Analysis of Numerical Methods

Assignment 2

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1.

$$
\begin{aligned}
(1.1) \qquad & \langle u, u_t \rangle + \langle u, Au_x \rangle = 0 \\
& \to \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|u\|^2 = -\int_0^1 u^T A u_x \,\mathrm{d}x \\
& = -\int_0^1 u_1(x)u_{2,x}(x)\,\mathrm{d}x - \int_0^1 u_2(x)u_{1,x}(x)\,\mathrm{d}x \\
& = -u_1(x)u_2(x)\Big|_0^1 + \int_0^1 u_{1,x}(x)u_2(x)\,\mathrm{d}x - \int_0^1 u_2(x)u_{1,x}(x)\,\mathrm{d}x \\
& = -u_1(x)u_2(x)\Big|_0^1 \\
& = 0
\end{aligned}
$$

Therefore the square of the norm of u is constant, and given by the square norm in the initial condition.

$$
\begin{aligned}
(1.2) \qquad & \|u(\cdot,t)\|^2 = \|u(\cdot,0)\|^2 = \|f(\cdot)\|^2 \\
& \to \|u(\cdot,t)\| = \|f(\cdot)\|
\end{aligned}
$$

2.