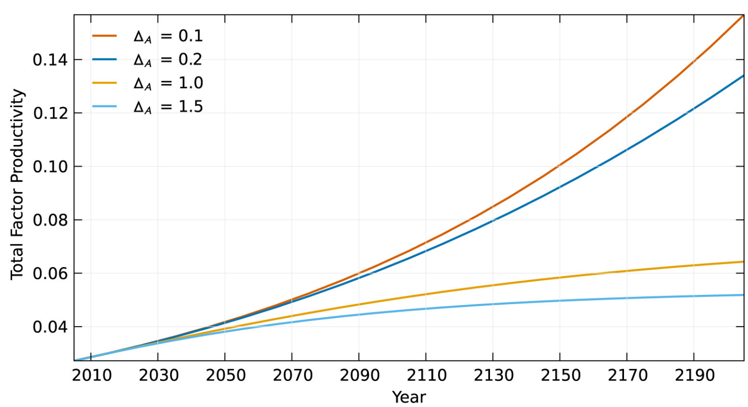

Figure 2: Pathway of productivity for choice of Δ_a

In webDICE, a portion of damages can be applied to TFP levels by choosing the “damages to productivity growth” damage function. (see section 3.5).

1.4 Time steps and periods

The model uses 10 year time steps. It shows output for 20 time steps or 200 years, starting in 2005. It runs for runs for 60 periods (600 years) in the background so that the end period does not influence the calculations of the social cost of carbon or the optimal emissions control rate.

2 Abatement costs

The abatement cost, $\Lambda(t)$, is the fraction of output spent on limiting emissions based on the specifications of a given climate policy. More specifically, it is the cost of reducing emissions of GHGs, measured as a fraction of total output. There are three components to abatement costs: the emissions control rate, $\mu(t)$, the participation fraction, $\varphi(t)$, and the cost of replacing all fossil fuels with carbon-free energy (the backstop technology), $\theta_1(t)$:

$$\Lambda(t) = \theta_1(t)\pi(t)\mu(t)^{\theta_2} \tag{11}$$

where,

$$\pi(t) = \varphi(t)^{1-\theta_2}, \tag{12}$$

therefore,

$$\Lambda(t) = \theta_1(t)\varphi(t)^{1-\theta_2}\mu(t)^{\theta_2}. \tag{13}$$

Users should expect abatement costs to increase with the emissions control rate, decrease as the participation fraction increases and decrease as the cost of the backstop technology decreases.

2.1 The emisisons control rate

Under the business as usual scenario the emissions control rate, $\mu(t)$, is zero; this scenario assumes no controls on emissions. When optimized or otherwise set by the user, μ(t) can take on any value between 0 and 1, a reflection of the fraction of total emissions that have been reduced. To represent increasing marginal costs, $\mu(t)$ is raised to the power θ_2. The default value of θ_2 is 2.8; users can adjust this parameter between 2 and 4.

Abatement costs will be positive in two circumstances: if the model is run in optimization mode or if the user sets emissions controls. In optimization mode, the model solves for the path of $\mu(t)$ that maximizes the discounted sum of utility over time. The user can indirectly set the emissions control rate by applying a climate treaty or a carbon tax:

2.1.1 Climate treaties

Users have the option of simulating a climate treaty by setting carbon caps in three years, 2050, 2100 and 2150, denoted by $e2050cap$, $e2100cap$, and $e2150cap$, respectively. The caps are specified as a percentage of emisssions in 2005 and apply in each time period until the next cap. For example, if $e2050cap = 100\%$, emissions in the each year between 2050 to 2100 can be no more than 2005 emissions, and if $e2100cap = 70\%$, emissions in each year between 2100 to 2150 can be no more than 70% of 2005 emissions. Users can set the caps anywhere from 100% (which caps emissions at the 2005 levels) to 0% (which reduces emissions to zero).

The model implements the caps by solving for $\mu(t)$ to produce the specified levels of emissions. Specifically, the model induces the controlled emissions rate $\mu(t) > 0$ from the carbon caps set by users whenever emissions of a year exceeds the cap in the threshold years of 2050-2100, 2100-2150, and 2150-$Tmax$. That is, given the equation for industrial emissions (see section 1.6) $E_{ind}(t) = \sigma(t)[1-\mu(t)] \cdot A(t)K(t)^{\gamma}L(t)^{1-\gamma}$, we set $\mu(t)$ as:

$$\mu(t) = \begin{cases} 0 & if \quad E_{ind}(t) < E_{ind}(2005) \times ecap \\ 1 - \frac{E_{ind}(2005) \times ecap}{\sigma(t)A(t)K(t)^{\gamma}L(t)^{1-\gamma}} & if \quad E_{ind}(t) > E_{ind}(2005) \times ecap \end{cases} \tag{14}$$

For instance, consider the year 2070. This year should obey the emissions cap of the year 2050. Thus, $\mu(2070) = 1 - \frac{E_{ind}(2005) \times e2050cap}{\sigma(2070)A(2070)K(2070)^{\gamma}L(2070)^{1-\gamma}}$ if $E_{ind)}(2070) > E_{ind}(2005) \times e2050cap$. If we have more emissions than the chosen amount for a given year, we would like to decrease it by controlling the reduction rate of $\mu(t)$.

One drawback is that the model uses step function setting caps in each threshold year and assuming that those caps apply for future years, until another threshhold year comes. Abrupt reduction of emissions might be more costly to implement than smooth reduction. The step function, however, reflects common practice in treaties which specify target years.

2.1.2 The participation fraction

The participation fraction, $\varphi(t) \in [0, 1]$ is the fraction of global emissions that are subject to the user-selected climate policy. If less than all countries participate in an emissions control regime or some industries are exempt, $\varphi(t) < 1$. This does not affect the emissions control level specified by the user, but rather affects the cost of reducing emissions to the selected level.

Suppose the emissions control level of the participating countries/sectors is called $\mu^P(t)$. This can be derived by inflating the global emission control rate, $\mu(t)$, by the participation fraction, $\varphi(t)$:

$$\mu^P(t) = \frac{\mu(t)}{\varphi(t)}, \tag{15}$$

and determing the fraction of output that affected by the emissions control regime. If users wish to approach the climate treaty from a negotiator's perspective they could multiply the emissions control rate of the among the participants by the participation fraction to get the global control rate set in webDICE.

$$Q^P(t) = Q(t)\varphi(t). \tag{16}$$

If $\psi^P(t)$ is the aggregate amount spent on abatement by the participating countries (which is equal to the total spent on abatement by all countries, $\psi(t)$) one can substitute in the above equalities:

$$\psi(t) = \psi^P(t) = Q^P(t) \cdot \theta_1(t) \cdot \mu^P(t)^{\theta_2} \tag{17}$$

$$= [Q(t)\varphi(t)]\theta_1(t)[\mu(t)/\varphi(t)]^{\theta_2} \tag{18}$$

$$= Q(t)\theta_2(t)\mu(t)^{\theta_2}\varphi(t)^{1-\theta_2} \tag{19}$$

where $\varphi(t)^{(1-\theta_2)}$ represents the increasing marginal costs of controlling emissions when participation is less than full (which, after adjusting for the sign, is the same marginal costs as the emissions control rate). We can think of $\varphi(t)^{1-\theta_2}$ as the participation markup in abatement costs.

Users specify the participation fractions for the years 2050, 2100, and 2150. The model interpolates an increasing fraction of participation for the years between these periods according to:

$$\varphi(t) = \begin{cases} \varphi(2050) + [\varphi(2010) - \varphi(2050)] \times exp(-0.25 \cdot t) & if \quad t = 2005, ..., 2045 \\ \varphi(2100) + [\varphi(2050) - \varphi(2100)] \times exp(-0.25 \cdot t) & if \quad t = 2055, .., 2095 \\ \varphi(2150) + [\varphi(2100) - \varphi(2150)] \times exp(-0.25 \cdot t) & if \quad t = 2105, ... \end{cases} \tag{20}$$

If users choose a participation fraction less than 1, the model still forces total emissions to comply with the chosen treaty. This implies that the base-line economic and emissions trajectory will not change significantly, if it changes at all. By decreasing the participation fraction, users are essentially increasing the abatement costs by increasing the cost of complying with the user-mandated climate treaty. For example, under the default settings, when participation is full, $\pi(t) = 1$. When only half of global emissions fall under the emissions control regime, $\pi(t) = (0.5)^{-1.8} \approx 3.5$. This means that abatement will be about 3.5 times as costly with only 50% participation.

2.1.3 User-chosen carbon tax

Alternatively, users can select a carbon tax to take effect in the same years as the emissions caps (2050, 2100 and 2150) (see section 2.1.2). A carbon tax can be set anywhere between \$0 and \$500 at each threshold. This tax rate is then used to calculate the emission control rate, $\mu(t)$:

$$\mu(t) = \left(\frac{\tau(t)}{BC(t)}\right)^{1/(\theta_2-1)} \tag{21}$$

where $\tau(t)$ is the carbon tax rate for a given time period and $BC(t)$ is the backstop cost as defined below. The carbon tax cannot be set at the same time as a climate treaty.

2.2 Backstop technology

The final component of the abatement cost is the cost of replacing all fossil fuels with zero-carbon energy. This cost, $\theta_1(t)$, reflects the cost of the technology (called the backstop technology) needed replace the last ton fossil carbon and the energy intensity of the economy. The backstop technology is assumed to start at a relatively high price, $\$1,170/tonC$, and to go down according to an exogenous (user chosen) path.

$$BC(t) = BC(0) \times \left[\frac{ratio - 1 + \exp(-BC_g(0) \times (t-1))}{ratio}\right], \tag{22}$$

where $BC(0)$ is the initial cost of the backstop technology and BC_g is the rate of decline in the cost of the backstop technology. $ratio$ is the ratio of the initial cost of the backstop technology, $BC(0)$, to the decline in price (in dollars) of the backstop technology between the current period and the final period, $BC(reduct)$.

$$ratio = BC(0)/BC(reduct) \tag{23}$$

Users can set both $ratio$ and the rate of decline, BC_g, based on their beliefs about how fast we will develop cheaper zero-carbon energy. The default value of $ratio$ is 2, indicating that the price of the backstop technology will decline by 50% by the final time period (a price of \$585/ton CO_2in the final period). $ratio$ can be adjusted between 1 and 4 depending on the user's beliefs. The default value of BC_g is 0.05; users can adjust the parameter between 0 and 0.2 depending on their beliefs.

Abatement costs are determined by:

$$\theta_1(t) = \left[\frac{BC(0) \times \sigma(t)}{\theta_2}\right] \times \left[\frac{ratio - 1 + \exp(-BC_g(0) \times (t-1))}{ratio}\right]. \tag{24}$$

where $\sigma(t)$ is the emissions intensity (see section 1.6) and $\theta_2$is the exponent on the emissions reduction rate in the abatement cost function (see section 1.5.1).

The marginal cost of abatement at emissions control rate of 100% (and 100% participation) is equal to the backstop price for each year:

$$\begin{aligned} BC(t) = MC(\Lambda)|_{\mu(t)=1} &= \frac{\partial \Lambda}{\partial \mu(t)}|_{\mu(t)=1} = \theta_1(t)\theta_2 \\ &= \underbrace{BC(0)}_{(1)} \times \underbrace{\sigma(t)}_{(2)} \times \underbrace{\left[\frac{ratio - 1 + \exp(-BC_g(0) \times (t-1))}{ratio}\right]}_{(3)}. \end{aligned}$$

The backstop cost in period t is proportional to the three factors. Factor (1) is a initial condition that can be based on data. Factor (2) is the carbon intensity, and is decreasing over time; as economy gets less carbon intense, the marginal cost of shifting all the fossil fuels into alternative energy decreases. Factor (3) represents improvement in the backstop technology, such as less expensive solar power.

Notice $\frac{\partial(3)}{\partial t} < 0$ so that (3) declines in time, reducing the cost of the backstop.

To understand how setting *ratio* determines the final price, note that at $t = 1$, $BC(1) = BC(0) \times \sigma(1)$, which is the current backstop price multiplied by the emissions intensity. As $t \to \infty$, $BC(t) \to [BC(0) - BC(reduct)] \times \sigma(t)$. The greater the reduction in costs, the lower the *ratio* . When $ratio = 1$, the final cost of the backstop technology is \$0. When $ratio = 2$, (as in the default), the final cost of the backstop technology is half the initial cost, and similarly for higher values of *ratio*. Figure 3 illustrates the pathway of the backstop price for different choices of *ratio* (when $BC_g = 0.05$) :

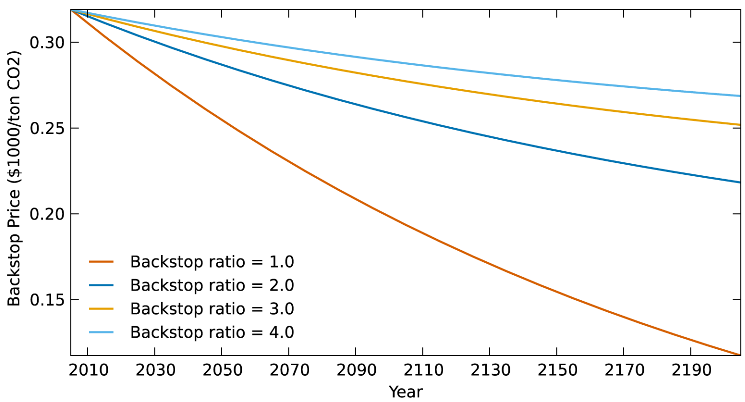

Figure 3: Pathway of backstop price for choice of *ratio* when BC_g set to default

3 Damages

The relationship between a warming climate and its effect on the economy is governed by the damage function. The assumptions made in the choice of the functional form of this relationship can have a significant effect on the model's estimates. webDICE allows users to choose the form of the relationship between increasing temperature and economic damages.

3.1 Default Damages

The default DICE damage function approximates a polynomial relationship between temperature and economic damages due to climate change. That is, the fraction of output lost in each period increases to the ϵ power of the increase in temperature. The functional form is modeled:

$$\Omega(t) = \frac{1}{1 + \pi_2 T_{AT}(t)^{\varepsilon}}. \tag{25}$$

The change in atmospheric temperature, T_{AT}, is guided by other assumptions in the model. Users can choose ϵ.

The exponent on temperature change, ϵ, represents the form of the relationship between an increasing temperature and damages. The default setting represents a quadratic relationship, that is $\epsilon = 2$. Users can choose to vary this relationship from linear to quartic (i.e. in the range $[1, 4]$). The coefficient on temperature change, π_2, is set to a default value of 0.0028. This coefficient calibrates the model so that 1.7% of GDP is lost when temperature increases by $2.5°C$ in the quadratic form. The users choice of ϵ applies to the other appropriate alternative damage functions

3.2 Environmental goods

The default damage function implicitly assumes that different types of consumption goods can be readily substituted for one another. For example, if agriculture represents 24% of GDP, a 1% loss to agriculture would correspond with a 0.24% loss in GDP and a 100% loss in the agricultural sector would yield only a 24% drop in global GDP. This specification of the damage function yields limited damages because it implicitly assumes that other goods can be substituted for food.

The "environmental goods" damage function modifies the damage function to allow some goods to have limited substitutability. As the price of those goods rise a growing portion of GDP will be devoted to that good.

This damage function is based on a model by Sterner and Persson[1] as modified by Weitzman. Sterner and Persson include relative price effects in a modified version of DICE by setting utility to be a CES function of the consumption of material goods and of an unspecified environmental good:

$$U\left(c(t)\right) = \frac{\left(\left[(1-b)\cdot c(t)^{1-1/\sigma} + b \cdot E^{1-1/\sigma}\right]^{\sigma/(1-\sigma}\right)^{1-\alpha}}{1-\alpha}, \tag{26}$$

where $c(t)$ and E represent the consumption of two goods taht will be affected differently by climate change, the elasticity of substitution beween them is given by σ, and b sets the share of overall consumption of the environmental good E. Sterner and Persson assume that only the consumption of the environmental good will be affected by changes in temperature, so that $E = E_0/\left(1 + \alpha T_{AT}(t)^2\right)$, where E_0 is the level of consumption of the environmental good in 2005 and α is a constant used for calibration.

The default damage function is set in terms of loss of consumption. To compare this to the Sterner and Persson damages, which reduce utility, set the elasticity of marginal utility, $\alpha = 2$. The default DICE utility function can be translated to the following disutility function (which determines utility

[1] Thomas Sterner and U. Martin Persson, "An Even Sterner Review," *Review of Environmental Economics and Policy* 2 (2008): 61-76, doi: 10.1093/reep/rem024

lost as a result of climate damages in each time period):

$$U(c(t)) = -[\frac{1}{c(t)} \times (1 + \pi_{2M} T_{AT}^{\epsilon})]. \tag{27}$$

If $\eta = 2$ and $\sigma = 1/2$, the Sterner and Persson CES utility function is equivalent to the disutility function:

$$U(c(t)) = -[\frac{1}{c(t)} + \left(1 + \pi_{2A} T^{2}\right)]. \tag{28}$$

where $\pi_{2A} = ab/[(1-b)E_0]$.

The only difference between these two disutility functions is the replacement of multiplication in the standard DICE utility function with with addition (as well as the value of the coefficient on temperature change to allow for identical calibration). Weitzman argues that no reasonable prior allows us to distinguish these two functional forms.

To translate the Sterner and Persson damage function into DICE, we compute the consumption that would produce equivalent utility using the CRRA utility function used in DICE. The resulting damage adjusted consumption is

$$\Omega(t) = \frac{1}{1 + c(t) \times \pi_{2A} T_{AT}^{\epsilon}}. \tag{29}$$

Compared to the default damages (25), this damage function includes a factor $c(t)$ in the demoninator.

From here, one can determine output and therefore damages from climate change to output. By setting multiplicative and additive damages equal to each other in the period when $T_{AT} \approx 2.5°$, webDICE approximates the same calibration as the default damage function. Using this calibration, the model applies damages to consumption and uses the same process as above to determine output and damages in each period.

Much like Sterner and Persson, the user will find that employing this damage function in the standard DICE model will yield "a far more stringent emissions policy than Nordhaus found with his multiplicive" form of the damage function. Note that the Sterner and Persson utility function allows one to choose the elasticity across the two types of goods. In the translation used here, this is set so that $\sigma = 1/2$, and cannot be modified.

3.3 Tipping point

Because of possible positive feedbacks in the climate system, once temperatures increase above a given point, damages may accelerate. For example, if warming is such that methane is released from the permafrost, this will increase warming, in turn causing more methane release. The tipping point damage function allows for this possibility.

webDICE uses the form of a tipping point damage function proposed by Martin Weitzman. With this damage function, damages drastically increase after temperatures have increased by around $6°C$.

$$\Omega = \frac{1}{1 + (\frac{T_{AT}}{20.46})^2 + (\frac{T_{AT}}{6.081})^{6.754}}. \tag{30}$$

3.4 Damages to productivity growth

The DICE model offers limited ways in which climate change can affect the economy. The damage functions discussed above imply that climate change reduces usable output but does not otherwise directly change the economy. (There will be indirect effects because less usable output translates into less capital in future years.) Climate change however, may not only affect output itself, but how we produce that output.

This "damages to productivity growth" specification of the damage function, derived from Moyer et al.,[2] directs a portion of damages from climate change to the growth of total factor productivty (TFP). This implies that climate change may not only affect what we produce, but how we produce it.

As discussed in section 1.3 total factor productivity (TFP), $A(t)$, in default DICE grows according to:

$$A(t) = \frac{A(t-1)}{1 - A_g(t-1)}. \tag{31}$$

Given the growth rate of $A(t)$, this damage function then allows a user-defined fraction of damages, f, to reduce the level of TFP instead of all damages being applied solely to output. More succinctly, this specification is defining a new path for TFP, which for purposes of this discussion we will call $A^*(t)$:

$$A^*(t) = (1 - f \cdot \Omega(t)) \times \frac{A^*(t-1)}{1 - A_g(t-1)}, \tag{32}$$

where $A(0) = A^*(0)$. The remainder of damages is applied to output as in the default damage function. Damages to output, $\Omega_Y(t)$, is given by:

$$\Omega_Y(t) = 1 - \frac{(1 - \Omega(t))}{[1 - f \cdot (1 - \Omega(t))]}. \tag{33}$$

Output in a given time period is equal to:

$$Y(t) = \Omega_Y(t) \cdot A^*(t) L(t)^{1-\gamma} K(t)^{\gamma} \tag{34}$$

This specification yields the same single period loss in consumption as the default DICE damage function but will significantly lower long term growth because productivity is reduce by climate change.

[2] Moyer, Elisabeth J. and Woolley, Mark D. and Glotter, Michael and Weisbach, David A., Climate Impacts on Economic Growth as Drivers of Uncertainty in the Social Cost of Carbon (July 31, 2013). RDCEP Working Paper No. 13-02. Available at SSRN: http://ssrn.com/abstract=2312770 or http://dx.doi.org/10.2139/ssrn.2312770

3.5 Comparison

Figure 4 illustrates the effect choice of damage function will have on the output of the model. Each of the following pathways uses the default webDICE parameter choices:

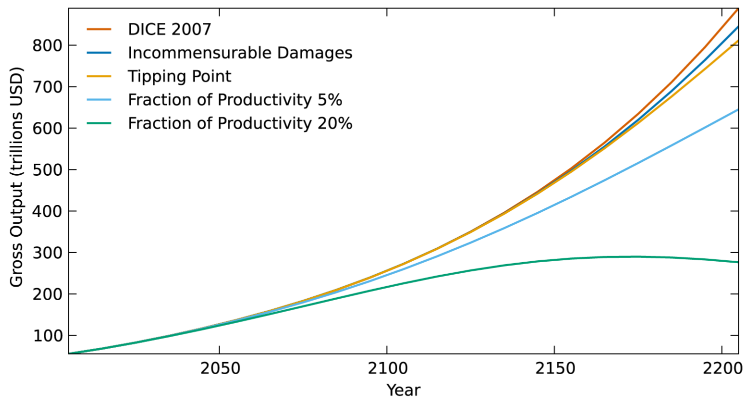

Figure 4: Comparison of total output after damages of default webDICE for choice of damage function

4 Carbon intensity and emissions

Total emissions are the sum of industrial emissions, E_{Ind}, and emissions from land-use change, :

$$E(t) = E_{Ind}(t) + E_{Land}(t) \tag{35}$$

4.1 Industrial emissions

Economic activity produces emissions. Industrial emissions, $E_{Ind}(t)$, in webDICE are equal to the carbon intensity of the economy, $\sigma(t)$ multiplied by output, and reduced by emissions controls $\mu(t)$.

$$E_{Ind}(t) = \sigma(t)[1 - \mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}. \tag{36}$$

In DICE2010, the carbon intensity of the economy is assumed to decline (even without a climate treaty or other policy) due to improvements in energy efficiency, according to:

$$\sigma(t) = \sigma(t-1) \times (1 - \sigma_g(t-1)) \tag{37}$$

where,

$$\sigma_g(t) = \sigma_g(t-1) \times (1 - \sigma_{d1}) \tag{38}$$

The initial carbon intensity of the economy, $\sigma(0)$, and the rate of decarbonization, $\sigma_g(t)$ are determined using historical data on GDP and emissions. The rate of decarbonization, $\sigma_g(t)$, is assumed to slow down at the user-chosen value of σ_{d1}. The default value of σ_{d1} is 0.003; but users can adjust the value of this rate based on their beliefs about the future timing of zero-carbon technologies. Figure 5 illustrates the evolution of carbon intensity for the choice of σ_{d1}.

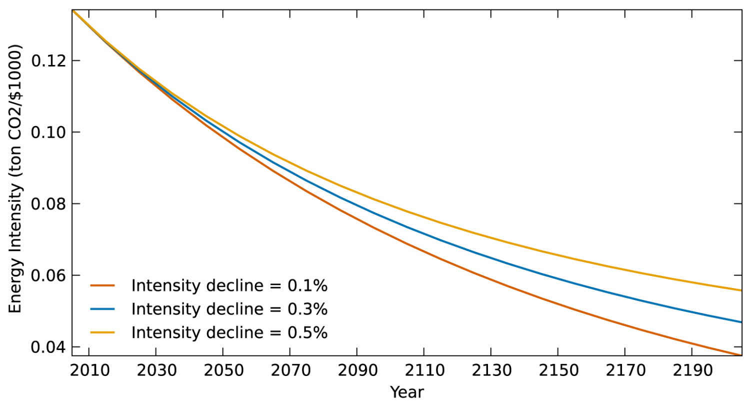

Figure 5: Pathway of energy intensity for choice of decline rate of energy intensity

In DICE2007, the carbon intensity of the economy evolves according to:

$$\sigma(t) = \frac{\sigma(t-1)}{1-\sigma_g(t)} \tag{39}$$

where,

$$\sigma_g(t) = -\sigma_g(0) \times exp(-\sigma_{d1} \times (t-1)). \tag{40}$$

The total available fossil fuel reserves limits the cumulative industrial emissions, $CCum$:

$$CCum \geq \sum_{t=0}^{Tmax} E_{Ind}(t) \tag{41}$$

This cap, $CCum$, is set to a default valule of 6,000 gigatons of carbon dioxide; users can increase this value based on their beliefs about the discoveries of additional fossil fuel reserves.

4.2 Emissions from land-use and land-use change

Emissions also arise from land-use change (e.g deforestation). Land-use change is currently exogenous to the model and is assumed to decline over time according to:

$$E_{Land}(t) = E_{Land}(0) \times (0.8)^{t-1}. \tag{42}$$

5 Carbon cycle and temperature change

5.1 DICE carbon cycle

DICE simulates the carbon cycle using a linear three-reservoir model where the three reservoirs are the deep ocean, the upper ocean and the atmosphere. Each of these reservoirs is well-mixed in the short run and a transition matrix governs the transfer of carbon among the reservoirs. If $M_i(t)$ is the mass of carbon (in gigatons) in reservoir i, then:

$$\begin{bmatrix} M_{AT}(t) \\ M_{UP}(t) \\ M_{LO}(t) \end{bmatrix} = \begin{bmatrix} \phi_{11} & \phi_{12} & 0 \\ 1-\phi_{11} & 1-\phi_{12}-\phi_{32} & \phi_{23} \\ 0 & \phi_{32} & 1-\phi_{23} \end{bmatrix} \begin{bmatrix} M_{AT}(t-1) \\ M_{UP}(t-1) \\ M_{L0}(t-1) \end{bmatrix} + \begin{bmatrix} E(t-1) \\ 0 \\ 0 \end{bmatrix}, \tag{43}$$

where the parameters $\phi_{i,j}$ represent the transfer rate from resevoir i to resevoir j (per time period), and $E(t)$ is emissions at time t. The model only includes CO_2 in its emissions factor and atmospheric carbon concentration. Other greenhouse gases are assumed to be exogenous and enter the forcing equation separately (see below).

5.2 Simplified BEAM carbon cycle

Glotter et al. demonstrate that the default DICE carbon cycle representation fails to accurately model oceanic carbon uptake.[3] In webDICE, the atmospheric temperature anomaly is a function of the concentration of carbon in the atmosphere. As was discussed in section 5.1, the atmospheric concentration of carbon in each time period in default DICE is determined by the concentration from the prior time period augmented by emissions and reduced by a constant fraction which is absorbed by the ocean. It is this constant fraction, $(1-\phi_{1,1})$ or $(\phi_{2,1})$, which accounts for the unphysical linear absorption of carbon by the oceans in the default DICE model.

Actual carbon uptake by the ocean is highly non-linear, characterized by a rapid initial uptake period followed by a long-tail equilibrium stage. The following graph compares the pathway of carbon mass in the atmosphere as prescribed by DICE versus BEAM (the Bolin and Eriksson Adjusted Model -

[3] Glotter, Michael and Pierrehumbert, Raymond T. and Elliott, Joshua and Moyer, Elisabeth J., A Simple Carbon Cycle Representation for Economic and Policy Analyses (September 1, 2013). RDCEP Working Paper No. 13-04. Available at SSRN: http://ssrn.com/abstract=2331074 or http://dx.doi.org/10.2139/ssrn.2331074

based on an established model first published in 1958)[4] given the same emissions trajectory (IPCC Scenario A2+).[5]

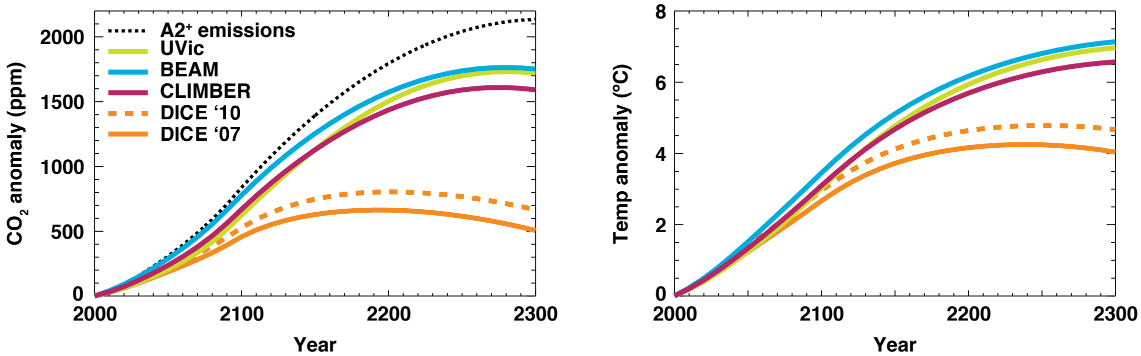

Figure 6: Comparison of DICE carbon cycle with BEAM and two other physical models for reference

[6]

The BEAM model does a much more complete job than DICE at capturing the relevant physics of oceanic carbon uptake. webDICE uses the simplified version of BEAM presented in Glotter et al. The authors demonstrate that their abbreviated version of the model, which excludes temperature-dependent coefficients, offers very similar results to the full BEAM model. webDICE utilizes simplified BEAM for ease of computation. Glotter et al. construct a similar three resevoir model to default DICE,

$$\frac{d}{dt}\begin{bmatrix} M_{AT}(t) \\ M_{UP}(t) \\ M_{LO}(t) \end{bmatrix} = \begin{bmatrix} -k_a & k_a \cdot A \cdot B & 0 \\ k_a & -(k_a \cdot A \cdot B) - k_d & \frac{k_d}{\delta} \\ 0 & k_d & -\frac{k_d}{\delta} \end{bmatrix} \begin{bmatrix} M_{AT} \\ M_{UP} \\ M_{LO} \end{bmatrix} + \frac{dE(t)}{dt}, \tag{44}$$

where A is the ratio of atmospheric carbon to upper oceanic dissolved CO_2 and B represents the partitioning of upper ocean dissolved CO_2 to total inorganic carbon and $\frac{dE(t)}{dt}$ is the emissions rate.

BEAM will estimate that carbon stays in the atmosphere longer than with the default carbon cycle. As a result, damages from climate change will persist for a longer period of time and be higher overall.

Technical details According to Henry's Law the partial pressure of atmospheric CO_2 must balance the concentration of CO_2 in the upper ocean: $A = \frac{AM}{OM/(\delta+1)}$, where AM is the number of moles in

[4]Bolin, Bert and Erik Eriksson, 1958. Changes in the Carbon Dioxide Content of the Atmosphere and Sea due to Fossil Fuel Combustion. In *The Atmosphere and the Sea in Motion: Scientific Contributions to the Rossby Memorial Volume*. Bert Bolin, ed. New York, Rockefeller Institute Press, 130–142.

[5]Montenegro, A., V. Brovkin, M. Eby, D. Archer, and A. J. Weaver (2007), Long term fate of anthropogenic carbon, Geophys. Res. Lett., 34, L19707, doi:10.1029/2007GL030905.

[6]Glotter, Michael and Pierrehumbert, Raymond T. and Elliott, Joshua and Moyer, Elisabeth J., A Simple Carbon Cycle Representation for Economic and Policy Analyses (September 1, 2013). RDCEP Working Paper No. 13-04. Available at SSRN: http://ssrn.com/abstract=2331074 or http://dx.doi.org/10.2139/ssrn.2331074

the atmosphere, $OM/(\delta+1)$ is the number of moles in the upper ocean, k_H represents the solubility of CO_2 in seawater.

$B = 1/(1+\frac{k_1}{[H^+]}+\frac{k_1 \cdot k_2}{[H^+]^2})$ where $[H^+]$ can be solved for by solving the following: $\left\{1 + \frac{k_1}{[H^+]} + \frac{k_1 \cdot k_2}{[H^+]^2}\right\} / \left\{\frac{k_1}{[H^+]} + \frac{2k_1 \cdot k_2}{[H^+\}^2}\right\} = \frac{M_{UP}}{Alk}$ where $Alk = 662.7\,Gt\,C$ the alkilinity.

$\frac{dE(t)}{dt} = E(t)/10$ in webDICE to adjust for different timescales.

5.3 Radiative Forcing

The relationship between greenhouse gases and radiative forcing $F(t)$ is given by

$$F(t) = \eta \frac{\ln\left(M_{AT}(t)\right) - \ln\left(M_{AT}^{1750}\right)}{\ln(2)} + F_{EX}(t) \tag{45}$$

where $F_{EX}(t)$ represents forcing from gases other than CO_2 in time period t and η represents the increase in forcing from the doubling of CO_2 in the atmosphere. These other forcings are estimated in a given period by a linear function of the difference between forcings from non-CO_2 today and an estimate of those forcings in the year 2100:

$$F_{EX}(t) = F_{EX}(0) + 0.1\left(F_{EX}(2100) - F_{EX}(0)\right) \cdot t. \tag{46}$$

Radiative forcing leads the warming in the atmosphere, which then warms the upper ocean, gradually warming the deep ocean. The model is:

$$\begin{aligned} T(t) &= T(t-1) + \xi_1\left[F(t) - \lambda T(t-1) - \xi_2\left(T(t-1) - T_{LO}(t-1)\right)\right] \\ T_{LO}(t) &= T_{LO}(t-1) + \xi_3\left(T(t-1) - T_{LO}(t-1)\right) \end{aligned}$$

where the ξ_i are the transfer coefficients reflecting the rates of flow and thermal capacities of the sinks. In particular, $1/\xi_1$ is the thermal capacity of the atmosphere and the upper oceans, $1/\xi_3$ is the transfer rate from the upper ocean to the deep ocean, and ξ_2 is the ratio of the thermal capacity of the deep oceans to the transfer rate from the shallow to deep ocean.
The key parameter is λ, or climate sensitivity, is a way of representing the equilibrium temperature from doubling the concentration of CO_2. If we solve the temperature equation for a constant temperature (i.e. equilibrium), we get $\Delta T/\Delta F = 1/\lambda$. If T_{2xCO_2} is the equilibrium impact of a doubling of CO_2 concentrations, we get $T_{2xCO_2} = \Delta F_{2xCO_2}/\lambda$ where ΔF_{2xCO_2} is the change in radiative forcing from a doubling of CO_2. Therefore, setting λ allows us to set the climate sensitivity.

5.4 Linear Carbon Model

Recent studies estimate that peak warming is linearly proportional to cumulative carbon emissions. [7] The linear carbon model uses this relationship to estimate temperature change as a function of cummulative emissions. The model does not include a carbon cycle or equations for radiative forcing, and as a result values for total carbon in the atmosphere or the upper and lower oceans are not computed. The default climate sensitivity is set to 3.2°, as in the other two carbon cycle models. This implies that we will increase global average temperature by two degrees Celsius for roughly every trillion tonnes of carbon that are emitted (since pre-industrial times).

6 Optimization

Users can run the model in either simulation mode or optimization. In simulation mode, the model projects outcomes based on the chosen parameter values. In optimization mode, the model finds the emissions control rate $\mu(t)$ that maximizes the discounted sum of utility given the assumptions in the model about output, emissions, and harm. It then translates these controls into the equivalent carbon tax rate, which is:

$$\tau(t) = BC(t)\mu(t)^{\theta_2 - 1}, \tag{47}$$

(assuming participation is complete, as it would be under an optimal control regime).

6.1 Objective Function

The objective function in the model is the discounted sum of utility:

$$W = \sum_{t=1}^{Tmax} U[c(t), L(t)]R(t). \tag{48}$$

Utility is

$$U\left[c(t), L(t)\right] = L(t)\left[c(t)^{1-\alpha}/(1-\alpha)\right]. \tag{49}$$

where α is the elasticity of marginal utility. α is set by default to be 2 but users can set it between 1 and 3. Higher values of α mean that marginal utility declines faster with increases in income. $c(t) = C(t)/L(t)$ is per capita consumption.

$R(t) = 1/[(1+\rho)^{(10\times(t-1))}]$ is the discount factor applied to utility. It is based on ρ, the pure rate of time preference or the utility discount rate. ρ is set by default to be 1.5% based on the value used in DICE, but users can set it to be any value between 0 and 4.

[7] Allen, M. R. et al (2009) Warming caused by cumulative carbon emissions towards the trillionth tonne, Nature, 458:1163-1166.

The discount rate is one of the most controversial components of climate policy as it in part determines the trade-offs between present and future generations. In addition, because it acts exponentially, small changes in its value can have large changes in policy. For this reason, we suggest users choose their preferred value but also to test how optimal policies change when they choose different values.

Our default values, $\rho = 1.5\%$ and $\alpha = 2$, combined with the implicit growth rate 2% using the default parameters, imply real return to capital of 5.5%. This can be changed by changing ρ and α as well as changing default parameters that affect growth.

Users should be aware that utilities are not comparable for different values of α, the elasticity of the marginal utility of consumption. Increasing or decreasing α may change utility for the same economic output even in a single period and comparisons of the raw utility values are not necessarily meaningful. It is not clear that there is a method of normalizing utility for different values of α that makes the comparisons meaningful.

7 Social Cost of Carbon

The social cost of carbon (SCC) is a commonly cited metric which estimates the "monitized damages associated with an incremental increase in carbon emissions in a given year."[8] U.S. government agencies are required to use this metric in cost-benefit evaluation of new programs. webDICE reports SCC in dollars per tonne of carbon dioxide.

This model estimates the SCC by first determining the consumption pathway implied by the user's choice of paramter values. Next, it adds a tonne of carbon to total emissions in year t and recalculates the consumption pathway after that year. webDICE then finds the difference in the two pathways in each year, applies the appropriate discount rate and sums the differences. The net present value of damages is then multiplied by 12/44 to covert from dollars per tonne of carbon to dollars per tonne of carbon dioxide. This process is repeated for each of the twenty time periods displayed in webDICE (although it calculates the SCC using the differences in damages over all $60 - t$ relevant time steps).

It is important to note that webDICE's estimates of the SCC will not match those the Interagency Working Group (IAWG) calculated using the DICE model. The IAWG specified exogenous emissions, GDP and populations pathways to enable comparison across three distinct models. webDICE uses Nordhaus' model of the economy and population from DICE 2010.

8 Computational considerations

The model is implemented in Python. We chose Python because it is widely used and free. The source code is available on the website. We use Amazon's EC2 to host the website, and are greatful to Amazon for a generous grant to support our use of EC2.

[8]Interagency Working Group. SCC. 2010

Nordhaus's original model was written in both GAMS and Excel. Our Python code has been verified against Nordhaus's GAMS code.

The model uses 10-year time steps and runs for 60 periods. We only display output for the first 20 time steps or 200 years. Simulating the economy for 200 years is highly speculative so displays of results this far in the future need to be understood as a way of understanding the possible effects of assumptions rather than a prediction about what will actually happen. The model runs for 600 years primarily so that in optimization mode, what happens in the final periods has little effect on earlier optimization choices.

8.1 Software dependencies

webDICE depends on python2. It was written with python 2.7, though will likely run with earlier versions up to 2.5 as well. Currently it will not run with python3, though this may be a feature in future versions.

It requires a handful of standard python packages. If you intend to run the software on your desktop, and not on the web, then you merely need versions of numpy and pandas. The web services in the code depend on flask, flask-beaker, and pyyaml. Each of these packages should be available through pip or easy_install.

8.2 Optimization

8.2.1 Installation

Optimization in webDICE depends on the IPOPT library[9] which interfaces with our python codebase through the pyipopt package[10]. IPOPT is capable of using several linear solvers—webDICE uses the MA57 solver from HSL[11]. IPOPT also depends on the LAPACK and BLAS libraries. This of course means that the user will have to install several libraries in order for the optimization to run successfully. General steps for installation on a Linux or Macintosh system are as follows (we have unfortunately not yet tested these steps in a Windows environment).

Installation from source code for IPOPT, LAPACK, Blas, and the HSL routines are all relatively straight-forward. A standard configure, make, and make install should suffice.

An updated pyipopt is not available through pip or easy_install. You should download the source code, or clone the repo from footnote 10, move into the directory, and python setup.py install. However, for installation to complete successfully, it will likely be necessary to modify the setup.py file. Adjust the IPOPT_DIR variable based on the values you used in configuring IPOPT (e.g. /usr/local).

[9] A. Wï¿œchter and L. T. Biegler, On the Implementation of a Primal-Dual Interior Point Filter Line Search Algorithm for Large-Scale Nonlinear Programming, Mathematical Programming 106(1), pp. 25-57, 2006

[10] Developed by Eric Xu, and available at: https://github.com/xuy/pyipopt

[11] HSL(2013). A collection of Fortran codes for large scale scientific computation. http://www.hsl.rl.ac.uk

Depending on the solver you're using and other environmental settings, one may need to adjust the extra_link_args, library_dirs, and libraries lists.

It is possible to install and run webDICE without installing any of the optimization libraries. Executing an optimized loop is of course not possible under such a scenario, and will result in a runtime exception. However running other portions of webDICE should succeed without trouble.

8.2.2 Execution

Efforts have been made to create an optimization routine that balances accuracy with the the swiftness that is required for a web-based application. However, calculating an optimized policy naturally takes longer than other policy choices. Currently, a user can expect this to take between 10 and 20 seconds.

Some features of webDICE increase the solve time of the optimization routine. For the time being, the added computation of the BEAM carbon model increases the necessary solve time to a point which is infeasible for web delivery. As such, optimizing a scenario that includes the BEAM model has been disabled.