Statistical Data Analysis, Assignment 3

Roel Deckers
Utrecht University

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1 Objective

To study the behaviour of gamma-triggered cosmic showers using Monte Carlo methods.

2 Theory

When a high energy photon strikes the atmosphere it will travel for some distance before splitting into a electron/positron pair by means of pair-production. This generate elctron/positron pair will travel for some distance before emmiting a Bremmstralung photon and changing direction. This process then repeats for the generated photon and electron/positron until their energy levels fall below 85 MeV.

2.1 distance travelled

Both photons and electrons will travel trough the atmosphere based on the density of the atmosphere and their interaction length. The density of the atmosphere as a function of height is given by

$$\rho(h) = \rho_0 \exp(-h/a) \tag{1}$$

where $\rho_0 = 1225kg/m3$ and $a = 8420m$. The interaction length of a photon to undergo pair-production is $X_{PP} = 380kg/m2$ and of an electron to undergo bremsstralung is $X_{brem} = 263kg/m2$ (for the following we will assume a photon, but the exact same formulas hold for the electron).

The real distance traveled by a photon follows an exponential distribution with a halflength given by its interaction length. The reason it follows an exponential distribution is that if a photon is known to have traveled a distance d without interacting, it is still expected to travel a further X_{PP}. That is, the process is continuous and memory-less which is a defining feature of the exponential distribution.

If we assume the photon travels straight down, we can determine its distance traveled by drawing a random number from the exponential distribution with mean X_{PP}, call this X_{real} and then solving:

$$X_{real} = \int_{h_0}^{h_1} \rho(h)\, \mathrm{d}h \tag{2}$$

for h_0[1]. If we let

$$R(h) = -a\rho_0 \exp(-h/a) \tag{3}$$
$$R^{-1}(x) = a\left(\log\left(-\frac{a\rho_0}{x}\right)\right) \tag{4}$$

then we can write this as

$$X_{real} = R(h_1) - R(h_0) \tag{5}$$
$$\rightarrow R^{-1}\left(R(h_1) - X_{real}\right) = h_0 \tag{6}$$

which gives us an easily computable function for h_0. We can then say that the distance traveled by the particle is $h_1 - h_0$. This means its final z-position is not always h_1 because (usually) a particle does not travel straight down, but this is a result of our previous approximation.

2.2 Energy ratio

When a particle splits in two, the ratio of energy carried by the first particle is given by

$$u_1 = E_1/E_{\text{parent}} \tag{7}$$

which is distributed between 0 and 1 according to

$$f(u) \propto 1 - \frac{4}{3}u(1-u). \tag{8}$$

The other particle gets a fraction of energy $u_2 = (1 - u_1)$. This distribution falls nicely in the unit square $[0,1] \times [0,1]$, therefore we sample this distribution using the acceptance-rejection method in out simulation.

2.3 Angles

The final degrees of freedom when simulating a particle splitting are the angles θ and ϕ the generated particles make w.r.t. the mother particle. for $theta_i$ we use the formula

$$\theta_i = m_e c^2/E_i \tag{9}$$

and for phi we generate a random number ϕ_r uniformly between 0 and 2π and we then set $\phi_1 = \phi_r$, $\phi_2 = \pi - \phi_r$.

This gives us everything we need to simulate a full cosmic shower using monte carlo methods.

[1] all particles travel downwards

3 Height of first split

First, we consider a photon approaching from infinity, this will be the initial trigger for our cosmic showers. Using equation 6 and noting that $R(\infty) = 0$ we can generate a probability distribution for the height-of-first-split, that is the height at which our initial photon will undergo pair-production, by repeatedly sampling X_{real} and using that to evaluate equation 6. The generated h_0 can then be used to generate a PDF of the height-of-first-split (after normalization). Such a distribution is shown in figure 1, the red line in the figure represents the height ($28\,km$) which corresponds to $X_{real} = 380\,kg/m2$, we see that it matches the peak of the distribution as one would expect.

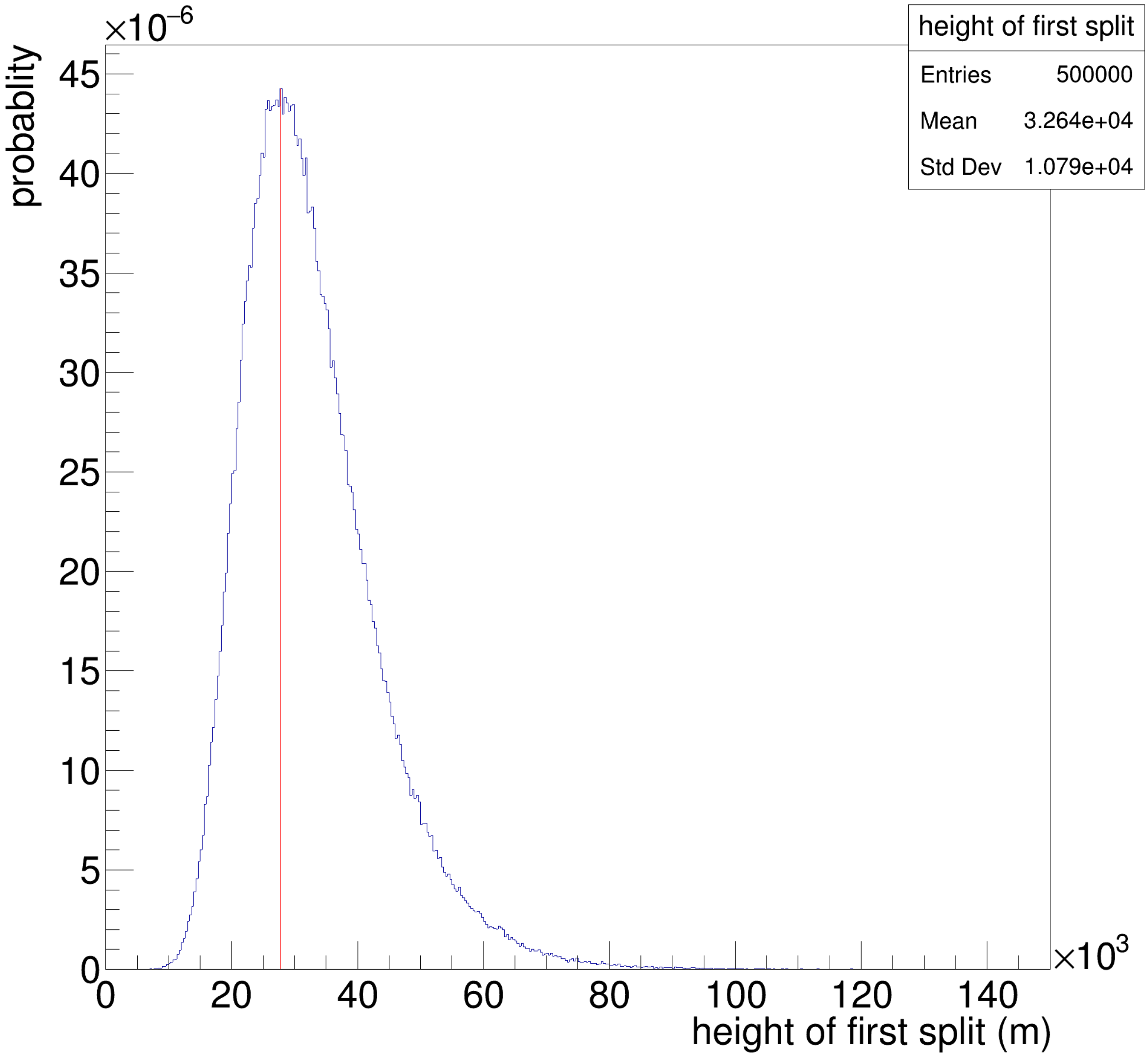

Figure 1: Simulated probability density function of the Height-of-first-split for a cosmic shower triggered by a photon. The red line corresponds to a height of $28\,km$ the theoretically expected most-likely value.

4 Shower Simulation

in figure 2 we have produced pictures of particle showers at 100 GeV, 1 TeV, and 10 TeV. At 100 GeV we can see the lightning-like structure we expect to see, with no particle moving in patently false patterns. At higher energies it becomes hard to see anything at all, but it displays the characteristic "fuzzy raindrop" shape that these kinds of particle collisions are supposed to have.

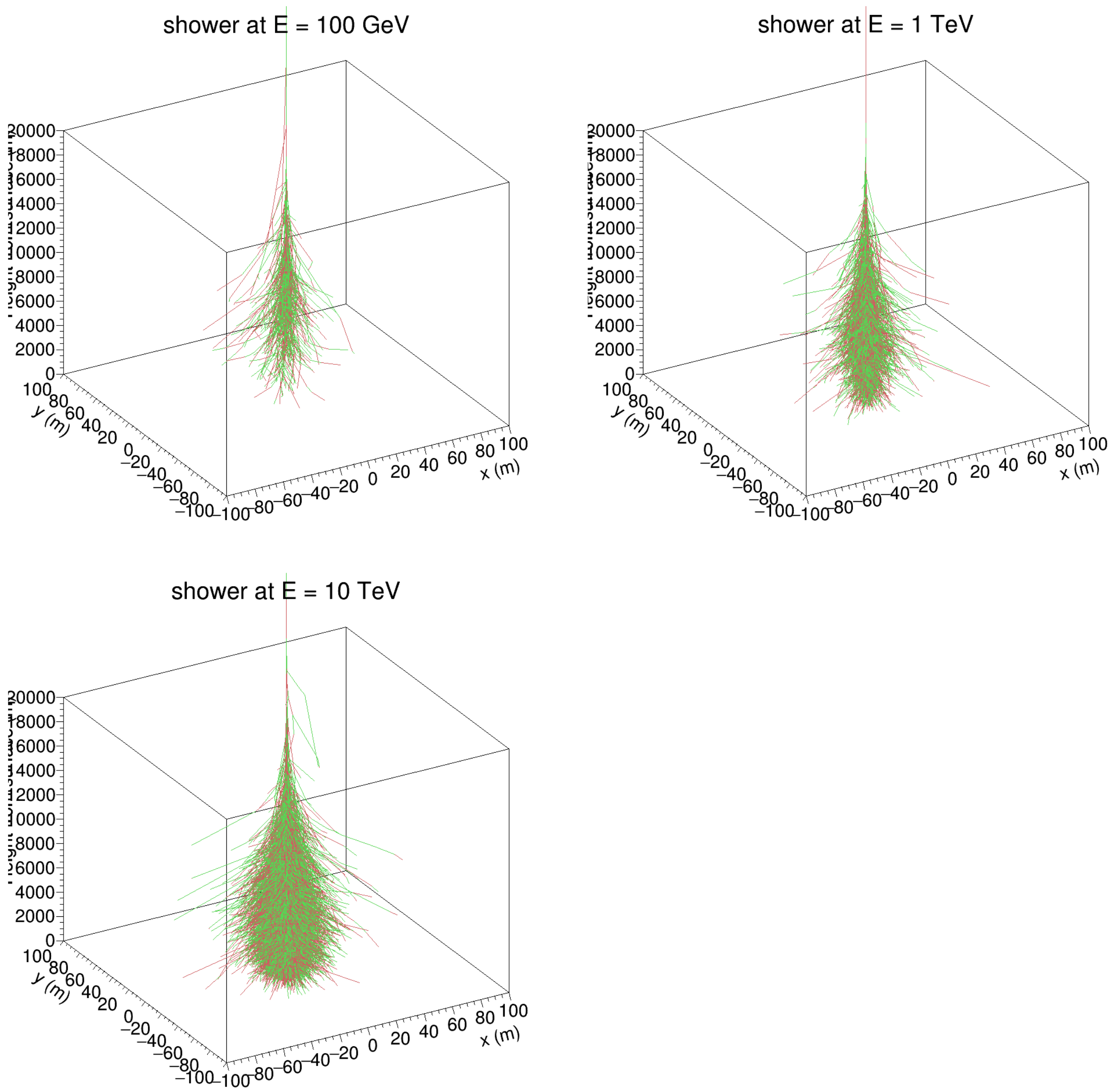

Figure 2: Monte Carlo simulations of cosmic showers at different initial gamma energies. Gammas are red, electrons are green.

5 Charged Particles vs. Height

Because figure 2 does not show much useful visual information we have instead plotted the number of charged particles as a function of height and energy in figure 3. Unsurprisingly, higher energy gammas produce more particles and the particles are distributed more sharply and closer to the earths surface. For the 10 TeV shower a large fraction of particles will make in fact make it to the surface.

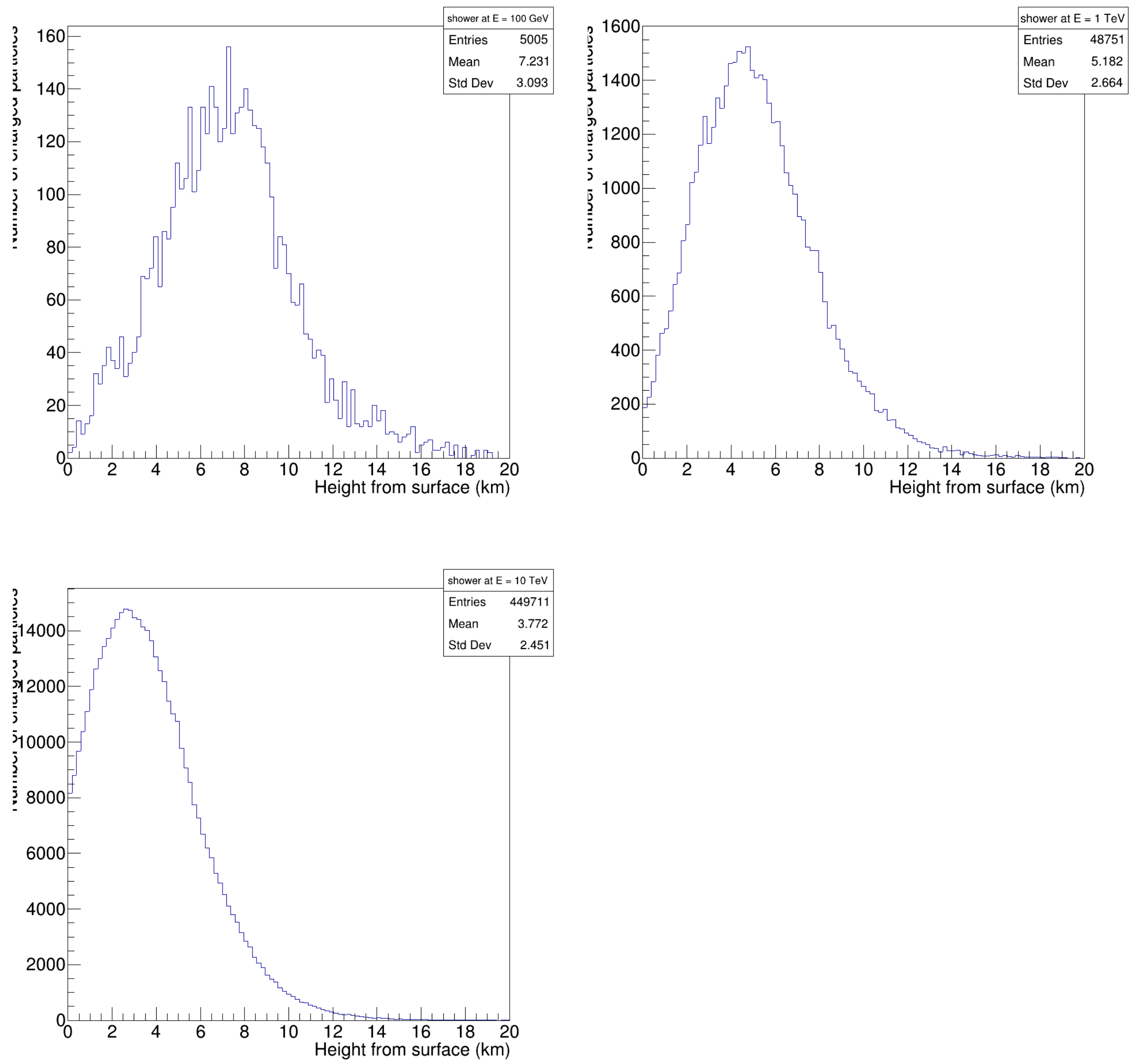

Figure 3:

6 Xmax vs. Energy

We observe no dependance.

7 Aerial Size

t.b.d.