

Question 1

Show that

$$w'X'Xw = w_0^2 \sum_{i=1}^n x_{i1}^2 + 2w_0w_1 \sum_{i=1}^n x_{i1}x_{i2} + w_1^2 \sum_{i=1}^n x_{i2}^2$$

where:

$$w = \begin{bmatrix} w_0 \\ w_1 \end{bmatrix}, \quad X = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \\ \vdots & \vdots \\ x_{n1} & x_{n2} \end{bmatrix}$$

Hint: Perform the multiplication step-by-step, starting with $X'X$, then $X'Xw$, and finally left-multiplying by w' .

Question 2

Using Python, write a script to compute the parameters w_0 and w_1 for a given dataset of pairs (x_i, t_i) , where x is the vector of years when Olympic games were held, and t is the winning time in seconds. You may find the data in the `.yaml` file on moodle.

- Implement linear regression to fit the best line $t = w_0 + w_1 \cdot x$.
- Compute the values of w_0 and w_1 using the normal equation.
- Plot the data points and the fitted line.