Statistical Data Analysis, Assignment 2

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1 Objective

To compute and visualize the coupling constants $(u_L^2, d_L^2, u_R^2, d_R^2)$ and their errors, starting from experimental data on neutrino and anti-neutrino collisions with protons and neutrons.

2 Given data

We observe 10 different events $(x_0, \ldots, x_9)$ each with their own number of events presented in table 1. For clarity we only list the shorthand notation. Each of these events are known to be statistically uncorrelated.

Event	number of occurences (stat.error)
x_0	812 (41)
x_1	1800 (50)
x_2	400 (33)
x_3	464 (35)
x_4	818 (33)
x_5	356 (22)
x_6	289 (27)
x_7	301 (29)
x_8	164 (19)
x_9	393 (24)

Table 1: Neutrino event counts and statistical errors. All events are uncorrelated.

3 Ratios

First, we wish to compute the ratios

$$R^{\nu p} = \frac{x_2}{x_0}, \tag{1}$$
$$R^{\nu n} = \frac{x_3}{x_1}, \tag{2}$$
$$R^{\bar{\nu} p} = \frac{x_6 - x_8 \cdot x_2/x_0}{x_4}, \tag{3}$$
$$R^{\nu p} = \frac{x_7 - x_9 \cdot x_3/x_1}{x_5}, \tag{4}$$

and their statistical errors. We can calculate the expectation value of R directly using the provided formula and expectation values of x_n. In order to compute the errors we will compute the covariance matrix $\boldsymbol{C}_R = \boldsymbol{R}_R \boldsymbol{C}_x \boldsymbol{R}_R^T$ which will give us the variance, and thus the square of the error, on the diagonal (we will need the off-diagonal elements later). Because R transforms x_n non-linearly this is an approximation. Here $\boldsymbol{C}_x$ is the covariance matrix of x_n which, because x_n is uncorrelated is just a diagonal matrix of the statistical errors in table 1 squared (i.e. the variance). And $\boldsymbol{R}_R$ is the derivative matrix of R w.r.t. x_n at the given values of x_n as shown in equation 3.45 of the course book.

For computing the $\boldsymbol{R}_R$ matrix we use a (provided) numerical derivative method.

We find that

$$\boldsymbol{C}_R = \begin{pmatrix} +2.270 \cdot 10^{-3} & & -4.552 \cdot 10^{-4} & \\ & +4.294 \cdot 10^{-4} & & -4.740 \cdot 10^{-4} \\ -4.552 \cdot 10^{-4} & & +1.417 \cdot 10^{-3} & \\ & -4.740 \cdot 10^{-4} & & +8.663 \cdot 10^{-3} \end{pmatrix} \tag{5}$$

which leads to

$$R^{\nu p} = 0.49 \pm 0.05, \tag{6}$$
$$R^{\nu n} = 0.26 \pm 0.02, \tag{7}$$
$$R^{\bar{\nu} p} = 0.25 \pm 0.04, \tag{8}$$
$$R^{\nu p} = 0.6 \pm 0.1. \tag{9}$$

4 Coupling Constants

Next we computed the coupling constants $c = (u_L^2, d_L^2, u_R^2, d_R^2)$ which are defined as

$$c = \begin{pmatrix} u_L^2 \\ d_L^2 \\ u_R^2 \\ d_R^2 \end{pmatrix} = \begin{pmatrix} 0.675 & -0.607 & -0.119 & 0.010 \\ -0.282 & 1.331 & 0.027 & -0.049 \\ -0.133 & 0.060 & 0.477 & -0.078 \\ 0.024 & -0.299 & -0.186 & 0.185 \end{pmatrix} \begin{pmatrix} R^{\nu p} \\ R^{\nu n} \\ R^{\bar{\nu} p} \\ R^{\nu p} \end{pmatrix} \tag{10}$$
$$= \boldsymbol{A} R \tag{11}$$

Despite the values of R being correlated, the expectation value of the the coupling constants are still given by straightforward application of this equation as it is a linear transformation (see book, 3.34). However, in order to calculate the variance we will need to use our previously computed covariance matrix $\boldsymbol{C}_R$. The new covariance matrix is given by

$$\boldsymbol{C}_c = \boldsymbol{A}\boldsymbol{C}_R\boldsymbol{A}^T = \begin{pmatrix} +1.292 \cdot 10^{-3} & -8.318 \cdot 10^{-4} & -4.831 \cdot 10^{-4} & +2.752 \cdot 10^{-4} \\ -8.318 \cdot 10^{-4} & +1.032 \cdot 10^{-3} & +2.843 \cdot 10^{-4} & -4.197 \cdot 10^{-4} \\ -4.831 \cdot 10^{-4} & +2.843 \cdot 10^{-4} & +4.790 \cdot 10^{-4} & -2.985 \cdot 10^{-4} \\ +2.752 \cdot 10^{-4} & -4.197 \cdot 10^{-4} & -2.985 \cdot 10^{-4} & +4.417 \cdot 10^{-4} \end{pmatrix}. \quad (12)$$

And c is then given by

$$c = \begin{pmatrix} u_L^2 \\ d_L^2 \\ u_R^2 \\ d_R^2 \end{pmatrix} = \begin{pmatrix} 0.15 \pm 0.04 \\ 0.18 \pm 0.03 \\ 0.03 \pm 0.02 \\ -0.01 \pm 0.02 \end{pmatrix} \quad (13)$$

5 Visualization

We have visualized the (co)variance of u_L^2 versus d_L^2 in figure 1. We plotted one standard-deviation contour using the provided function and also superimposed the central point corresponding to the two expectation values with error bars showing the uncorrelated errors. We can see that if we were to ignore the covariance completely and only look at the variance of the graph we would get a whole different (and much less complete) picture.

6 Conclusion

When calculating error propagations it is important to take covariance into account. If a variable transforms non-linearly it is possible to use Taylor expansion to get a linear approximation of the non-linear function and to use that to compute the propagated covariance matrix instead. When possible, Visualizations should also take the covariance into account (e.g. by making a contour plot) as it provides a much clearer picture.

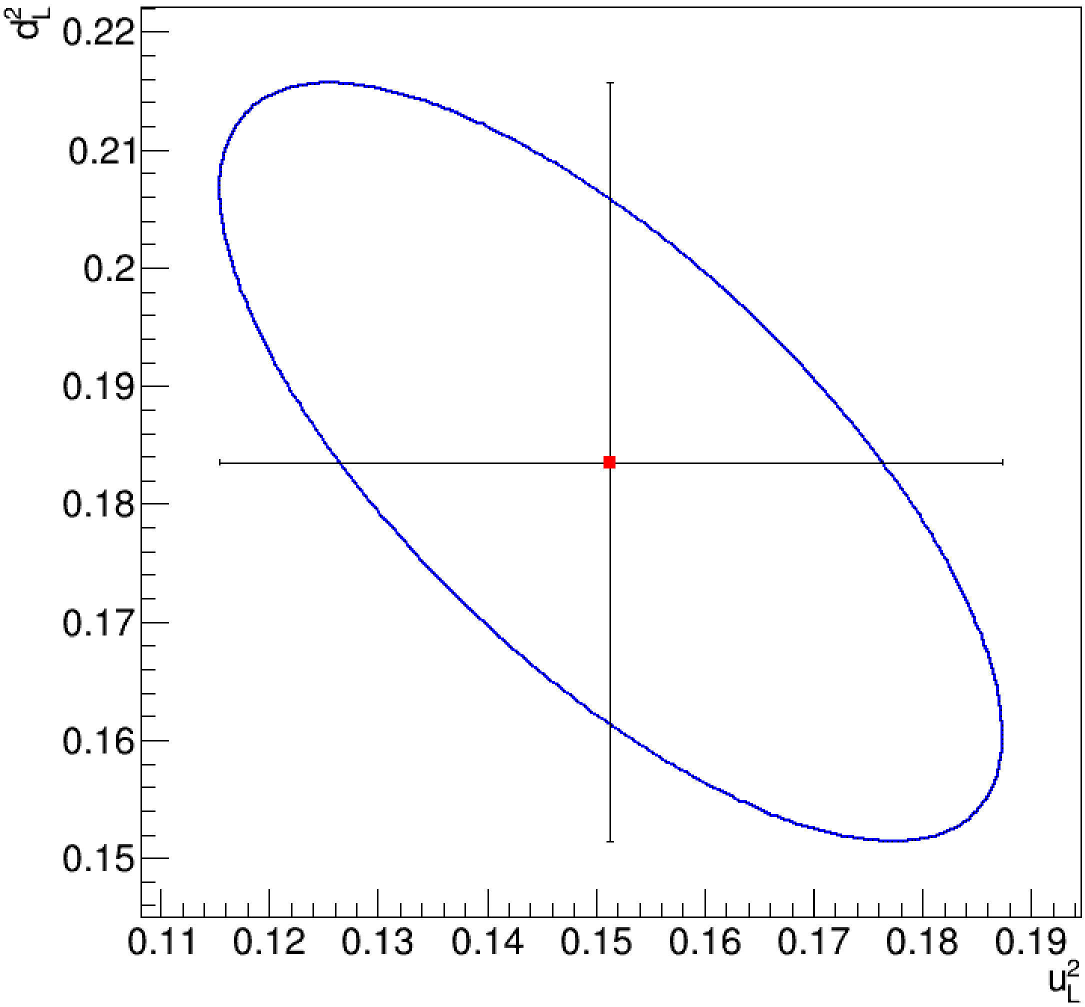

Figure 1: u_L^2 versus d_L^2. The red point marks the expectation value and the error bars show the standard deviation of each variable. The blue line is the one-standard deviation contour line.