Optimisation: Ice on Mars

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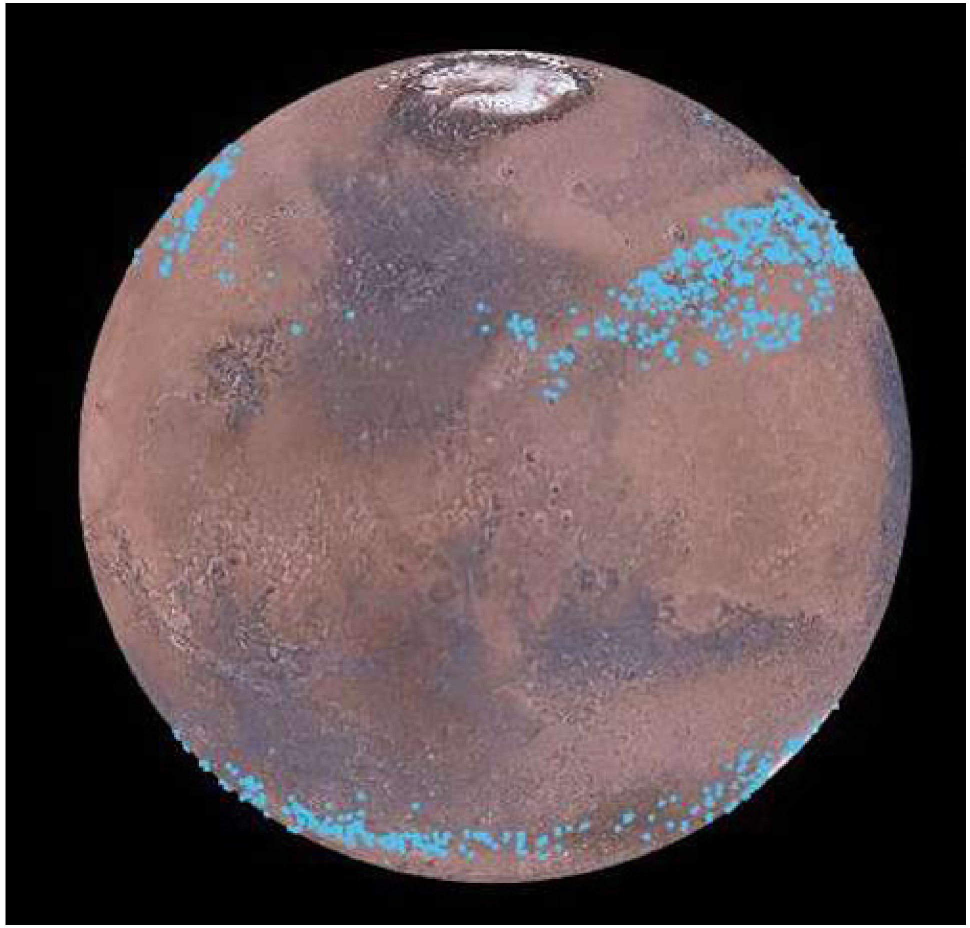
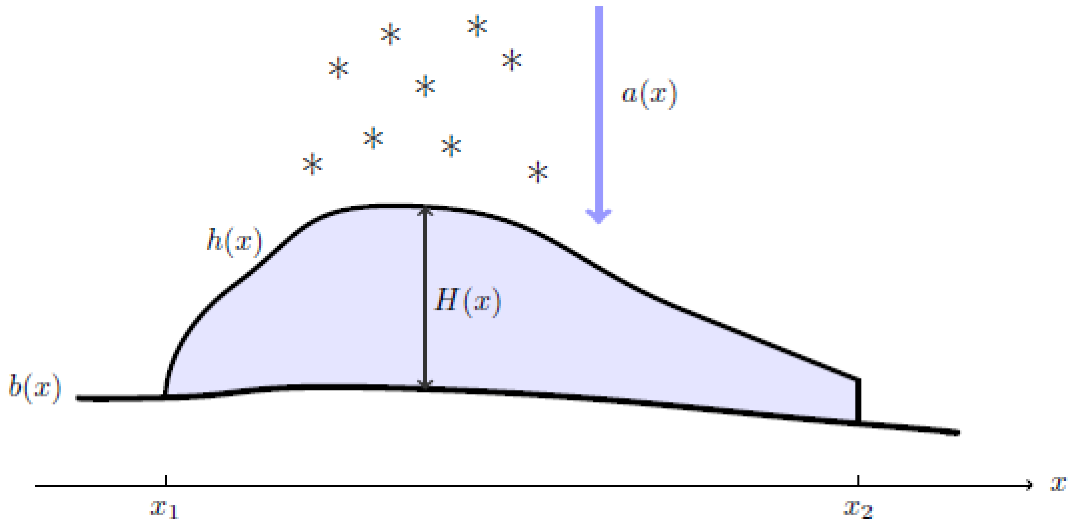


Researchers have identified thousands of glacier-like formations on the planet. The ice in the glaciers is equivalent to over 150 billion cubic meters of ice — that much ice could cover the entire surface of Mars with 1.1 meters of ice. The ice at the mid-latitudes is therefore an important part of Mars' water reservoir.

NASA/Levy et al./Nanna Karlsson

Shallow Ice Approximation



A nonlinear relationship can be established between the thickness $H(x)$ and the mass balance function on the surface $a(x)$ that

$$a(x) = -\frac{2A}{n+2}(\rho g)^n H(x)^{n+2} \left|\frac{\mathrm{d}h}{\mathrm{d}x}\right|^{n-1} \frac{\mathrm{d}h}{\mathrm{d}x}, \qquad (1)$$

Objective Function

- $$\min_{n} \|H(x) - H_{obs}(x)\|_2^2.$$

$$H(x) = \left(\frac{-a(x)(n+2)}{2A(\rho g)^n \left|\frac{dh}{dx}\right|^{n-1} \frac{dh}{dx}} \right)^{\frac{1}{n+2}}$$

In Matlab,

```
diff = ((-a.*(n+2)./(2*A*(rho*g).^n.*abs(dhdx).^(n-1).*dhdx)).^(1/(n+2)))-H_obs;
dH = sum(diff.^2);
```

Optimizing the optimization

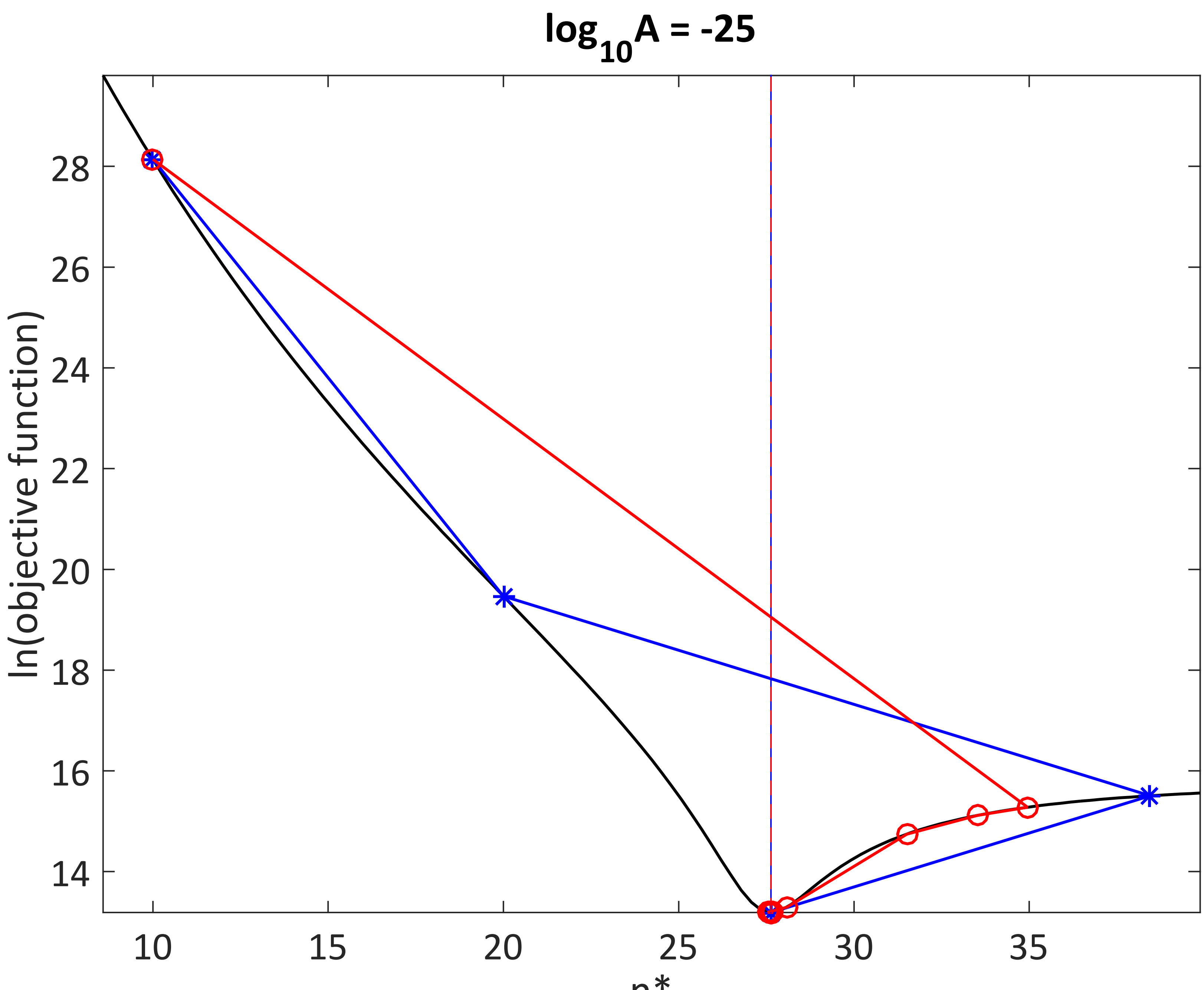
- Rescale the domain
  - n = 0.1*x0(1)
  - A -> 10^x0(2)
- Compute the gradient?
  - Horrible expression, much more expensive to evaluate than finite differences.

Set A = 1e-25, initial n = 3

Solver	fminunc	lsqnonlin
n	2.76327	2.76303
Iterations	5	12
Function calls	18	34
Objective function	13.1875	13.1875

Set A = 1e-25, initial n = 1

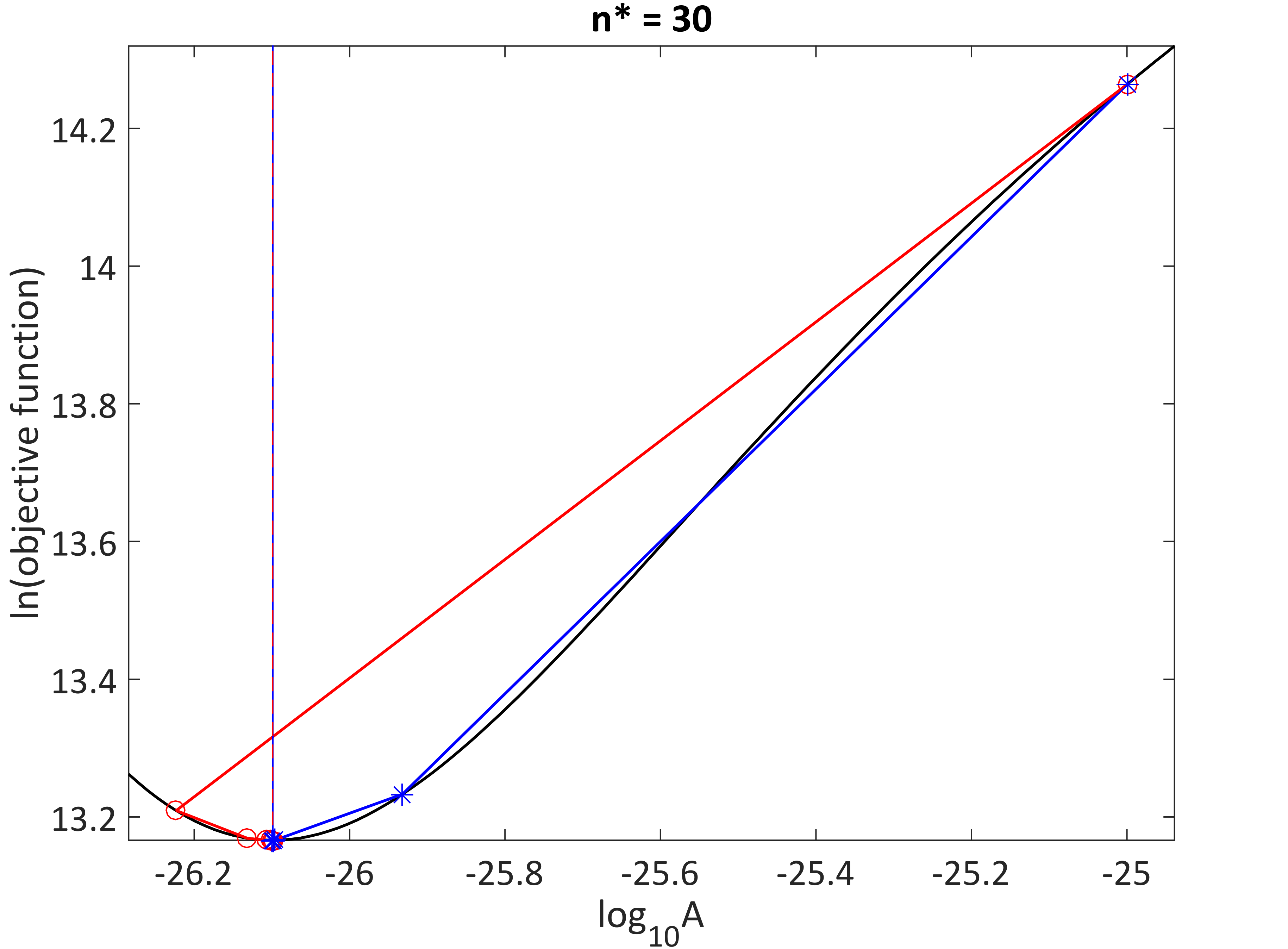
Solver	fminunc	lsqnonlin
n	2.76327	2.76306
Iterations	4	13
Function calls	30	37
ln(Objective function)	13.1875	13.1875



log$_{10}$A = -25
ln(objective function)
n*
28
26
24
22
20
18
16
14
10
15
20
25
30
35

Set n = 3, initial A = 1e-25

Solver	fminunc	lsqnonlin
log10(A)	-26.0987	-26.0989
Iterations	5	6
Function calls	16	20
ln(Objective function)	13.1661	13.1661



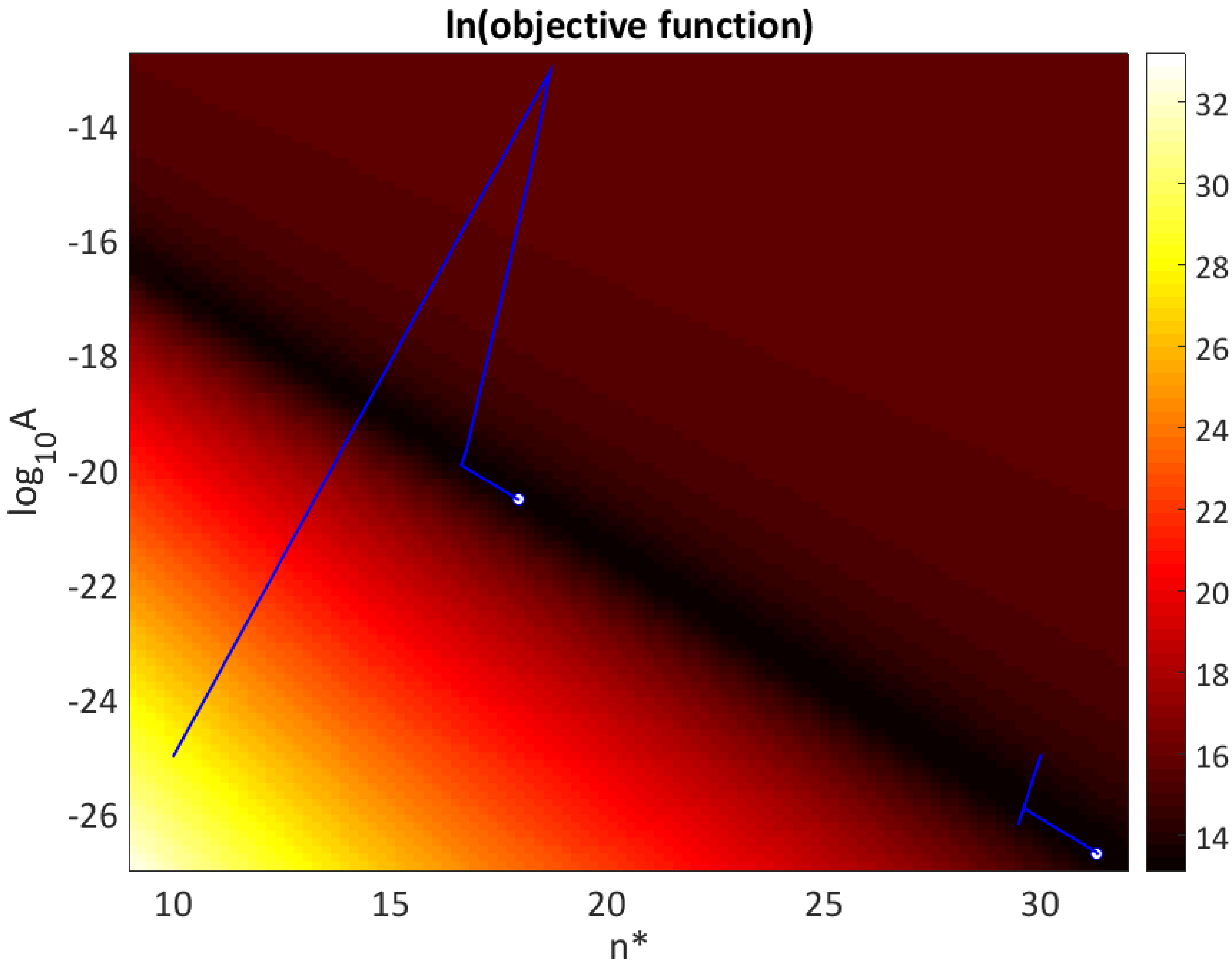
n* = 30
ln(objective function)
14.2
14
13.8
13.6
13.4
13.2
-26.2
-26
-25.8
-25.6
-25.4
-25.2
-25
$\log_{10}A$

Initial n = 1, A = 1e-25

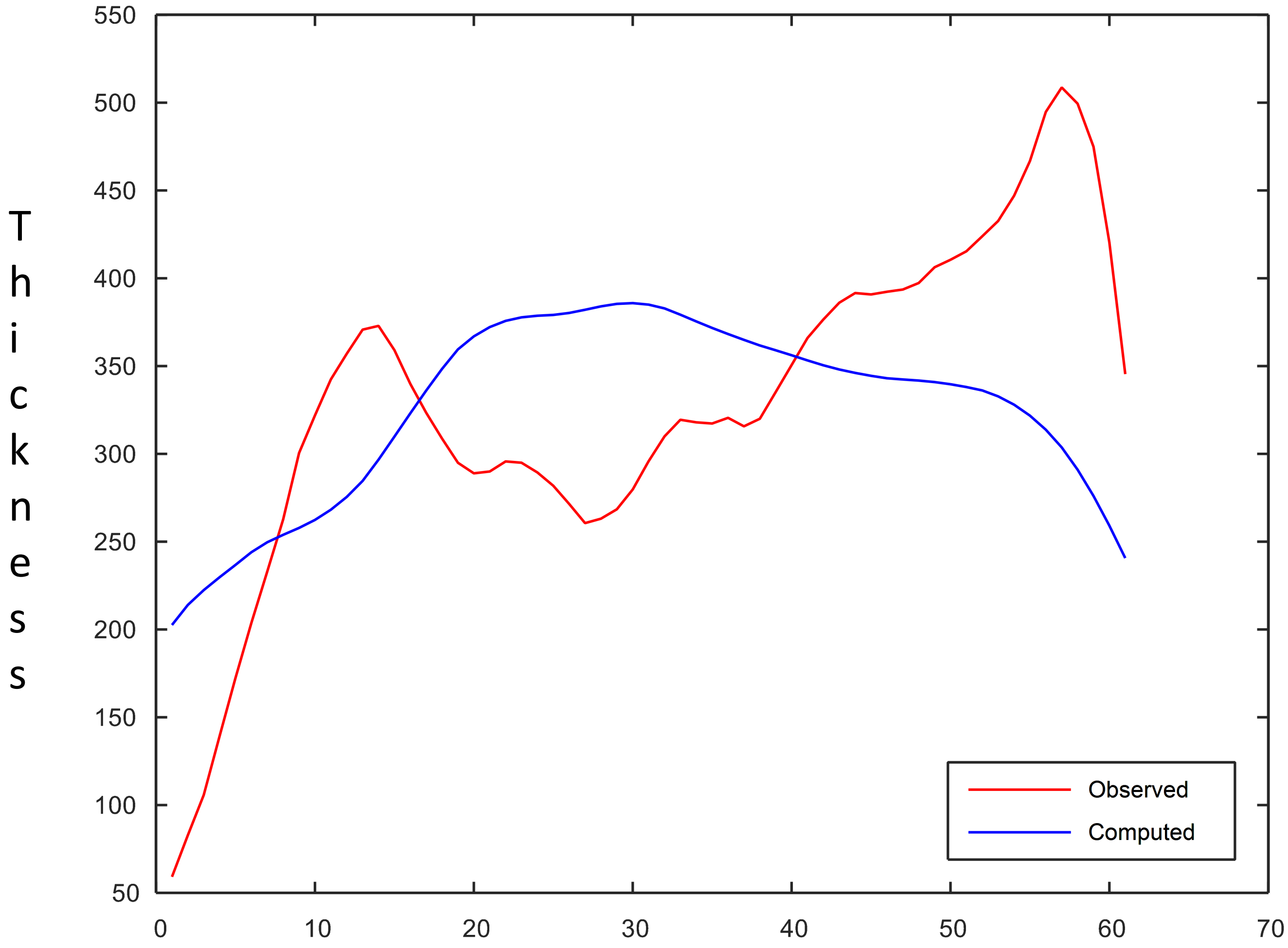
Solver	fminunc	lsqnonlin
n	66	1.8
log10(A)	-320	-20.5
Iterations	20	210
Function calls	243	772
ln(Objective function)	12.9	13.3

Initial n = 3, A = 1e-25

Solver	fminunc	lsqnonlin
n	65.8	3.13
log10(A)	-320	-26.7
Iterations	21	209
Function calls	289	755
ln(Objective function)	12.9	13.2

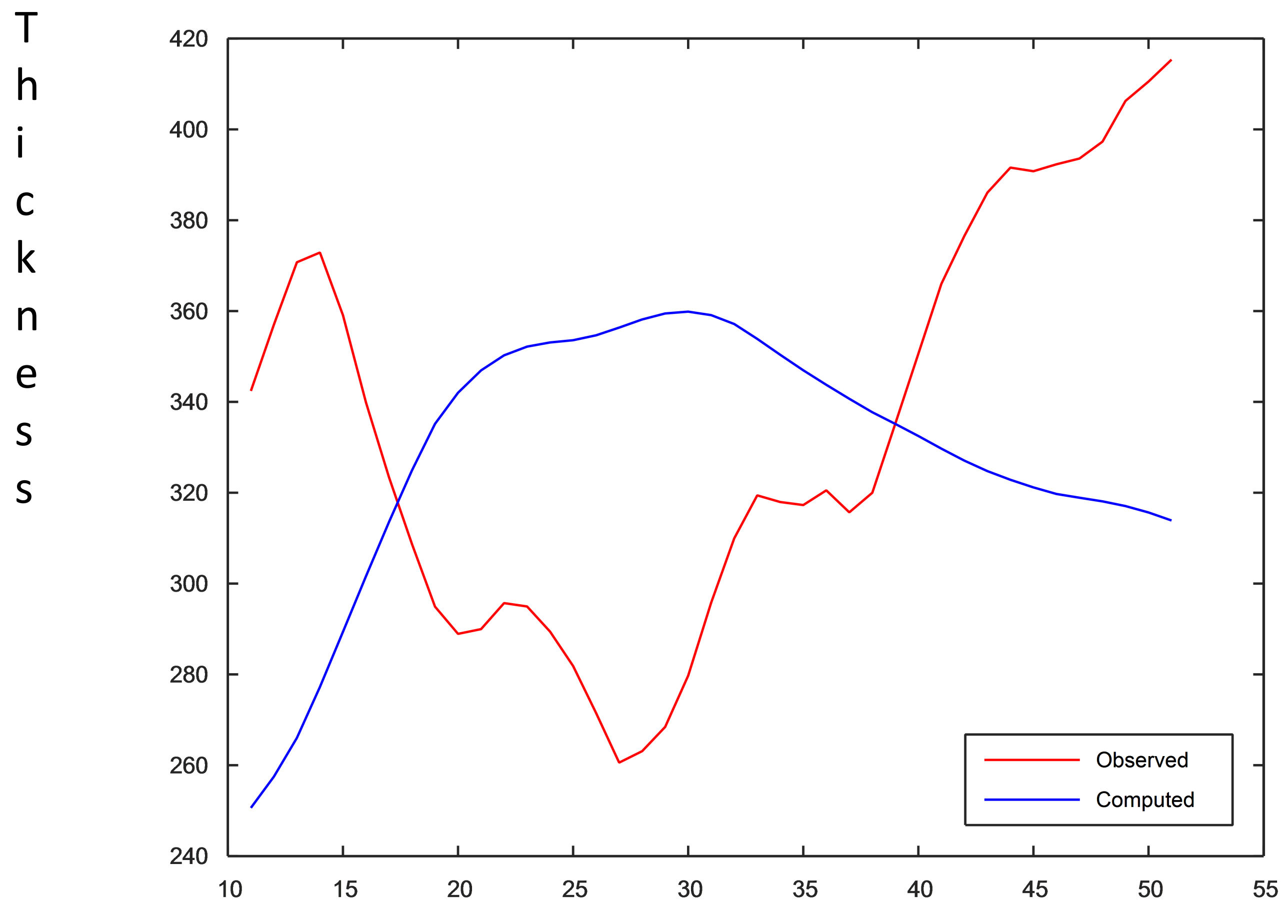


ln(objective function)
log10A
n*
-14
-16
-18
-20
-22
-24
-26
10
15
20
25
30
32
30
28
26
24
22
20
18
16
14



Thickness
550
500
450
400
350
300
250
200
150
100
50
0
10
20
30
40
50
60
70
Observed
Computed

Consider part of the observation



n = 2.8042

A = -25.0447

Obj = 1.807e+5