1 EQUATIONS

Objective function

1. $W = \sum_{t=1}^{Tmax} U[c(t), L(t)]R(t)$

2. $R(t) = 1/[(1+\rho)^{(10\times(t-1))}]$

3. $U[c(t), L(t)] = L(t)[c(t)^{1-\alpha}/(1-\alpha)]$

4. $c(t) = C(t)/L(t)$

Population function

5. $L(t) = L(0) \times \left(1 - \frac{e^{L_g\times(t-1)}-1}{e^{L_g(t-1)}}\right) + L(Tmax) \times \left(\frac{e^{L_g\times(t-1)}-1}{e^{L_g(t-1)}}\right)$

Production function

6. $Q(t) = \Omega(t)[1-\Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$

7. $Q(t) = C(t) + I(t)$

8. $I(t) = s \times Q(t)$

9. $K(t) = I(t) + (1-\delta_K)K(t-1)$

Total Factor Productivity

10. $A_g(t) = A_g(0) \times exp(-\Delta_a \times (t-1))$

11. $A(t) = \frac{0.95\times A(t-1)}{1-A_g(t-1)}$ for $t \geq 1$ given $A(0)$

Climate damage function

12. $\Omega(t) = 1/[1+\pi_2 T_{AT}(t)^{\epsilon}]$

Abatement cost function

13. $\Lambda(t) = \pi(t)\theta_1(t)\mu(t)^{\theta_2}$

14. $\pi(t) = \varphi(t)^{1-\theta_2}$

15. $$\varphi(t) = \begin{cases} \varphi(1) & if \quad t=1 \\ \varphi(21) + [\varphi(2)-\varphi(21)] \times exp(-DFE \times (t-2)) & if \quad t=2,..,24 \\ \varphi(21) & if \quad t=25,... \end{cases}$$

16. $\theta_1(t) = \left[\frac{BC(0)\times\sigma(t)}{\theta_2}\right] \times \left[\frac{BC(0)}{BC(T\max)} - 1 + \frac{\exp(-BC_g(0)\times(t-1))}{BC(0)/BC(T\max)}\right]$

Emissions

17. $E(t) = E_{Ind}(t) + E_{Land}(t)$

18. $E_{Ind}(t) = \sigma(t)[1-\mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$

19. $\sigma(t) = \frac{\sigma(t-1)}{1-\sigma_g(t)}$ for $t \geq 1$ given $\sigma(0)$

20. $\sigma_g(t) = \sigma_g(0) \times exp(-\sigma_{d1} \times (t-1))$

21. $CCum \geq \sum_{t=0}^{Tmax} E_{Ind}(t)$

22. $E_{Land}(t) = E_{Land}(0) \times (1-0.1)^{t-1}$

Carbon tax

23. $P(t) = \varphi(t)\theta_1(t)\mu(t)^{\theta_2 - 1}$

Carbon cycle and Climate Model

24. $M_{AT}(t) = E(t) + \phi_{11} M_{AT}(t-1) + \phi_{21} M_{UP}(t-1)$

25. $M_{UP}(t) = \phi_{12} M_{AT}(t-1) + \phi_{22} M_{UP}(t-1) + \phi_{32} M_{LO}(t-1)$

26. $M_{LO}(t) = \phi_{23} M_{UP}(t-1) + \phi_{33} M_{LO}(t-1)$

27. $F(t) = \eta\{log_2[M_{AT}(t)/M_{AT}(1750)]\} + F_{EX}(t)$

28. $T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \xi_2 T_{AT}(t-1) - \xi_3[T_{AT}(t-1) - T_{LO}(t-1)]\}$

29. $T_{LO}(t) = T_{LO}(t-1) + \xi_4\{T_{AT}(t-1) - T_{LO}(t-1)\}$

2 VARIABLES

t: time in decades from 2001-2010, 2011-2020, ..., 2590-2600. The last time period is 60 and this period 60 is denoted $Tmax$.

$C(t)$: total consumption

$c(t)$: per capita consumption

$L(t)$: population in millions

$I(t)$: investment

$K(t)$: capital

$R(t)$: social time preference discount factor

$\rho(t)$: social time preference rate per year

$A(t)$: total factor productivity

$A_g(t)$: growth rate of total factor productivity per decade

$E(t)$: total carbon emissions (billions of metric tons of carbon per period)

$E_{Land}(t)$: carbon emissions from land use, that is, deforestation (billions of metric tons of carbon per period)

$E_{Ind}(t)$: industrial carbon emissions (billions of metric tons of carbon per period

$T_{AT}(t)$: global mean surface temperature ($°C$ increase from 1900)

$\sigma(t)$: ratio of uncontrolled industrial emissions to output (metric tons of carbon per output in 2005 prices)

$\sigma_g(t)$: rate of decline of carbon intensity per decade > 0

$\Omega(t)$: damage function; $1 - \Omega(t)$ is the percentage of output that vanish due to higher mean surface temperature

$\Lambda(t)$: abatement cost function; the percentage of output that vanish in order to keep the emission level under policy

3 Parameters

3.1 PARAMETERS THAT USERS CHOOSE

α: elasticity of marginal utility of consumption $\in [1, 3]$. Default $= 2$.

ρ: social time preference rate "per year". We will average it over 10 years according to equation 2. This value is in the range of $[0, 4]$. Default $= 0.015$.

$L(Tmax)$: asymptotic population in millions in the last period $\in [8000, 12000]$. Default $= 8600$ million.

Δ_a : decline rate of technological change per decade $\in [0.05, 0.15]$. Default $= 0.1$.

δ_K: depreciation rate of technological change per decade. The range is $[0.08, 0.2]$ Default $= 0.1$.

σ_{d1}: decline rate of decarbonization per decade $\in [0, 0.06]$ Default $= 0.003$.

ϵ: damage exponent in climate damage function $\in [1, 3]$. Default $= 2$.

π_2: coefficient on the damage exponent term, $T_{AT}(t)^\epsilon$ in climate damage function $\in [0.002, 0.0035]$. Default $= 0.0028$.

$BC_g(0)$: inital cost decline in backstop technology % per decade $\in [0, 0.2]$. Default $= 0.05$.

$BC(0)/BC(Tmax)$: ratio of initial to final backstop cost, that is, the cost of replacing all emissions in today's $ per ton of CO_2, relative to future cost $\in [0.5, 4]$. Default $= 2$.

θ_2: exponent of emission reduction rate in abatement cost function $\in [2, 4]$. Default $= 2.8$.

$CCum$: fossil fuels remaining, measured in CO_2 emissions; maximum consumption of fossil fuels (billions of metric tons of cabon) $\in [6000, 9000]$. Default $= 6000$.

s: savings rate $\in [0.15, 0.25]$. Default $= 0.2$.

$e2050cap$: the mandated decrease in emissions by 2050 as a share of 2005 year emissions; emissions cap by 2050 $\in [0, 1]$. Default $= 0$.

$e2100cap$: the mandated decrease in emissions by 2100 as a share of 2005 year emissions; emissions cap by 2100 $\in [0,1]$. Default = 0.

$e2150cap$: the mandated decrease in emissions by 2150 as a share of 2005 year emissions; emissions cap by 2150 $\in [0,1]$. Default = 0.

DFE: decline rate of participation

$\varphi(2)$: fraction of emissions under control in 2015. Default = 1

$\varphi(21)$: fraction of emissions uner control in 2205. Default = 1

3.2 PARAMETERS FROM DATA

$\mu(t)$: emissions reduction rate as a percent of total emissions. Set to zero unless emissions controls are imposed.

$L(0)$: 2005 world populations in millions = 6514

L_g: growth rate of population per decade =0.35

$A(0)$: inital level of total factor productivity =0.02722

$A_g(0)$: initial growth rate of TFP per decade =0.092

γ : capital elasticity of output in production function =0.300

$Q(0)$: 2005 world gross output in 2005 US dollars in trillions=61.1

$K(0)$: 2005 capital value in 2005 US dollars in trillions=137

$\sigma(0)$: 2005 effective carbon intensity, that is, CO_2-equivalent emissions and GNP ratio in 2005 = 0.13418

$\sigma_g(0)$: inital rate of decline of carbon intensity per decade = 0.0730

$E_{Land}(0)$: carbon emissions from deforestation 2005 (GtC per decade) =11

$E(0)$: total emissions in year 2005 =84.1910

$BC(0)$: cost of backstop technology in 2005 = 1.17

$\varphi(1)$: fraction of emissions under control in 2005