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Lab — Iterative Methods

Memory requirements for LU factorization of sparse, banded matrices

Introduction

Large linear systems of equations arise when solving PDEs numerically. Here, we let the computational domain be $\Omega = [0\ 1] \times [0\ 1]$ (the unit square), and solve the problem using finite differences. The PDE is given by

$$\begin{cases} \Delta u(x,y) &= 0, & (x,y) \in \Omega, \\ u(x,y) &= r(x,y), & (x,y) \in \partial\Omega. \end{cases} \tag{1}$$

Let $h_x = \frac{1}{N_x+1}$ and $h_y = \frac{1}{N_y+1}$. Then the discrete problem is

$$\begin{cases} \frac{u_{i+1,j}-2u_{i,j}+u_{i-1,j}}{h_x^2} + \frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{h_y^2} = 0, & 1 \le i \le N_x,\ 1 \le j \le N_y, \\ u_{i,j} = r(x_i, y_j), & x_i = 0, 1 \text{ or } y_j = 0, 1. \end{cases}$$

By ordering the unknown values $u_{i,j}$ for example row by row, we can write down a linear system of equations $Au = b$. So far, we have solved all linear systems by computing the LU-factorization of A and then computing u through forward and backward substitution. Now we will explore the effects of this in terms of memory requirements.

When you have read the introduction above you should:

- Compute the matrix A through `A=soapfilm(Nx,Ny);`. To compute the soap film surface accurately, we would need large values of N_x and N_y. However, the purpose of the present experiment is not to solve the PDE but to study the sparsity pattern of the matrices involved. For a good visualization of the sparsity pattern it is appropriate to use $N_x = N_y = 10$.
- Run the script `myLU` to study the sparsity patterns of A, L, and U. How does the pattern depend on N_x and N_y respectively? Try cases where $N_x \neq N_y$ to study this!
- Solve the problem for different choises of N_x and N_y (especially cases where $N_x \neq N_y$), run [A,b,x,y,u] = soapfilm(Nx,Ny), and study how the run time depends on the sparsity pattern (= matrix bandwidth) and matrix size.

Pay attention to:

- The number of nonzero elements in A, L and U respectively. (In the figures, `nz` is the number of nonzero elements.)
- How many nonzero diagonals A have.
- How many nonzero diagonals do L and U have together.
- How the bandwidth depends on N_x and N_y respectively.
- How the run time depends on the bandwidth.