

Multivariate Analysis of House prices

Introduction

Objective and Data Set Description:

The dataset has 79 explanatory variables and 1460 observations, describing (almost) every aspect of residential homes (dimensions, neighborhoods, sale prices etc.) in Ames, Iowa. The data set is multivariate where the final dimensions will be selected afterwards by reducing. Predicting the final price of each home is also possible with this data set using regression.

For details of variable the link to the data set is below: <https://www.kaggle.com/c/house-prices-advanced-regression-techniques/data>

Motivation: To address following Business Questions:

1. Dimensionality reduction using PCA and Factor analysis.
2. Clustering on the data set to find the clusters of different types houses. For improved price assessment and marketing based on different classes of houses identified.
3. Predicting the house prices in Ames, Iowa using regression model and principle component regression.

Variable of our analysis and their descriptions (16 variables):

1. LotArea :Lot size in square feet
 2. MasVnrArea :Masonry veneer area in square feet
 3. BsmtFinSF1 :Basement type 1 finished square feet
 4. BsmtUnfSF :Unfinished square feet of basement area
 5. TotalBsmtSF :Total square feet of basement area
 6. X1stFlrSF :First Floor square feet
 7. X2ndFlrSF :Second floor square feet
 8. GrLivArea :Above grade (ground) living area square feet
 9. BsmtFullBath :Basement full bathrooms
 10. FullBath :Full bathrooms above grade
 11. BedroomAbvGr :Number of bedrooms above basement level
 12. KitchenAbvGr :Number of kitchens
 13. TotRmsAbvGrd :Total rooms above grade (does not include bathrooms)
 14. GarageArea :Size of garage in square feet
 15. WoodDeckSF :Wood deck area in square feet
 16. OpenPorchSF :Open porch area in square feet
-

Data Preprocessing and Cleaning

We have narrowed our data set to 16 variables. For further dimension reduction and missing value analysis we are going to do correlation analysis and visualization as below:

a) Import data set into R environment

```
#data cleaning
Housing<-read.csv("C:/Education/Multivariate Analysis/Project/Housing/Data/Fu
ll_Housing_New.csv")
Housing<-Housing[, -c(1,18)] #Excluding ID and SalePrice
str(Housing)

## 'data.frame':    2919 obs. of  16 variables:
## $ LotArea      : int  8450 9600 11250 9550 14260 14115 10084 10382 6120 74
20 ...
## $ MasVnrArea   : int   196  0 162  0 350  0 186 240  0  0 ...
## $ BsmtFinSF1   : int   706 978 486 216 655 732 1369 859  0 851 ...
## $ BsmtUnfSF    : int   150 284 434 540 490 64 317 216 952 140 ...
## $ TotalBsmtSF  : int   856 1262 920 756 1145 796 1686 1107 952 991 ...
## $ X1stFlrSF    : int   856 1262 920 961 1145 796 1694 1107 1022 1077 ...
## $ X2ndFlrSF    : int   854  0 866 756 1053 566  0 983 752  0 ...
## $ GrLivArea    : int  1710 1262 1786 1717 2198 1362 1694 2090 1774 1077 ..
.
## $ BsmtFullBath: int    1  0 1 1 1 1 1 1 0 1 ...
## $ FullBath     : int    2  2 2 1 2 1 2 2 2 1 ...
## $ BedroomAbvGr: int    3  3 3 3 4 1 3 3 2 2 ...
## $ KitchenAbvGr: int    1  1 1 1 1 1 1 1 2 2 ...
## $ TotRmsAbvGrd: int    8  6 6 7 9 5 7 7 8 5 ...
## $ GarageArea   : int   548 460 608 642 836 480 636 484 468 205 ...
## $ WoodDeckSF   : int    0 298  0  0 192 40 255 235 90  0 ...
## $ OpenPorchSF  : int    61  0 42 35 84 30 57 204  0 4 ...
```

We have imported the complete data set (training and test) and excluded the ID and SalePrice variables, for the purpose of further multivariate analysis.

b) Dealing with Missing Values in Original data set.

Let us observe our data set for missing values as follows:

```
sum(is.na(Housing)) #Count NA values across variables
## [1] 29

sapply(Housing, function(x) sum(is.na(x)))# number of nas
##      LotArea    MasVnrArea    BsmtFinSF1    BsmtUnfSF    TotalBsmtSF
##          0          23          1          1          1
##    X1stFlrSF    X2ndFlrSF    GrLivArea BsmtFullBath    FullBath
##          0          0          0          2          0
## BedroomAbvGr KitchenAbvGr TotRmsAbvGrd    GarageArea    WoodDeckSF
##          0          0          0          1          0
## OpenPorchSF
##          0
```

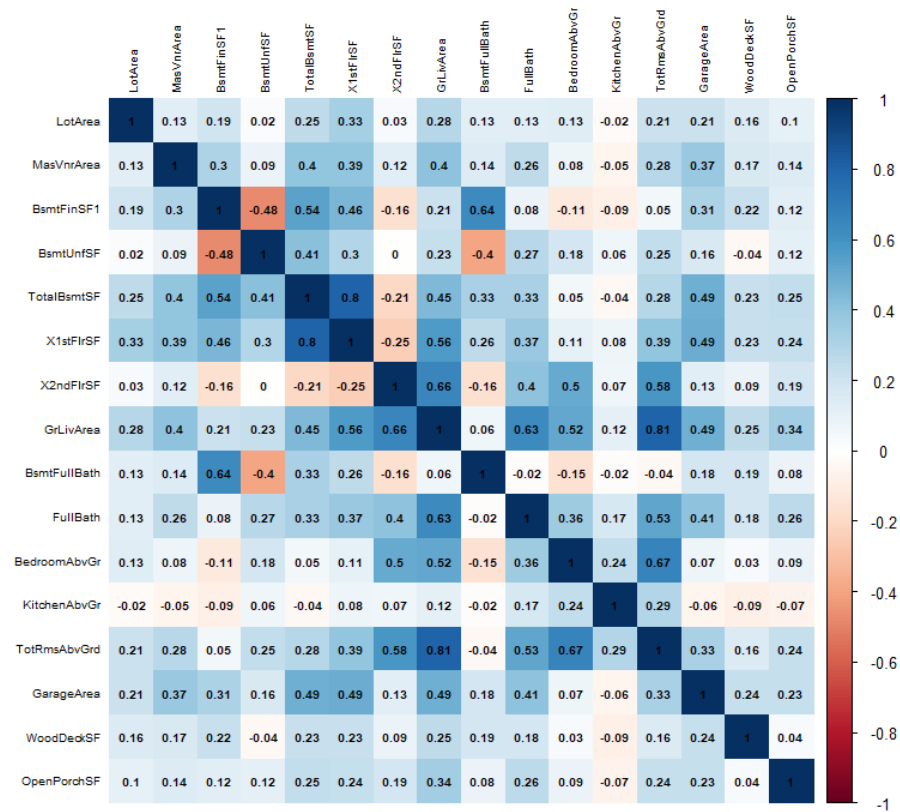
We observe 29 NA values in our dataset. Considering these missing values to be completely random, we replace/impute these values with their respective column mean values.

```
imp<-apply(Housing,2,mean,na.rm= T)
house<- Housing
for(i in 1:ncol(Housing)){
  house[is.na(house[,i]),i]<-imp[i]
}
sapply(house, function(x) sum(is.na(x)))#imputing nas with col means
##      LotArea      MasVnrArea      BsmtFinSF1      BsmtUnfSF      TotalBsmtSF
##           0              0              0              0              0
##      X1stFlrSF      X2ndFlrSF      GrLivArea      BsmtFullBath      FullBath
##           0              0              0              0              0
## BedroomAbvGr KitchenAbvGr TotRmsAbvGrd      GarageArea      WoodDeckSF
##           0              0              0              0              0
##      OpenPorchSF
##           0
#str(house)
```

c) Correlation analysis

Let us observe the correlations between the variables of our dataset, as we understand that there needs to be correlation between the variables for proceeding with the multivariate analysis.

```
library(corrplot)
corrplot(cor(house), method="color", addCoef.col = "black",
         tl.col="black", tl.cex=0.6, number.cex=0.6)
```

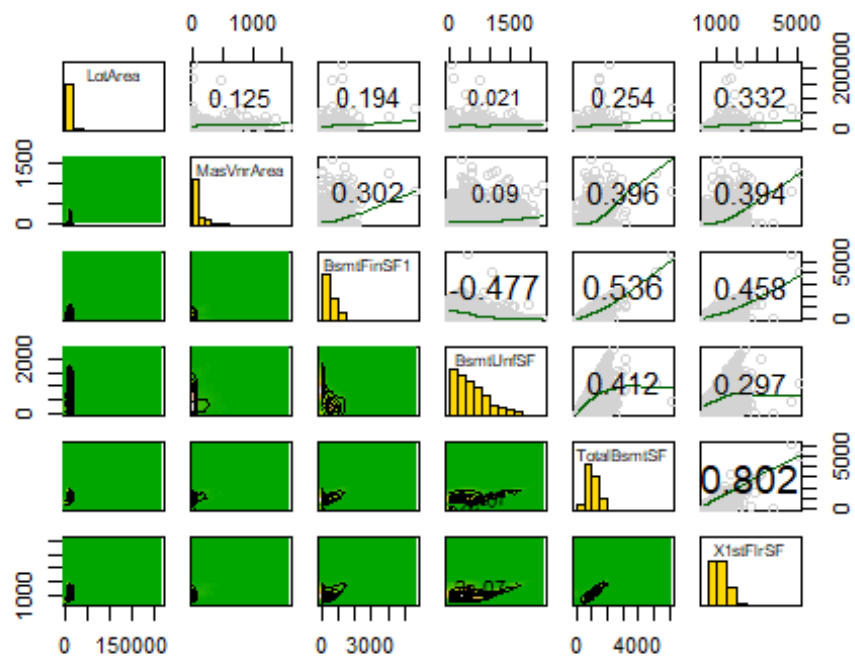


From the above corellogram, we observe weak correlation of following variables with rest of the variables:

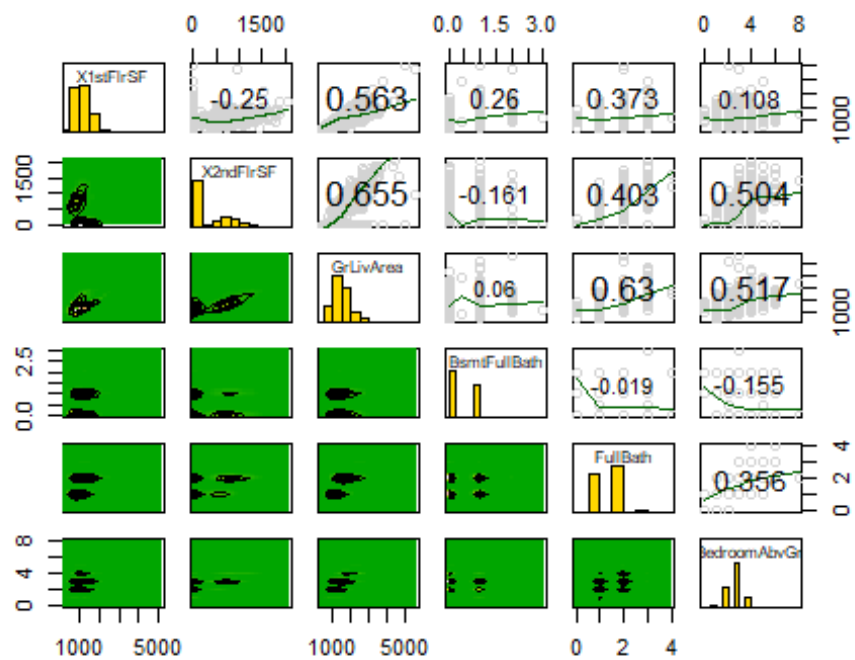
1. KitchenAbvGrd
2. WoodDeckSF
3. OpenPorchSF
4. LotArea

Let us confirm the same from further correlation visualizations.

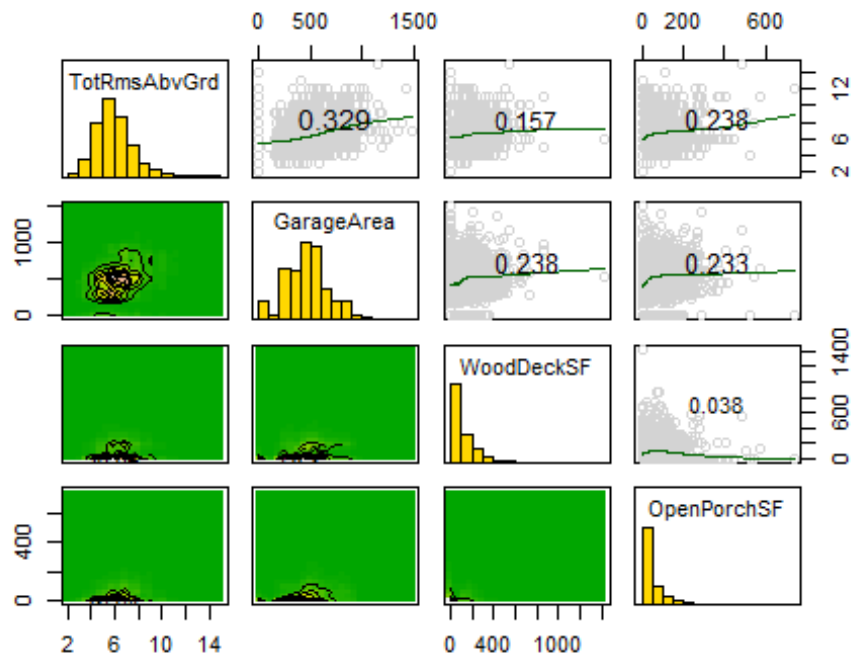
```
library(ResourceSelection)
#str(house)
#dim(house)
#cor(house)#correltion matrix
kdepairs(house[,1:6])
```



```
kdepairs(house[,6:11])
```



```
kdepairs(house[,13:16])##12 is the kitchen above the garage . most of the values 1 or 2 , so it acts as binary variable
```



#multivariate normality check

From the above visualizations, we confirm that the 3 variables above mentioned express weak correlation with the rest of the variables and hence we exclude them from further multivariate analysis.

#New dataset

```
house.new<-house[, -c(1,12,15,16)]
str(house.new)
```

```
## 'data.frame': 2919 obs. of 12 variables:
## $ MasVnrArea : num 196 0 162 0 350 0 186 240 0 0 ...
## $ BsmtFinSF1 : num 706 978 486 216 655 ...
## $ BsmtUnfSF : num 150 284 434 540 490 64 317 216 952 140 ...
## $ TotalBsmtSF : num 856 1262 920 756 1145 ...
## $ X1stFlrSF : num 856 1262 920 961 1145 ...
## $ X2ndFlrSF : num 854 0 866 756 1053 ...
## $ GrLivArea : num 1710 1262 1786 1717 2198 ...
## $ BsmtFullBath: num 1 0 1 1 1 1 1 1 0 1 ...
## $ FullBath : num 2 2 2 1 2 1 2 2 2 1 ...
## $ BedroomAbvGr: num 3 3 3 3 4 1 3 3 2 2 ...
## $ TotRmsAbvGrd: num 8 6 6 7 9 5 7 7 8 5 ...
## $ GarageArea : num 548 460 608 642 836 480 636 484 468 205 ...
```

d) Outlier Analysis

In the previous visualizations we observe outliers in the dataset. Here we attempt to identify and exclude the outliers using the Mahalanobis distances.

We perform our outlier analysis, considering only the training dataset. Also, we perform scaling to standardize the range of independent variables.

```
house.new <- house.new[1:1460,] #Train data
house.scale<- scale(house.new)
#head(house.scale)
summary(house.scale)
```

##	MasVnrArea	BsmtFinSF1	BsmtUnfSF	TotalBsmtSF
##	Min. :-0.5742	Min. :-0.9727	Min. :-1.2837	Min. :-2.4103
##	1st Qu.:-0.5742	1st Qu.:-0.9727	1st Qu.:-0.7791	1st Qu.:-0.5965
##	Median :-0.5742	Median :-0.1319	Median :-0.2031	Median :-0.1503
##	Mean : 0.0000	Mean : 0.0000	Mean : 0.0000	Mean : 0.0000
##	3rd Qu.: 0.3355	3rd Qu.: 0.5889	3rd Qu.: 0.5449	3rd Qu.: 0.5489
##	Max. : 8.2867	Max. :11.4018	Max. : 4.0029	Max. :11.5170
##	X1stFlrSF	X2ndFlrSF	GrLivArea	BsmtFullBath
##	Min. :-2.1434	Min. :-0.7949	Min. :-2.24835	Min. :-0.8197
##	1st Qu.:-0.7259	1st Qu.:-0.7949	1st Qu.:-0.73450	1st Qu.:-0.8197
##	Median :-0.1956	Median :-0.7949	Median :-0.09794	Median :-0.8197
##	Mean : 0.0000	Mean : 0.0000	Mean : 0.00000	Mean : 0.0000
##	3rd Qu.: 0.5914	3rd Qu.: 0.8728	3rd Qu.: 0.49723	3rd Qu.: 1.1074
##	Max. : 9.1296	Max. : 3.9356	Max. : 7.85288	Max. : 4.9617
##	FullBath	BedroomAbvGr	TotRmsAbvGrd	GarageArea
##	Min. :-2.8408	Min. :-3.5137	Min. :-2.7795	Min. :-2.21220
##	1st Qu.:-1.0257	1st Qu.:-1.0621	1st Qu.:-0.9338	1st Qu.:-0.64769
##	Median : 0.7895	Median : 0.1637	Median :-0.3186	Median : 0.03283
##	Mean : 0.0000	Mean : 0.0000	Mean : 0.0000	Mean : 0.00000
##	3rd Qu.: 0.7895	3rd Qu.: 0.1637	3rd Qu.: 0.2967	3rd Qu.: 0.48184
##	Max. : 2.6046	Max. : 6.2928	Max. : 4.6033	Max. : 4.42001

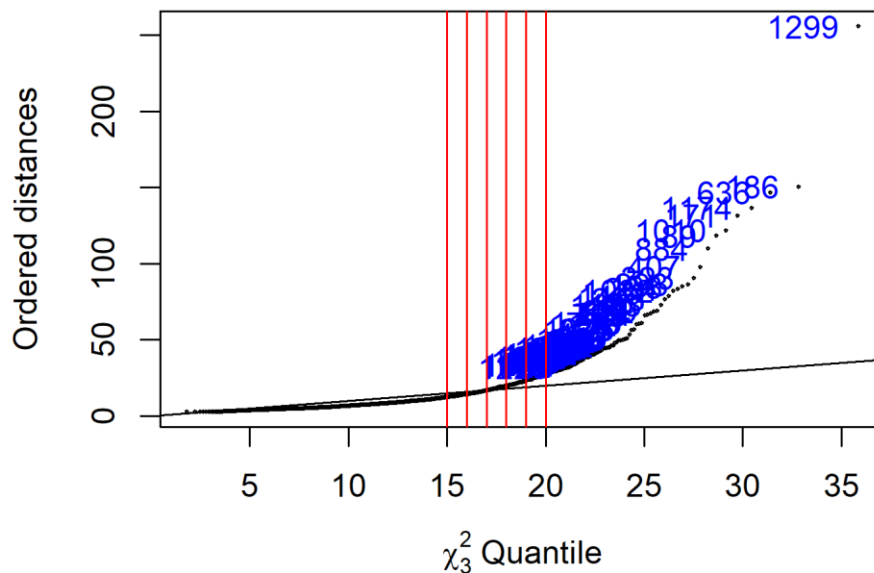
```
#str(house.scale)
```

From the above summary we observe that the max values of most variables even after scaling are above value 3, i.e. we notice significant outliers in the dataset. Following we visualize the distances of observations using the Chi-plot.

Chi-Plot

```
cm <-colMeans(house.scale)
S <-cov(house.scale)
d<-mahalanobis(house.scale,(cm),S)
plot(qc<-qchisq((1:nrow(house.scale) -1/2) /nrow(house.scale), df =ncol(house
.scale)), sd<-sort(d),xlab =expression(paste(chi[3]^2, " Quantile")), ylab ="
Ordered distances", cex = .2)
oups <- which(rank(abs(qc - sd), ties = "random") > nrow(house.scale)-70)
text(qc[oups], sd[oups] - 1.5, names(oups),pos = 2, col = "blue")
```

```
abline(a =0, b =1)
abline(v=c(15:20), col="red")
```



We observe the distance do not lie normal on the chi-plot and hence the observations are not multivariate normally distributed. We exclude the observations having distance greater than 16, as we observe a deviation of distances in the chi-plot from quantile point 16. Hence, we proceed further excluding observations having distances greater than 16.

```
library(plyr)
library(dplyr)
m_dist <- mahalanobis(house.new, colMeans(house.new), cov(house.new))# getting m-dist
house.new$MD <- round(m_dist, 2)# rounding off and md col

house.new$outlier <- "No" #adding column for outlier
house.new$outlier[house.new$MD > 16] <- "Yes"
house.out <- house.new %>% filter(house.new$outlier=="No") # filter out outliers
dim(house.out)#dimension

## [1] 1224 14

house.reqd <- house.out[,1:12]#final data set for further analysis
```


We utilize the knowledge gained from this course to implement the PCA, Factor analysis for dimension reductions.

Principle Component Analysis

In this section, we perform the PCA to reduce the number of variables in the data set while accounting for as much original variation in the data set as possible.

```
# principal component analysis
house.pca<-princomp(house.reqd, cor = T)
summary(house.pca, loadings = T)

## Importance of components:
##
##          Comp.1    Comp.2    Comp.3    Comp.4    Comp.5
## Standard deviation  2.0641119 1.6855247 1.3693981 0.89410597 0.84324487
## Proportion of Variance 0.3550465 0.2367495 0.1562709 0.06661879 0.05925516
## Cumulative Proportion 0.3550465 0.5917960 0.7480669 0.81468568 0.87394084
##
##          Comp.6    Comp.7    Comp.8    Comp.9
## Standard deviation  0.68624990 0.63458412 0.56462230 0.42238153
## Proportion of Variance 0.03924491 0.03355808 0.02656653 0.01486718
## Cumulative Proportion 0.91318575 0.94674383 0.97331036 0.98817754
##
##          Comp.10    Comp.11    Comp.12
## Standard deviation  0.35627176 0.121771691 1.056499e-02
## Proportion of Variance 0.01057746 0.001235695 9.301588e-06
## Cumulative Proportion 0.99875500 0.999990698 1.000000e+00
##
## Loadings:
##
##          Comp.1 Comp.2 Comp.3 Comp.4 Comp.5 Comp.6 Comp.7 Comp.8
## MasVnrArea  -0.246  0.132          0.711  0.616          -0.165
## BsmtFinSF1          0.456  0.382          0.173  0.320  0.191
## BsmtUnfSF    -0.175 -0.159 -0.622          -0.352 -0.104
## TotalBsmtSF  -0.311  0.355 -0.274 -0.170
## X1stFlrSF    -0.323  0.333 -0.255 -0.229  0.114  0.112          0.178
## X2ndFlrSF    -0.206 -0.420  0.343  0.154 -0.130          -0.222  0.245
## GrLivArea    -0.439 -0.146  0.131          0.118 -0.167  0.366
## BsmtFullBath          0.402  0.366 -0.139 -0.108 -0.116 -0.711 -0.383
## FullBath     -0.357          0.184 -0.517  0.522  0.189 -0.493
## BedroomAbvGr -0.272 -0.272  0.181 -0.444  0.358 -0.245  0.306 -0.519
## TotRmsAbvGrd -0.390 -0.221  0.140 -0.219  0.136          -0.110  0.185
## GarageArea   -0.328  0.158          0.282 -0.399 -0.762  0.211
##
##          Comp.9 Comp.10 Comp.11 Comp.12
## MasVnrArea
## BsmtFinSF1    0.166 -0.290 -0.589
## BsmtUnfSF     0.167 -0.197 -0.592
## TotalBsmtSF   0.374 -0.463  0.548
## X1stFlrSF    -0.186  0.582          0.481
## X2ndFlrSF     0.379          0.603
## GrLivArea     0.219  0.372          -0.637
## BsmtFullBath
## FullBath     -0.115
```

```
## BedroomAbvGr  0.247
## TotRmsAbvGrd -0.707 -0.408
## GarageArea
```

We observe that the first 3 principle components captures 75 percent of variation in the dataset. Also, the first 3 PCs have standard deviation of greater than 1. Hence we consider the first 3 PCs.

We can interpret the three principle components based on the direction and magnitude of loadings as follows:

First PC

```
house.pca$loadings[,1]
```

```
##  MasVnrArea  BsmtFinSF1  BsmtUnfSF  TotalBsmtSF  X1stFlrSF
## -0.24632739 -0.09883743 -0.17534231 -0.31104675 -0.32300770
##   X2ndFlrSF  GrLivArea BsmtFullBath  FullBath BedroomAbvGr
## -0.20624749 -0.43853878 -0.04329910 -0.35675277 -0.27218333
## TotRmsAbvGrd  GarageArea
## -0.38978554 -0.32818719
```

We observe that all the variables are in the same direction for PC1 and has more weightage for TotalBsmtSF, X1stFlrSF, GrLivArea, FullBath, TotRmsAbvGrd and GarageArea. Hence we can consider the PC1 to represent overall characteristics of the house.

PC1 - "Overall Feature"

Second PC

```
house.pca$loadings[,2]
```

```
##  MasVnrArea  BsmtFinSF1  BsmtUnfSF  TotalBsmtSF  X1stFlrSF
##  0.1318088  0.4558169  -0.1592894   0.3552447   0.3330759
##   X2ndFlrSF  GrLivArea BsmtFullBath  FullBath BedroomAbvGr
## -0.4200702 -0.1458787  0.4024352  -0.0737999  -0.2715232
## TotRmsAbvGrd  GarageArea
## -0.2205623   0.1575859
```

We observe that PC2 gives more weightage to X2ndFlrSF in the same direction of PC1 and in opposite direction to BsmtFinSF1, TotalBsmtSF, X1stFlrSF and BsmtFullBath. Hence, we can consider this component to be describing the Second Floor area in contrast to the Basement and First Floor areas.

PC2 - "Second Floor Feature"

Third PC

```
house.pca$loadings[,3]
```

```
##  MasVnrArea  BsmtFinSF1  BsmtUnfSF  TotalBsmtSF  X1stFlrSF
##  0.06240068  0.38151464 -0.62188303 -0.27435805 -0.25544703
```

```
##      X2ndFlrSF      GrLivArea BsmtFullBath      FullBath BedroomAbvGr
##      0.34291252      0.13082062      0.36572903     -0.03603942      0.18144872
## TotRmsAbvGrd      GarageArea
##      0.13954785     -0.03364352
```

We observe that PC3 gives more weightage to BsmtUndSF in the same direction of PC1 and in opposite direction to BsmtFinSF1, X2stFlrSF and BsmtFullBath. Hence, we can consider this component to be describing the Basement unfinished area in contrast to the finished basement area and Second Floor areas.

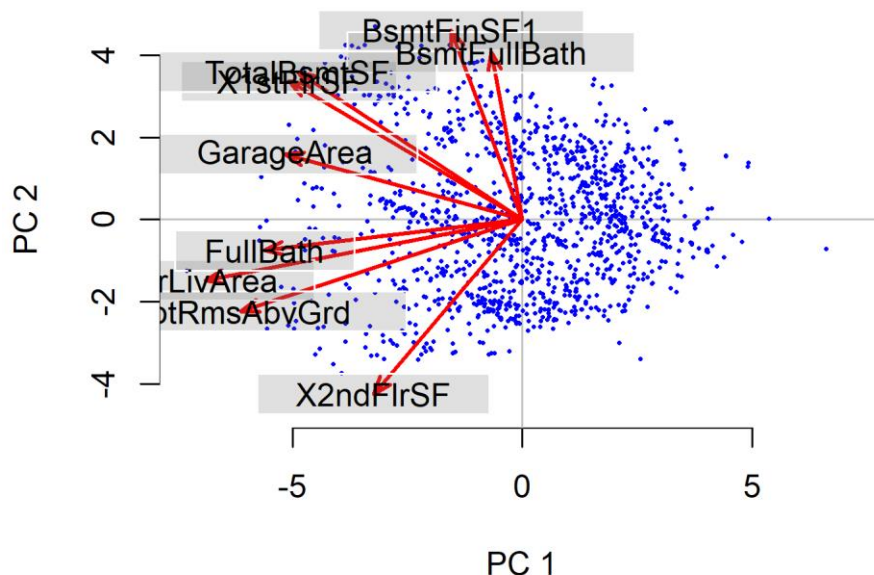
PC3 - "Unfinished Basement area"

Bi-plot

Here we attempt to plot the bi-plot which represents the variables and observations on to a single plot of PC components.

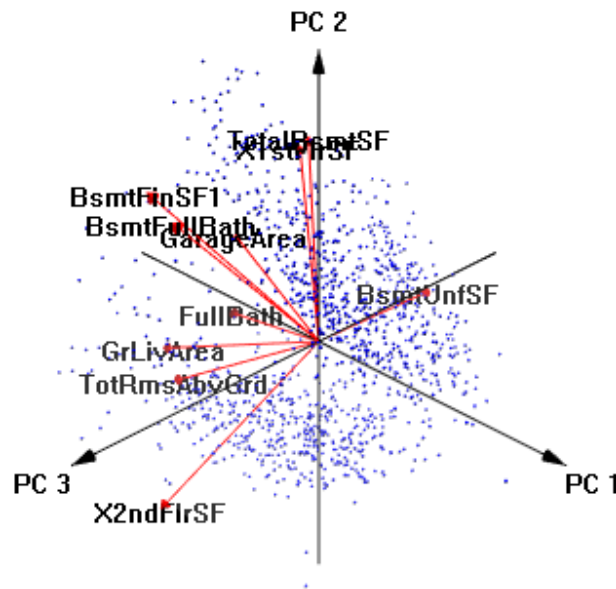
(We make use of the pca3d package, which essentially is a shortcut to RGL graphics library and get the pca visualization done.)

```
#2d representation
library(pca3d)
pca2d(house.pca, biplot=T, col="blue", radius = 0.3, title = "2D bi-plot")
```



As we have considered 3 PCs, we attempt for a 3 dimensional plot and interpret accordingly. (We here do not label the observartions as it would turn complete mess)

```
pca3d(house.pca, biplot = T, radius= 0.3, col="blue", axes.color = "black")
snapshotPCA3d(file="3d.png")
```



From the above bi-plot we observe:

1. BsmtFinSF1 and BsmtFullBath are closely correlated and hence have similar profiles.
2. TotalBsmtSF and X1stFlrSF are closely and hence have similar profiles.
3. FullBath, GrLivArea and TotRmsAbvGrd are correlated to each other in the same direction.
4. We observe the BsmtUnfSF is oppositely correlated to BsmtFinSF, which was captured by the PC3 and we conclude that they have opposite profiles which is reasonable to understand as they represent unfinished and finished areas of the house.
5. Further we observe nearly uncorrelated group of variables which are at 90 degrees to each other.

Note: We here are not interpreting anything about the observations and conclude only saying that points close to each other have similar scores on the PCs.

Exploratory Factor Analysis

We perform the Exploratory factor analysis, as a dimension reduction technique and observe the latent variables of the housing dataset.

```
# Scaling the data
house.reqd.scale<-scale(house.reqd)
# Factor analysis
(house.fa <- factanal(house.scale, factors = 3))
```

```
##
## Call:
## factanal(x = house.scale, factors = 3)
##
## Uniquenesses:
##      MasVnrArea    BsmtFinSF1    BsmtUnfSF    TotalBsmtSF    X1stFlrSF
##      0.804        0.005        0.385        0.294        0.005
##      X2ndFlrSF    GrLivArea    BsmtFullBath    FullBath    BedroomAbvGr
##      0.005        0.005        0.565        0.587        0.662
## TotRmsAbvGrd    GarageArea
##      0.300        0.693
##
## Loadings:
##              Factor1 Factor2 Factor3
## MasVnrArea    0.285    0.308    0.141
## BsmtFinSF1      0.447    0.892
## BsmtUnfSF      0.118    0.311   -0.710
## TotalBsmtSF      0.817    0.173
## X1stFlrSF      0.126    0.990
## X2ndFlrSF      0.943   -0.324
## GrLivArea      0.886    0.459
## BsmtFullBath      0.257    0.600
## FullBath       0.553    0.314
## BedroomAbvGr    0.558      -0.150
## TotRmsAbvGrd    0.766    0.318   -0.108
## GarageArea     0.298    0.455    0.105
##
##              Factor1 Factor2 Factor3
## SS loadings    3.097    2.830    1.763
## Proportion Var 0.258    0.236    0.147
## Cumulative Var 0.258    0.494    0.641
##
## Test of the hypothesis that 3 factors are sufficient.
## The chi square statistic is 2446.04 on 33 degrees of freedom.
## The p-value is 0

print(house.fa$loadings, cut= 0.5)

##
## Loadings:
##              Factor1 Factor2 Factor3
## MasVnrArea
## BsmtFinSF1      0.892
## BsmtUnfSF      -0.710
## TotalBsmtSF      0.817
## X1stFlrSF      0.990
## X2ndFlrSF      0.943
## GrLivArea      0.886
## BsmtFullBath      0.600
## FullBath       0.553
```

```
## BedroomAbvGr  0.558
## TotRmsAbvGrd  0.766
## GarageArea
##
##               Factor1 Factor2 Factor3
## SS loadings    3.097   2.830   1.763
## Proportion Var  0.258   0.236   0.147
## Cumulative Var  0.258   0.494   0.641
```

We observe from the cumulative variance of factors, we observe that the 3 factors captures 64 percent variance in the dataset.

Further we observe that 3 factors were sufficient to explain the variability in the dataset. As we observe the factor loadings, we make notice of the following:

1. Factor1 gives more weightage to the X2ndFlrSF, GrLivArea and TotRmsAbvGrd and may be considered to represent the latent variable "Above Grade rooms and 2nd floor area".
2. Factor2 gives more weightage to the TotalBsmtSF and X1stFlrSF and may be considered to represent the latent variable "Basement and 1st floor area".
3. Factor3 gives more weightage to the BsmtFinSF1 and BsmtFullBath in direction of PC1 and BsmtUnfSF in opposite direction and hence may be considered to represent the latent variable "Impact of finished area and Basement full bathrooms".

Confirmatory factor analysis

We perform the confirmatory analysis to confirm the similarity between restricted covariance matrix (obtained through factor analysis) and non-restricted covariance matrix (obtained from raw data).

```
#chouse<-(house.new[, -c(1,3,9,10,12)])
chouse<-house.reqd
library(sem)
real_model <- specifyModel(file = "realestate_model1.txt")
real_model

##      Path                                Parameter StartValue
## 1  Factor1      -> X2ndFlrSF                lambda1
## 2  Factor1      -> GrLivArea                 lambda2
## 3  Factor1      -> TotRmsAbvGrd              lambda3
## 4  Factor2      -> TotalBsmtSF               lambda4
## 5  Factor2      -> X1stFlrSF                 lambda5
## 6  Factor1      <-> Factor2                  corr
## 7  X2ndFlrSF    <-> X2ndFlrSF                 theta1
## 8  GrLivArea    <-> GrLivArea                 theta2
## 9  TotRmsAbvGrd <-> TotRmsAbvGrd              theta3
## 10 TotalBsmtSF  <-> TotalBsmtSF               theta4
## 11 X1stFlrSF    <-> X1stFlrSF                 theta5
```

```
## 12 Factor1      <-> Factor1      <fixed>    1
## 13 Factor2      <-> Factor2      <fixed>    1

opt <- options(fit.indices = c("GFI", "AGFI", "SRMR"))
real_sem <- sem(real_model, cor(chouse), nrow(chouse)) #cor or cov same result.
real_sem

##
## Model Chisquare = -2013151107   Df = 4
##
##      lambda1      lambda2      lambda3      lambda4      lambda5
## 0.144174251  3.569625104 -0.020333439 -0.266278082 -0.183941670
##      corr      theta1      theta2      theta3      theta4
## -0.250321413  1.127602682 -11.925361089 -0.011854047  0.163091876
##      theta5
## -0.006216737
##
## Iterations = 670

#summary(real_sem)
#MasVnrArea, BsmtFinSF1, BsmtUnfSF, BsmtFullBath, FullBath, BedroomAbvGr, GarageArea
#MasVnrArea, BsmtUnfSF, FullBath, BedroomAbvGr, GarageArea
```

We attempted confirmatory factor analysis using both 2 and 3 factors. However, in both the cases the result is “coefficient covariances cannot be computed”. Here, R is unable to calculate the factor loadings to show the summary. Also we found lambda values more than one and theta values negative. So, it a bad model too. Hence, we can conclude that it is not possible to get the latent variables with every confirmatory factor analysis.

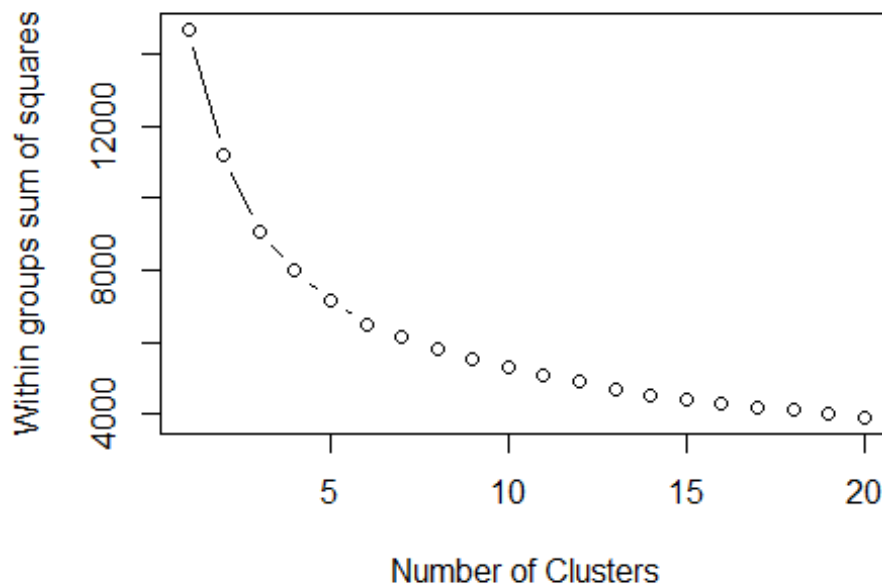
Clustering Analysis: K-means

```
# For K-means clustering scalling is important as it calcualtes the distance from centroids so we will use scalled data

#Within groups sum of squares
plot.wgss = function(mydata, maxc) {
  wss = numeric(maxc)
  for (i in 1:maxc) wss[i] = kmeans(mydata,centers=i, nstart = 20)$tot.withinss
  plot(1:maxc, wss, type="b", xlab="Number of Clusters",
       ylab="Within groups sum of squares", main="Scree Plot") }

#Scree plot to decide the number of clusters
plot.wgss(house.reqd.scale, 20)
```

Scree Plot



The scree plot analysis shows we should pick 5 clusters as per elbow test.

#building k-means clusters

```
km2 <- kmeans(house.reqd.scale, 5)
```

NOTE: We cannot plot Multivariate data on 2 axis so better to plot clusters on Principle components, using non scaled data set and cor=true.

```
pca <- princomp(house.reqd,cor=T)
```

```
pca$loadings[,1:3] # how you name pc1, pc2, and pc3?
```

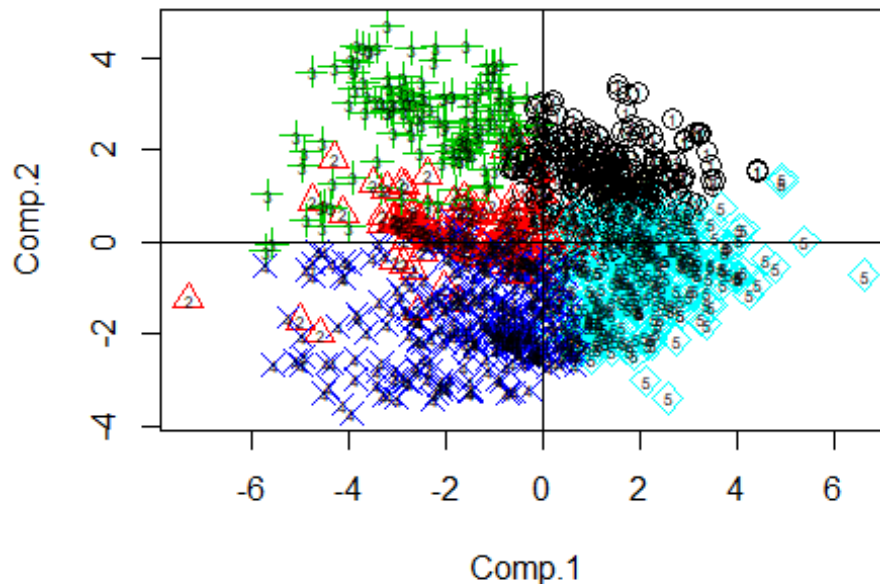
##	Comp.1	Comp.2	Comp.3
## MasVnrArea	-0.24632739	0.1318088	0.06240068
## BsmtFinSF1	-0.09883743	0.4558169	0.38151464
## BsmtUnfSF	-0.17534231	-0.1592894	-0.62188303
## TotalBsmtSF	-0.31104675	0.3552447	-0.27435805
## X1stFlrSF	-0.32300770	0.3330759	-0.25544703
## X2ndFlrSF	-0.20624749	-0.4200702	0.34291252
## GrLivArea	-0.43853878	-0.1458787	0.13082062
## BsmtFullBath	-0.04329910	0.4024352	0.36572903
## FullBath	-0.35675277	-0.0737999	-0.03603942
## BedroomAbvGr	-0.27218333	-0.2715232	0.18144872
## TotRmsAbvGrd	-0.38978554	-0.2205623	0.13954785
## GarageArea	-0.32818719	0.1575859	-0.03364352

#plotting the clusters against the PC1 and PC2

```
plot(pca$scores[, c(1:2)], pch = km2$cluster, col=km2$cluster, cex=1.5)
```



```
abline(h=0,v=0)
text(pca$scores[, c(1:2)], labels = km2$cluster ,cex=0.5)
```

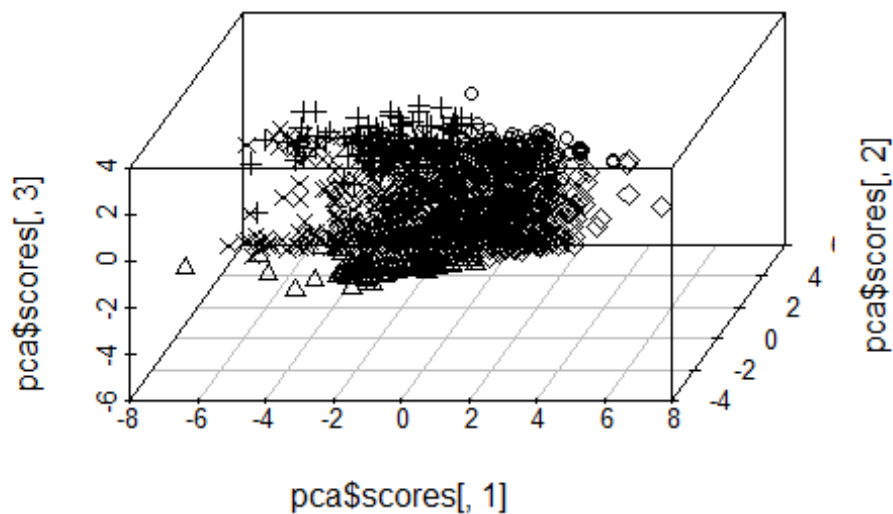


The 2D plot of PC1 scores and PC2 scores for our five clusters shows following:

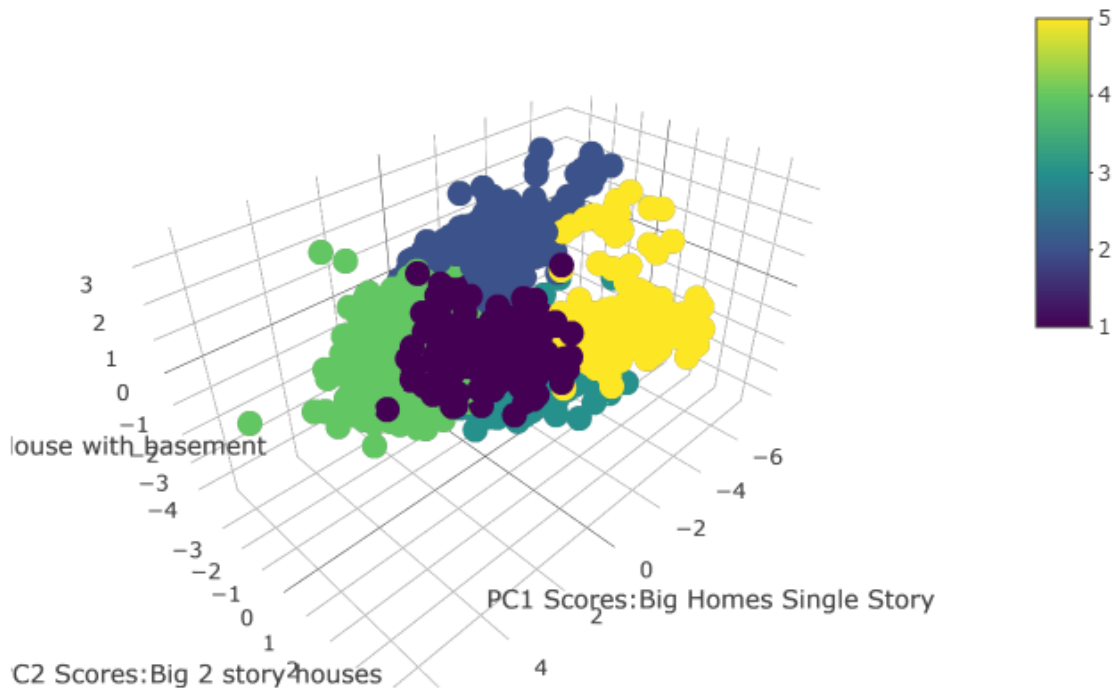
- Cluster 1:(black): Average houses having average surface area and basement features.
- Cluster 2(Red): Shows more weight towards positives of PC2, so they are the houses having big basements finished surface area and basement full baths.
- Cluster 3(Green): Shows big houses with high surface areas of both 1st and 2nd floors.
- Cluster 4(D.Blue): Houses having big 2nd floors areas.
- Cluster 5(L.Blue): Houses having big 1st floor,basement and garage areas.

**** 3D Representation plot of 3 Principal Components ****

```
#3D plot to show all clusters mapped on 3 Principal components we created in the earlier section
library("scatterplot3d")
scatterplot3d(x=pca$scores[, 1],y=pca$scores[, 2],z=pca$scores[, 3], pch = km2$cluster, angle=60)
```



```
# A better 3D Representation using 3D plot.
#install.packages("plotly")
library(plotly)
scoresDF<-as.data.frame(pca$scores)
plot_ly(scoresDF, x=~Comp.1, y=~Comp.2, z=~Comp.3, text = km2$cluster, color=
km2$cluster ) %>% #(color range if required) colors = c('#BF382A', '#0C4B8E')
  add_markers() %>%
  layout(scene = list(xaxis = list(title = 'PC1 Scores:Big Homes Single Story
'),
                      yaxis = list(title = 'PC2 Scores:Big 2 story houses'),
                      zaxis = list(title = 'PC3 Scores:House with basement'))))
```



The second 3D plot is an interactive plot, best visible when seen as html. Please check the attachment (html file) for this visualization). When hovering over an observation point on the 3d plot we can see the cluster it belongs to and what are the PC scores in three dimensions. The 5 clusters are color encoded as per legend on the plot. Further analysis of centroids is explained in the next sections.

#Centers of the clusters

km2\$centers

##	MasVnrArea	BsmtFinSF1	BsmtUnfSF	TotalBsmtSF	X1stFlrSF	X2ndFlrSF
## 1	-0.20830058	0.6461489	-0.72626585	-0.01696165	-0.05500583	-0.7278313
## 2	0.18721480	-0.8670502	1.91440915	1.20115179	1.18790599	-0.7145118
## 3	1.01301706	1.6480792	-0.26739414	1.56728943	1.57484276	-0.4996277
## 4	0.08243948	-0.3130742	-0.01984283	-0.41768034	-0.38850755	1.2759631
## 5	-0.44455857	-0.5470806	-0.08226440	-0.73766557	-0.73559589	-0.2642605

##	GrLivArea	BsmtFullBath	FullBath	BedroomAbvGr	TotRmsAbvGrd	GarageArea
## 1	-0.7304251	1.0083194	-0.7492175	-0.44730047	-0.6525119	-0.09904256
## 2	0.2212630	-0.6783678	0.7186261	-0.05625971	0.1474209	0.62165997
## 3	0.7166975	1.0545109	0.8047877	-0.09080379	0.3839153	0.90717882
## 4	0.9122445	-0.2456174	0.7407799	0.80303540	0.9049115	0.22354653
## 5	-0.8037111	-0.7421556	-0.8536456	-0.44372898	-0.6818884	-0.83105170

Cluster Centroid Analysis

Based on the centers above we reach to the following conclusions:

- Cluster 1: Houses having high, **basement** finished surface area.
- Cluster 2: Houses having high, **second floor + big house** surface area.
- Cluster 3: Houses having average, **basement surface areas garage and full baths**.
- Cluster 4: Houses having small areas in all variable representative of **small houses**.
- Cluster 5: **Big houses have high quality** and higher percentage of finished area.

Total houses in each cluster:

```
table(km2$cluster) # number of observations in a cluster
```

```
##
##  1  2  3  4  5
## 268 139 142 353 322
```

**** Houses in a particular Cluster **** To check the details about the houses in each cluster we can find the cluster data points.

#One can check and trace back the house IDs which are in cluster 1. This may be helpfull to further study the cluster with attributes which were not accounted for but were available externally i.e. in master data.

```
h <- subset(house.reqd.scale, km2$cluster ==1)
head(h)
```

```
##      MasVnrArea BsmtFinSF1  BsmtUnfSF TotalBsmtSF  X1stFlrSF  X2ndFlrSF
## [1,] -0.6436690  1.3333996 -0.7059280  0.65658120  0.46869059 -0.8114359
## [2,] -0.6436690  0.7300189 -1.2388697 -0.67597198 -0.95597175  0.5719837
## [3,] -0.6436690  1.0218982 -1.0547626 -0.11835853 -0.09689424 -0.8114359
## [4,] -0.6436690  1.1568003 -1.0692973  0.02175973 -0.21001121 -0.8114359
## [5,] -0.6436690  0.7422827 -0.9699764 -0.34426346 -0.60133477 -0.8114359
## [6,]  0.9454894  0.7324716 -0.1342270  0.63084519  0.44117565 -0.8114359
##      GrLivArea BsmtFullBath   FullBath BedroomAbvGr TotRmsAbvGrd
## [1,] -0.4137501  -0.8237298  0.9008189    0.2470505   -0.2244694
## [2,] -0.1827576  1.1968018 -1.0472417   -2.6605443   -0.9325892
## [3,] -0.8410860  1.1968018 -1.0472417   -1.2067469   -0.9325892
## [4,] -0.9265532  1.1968018 -1.0472417    0.2470505   -0.9325892
## [5,] -1.2222235  1.1968018 -1.0472417   -1.2067469   -1.6407090
## [6,] -0.4345394  1.1968018 -1.0472417   -1.2067469   -0.9325892
##      GarageArea
## [1,] -0.01090519
## [2,]  0.09333495
## [3,] -1.33996695
## [4,] -0.40701771
## [5,] -0.57380193
## [6,] -0.57380193
```

Predictive Analysis: Regression

Initially we consider all the variables of our considered dataset to perform linear regression to predict SalePrice of houses. Following we reconstruct our required dataset.

```
#data cleaning
Housing<-read.csv("C:/Education/Multivariate Analysis/Project/Housing/Data/Full_Housing_New.csv")
str(Housing)

## 'data.frame':    2919 obs. of  18 variables:
## $ Id           : int  1 2 3 4 5 6 7 8 9 10 ...
## $ LotArea      : int  8450 9600 11250 9550 14260 14115 10084 10382 6120 74
## 20 ...
## $ MasVnrArea   : int  196 0 162 0 350 0 186 240 0 0 ...
## $ BsmtFinSF1   : int  706 978 486 216 655 732 1369 859 0 851 ...
## $ BsmtUnfSF    : int  150 284 434 540 490 64 317 216 952 140 ...
## $ TotalBsmtSF  : int  856 1262 920 756 1145 796 1686 1107 952 991 ...
## $ X1stFlrSF    : int  856 1262 920 961 1145 796 1694 1107 1022 1077 ...
## $ X2ndFlrSF    : int  854 0 866 756 1053 566 0 983 752 0 ...
## $ GrLivArea    : int  1710 1262 1786 1717 2198 1362 1694 2090 1774 1077 ..
##
## $ BsmtFullBath: int  1 0 1 1 1 1 1 1 0 1 ...
## $ FullBath     : int  2 2 2 1 2 1 2 2 2 1 ...
## $ BedroomAbvGr: int  3 3 3 3 4 1 3 3 2 2 ...
## $ KitchenAbvGr: int  1 1 1 1 1 1 1 1 2 2 ...
## $ TotRmsAbvGrd: int  8 6 6 7 9 5 7 7 8 5 ...
## $ GarageArea   : int  548 460 608 642 836 480 636 484 468 205 ...
## $ WoodDeckSF   : int  0 298 0 0 192 40 255 235 90 0 ...
## $ OpenPorchSF  : int  61 0 42 35 84 30 57 204 0 4 ...
## $ SalePrice    : int  208500 181500 223500 140000 250000 143000 307000 200
## 000 129900 118000 ...

Housing<-Housing[,-c(1,2,13,16,17)]
#head(Housing)
#str(Housing)
#sum(is.na(Housing))
#sapply(Housing, function(x) sum(is.na(x)))# number of nas
house.train<-Housing[1:1460,]
str(house.train)

## 'data.frame':    1460 obs. of  13 variables:
## $ MasVnrArea   : int  196 0 162 0 350 0 186 240 0 0 ...
## $ BsmtFinSF1   : int  706 978 486 216 655 732 1369 859 0 851 ...
## $ BsmtUnfSF    : int  150 284 434 540 490 64 317 216 952 140 ...
## $ TotalBsmtSF  : int  856 1262 920 756 1145 796 1686 1107 952 991 ...
## $ X1stFlrSF    : int  856 1262 920 961 1145 796 1694 1107 1022 1077 ...
## $ X2ndFlrSF    : int  854 0 866 756 1053 566 0 983 752 0 ...
## $ GrLivArea    : int  1710 1262 1786 1717 2198 1362 1694 2090 1774 1077 ..
##
## $ BsmtFullBath: int  1 0 1 1 1 1 1 1 0 1 ...
```

```
## $ FullBath      : int  2 2 2 1 2 1 2 2 2 1 ...
## $ BedroomAbvGr: int  3 3 3 3 4 1 3 3 2 2 ...
## $ TotRmsAbvGrd: int  8 6 6 7 9 5 7 7 8 5 ...
## $ GarageArea   : int  548 460 608 642 836 480 636 484 468 205 ...
## $ SalePrice    : int  208500 181500 223500 140000 250000 143000 307000 200
000 129900 118000 ...
```

```
house.test<-Housing[1461:2919,1:12]
str(house.test)
```

```
## 'data.frame':    1459 obs. of  12 variables:
## $ MasVnrArea   : int  0 108 0 20 0 0 0 0 0 0 ...
## $ BsmtFinSF1   : int  468 923 791 602 263 0 935 0 637 804 ...
## $ BsmtUnfSF    : int  270 406 137 324 1017 763 233 789 663 0 ...
## $ TotalBsmtSF  : int  882 1329 928 926 1280 763 1168 789 1300 882 ...
## $ X1stFlrSF    : int  896 1329 928 926 1280 763 1187 789 1341 882 ...
## $ X2ndFlrSF    : int  0 0 701 678 0 892 0 676 0 0 ...
## $ GrLivArea    : int  896 1329 1629 1604 1280 1655 1187 1465 1341 882 ...
## $ BsmtFullBath: int  0 0 0 0 0 0 1 0 1 1 ...
## $ FullBath     : int  1 1 2 2 2 2 2 2 1 1 ...
## $ BedroomAbvGr: int  2 3 3 3 2 3 3 3 2 2 ...
## $ TotRmsAbvGrd: int  5 6 6 7 5 7 6 7 5 4 ...
## $ GarageArea   : int  730 312 482 470 506 440 420 393 506 525 ...
```

```
imp<-apply(house.train,2,mean,na.rm= T)
house.reg<- house.train
for(i in 1:ncol(house.train))
{house.reg[is.na(house.reg[,i]),i]<-imp[i]}
sapply(house.reg, function(x) sum(is.na(x)))#imputing nas with col means
```

```
## MasVnrArea BsmtFinSF1 BsmtUnfSF TotalBsmtSF X1stFlrSF
## 0 0 0 0 0
## X2ndFlrSF GrLivArea BsmtFullBath FullBath BedroomAbvGr
## 0 0 0 0 0
## TotRmsAbvGrd GarageArea SalePrice
## 0 0 0
```

```
house.train<-house.reg
```

Now that we have obtained our required train and test data sets, we perform linear regression as follows:

```
library(MASS)
model1<- lm(SalePrice ~ .,data = house.train )
#model2 <- stepAIC(model1)
summary(model1)

##
## Call:
## lm(formula = SalePrice ~ ., data = house.train)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -632865  -17748       792   17766  270896
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -5719.269   5814.129  -0.984  0.32544
## MasVnrArea     47.016     7.138    6.587 6.26e-11 ***
## BsmtFinSF1     11.903     7.398    1.609  0.10786
## BsmtUnfSF       6.275     7.397    0.848  0.39638
## TotalBsmtSF    34.856     8.409    4.145 3.59e-05 ***
## X1stFlrSF     57.582    24.239    2.376  0.01765 *
## X2ndFlrSF     69.282    23.829    2.908  0.00370 **
## GrLivArea     -5.529    23.570   -0.235  0.81458
## BsmtFullBath  12143.528  2986.478   4.066 5.04e-05 ***
## FullBath      21510.271  2728.546   7.883 6.22e-15 ***
## BedroomAbvGr -17210.216  1955.078  -8.803 < 2e-16 ***
## TotRmsAbvGrd  4528.538  1443.424   3.137  0.00174 **
## GarageArea    72.826     6.663   10.929 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 42830 on 1447 degrees of freedom
## Multiple R-squared:  0.7118, Adjusted R-squared:  0.7094
## F-statistic: 297.8 on 12 and 1447 DF,  p-value: < 2.2e-16
```

We use the above built model to predict the SalePrices in the test dataset.

```
x <- predict(model1, house.test)
summary(x)

##      Min. 1st Qu.  Median      Mean 3rd Qu.      Max.      NA's
##   38368 128170 169124 178159 214849 721882        18
```

We have submitted these results to the Kaggle Competition and obtained a score of 0.21 with a rank of 4525 on 04.24.2018.

Following we perform the principle component regression, which provides us with reduced dimensions keeping most of the variability, avoids multicollinearity between the predictor variables and reduces the risk of overfitting.

However, doing so the interpretation becomes difficult and also some deviation of accuracy from the complete model prediction.

```
# creating dataset for predictions using pca
p.house.new <- house.new
a <- row.names(house.new[house.new$outlier=="Yes",])
p.house.train <- house.train
p.house.train[-c(as.numeric(a)),] -> p.house.train.new
```

We build the training dataset using the 3 PC scores and append the SalePrice variable.

```
#Creating training dataset
p.house.train.data <- data.frame(house.pca$scores[,1:3], SalePrice = p.house.
train.new$SalePrice)

#Test dataset pca
y <- predict(house.pca, newdata = house.test)

#PCR
mf1 <- lm(SalePrice ~ ., data = p.house.train.data)
results <- predict(mf1, as.data.frame(y[,1:3]))
```

We have submitted these results to the Kaggle competition as well and obtained a score of 0.23 and rank 4585, which is lower than the complete linear regression model as expected, since our 3 principle components captured only 75 percent of variation in the data set.

Future Work and Analysis

- The clustering features show that we have distinct classes of houses in the market for sale. We can relate the prices of the houses sold with their classes to find further meaningful patterns. i.e., to answer questions like:
 - Whether houses with big basement do get sold at above average price or lower price?
 - What about the actual sale prices of houses having full baths in basements vs. without full baths?
 - Acquiring further information about the customers demographics would be helpful in understanding what cluster(type) would they prefer to purchase.
 - Extensive regression techniques may be utilized along with considering other variables in the original dataset to ensure accuracy in predictions of Houses Sale Price.
 - Inclusion of categorical variables (like Neighbourhood) in analysis would justify or contradict the outliers excluded.
-

References

Dataset

- House prices advance regression techniques, Kaggle.com (<https://www.kaggle.com/c/house-prices-advanced-regression-techniques>)

Textbook

- https://ttu.blackboard.com/bbcswebdav/pid-3383258-dt-content-rid-25030997_1/courses/201857-ISQS-6350-001/An%20Introduction%20to%20Applied%20Multivariate%20Analysis-Everet.pdf

Outlier Analysis

- Outlier Detection with Mahalanobis Distance. (2016, December 08). Retrieved from <https://www.r-bloggers.com/outlier-detection-with-mahalanobis-distance/>

PCA and Factor Analysis

- Factor Analysis and Principal Components. (2002, May 07). Retrieved from <https://www.sciencedirect.com/science/article/pii/S0047259X8571069X>
- Use of Exploratory Factor Analysis in Published Research. (n.d.). Retrieved from <http://journals.sagepub.com/doi/abs/10.1177/0013164405282485>

Multivariate Regression +

- Manjula, R., Jain, S., Srivastava, S., & Kher, P. R. (2017). Real estate value prediction using multivariate regression models. IOP Conference Series: Materials Science and Engineering, 263, 042098. doi:10.1088/1757-899x/263/4/042098

Principle component regression

- Bansal, S., Dar, P., Jain, K., Jain, S., & Analytics Vidhya Content Team. (2016, July 27). Practical Guide to Principal Component Analysis (PCA) in R & Python. Retrieved from <https://www.analyticsvidhya.com/blog/2016/03/practical-guide-principal-component-analysis-python/>
 - Jolliffe, I. (1982). A Note on the Use of Principal Components in Regression. Journal of the Royal Statistical Society. Series C (Applied Statistics), 31(3), 300-303. doi:10.2307/2348005
-