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Assignment - Heat Conduction

Heat equation

Heat is transferred spontaneously from higher to lower temperature. The heat transport can take place by three different ways, through heat conduction (electrons), convection (flow of media), and by radiation (photons). We will only study heat conduction and disregard the other phenomena.

The heat equation is a parabolic PDE:

$$\rho C \frac{\partial T}{\partial t} - \nabla \cdot (k \nabla T) = Q + h(T_{\text{ext}} - T) \tag{1}$$

It describes the heat transfer process for plane and axisymmetric cases, and uses the following parameters:

- Density ρ
- Heat capacity C
- Coefficient of heat conduction k
- Heat source Q
- Convective heat transfer coefficient h
- External temperature T_{ext}

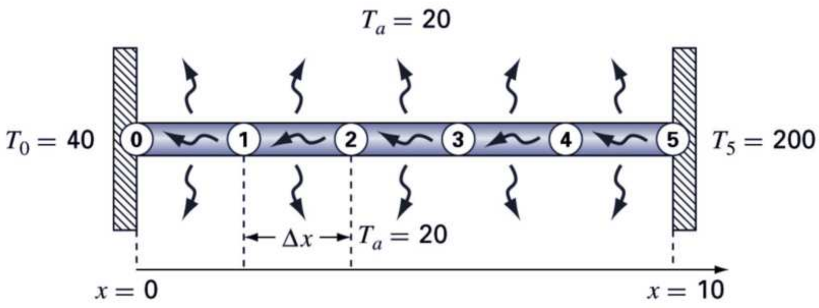
The term $h(T_{\text{ext}} - T)$ is a model of transversal heat transfer from the surroundings, and it may be useful for modeling heat transfer in thin cooling plates etc. The boundary conditions can be of Dirichlet type, where the temperature on the boundary is specified, or of Neumann type where the heat flux, $\mathbf{n}\cdot(k\nabla T)$, is specified. Together with boundary conditions the heat equation above describes the temperature distribution in a homogeneous isotropic solid body.

Model Problem

Consider heat conduction in a thin rod positioned between two walls with constant temperature, see Figure 1. At the left wall we have a temperature 40C and at the right wall we have 200C. Heat flows through the rod as well as between the rod and the surrounding air where the temperature is 20C. This can be modeled with the one dimensional steady state heat equation,

$$-k\frac{d^2T}{dx^2} = Q + h(T_{\text{ext}} - T). \tag{2}$$

Solve the steady state problem using the FEM with an equidistant grid from x=0 to x=10 and compute the temperature distribution. Use k=1, h=1 and Q=0. On the boundaries you can use Dirichlet condition setting the temperature to the given values. Derive the corresponding linear system of equations in detail and solve it in MATLAB. In this case we can derive the analytical solution to the heat equation. Compare your numerical solution to the analytical solution and do convergence studies. How fast does the error decrease when we decrease the step size between the nodes?



Figur 1: Model problem, a noninsulated uniform rod positioned between two walls of constant but different temperature. The domain is discretized into 6 nodes.

Solving an Application

As a new employed engineer you have been assigned a task to control the temperature in a critical part of a machine. It is important that temperature does not exceed a certain threshold in the part. To calculate the temperature you will need to solve the heat equation in 2D.

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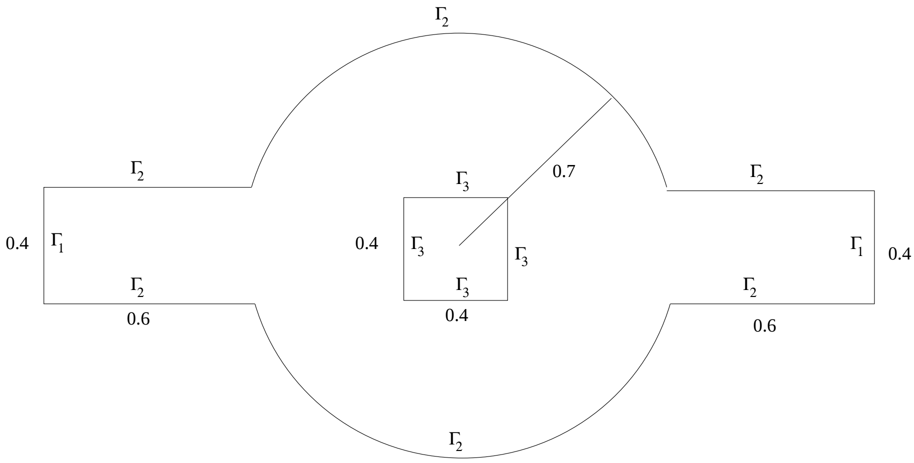
The Problem

We will solve the heat equation under stationary conditions (we have temperature equilibrium, i.e. no time dependency) and without any heat sources inside the body. Figure 2 defines the geometry of the body (consider a thin plate). On the surface and on the boundaries Γ_1 we have heat ex-



Figur 2: Computational domain with boundaries.

change with the outside media, e.g. cooling by convection, airflow with temperature T_0. The boundary conditions on Γ_1 becomes

$$n \cdot (k\nabla T) = h(T_0 - T).$$

On the boundaries Γ_3 we will have the same boundary condition but with the external temperature T_1. The boundaries Γ_2 are perfect insulators, the normal component of the heat flow is zero,

$$n \cdot \nabla T = 0.$$

Task 1, stationary solution

Solve the linear stationary heat equation with MATLAB's PDEtoolbox. Model a body exactly as described in Figure 2. In the middle of the body, on boundary Γ_3, we have a heat flow with temperature $T_1 = 400$K. Outside the body the external temperature is $T_{\text{ext}} = 290$K and on boundary Γ_1, we have cooling with temperature $T_0 = 200$K. What is the maximum temperature in the body? Use the parameters $k = 3$ and $h = 2$. Make sure that your answer does not depend on the discretization of the domain.

Task 2, time dependent solution

Start from an initial constant temperature $T(t_0) = 290$K on the plate and compute the solution for different time steps. When will the solution reach its steady state? (You can use the scaled parameters $C = 1, \rho = 1$ or look up material specific constants to make it more realistic.)

Task 3, problem of own choice

Model heat conduction on a problem or geometry of your own choice. The geometry should be realistic and non-trivial to model.

Report

No formal report is required but present your theoretical derivations and your numerical experiments with relevant figures, tables, reflections and explaining text in a form of a diary entry, i.e., a short informal report and submit in Studentportalen.