Project
Computational Physics

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1. *Task 1*

1a) *Analytical Solution to the Potential Well/Barrier Problem*

start with

$$\int \frac{\mathrm{d}r}{r^2\sqrt{1-\frac{b^2}{r^2}-a}} \tag{1.1}$$

and subsitute $x = b/r,\ \mathrm{d}x = -b/r^2\,\mathrm{d}r$ giving:

$$\begin{aligned}&\int \frac{\mathrm{d}x}{-b\sqrt{1-x^2-a}} \\ =&\int \frac{\mathrm{d}x\sqrt{\frac{1}{1-a}}}{-b\sqrt{1-a-x^2}\sqrt{\frac{1}{1-a}}} \\ =&-\frac{1}{b\sqrt{1-a}}\int \frac{\mathrm{d}x}{\sqrt{1-\left(\frac{x}{\sqrt{1-a}}\right)^2}}\end{aligned} \tag{1.2}$$

now subsitute $u = x/\sqrt{1-a}$ and $\mathrm{d}u = \mathrm{d}x/\sqrt{1-a}$:

$$\begin{aligned}&-\frac{1}{b}\int \frac{\mathrm{d}u}{\sqrt{1-u^2}} \\ =&-\frac{1}{b}\arcsin u\end{aligned} \tag{1.3}$$

Now all that is left is backsubsitution of $u \to x \to r$, which leads to:

$$\begin{aligned}&-\frac{1}{b}\arcsin u \\ =&-\frac{1}{b}\arcsin\frac{x}{\sqrt{1-a}} \\ \int \frac{\mathrm{d}r}{r^2\sqrt{1-\frac{b^2}{r^2}-a}} =&-\frac{1}{b}\arcsin\frac{b}{r\sqrt{1-a}}\end{aligned} \tag{1.4}$$