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Workout — Iterative Methods

Mandatory exercises

1. You are going to solve the stationary heat equation for a steel bar whose endpoints are kept at different fixed temperatures. Your discretization leads to the following linear system of equations

$$\underbrace{\begin{pmatrix} -2 & 1 & & & \\ 1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ & & & 1 & -2 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_{N-1} \\ u_N \end{pmatrix}}_{u} = \underbrace{\begin{pmatrix} \alpha \\ 0 \\ \vdots \\ 0 \\ \beta \end{pmatrix}}_{b}$$

In this exercise you will apply Jacobi's method to this problem.

   (a) For the case of $N = 3$ and $u^{(0)} = [0 \quad 0 \quad 0]^T$: compute $u^{(1)}$ and $u^{(2)}$ by hand, to become familiar with the steps in Jacobi's method.

   (b) Write pseudo-code for Jacobi's method applied to the problem above, for general N and $u^{(0)}$.

2. Stationary iterative methods can be formally expressed as $u^{(k+1)} = Gu^{(k)} + c$. Does the iterative method converge for all $u^{(0)}$ if the matrix G has eigenvalues

   (a) $\lambda = \{-0.35,\ 0.9,\ -0.1,\ 0,\ 0.5\}$?

   (b) $\lambda = \{0.1,\ 0.2,\ 0.3,\ 0.4,\ 0.5,\ 0.6,\ 0.7,\ 0.8,\ 0.9,\ 1\}$?

3. Carry out the steps in two iterations of the Power Method applied to the following matrix:

$$A = \begin{pmatrix} -0.4950 & -0.5050 \\ -0.5050 & -0.4950 \end{pmatrix}$$

Use $[1 \quad 0]^T$ as initial vector. What is the resulting approximate eigenvalue?

Non-mandatory exercises

5. Repeat Exercise 1 for Gauss-Seidel's method.

6. Does the stationary iterative method $u^{(k+1)} = Gu^{(k)} + c$ converge for all $u^{(0)}$ if the matrix Gs has eigenvalues $\lambda = \{\cos(\pi j/(N+1))\}_{j=1}^{N}$?

7. A matrix A has 5 non-zero diagonals (including the main diagonal). Due to fill-in, the L and U factors of A each have 201 non-zero diagonals. The matrix A is $N \times N$, where $N = 200\,000$. Roughly how many floating point numbers need to be stored when using

   (a) Jacobi's method?

   (b) LU factorization?

   You may assume that each diagonal has the same length and you can neglect everything except the matrices involved.

8. For the same matrix as in the previous exercise, let's say that you have decided to use the Power Method to compute its largest magnitude eigenvalue. What will the execution time per iteration be if the average time per floating point operation is 10^{-9} seconds?