1 EQUATIONS

Note on time steps: webDICE, following DICE, is implemented in 10-year time steps. The equations are presented for arbitrary intervals for ease of readabilty. Most of the data is in annual time steps and is converted to 10-year units within the model.

Objective function

1. $W = \sum_{t=1}^{Tmax} U[c(t), L(t)]R(t)$
2. $R(t) = 1/[(1+\rho)^{(t-1)}]$
3. $U[c(t), L(t)] = L(t)[c(t)^{1-\alpha}/(1-\alpha)]$
4. $c(t) = C(t)/L(t)$

Population function

5. $L(t) = L(0) \times \left(1 - \frac{e^{L_g \times (t-1)} - 1}{e^{L_g \times (t-1)}}\right) + L(Tmax) \times \left(\frac{e^{L_g \times (t-1)} - 1}{e^{L_g \times (t-1)}}\right)$

Production function

6. $Q(t) = \Omega(t)[1 - \Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$
7. $Q(t) = C(t) + I(t)$
8. $I(t) = s \times Q(t)$
9. $K(t) = I(t) + (1 - \delta_K)K(t-1)$

Total Factor Productivity

10. $A_g(t) = A_g(0) \times exp(-\Delta_a \times (t-1))$
11. $A(t) = \frac{A(t-1)}{1 - A_g(t-1)}$ for $t \geq 1$, given $A(0)$

Climate damage function

12. $\Omega(t) = 1/[1 + \pi_2 T_{AT}(t)^{\epsilon}]$

Abatement cost function

13. $\Lambda(t) = \pi(t)\theta_1(t)\mu(t)^{\theta_2}$
14. $\pi(t) = \varphi(t)^{1-\theta_2}$
15. $\varphi(t) = \begin{cases} \varphi(1) & if \quad t = 1 \\ \varphi(21) + [\varphi(2) - \varphi(21)] \times exp(-DFE \times (t-2)) & if \quad t = 2, .., 24 \\ \varphi(21) & if \quad t = 25, ... \end{cases}$
16. $\theta_1(t) = \left[\frac{BC(0) \times \sigma(t)}{\theta_2}\right] \times \left[\frac{ratio - 1 + \exp(-BC_g(0) \times (t-1))}{ratio}\right]$, where $ratio = BC(0)/BC(reduct)$
17. $\mu(t) = \begin{cases} 0 & if \quad E_{ind}(t) < E_{ind}(2005) \times e(t)cap \\ 1 - \frac{E_{ind(2005)} \times ecap}{\sigma(t)A(t)K(t)^{\gamma}L(t)^{1-\gamma}} & if \quad E_{ind}(t) > E_{ind}(2005) \times e(t)cap \end{cases}$

Emissions

18. $E(t) = E_{Ind}(t) + E_{Land}(t)$

19. $E_{Ind}(t) = \sigma(t)[1 - \mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$

20. $\sigma(t) = \frac{\sigma(t-1)}{1-\sigma_g(t)}$ for $t \geq 1$ given $\sigma(0)$

21. $\sigma_g(t) = -\sigma_g(0) \times exp(-\sigma_{d1} \times (t-1))$

22. $CCum \geq \sum_{t=0}^{Tmax} E_{Ind}(t)$

23. $E_{Land}(t) = E_{Land}(0) \times (1 - 0.1)^{t-1}$

Carbon tax

24. $P(t) = \theta_1(t)\mu(t)^{\theta_2 - 1}$, for $\varphi = 1$.

Carbon cycle and Climate Model

25. $M_{AT}(t) = E(t) + \phi_{11}M_{AT}(t-1) + \phi_{21}M_{UP}(t-1)$

26. $M_{UP}(t) = \phi_{12}M_{AT}(t-1) + \phi_{22}M_{UP}(t-1) + \phi_{32}M_{LO}(t-1)$

27. $M_{LO}(t) = \phi_{23}M_{UP}(t-1) + \phi_{33}M_{LO}(t-1)$

28. $F(t) = \eta\{log_2[M_{AT}(avg)/M_{AT}(1750)]\} + F_{EX}(t)$, where

$M_{AT}(avg) = [M_{AT}(t) + M_{AT}(t+1)]/2$, and

$F_{EX}(t) = F_{EX}(0) + 0.1\,(F_{EX}(21) - F_{EX}(0)) \cdot t$

29. $T_{AT}(t) = T_{AT}(t-1) + \xi_1\{F(t) - \lambda T_{AT}(t-1) - \xi_2[T_{AT}(t-1) - T_{LO}(t-1)]\}$

30. $T_{LO}(t) = T_{LO}(t-1) + \xi_3\{T_{AT}(t-1) - T_{LO}(t-1)\}$

2 VARIABLES

t: time in decades from 2001-2010, 2011-2020, ..., 2590-2600. The last time period is 60 and this period 60 is denoted $Tmax$.

$C(t)$: total consumption

$c(t)$: per capita consumption

$L(t)$: population in millions

$I(t)$: investment

$K(t)$: capital

$R(t)$: social time preference discount factor

$\rho(t)$: social time preference rate

$A(t)$: total factor productivity

$A_g(t)$: growth rate of total factor productivity

$E(t)$: total carbon emissions (billions of metric tons of carbon per period)

$E_{Land}(t)$: carbon emissions from land use; that is, deforestation (billions of metric tons of carbon per period)

$E_{Ind}(t)$: industrial carbon emissions (billions of metric tons of carbon per period)

$T_{AT}(t)$: global mean surface temperature ($^{\circ}C$ increase from 1900)

$\sigma(t)$: ratio of uncontrolled industrial emissions to output (metric tons of carbon per output in 2005 prices)

$\sigma_g(t)$: rate of decline of carbon intensity per decade, expressed as a *positive* number

$\Omega(t)$: damage function; $1 - \Omega(t)$ is the percentage of unsable output due to harms from climate change

$\Lambda(t)$: abatement cost function; the percentage of output spent on reducing emissions

3 Parameters

3.1 PARAMETERS THAT USERS CHOOSE

α: elasticity of marginal utility of consumption. $\alpha \in [1, 3]$. Default = 2.

ρ: social time preference rate. $\rho \in [0, 4]$. Default = 0.015.

$L(Tmax)$: asymptotic population in millions in the last period. $L(Tmax) \in [8000, 12000]$. Default = 8600 million.

Δ_a : decline rate of technological change. $\Delta_a \in [0.05, 0.15]$. Default = 0.1.

δ_K: depreciation rate of technological change. $\delta_k \in [0.08, 0.2]$ Default = 0.1.

σ_{d1}: decline rate of decarbonization. $\sigma_{d1} \in [0, 0.06]$ Default = 0.003.

ϵ: damage exponent in climate damage function. $\epsilon \in [1, 3]$. Default = 2.

π_2: coefficient on the damage exponent term, $T_{AT}(t)^{\epsilon}$ in climate damage function. $\pi_2 \in [0.002, 0.0035]$. Default = 0.0028.

$BC_g(0)$: inital cost decline in backstop technology in percent. $BG_g(0) \in [0, 0.2]$. Default = 0.05.

$ratio$: the ratio of initial backstop cost, $BC(0)$, to the reduction in costs, $BC(reduct)$, of replacing all emissions in \$ per ton of CO_2. $ratio \in [1, 4]$. Default = 2. $ratio = 1$ implies a final backstop cost of \$0.

θ_2: exponent of emission reduction rate in abatement cost function. $\theta_2 \in [2, 4]$. Default = 2.8.

$CCum$: fossil fuels remaining, measured in CO_2 emissions; maximum consumption of fossil fuels (billions of metric tons of cabon). $CCum \in [6000, 9000]$. Default = 6000.

s: savings rate. $s \in [0.15, 0.25]$. Default = 0.2.

$e2050cap$: the mandated decrease in emissions by 2050 as a share of 2005 year emissions; emissions cap by 2050. $e2050cap \in [0, 1]$. Default = 0.

$e2100cap$: the mandated decrease in emissions by 2100 as a share of 2005 year emissions; emissions cap by 2100. $e2100cap \in [0, 1]$. Default = 0.

$e2150cap$: the mandated decrease in emissions by 2150 as a share of 2005 year emissions; emissions cap by 2150 $e2150cap \in [0, 1]$. Default = 0.

$\varphi(2)$: fraction of emissions under control in 2015. Default = 1

$\varphi(21)$: fraction of emissions uner control in 2205. Default = 1

DFE: rate of increase in participation in emissions control regime. Default = 0.

T_{2xCO_2}: temperature increase from a doubling of CO_2. Default = 3.

3.2 PARAMETERS FROM DATA

3.2.1 Economics model

$\mu(t)$: emissions reduction rate as a percent of total emissions. Set to zero unless emissions controls are imposed.

$L(0)$: 2005 world populations in millions = 6514

L_g: growth rate of population per decade =0.35

$A(0)$: inital level of total factor productivity =0.02722

$A_g(0)$: initial growth rate of TFP per decade =0.092

γ : capital elasticity of output in production function =0.300

$Q(0)$: 2005 world gross output in 2005 US dollars in trillions=61.1

$K(0)$: 2005 capital value in 2005 US dollars in trillions=137

$\sigma(0)$: 2005 effective carbon intensity; that is, CO_2-equivalent emissions and GNP ratio in 2005 = 0.13418

$\sigma_g(0)$: iniital rate of decline of carbon intensity per decade = 0.0730

$E_{Land}(0)$: carbon emissions from deforestation 2005 (GtC per decade) =11

$E(0)$: total emissions in year 2005 =84.1910

$BC(0)$: cost of backstop technology in 2005 = 1.17

$\varphi(1)$: fraction of emissions under control in 2005, set to be 1.

3.2.2 Climate model

$M_{AT}(2000)$: mass of carbon in the atmosphere in 2005 in GtC = 808.9

$M_U(2000)$: mass of carbon in the upper ocen in 2005 in GtC = 1255

$M_{LO}(2000)$: mass of carbon in the lower ocean in 2005 in GtC = 18365

M_{PI}: preindustrial concentration of carbon in the atmosphere in GtC = 592

Transition matrix for carbon, where $\phi_{i,j}$ is the transfer rate of carbon from resevoir i to resevoir j for i, $j = AT$, U and LO:

$$\begin{bmatrix} 0.810712 & .0189288 & 0 \\ 0.097213 & 0.852787 & .05 \\ 0 & 0.003119 & 0.996881 \end{bmatrix}$$

F_{2xCO_2}: forcing from a double of CO_2 = 3.8

λ: ratio of temperature change to forcing in equilibrium = 0.3

ξ_1: inverse of therma capacity of the atmosphere and the upper ocean =

ξ_2: ratio of the therma capacity of the deep oceans to the transfer rate from the shallow ocean to the deep ocean = 0.

ξ_3:transfer rate of heat from the upper ocean to the deep ocean = 0.50

$T_{AT}(0)$: temperature change from 1900 until 2000: 0.7307

$T_{LO}(0)$: temperature change in the lower ocean from1900 until 2000: 0.0068

$F_{EX}(0)$: non-CO_2 forcings in 2005 = -0.06

$F_{EX}(21)$: estimate of non-CO_2 forcings in 2100 = 0.30