S. Leupold **October – December 2015**

Hadron and Quark Physics

Hand-in problems 4

1. **Time-like and space-like four-vectors:** *5 points*

   (a) Given a four-vector p with $p^2 > 0$, i.e. $(p^0)^2 - \vec{p}^{\,2} > 0$. Such a four-vector is called time-like. Show that one can find a Lorentz transformation (boost operation) $p \to p'$ such that the three-momentum $\vec{p}\,'$ vanishes in the new frame.
   Hint: Use of rotations is allowed. Feel free to choose your coordinate system such that $\vec{p}$ points along one of the axes of your coordinate system.

   (b) Show that it is not possible for $p^2 < 0$ (space-like four-vector) to find a Lorentz transformation such that the three-momentum vanishes in the new frame. Show that instead one can find a Lorentz transformation such that the energy vanishes in the new frame.

   (c) Given two four-vectors p_1 and p_2 of massive particles with masses $m_1, m_2 \neq 0$, i.e. $p_i^0 = \sqrt{m_i^2 + \vec{p}_i^{\,2}}$, $i = 1, 2$. Show that $p := p_1 + p_2$ satisfies $p^2 > 0$. Do <u>not</u> use that one might choose a frame $\vec{p}_1 + \vec{p}_2 = 0$, because this would require $p^2 > 0$.
   Hint: Make use of result (a), i.e. for any four-vector k where you know that $k^2 > 0$ holds you can choose a frame with $\vec{k} = 0$.

   (d) Given two four-vectors q_1 and q_2 of massive particles with the same mass $m \neq 0$. Show that $q := q_1 - q_2$ satisfies $q^2 \leq 0$.

   (e) Show that for a scattering reaction of two particles to two particles the Mandelstam variable s satisfies $s \geq 0$ and that for elastic scattering, i.e. $m_1 = m_3$ and $m_2 = m_4$, the Mandelstam variable t satisfies $t \leq 0$.

2. **Lie algebras:** *5 points*

   Solve task 1 from section 4.6 of the Lecture Notes

3. **A downgraded version of the weak theory:** *10 points*

   Solve task 1 from section 5.4 of the Lecture Notes

To be handed in at 7.12.2015 at the latest.