EQUATIONS and VARIABLES TAB: DICE-2007 MODEL

1 EQUATIONS

- get rid of all 10s except for ρ. Is it consistent with matlab code?

- (Eq)s are my new notation for equations and (A)s are Nodhaus'. Newly added equations do not have (A)s.

Objective function

(Eq1) (A.1) $W = \sum_{t=1}^{Tmax} U[c(t), L(t)]R(t)$

(Eq2) (A.2) $R(t) = 1/[(1+\rho)^{(10\times(t-1))}]$

(Eq3) (A.3) $U[c(t), L(t)] = L(t)[c(t)^{1-\alpha}/(1-\alpha)]$

(Eq4) (A.8) $c(t) = C(t)/L(t)$

Population function

(Eq5) $L(t) = L(0) \times \left(1 - \frac{e^{L_g \times (t-1)}-1}{e^{L_g (t-1)}}\right) + L(Tmax) \times \left(\frac{e^{L_g \times (t-1)}-1}{e^{L_g (t-1)}}\right)$

Production function

(Eq6) (A.4) $Q(t) = \Omega(t)[1-\Lambda(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$

(Eq7) (A.7) $Q(t) = C(t) + I(t)$

(Eq8) $I(t) = s \times Q(t)$

(Eq9) (A.9) $K(t) = I(t) + (1-\delta_K)K(t-1)$

Total Factor Productivity

(Eq10) $A_g(t) = A_g(0) \times exp(-\Delta_a \times (t-1))$

(Eq11) $A(t) = \frac{0.95 \times A(t-1)}{1-A_g(t-1)}$ for $t \geq 1$ given $A(0)$

Climate damage function

(Eq12) (A.5) $\Omega(t) = 1/[1 + \pi_2 T_{AT}(t)^{\epsilon}]$

Abatement cost function

(Eq13) (A.6) $\Lambda(t) = \theta_1(t)\mu(t)^{\theta_2}$

(Eq14) $\theta_1(t) = \left[\frac{BC(0)\times\sigma(t)}{\theta_2}\right] \times \left[\frac{BC(0)}{BC(Tmax)} - 1 + \frac{exp(-BC_g(0)\times(t-1))}{BC(0)/BC(Tmax)}\right]$

Emission function

(Eq15) (A.12) $E(t) = E_{Ind}(t) + E_{Land}(t)$

(Eq16) (A.11) $E_{Ind}(t) = \sigma(t)[1-\mu(t)]A(t)K(t)^{\gamma}L(t)^{1-\gamma}$

(Eq17) $\sigma_g(t) = \sigma_g(0) \times exp(-\sigma_{d1} \times (t-1))$ *** why not add minus sign in front of $\sigma_g(0)$ and let $\sigma_g(o)$ >0 so that $\sigma_g(t) = -\sigma_g(0) \times exp(-\sigma_{d1} \times (t-1))$?

(Eq18) $\sigma(t) = \frac{\sigma(t-1)}{1-\sigma_g(t)}$ for $t \geq 1$ given $\sigma(0)$

(Eq19) (A.12) $CCum \geq \sum_{t=0}^{Tmax} E_{Ind}(t)$

(Eq20) $E_{Land}(t) = E_{Land}(0) \times (1-0.1)^{t-1}$

Social Cost of Carbon / Carbon tax / Carbon price

(Eq21) $P(t) = \theta_1(t)\mu(t)^{\theta_2 - 1}$

CARBON CYCLE AND CLIMATE MODEL EQUATIONS ... TO BE ADDED LATER...

2 VARIABLES

t: time in decades from 2001-2010, 2011-2020, ..., 2590-2600. The last time period is 60 and this period 60 is denoted $Tmax$.

$C(t)$ total consumption

$c(t)$ per capita consumption

$L(t)$ population in millions

$I(t)$ investment

$K(t)$ capital

$R(t)$ social time preference discount factor

$\rho(t)$ social time preference rate per year

$A(t)$ total factor productivity

$A_g(t)$ growth rate of total factor productivity per decade

$E(t)$ total carbon emissions (billions of metric tons of carbon per period)

$E_{Land}(t)$ carbon emissions from land use/ deforestation (billions of metric tons of carbon per period)

$E_{Ind}(t)$ industrial carbon emissions (billions of metric tons of carbon per period

$T_{AT}(t)$ global mean surface temperature ($°C$ increase from 1900)

$\sigma(t)$ ratio of uncontrolled industrial emissions to output (metric tons of carbon per output in 2005 prices)

$\sigma_g(t)$ rate of decline of carbon intensity per decade

$\Omega(t)$ damage function; $1 - \Omega(t)$ is the percentage of output that vanish due to higher mean surface temperature

$\Lambda(t)$ abatement cost function; the percentage of output that vanish in order to keep the emission level under policy

2.1 PARAMETERS THAT USERS CHOOSE

determine the appropriate range for each parameter

α: elasticity of marginal utility of consumption

ρ: social time preference rate "per year". We will average it over 10 years according to (Eq2) $\in [0, 1]$

$L(Tmax)$: asymptotic population in millions in the last period

Δ_a : decline rate of technological change per decade $>\mathbf{0}$

*** change the matlab code: it assumes Δ_a is annual decline rate and multiplies 10. Maybe that's why we had decreasing TFP for large Δ_a?

δ_K: depreciation rate of technological change per decade

σ_{d1}: decline rate of decarbonization per decade

ϵ: damage exponent in climate damage function

π_2: coefficient on the damage exponent term, $T_{AT}(t)^\epsilon$ in climate damage function

$BC_g(0)$: inital cost decline in backstop technology % per decade =0.05

θ_2: exponent of emission reduction rate in abatement cost function >1

$CCum$**:** maximum cumulative extraction fossil fuels ?????

Or maximum consumption of fossil fuels (billions of metric tons of cabon) in Nordhaus' dice-2007 appendix. should be equivalent?

s: savings rate $\in [0, 1]$

carbon policy variables (ecaps)—–> How does it affect $\mu(t)$?

param.e2005cap is the emissions cap as a percentage of year 2005 emissions

emissions cap by 2050 $\in [0, 1]$

emissions cap by 2100 $\in [0, 1]$

emissions cap by 2150 $\in [0, 1]$

2.2 PARAMETERS FROM DATA ARE..

delete?: number of scenarios to run?=1

delete?: $\mu(t)$: emission reduction rate, It represents the fractional reduction of emissions relative to uncontrolled emissions. fraction of controlled emission in 2005 set to =0 for now. .

$L(0)$: 2005 world populations in millions = 6514

L_g: growth rate of population per decade =0.35 ??? TOO HIGH?

$A(0)$: inital level of total factor productivity =0.02722

$A_g(0)$: initial growth rate of TFP per decade =0.092

γ : capital elasticity of output in production function =0.300

$Q(0)$: 2005 world gross output in 2005 US dollars in trillions=61.1

$K(0)$: 2005 capital value in 2005 US dollars in trillions=137

$\sigma(0)$: 2005 effective carbon intensity / CO_2-equivalent emissions and GNP ratio in 2005 =0.13418

$\sigma_g(0)$: initial growth of carbon intensity per decade =-0.0730 <0

initial rate of decline of carbon intensity per decade = 0.0730 >0

$E_{Land}(0)$: carbon emissions from deforestation 2005 (GtC per decade) =11

$E(0)$: total emissions in year 2005 =84.1910

π_1: damage intercept =0.0000

$BC(0)$: Backstop tech Cost /cost of backstop technology in 2005 = 1.17

$BC(0)/BC(Tmax)$: ratio of initial to final backstop cost =2

What about letting users choose $BC(Tmax)$ (or equivalently the ratio $BC(0)/BC(Tmax)$) depending on their extent of optimism on future tech growth?