

Question 1

Suppose we wanted to estimate the probability of an event y_i occurring, given some observations. We could use a logit regression model, where the pdf of y_i is:

$$f(y_i; \boldsymbol{\beta}) = \mu_i^{y_i} (1 - \mu_i)^{1-y_i}, \quad y_i = 0, 1$$
$$\text{where } \mu_i = \frac{1}{1 + \exp(-\mathbf{x}_i' \boldsymbol{\beta})}$$

The function $\sigma(z) = \frac{1}{1+\exp(-z)}$ represents the logistic function and constrains the predicted y_i to be between 0 and 1 (as required for a probability). $\boldsymbol{\beta}$ is a vector of coefficients. Following the example in the lab,

- i) Find the log-likelihood function and derive the gradient and Hessian.
- ii) Using Python, write a class to represent the Logit model.
- iii) Use the following dataset and initial values of $\boldsymbol{\beta}$ to estimate the MLE with the Newton-Raphson algorithm developed in class:

$$\mathbf{X} = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 1 & 1 \\ 1 & 4 & 3 \\ 1 & 5 & 6 \\ 1 & 3 & 5 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \quad \boldsymbol{\beta}_{(0)} = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

Verify your results with `statsmodels` – you can import the Logit function with the following import statement:

```
from statsmodels.api import Logit
```