Pricing of Barrier Options using Finite Difference

For up-and-out options

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Barrier Options

- Barrier options are options which either come into play or expire worthelessly if the underlying asset hits a predetermined value B.
- *Up-an-out options* are which expire worthlessly when they exceed a certain barrier B.

Methodology

We start with the standard Black-Scholes equation for pricing options

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} - rV = 0 \tag{0.1}$$

and then discretize in S space using a standard equidistant grid and the standard second-order central finite-difference methods for the derivatives.

Methodology

$$\frac{\partial V}{\partial S} = \frac{V(t, S + \Delta S) - V(t, S - \Delta S)}{2\Delta S}, \tag{0.2}$$
$$\frac{\partial^2 V}{\partial S^2} = \frac{V(t, S + \Delta S) - 2V(t, S) + V(t, S - \Delta S)}{\Delta S^2}$$

Methodology

We Discretize in time by making the change of variable $t \to \tau, \tau = T - t$ and integrate using the BDF-2 scheme. The BDF-2 scheme is given by

$$V^+ - \frac{4}{3}V + \frac{1}{3}V^- = \frac{2}{3}\Delta t A V^+ \tag{0.3}$$
$$\to (3I - 2\Delta t A)V^+ = 4V - V^-$$

Where A is a tridiagonal matrix constructed using the finite-difference scheme we showed before.

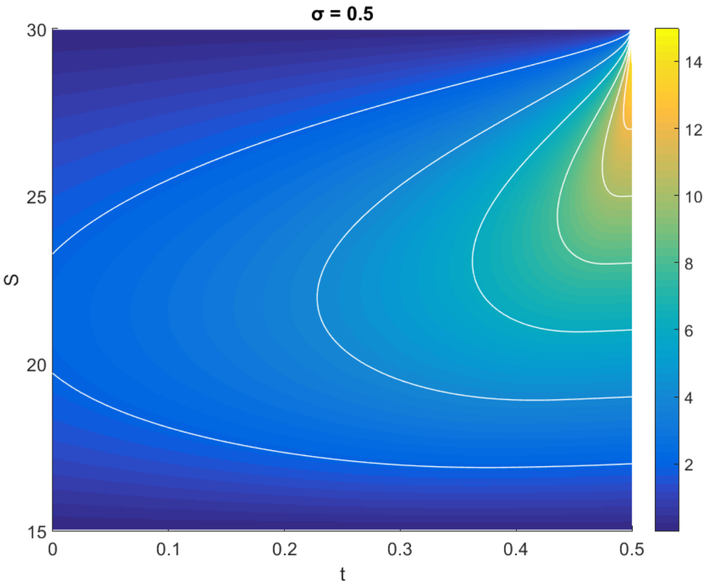
Boundary Conditions

The boundary conditions on a up-and-out barrier option are given by

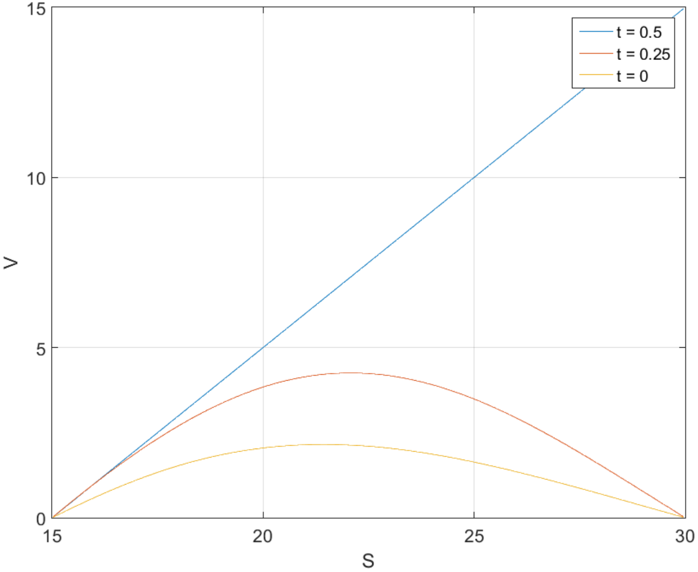
$$V(S > \text{barrier}) = V(S < K) = 0. \tag{0.4}$$

This is easy to understand intuitively, as these are the conditions imposed on us by the definition of the barrier and the pay-off function respectively.

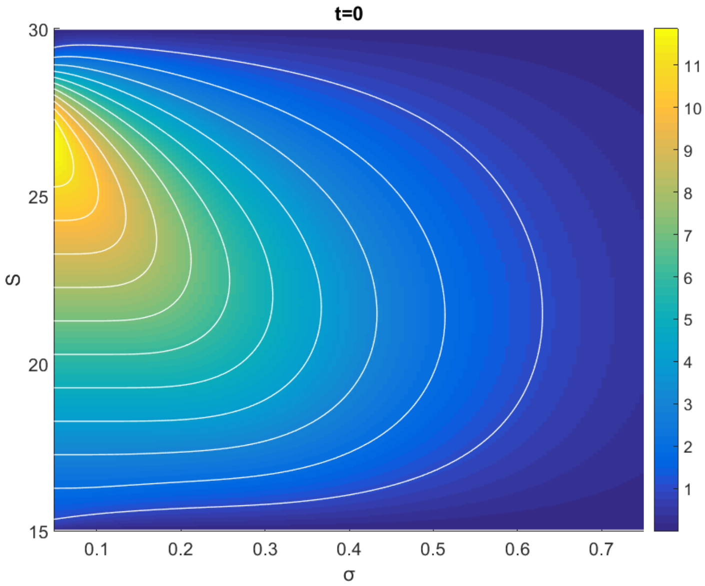
Results



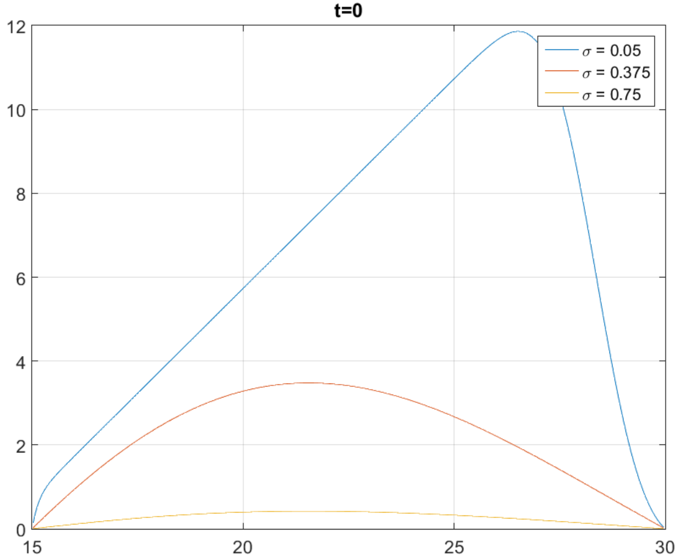
Results



Results



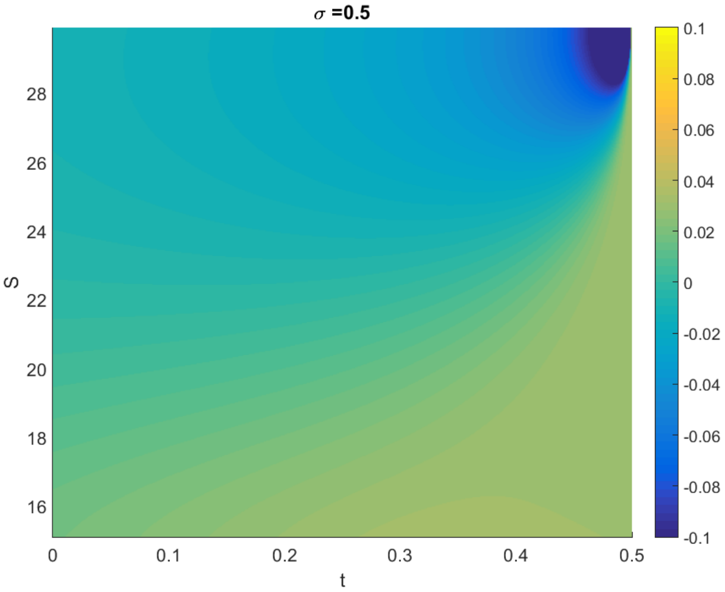
Results



Greeks

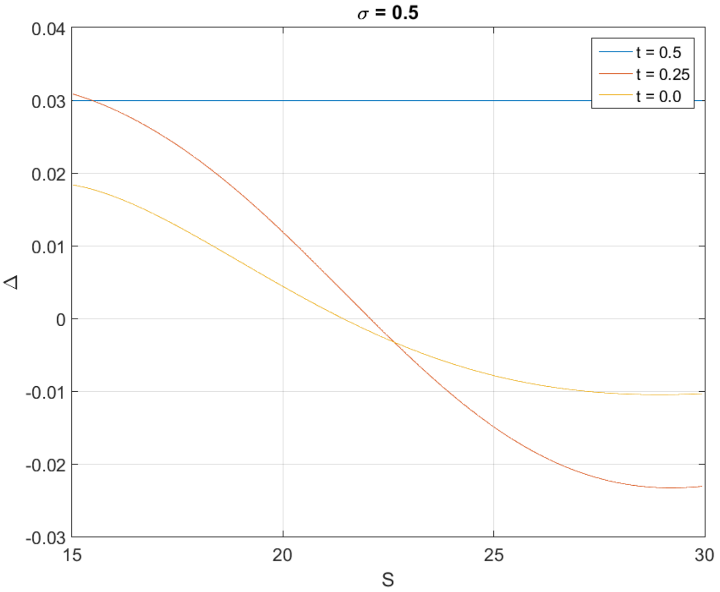
- Δ is a measure of the rate of change in the value V as function of S. It is given by $\Delta = \frac{\partial V}{\partial S}$.
- ν (Vega) is a measure of how the value of an option changes with the implied volatility of the underlying asset. It is defined as $\nu = \frac{\partial V}{\partial \sigma}$.

Δ

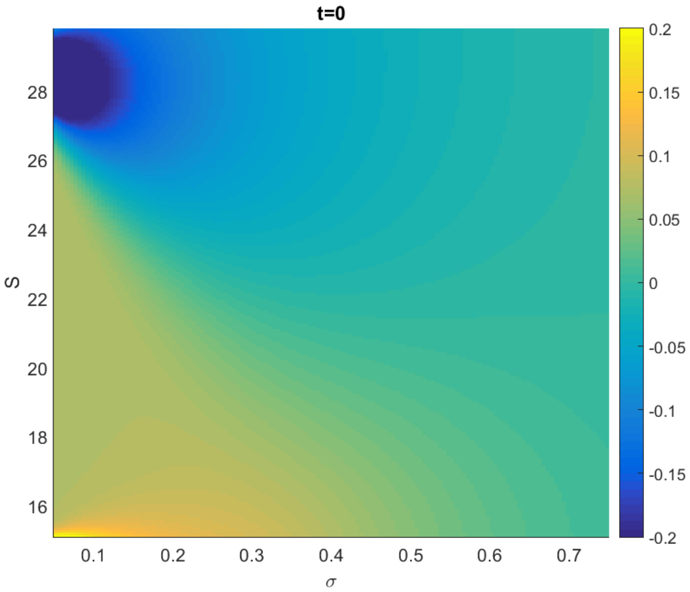


σ =0.5
S
t
0
0.1
0.2
0.3
0.4
0.5
16
18
20
22
24
26
28
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1

Results

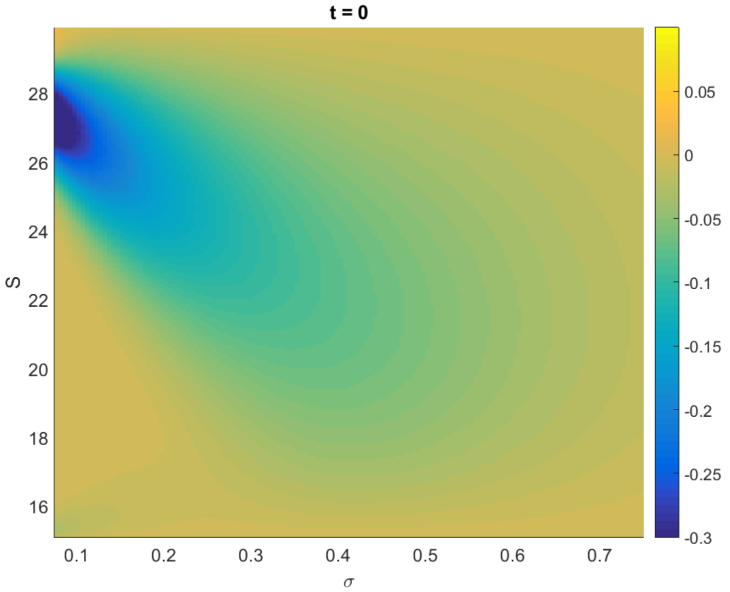


Δ



t=0
28
26
24
22
20
18
16
S
0.1
0.2
0.3
0.4
0.5
0.6
0.7
σ
0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
-0.2

ν


t = 0
28
26
24
22
20
18
16
S
0.1
0.2
0.3
0.4
0.5
0.6
0.7
σ
0.05
0
-0.05
-0.1
-0.15
-0.2
-0.25
-0.3