# Workout 3

## Scientific Computing III

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### 1.

This method can be described (ignoring the boundary conditions) as

$$D_{+,t}u = D_{0,x}u \tag{1.1}$$

or equivalently The error of $D_{+,t}$ can be determined using Taylor expension in the following fashion (we expand around $\bar{x}$):

$$u(\bar{x} + \Delta) \approx u(\bar{x}) \pm \Delta u'(\bar{x}) + \frac{1}{2}\Delta^2 u''(\bar{x}) \pm \frac{1}{6}\Delta^3 u'''(\bar{x}) + \mathcal{O}(\Delta)^4 \tag{1.2}$$

From this we find that

$$\begin{aligned} D_{+,t}u &= \frac{u(x, \bar{t} + \Delta t) - u(x, \bar{t})}{\Delta t}, \\ &= \frac{\Delta t \frac{\partial u}{\partial t} + \frac{1}{2}\Delta t^2 \frac{\partial^2 u}{\partial t^2} + \mathcal{O}(\Delta t^3)}{\Delta t}, \\ &= \frac{\partial u}{\partial t} + \frac{1}{2}\Delta t \frac{\partial^2 u}{\partial t^2} + \mathcal{O}(\Delta t^2). \end{aligned} \tag{1.3}$$

And

$$\begin{aligned} D_{0,x}u &= \frac{u(\bar{x} + \Delta x, t) - u(\bar{x} - \Delta x, t)}{2\Delta x}, \\ &= \frac{2\Delta x \frac{\partial u}{\partial x} + 2\frac{1}{6}\Delta x^3 \frac{\partial^3 u}{\partial x^3} + \mathcal{O}(\Delta x^4)}{2\Delta x}, \\ &= \frac{\partial u}{\partial x} + \frac{1}{6}\Delta x^2 \frac{\partial^3 u}{\partial x^3} + \mathcal{O}(\Delta x^3). \end{aligned} \tag{1.4}$$

Now we can compute the local truncation error by subtracting the rhs from the lhs of the approximate model (dropping the big-oh terms):

$$\begin{aligned} \tau(x,t) &= D_{+,t}u - D_{0,x}u \\ &= \left(\frac{\partial u}{\partial t} + \frac{1}{2}\Delta t \frac{\partial^2 u}{\partial t^2}\right) - \left(\frac{\partial u}{\partial x} + \frac{1}{6}\Delta x^2 \frac{\partial^3 u}{\partial x^3}\right) \\ &= \frac{\partial u}{\partial t} - \frac{\partial u}{\partial x} + \left(\frac{1}{2}\Delta t \frac{\partial^2 u}{\partial t^2} - + \frac{1}{6}\Delta x^2 \frac{\partial^3 u}{\partial x^3}\right) \end{aligned} \tag{1.5}$$

### 2.

The above equation shows that the order-of-accuracy is $\mathcal{O}\left(\Delta t + \Delta x^2\right)$

### 3.

A method is said to be consistent if $\tau(x,t) \to 0$ as $\Delta x, \Delta t \to 0$. This is the case here (the 'real' derivatives are equal by the definition) so the method is consistent.