

# **ARM Constant Multiplication**

Time limit: 1000 ms Memory limit: 256 MB

In this challenge, we will explore how a compiler for a 3232-bit ARM processor may implement multiplication by an unsigned integer constant without using a multiply instruction, which would consume relatively a lot of energy. You are allowed to use the four instructions MOV, ADD, SUB, and RSB operating on registers where the last register may be logically shifted left or right (LSL or LSR). The allowed instructions have the following syntax:

- MOV Rd, Ra, LSL/LSR #i: Move the shifted value of Ra to Rd.
- ADD Rd, Rb, Ra, LSL/LSR #i: Add the shifted value of Ra to Rb, and save the result in Rd.
- SUB Rd, Rb, Ra, LSL/LSR #i: Subtract the shifted value of Ra from Rb, and save the result in Rd.
- RSB Rd, Rb, Ra, LSL/LSR #i: Subtract Rb from the shifted value of Ra, and save the result in Rd. RSB stands for Reverse SuBtract.

Rd is the destination register where the result of the instruction is saved. Rb and Ra are operand registers, and their values do not change during the instruction, though Rd may be the same as Rb and/or Ra in which case the value of the register may change. The value of Ra is shifted left (LSL) or right (LSR) by i bits before it is used by the instruction. That is, Ra LSL #i means the value of Ra << i, and Ra LSR #i means the value of Ra >> i. For LSL, the value of i must be between 00 and 3131 inclusive. For LSR, the value of i must be between 11 and 3232 inclusive. Note that the range of i is different for LSL and LSR. This is because the shifting amount ii must fit into 55 bits in an instruction. All registers store 3232-bit integers. Bits are discarded when shifted outside the 3232-bit range. If you logically shift right by 3232 (LSR #32), you end up with 00 in the register. Overflow bits are discarded in addition, and underflow bits are discarded in subtraction.

For each test case you will be given a single constant unsigned integer multiplier CC. You should return a sequence of ARM instructions that implement the multiplication by this constant multiplier. Assume the multiplicand is in the register R0, and the product should end up in the same register R0 afterwards. That is, if the value of the register R0 is XX in the beginning, its value should become  $C \cdot Cdot \cdot XC \cdot X$  truncated to 3232 bits in the end. You are allowed to use 1212 additional registers R1, R2, ... R12 to save any intermediate results. The values in R1, ... R12 are initially all zeros. You are asked to minimize the number of instructions so that the multiplication can be executed as quickly as possible.

# Standard input

The input has a single integer TT on the first line, the number of test cases.

Each test case has a single line with one unsigned 3232-bit integer, the constant unsigned integer multiplier CC.

## Standard output

For each test case, output a sequence of instructions to perform the multiplication, one instruction per line. Print a single line with END to mark the end of the output for each test case. Each instruction should be one of the four classes given above and follow the provided formats. Note that the shift operation is required and you should set i to zero for LSL to use the value of Ra without any shift.

### Scoring

For each test case, your solution will be scored based on kk, the number of instructions used ( $k \neq 0$ ). It can be shown that for any CC, it is possible to implement the multiplication using no more than 3232 instructions. Your submission will get a score of  $(1.0 - k/33)^2(1.0-k/33)$ 2 if it correctly performs multiplication and satisfies  $k \neq 32k \leq 32$ . A submission that performs multiplication incorrectly or has k > 32k > 32 will receive zero points.

A test file will only receive a positive final score if **every** test case in the test file receives a positive score, in which case the final score of the test file is the average of the scores your solution receives for all the test cases in this test file times 55.

#### Constraints and notes

END

- T = 100T = 100 for all the hidden test files.
- Each CC is an unsigned 3232-bit integer.

Input	Output	Explanation
1	MOV R0, R0, LSL #3	RØ is shifted to the left by $33$ bits, getting RØ $\ll$ 3. This equals 8 * RØ and is the product saved back to RØ.
8	END	
1	ADD R0, R0, R0, LSL #3	The ADD instruction adds 88 times R0 to R0, getting 9 * R0, and saves it back to R0.
9	END	
2	ADD R1, R0, R0, LSL #1	Let the initial value in R0 be $XX$ The ADD instruction adds $2X2X$ from R0, LSL #1 and $XX$ from R0, getting
115	SUB R1, R1, R0, LSL #4	saves it to R1. The SUB instruction gets R1 - R0 * 16 which is $3X - 16X = -13X3X - 16X = -13X$ and
105	ΔDD R0 R1 R0 ISI #7	

The last ADD instruction obtains R1 + 128 \* R0 which is -13X + 128X = 115X - 13X + 128X = 115X

RSB	R1,	R0,	R0,	LSL	#2
MOV	R0,	R0,	LSL	#5	
ADD	R0,	R0,	R1,	LSL	#0
RSB	R0,	RØ,	R0,	LSL	#2

Output

END

Input

#### Explanation

Note that in the last test case you can do better using only two instructions:

- RSB R0, R0, R0, LSL #4
- RSB R0, R0, R0, LSL #3

The first RSB places 15  $\,^*$  R0 in R0. The second RSB calculates 7  $\,^*$  R0. Before the second instruction

, R0 has 15X15X,

and therefore the second RSB will give us 105X105X in R0 as a result.