UPPSALA
UNIVERSITET

Institutionen för informationsteknologi
Beräkningsvetenskap

Besöksadress:
MIC hus 2, Polacksbacken
Lägerhyddvägen 2

Postadress:
Box 337
751 05 Uppsala

Telefon:
018–471 0000 (växel)

Telefax:
018–51 19 25

Hemsida:
http:/www.it.uu.se/

Department of Information Technology
Scientific Computing

Visiting address:
MIC bldg 2, Polacksbacken
Lägerhyddvägen 2

Postal address:
Box 337
SE-751 05 Uppsala
SWEDEN

Telephone:
+46 18–471 0000 (switch)

Telefax:
+46 18–51 19 25

Web page:
http:/www.it.uu.se/

Lab — Iterative Methods

Convergence of iterative methods

Iteration matrices of the soap film matrix

We will now investigate the iteration matrices for Jacobi's method and Gauss-Seidel's method, respectively. Each of these methods can mathematically be expressed as:

$$x^{(k+1)} = Gx^{(k)} + d$$

where G is a matrix and d a vector. The rate of convergence for an iterative method will depend on the eigenvalues of its *iteration matrix* G. More precisely, the successive approximations $x^{(k)}$ will converge if the *spectral radius* of G is less than one. The spectral radius is the maximum absolute value of the eigenvalues of G and is denoted by $\rho(G)$.

When you have read the introduction above you should:

- Use `soapfilm` to generate the coefficient matrix A for the soap film problem. The command `A=soapfilm(Nx,Ny);` with $N_x = N_y = 16$ will generate the matrix for the case that you used in the experiments with Jacobi's and Gauss-Seidel's method in the previous part of this lab session.
- Next, use the command `GJ = JacobiMatrix(A)` to generate the iteration matrix for Jacobi's method in this case. Then, use the command `GGS = GaussSeidelMatrix(A)` to generate the iteration matrix for Gauss-Seidel's method.
- To compute the spectral radius of `GJ`, do `max(abs(eig(GJ)))` (read Matlab's Help to learn about the different commands in this expression). You can compute the spectral radius of `GGS` analogously, but now you should use `eigs` instead of `eig` since `GGS` is stored in so called sparse format.

Pay attention to:

- What is the relation between the rate of convergence for Jacobi's method and the rate of convergence for Gauss-Seidel?

UPPSALA
UNIVERSITET

- Does the result in this experiment agree with your observations about number of iterations for Jacobi and Gauss-Seidel in the previous part of the lab session?