Quark and Hadron Physics

Assignment 1

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1. *Relativity and Electrodynamics*

1a) *Four-derivative*

Using Feynmann subscript notation:

$$\begin{aligned} \partial^\mu (x \cdot b) &= \partial^\mu (x^\nu b_\nu), \\ &= \partial^\mu_x (x \cdot b) + \partial^\mu_b (x \cdot b), \\ &= (1, -\vec{1}) \cdot (b^0, \vec{b}) + (\frac{\partial b^0}{\partial t}, -\nabla \vec{b}) \cdot (t, \vec{x}), \\ &= (b^0, -\vec{b}) + 0 \cdot x, \\ &= b^\mu. \end{aligned} \tag{1.1}$$

Note that we have had to assume that b is independent of t and x to obtain this result. (only b independent of x was given).

1b) *Relativistic formulation of the Maxwell equations*

If we expand

$$\partial_\mu F^{\mu\nu} = j^\nu \tag{1.2}$$

into it's matrix form we obtain

$$\nabla \cdot E = \rho \tag{1.3}$$

In the first component, which is Guass' law. We now turn our attention to the three other components, expanded they read

$$\begin{aligned} -\frac{\partial E^1}{\partial t} + \frac{\partial B^3}{\partial x_2} - \frac{\partial B^2}{\partial x_3} &= j^1 \\ -\frac{\partial E^2}{\partial t} + \frac{\partial B^1}{\partial x_3} - \frac{\partial B^3}{\partial x_1} &= j^2 \\ -\frac{\partial E^3}{\partial t} + \frac{\partial B^2}{\partial x_1} - \frac{\partial B^1}{\partial x_2} &= j^3 \end{aligned} \tag{1.4}$$

which can be written simultanously in vector form as

$$\frac{\partial}{\partial t}\vec{E} = \nabla \vec{B} - \vec{j}, \tag{1.5}$$

which is Ampere's law.