14.310x Flipped Classroom materials

——W10——

rduranl

Course Guide

14.310x Flipped Classroom

Week 1 Instructions

R for data analysis and working in the cloud

Checklist

- ☐ Complete the INTRO TO R interactive course from the `Swirl` package[1] (Requirement)
- ☐ Watch the Getting started with Google Colab notebooks video tutorial (Requirement)
- ☐ Create or set up a personal Google account (you must be able to use Drive and Colab). (Requirement)
- ☐ Create your first Colab notebook (Session 1)
- ☐ Complete Coding Lab 1. (Session 1)
- ☐ Complete Coding Lab 2 (Session 2)

C.1.0

Your first Colab notebook

Instructions: Read and follow the steps below **before proceeding with the activity**. After reading the instructions, access the notebook link and complete the exercises in Colab. This is an individual task, but you will collaborate on the final question.

Notebook: Your first Colab notebook

[1] **Module 1** > **Introduction to R** in the online component of the course.

We will cover the essentials of working with Jupyter Notebooks on Google Colab — this resource will be an important tool throughout. Before you can begin working on the coding labs in Colab, make sure to:

- **Create or use a personal Google account**. While it is possible for you to download the Jupyter Notebooks we will work on, and manage them locally in your own machine, we strongly advise you to work on Colab since it will make it easier for your classmates and the instructor to interact with your work when needed. Colab has many features Google Drive already offers: comments, real-time colaboration in the same document or folder, etc.
- **Save the shared notebooks and documents to your own drive.** The notebooks you will have access to are read-only; in order to work on them you will have to copy them to your drive. It is recommended that you maintain a per-week structure in your folders as this will make it easier to follow instructions, especially when reading files or collaborating with others. To save the notebook to your Drive, go to the File menu: `File > Save a copy in Drive`.
- **(When creating a new notebook) Change Colab Notebook's runtime type to R.** By default, when you create a new notebook in Colab, the virtual machine Colab sets up for you is a `Python` installation. This means all the cells in the notebook will only recognize and run `Python` syntax or commands. To switch to an `R` language setup:
  1. Open the notebook menu and go to `Runtime > Change runtime type`.
     - Or click the toggle in the upper-right corner (as shown below) and select `Change runtime type`
  2. In the dropdown labeled `Runtime type`, select `R`.
  3. Click `Save`.

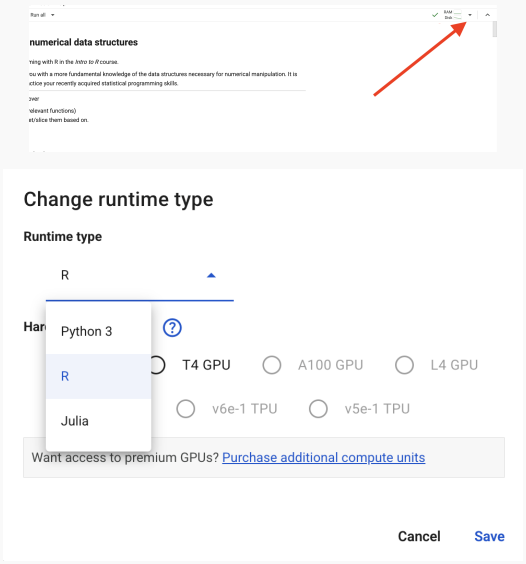

Fig.Changing the runtime type

You may now create your first Colab notebook

C.1.1

Coding Lab 1— Numeric data structures in R

Instructions: Work individually. Solve all exercises in the corresponding Colab notebook. Record your answers and/or code in your copy of the notebook.

Notebook: Coding Lab 1

Submit your work in the format required by the instructor.

C.1.2

Coding Lab 2 — Data manipulation with `dplyr`

Instructions: Work individually. Solve all exercises (sections 0 to 3) in the corresponding Colab notebook; record your answers and/or code in your own copy.

Notebook: Coding Lab 2

Optional **Section 4** allows work in pairs or teams of 3.

Upon completion, submit your work in the format required by the instructor.

14.310x Flipped Classroom
Week 2 Instructions

Discrete random variables, joint and conditional distributions

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Checklist

- ☐ Complete the ADVANCED R interactive course from the `Swirl` package and watch the `ggplot` tutorial[2] (Requirement)
- ☐ Watch the **Import Data** tutorial[3] (Requirement)
- ☐ Read the note on importing `aiwars` and other datasets (Requirement)
- ☐ Complete Coding Lab 3 (Session 1)
- ☐ Complete Guided Case 1 (Session 2)

Importing `aiwars` and other datasets

Depending on flipped classroom logistics of your group, your access to the sessions' assets (datasets, figures, scripts, etc.) will come in one of two forms:

- **Through a direct URL** for instance, the original `aiwars.csv` dataset lives directly in this URL:
  `https://docs.google.com/spreadsheets/d/1NeZZWI2fT71M9QD8zjnz817T1B3XBM6lolqArSe9CwQ/export?format=csv`
- **Your instructor will provide them to you privately** and they will either:
  - Share them via a URL similar to the one above.
  - Share the files through other means.

[2] **Module 2 > R Course and R Tutorial: ggplot** in the online component of the course.
[3] **Module 3 > R Tutorials: Basic Functions** in the online component of the course.

As they may want to slightly modify the original files for grading purposes, or have any other goal in mind.

If a dataset's URL is provided, you can read the data directly into R with the URL and the appropriate reading function; simply provide the URL as the path. For instance, `aiwars` is in `.csv` format:

```
URL_aiwars <- "https://docs.google.com/spreadsheets/d...."

aiwars <- read.csv(URL_aiwars)
```

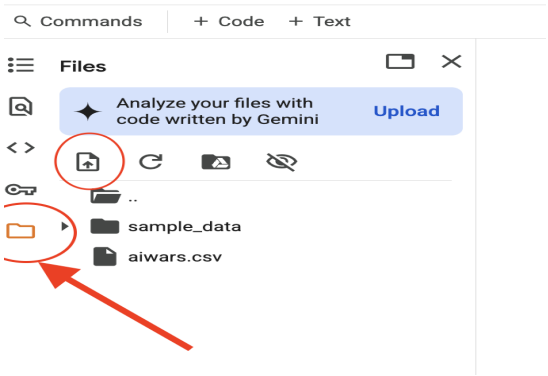
And similarly for other formats. Suppose we had a single Excel sheet:

```
install.packages("readxl")
library(readxl)

URL_aiwars_xl <- "https://...some-URL"

aiwars <- read_excel(URL_aiwars_xl)
```

If the file is shared in any other way, we will first need to upload it to the virtual machine's disk in Colab and read it from there, providing the `path` to it — as you would do if you were reading data to R in your own computer. To upload a file in Colab, click the folder icon in the leftmost menu bar, as shown in the screenshot. Then click the upload icon (circled in red); as you can see, we already uploaded the file.



Once uploaded the file can be read as a if in your machine:

```
aiwars <- read.csv("aiwars.csv")
```

Important: Colab runtime limitations

While your Colab notebook — including all code cells, text, and outputs — will remain saved, the underlying *runtime* (i.e., the virtual machine that executes your code) is temporary. After a period of inactivity or when a time limit is reached, the runtime will automatically disconnect.

When this happens:

- All variables and objects stored in memory (e.g., your R data frames, models, vectors) will be lost.
- Any files you uploaded manually will be erased.

When reconnecting, a fresh virtual machine will be started, and you'll have to re-run your code, and re-upload any needed files. Disconnects may happen occasionally as you work in off-notebook tasks; re-reading or re-uploading files should not take over 30 seconds.

Coding exercises in case studies

Throughout the course, the case studies you will work on contain a mix of conceptual and coding questions. To simplify your workflow, all the questions where you're required to write code are specially labeled. They appear in this format:

Question 5. [2.3] calculate the probability of event A...

The small icon signals that the question requires coding. The number inside the brackets (2.3 in this example) is a numbering **within the Session's dedicated Colab notebook**, which will often be different from the overall question number for the activity.

Further, if you click on the orange icon, it will take you directly to the question's cell within the Notebook by opening a new browser tab – remember you must work on your own copy of the notebook. These links are simply provided as a convenience to help you locate and visualize the relevant task quickly.

Apart from being the place where you are expected to code your answers, the corresponding cell usually includes a placeholder for your code, often accompanied by additional hints, extended context, or partially completed code to help you get started. When working on a case study, keep both this PDF and Colab notebook open and use these references to move smoothly between the written materials and your code.

C.2.3

Coding Lab 1 — Web Scraping in R: step by step guide

Instructions: Work individually. Solve all exercises in the corresponding Colab notebook. Record your answers and/or code in your copy of the notebook, or in the format required by the instructor.

Notebook: Coding Lab 1

Case study G.2.1

Random variables in the wild: *Reddit* posts and empirical distributions

Instructions: Work in pairs or groups of three. Answer the questions

Notebook: Case Study 1

Dataset: aiwars.csv

One of you must share editor access to their notebook with the rest.

Scenario: We will cover this week's lecture contents using a real-world dataset from Reddit. You will put your data manipulation (with dplyr and base R) skills to practice

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Context: The `AIwars` dataset consists of a collection of posts scraped from the `r/AIwars` subreddit, a forum where users debate the societal implications of AI. These range from predictions about AI-driven job loss and technological conflict to satire, trolling, and speculation. The dataset captures a rich period of discussion and polarization. Each observation corresponds to a single post, with information about its author, contents and engagement.

Key Variables:

- `post_index` — Unique identifier for each post (starts at 1 and consecutively numerates all posts).
- `author` — Reddit username of the post author (may be `[deleted]`).
- `post_date` — an R `Date`
- `fulltext` — Full text of the Reddit post in format `TITLE: [some post title] TEXT: [some post text]`
- `post_length` — the net number of characters in `fulltext` (excluding the TITLE and TEXT headers).
- `post_upvotes` — Number of upvotes (user endorsements or *likes*) the post received.
- `comments_number` — Total comments the post received (includes replies to other comments).

Part 1. Basic dataset facts

1.1 [1.1] Install and load the `tidyverse` packages. Create `aiwars_URL`, a `character` with the download URL for `aiwars.csv`. Then read in the dataset as `aiwars`, a data frame. Use `glimpse()` to answer:

(a) How many posts are we working with?
(b) How many variables does the dataset have?
(c) How many are *true* `character` types?
(d) How many are numeric?
(e) Which characters are better described as factors? Why?

1.2 [1.1.1] For the entire case study, will use only the **Key Variables** described above. While `select()`ing columns from the current data frame is possible, a memory-efficient alternative is to read in only those columns we need.
Use the `col_select` argument in `read_csv()` to create `aiwars_short`, a data frame containing only the columns described as **Key Variables**.

1.3 [1.1.2] Use `select()` to create `aiwars_short2`, a dataframe with only the **key variables**.

1.4 Moving forward we will work with `aiwars_short` only. You may delete `rm(aiwars, aiwars_short2)`.

1.5 Use `summary()` or other summarizing methods to answer the following questions:

(a) What is the time span of the posts?
(b) On average, how long (in words) is a post on this subreddit?
(c) How many posts don't have any replies?
(d) What is the largest number of upvotes in a post?
(e) Which author has posted the most? (Consider `as.factor()`).
(f) How many distinct users have posted in this subreddit? (Consider `unique()`).

Part 2. Counting posts

2.1 [2.1] Create the following variables in `aiwars_short`:

(a) `popular`, takes value 1 when the post has 29 or more upvotes. Takes value 0 when the post has less than 29 likes.
(b) `text_classification`, which takes value `mostly title` if it is less than 70 characters long, `short` if it has 70 or more but less than 110 characters, `common` if it has 110 or more but less than 900 characters, and `long` if the post has over 900 characters.

2.2 How many `common` posts are there?

2.3 If we draw a post at random, it will be a popular one with what probability?

2.4 If we instead sample 10 posts at random **with replacement**, what is the probability 3 of them are popular?

2.5 [2.2] Suppose random variable X; $\Omega_X = \{0, 1, 2, ..., N\}$ represents the number of popular posts in $N = 10$ draws with replacement from `aiwars_short`. For the following probability statements describing various events, first write them down as a probability and then use R to compute them.

(a) What is the probability of getting at most 4 popular posts?
(b) What is the probabililty of getting at least one but at most 3?
(c) What is the probability of getting a number of popular posts that is not 0 or 4?

2.6 Similarly, we can treat the texts' classification as random variable Y. Is it continuous or discrete? What is Ω_Y?

2.7 [2.3] Express $f_Y(y)$ as a data frame `f_Y` with as many rows as elements in

Ω_Y and two columns: `y`, the value taken by Y and `p` its mass or density.

2.8 What is the probability a post is not `mostly title` ?

2.9 What is the probability a post is either `common` or `long`?

Are popular posts different?

2.10 What is the probability a post is popular **AND** `long`?

As we learned from the lecture, if two events are **not independent**, the additional information one of them provides should *update* the probabiility of the other. We will examine this fact by calculating the probability of a post being long **GIVEN that we know it is popular**:

$$P(Y = long \mid popular = 1)$$

As a reminder:

$$P(long \mid popular = 1) = \frac{P(long \cap popular = 1)}{P(popular = 1)}$$

2.11 Use your answers to questions 2.3 and 2.10 to compute the probability a post is long GIVEN it is popular. How does this probability compare to the unconditional probabiity $P(Y = long)$?

2.12 We now know that the event $(Y = long)$ has two mutually exclusive and exhaustive partitions: $(popular = 1)$ and $(popular = 0)$. Following the lecture, Show that the Law of Total Probability verifies for this event.

2.13 What is $P(popular = 1 \mid long)$? Is your answer the same as in 2.11 ? Should it be? Why?

2.14 [2.4] Create `joint_pmf`, a data frame similar to the one you created for 2.7; in this case, it should display the probability for all possible values in $Y \cap Popular$. This is the ordered pair $\Omega_Y \times \Omega_{Popular} = (long, 1), (long, 0), ..., (\text{mostly title}, 1), (\text{mostly title}$

2.15 [2.4.1] Use `joint_pmf`, `p` (or $f_{popular}$) and `f_Y` to compute the following in R:

(a) $P(Y \mid Popular = 1)$, a vector of length 4.

(b) $P(Popular \mid Y = common)$. What is its length? .

14.310x Flipped Classroom

Week 3 Instructions

Continuous random variables; conditional and joint pdfs. More advanced data manipulation and visualization in R

Checklist

- ☐ Complete Guided Case Study 2 (Sessions 1 & 2)
  - ☐ We will continue to work with `aiwars`
  - ☐ Get the `aiwars_embeddings` dataset here
- ☐ Complete Open Case Study 1 (Session 2)

Case study G.3.2

Analyzing text similarity:
language semantics and random variables

Instructions: Work on your own for Session 1, and in pairs or groups of 3 for Session 2.

Notebook: Case Study 2

Dataset: `aiwars_embeddings.csv`

Part 1 Part 2 Part 3

Dataset: `speech_anchors.csv`

The `aiwars_embeddings` dataset (we split it in 3) contains 1024 variables associated with each post's `fulltext` in `aiwars`. You will learn what these embedding variables `V1`, `V2`, ..., `V1024` represent below, but keep in mind `post_index` in both datasets uniquely links each post in `aiwars` to its embedding in `aiwars_embeddings`. Dataset `speech_anchors` is a 2-row data frame with the same structure but `label`s instead of post indices.

Part 0. Required data and packages

0.1 [0.1] Load/install the `tidyverse` and the `geometry` packages.

0.2 [0.1] Read in `aiwars`, the same dataset as last week. Also, read in `speech_anchors.csv` as `anchors`. This should be a data frame with two rows and 1025 columns.

0.3 [0.1] Read in parts 1,2 and 3 of `aiwars_embeddings` as `part1`, `part2`,`part3` respectively. Then use `rbind()` to vertically merge the parts into `aiwars_embeddings`. Optionally, use `arrange()` to sort the dataset in `post_index`- ascending order.

Scenario:

Natural Language Processing (NLP) focuses on enabling machines to understand and work with human language. One of the key advances in NLP has been the development of methods that represent text — such as words, sentences, or entire posts — as vectors. These vector representations, often referred to as *embeddings*, capture aspects of a text's semantic content: its meaning, tone, and topical focus.

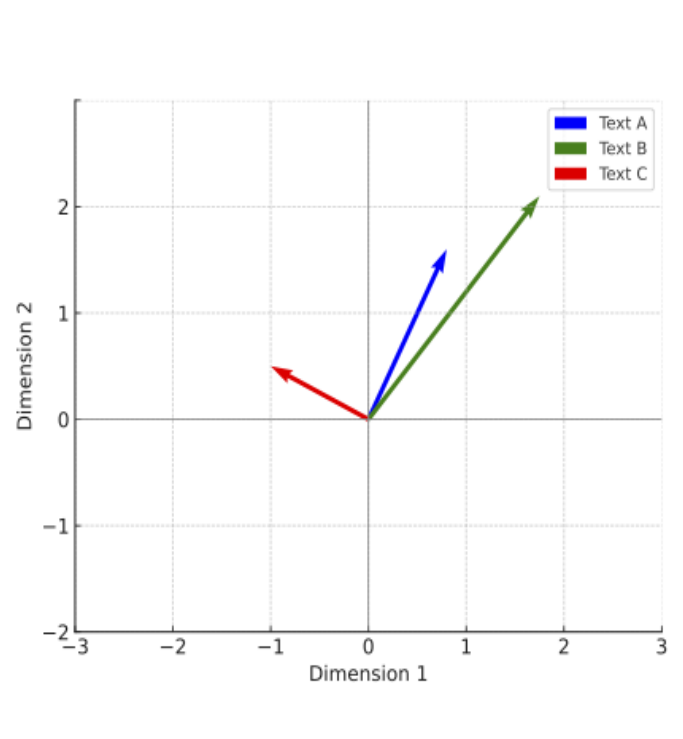
To illustrate the idea, suppose we have the following texts:

- Text A: "I love how these new AI tools can generate art!"
- Text B: "AI-generated images are cool, but I'm worried about copyright."
- Text C: "I painted this myself — no software involved."

We embed these texts in 2-dimensional[0] vectors from the origin, plotted below:

- $\vec{a} = \begin{pmatrix} x_a \\ y_a \end{pmatrix} = \begin{pmatrix} 0.8 \\ 1.6 \end{pmatrix}$
- $\vec{b} = \begin{pmatrix} x_b \\ y_b \end{pmatrix} = \begin{pmatrix} 1.7 \\ 2 \end{pmatrix}$
- $\vec{c} = \begin{pmatrix} -1 \\ 0.3 \end{pmatrix}$

Figure 1: **2D text embeddings (vector from the origin)**



Despite their differences in tone, $\vec{a}$ and $\vec{b}$ in Figure 1 are close to each other because the texts they represent use AI-related language, whereas $\vec{c}$ is far removed given the lexical differences. In part 2, we will explain what "close to each other" means for vectors, with two common semantic similarity measures: **direction** (i.e., the *angle* they form) and **distance** between them.

Part 1. Coding vector operations

You will start by programming functions necessary for this analysis, such as the *magnitude* of a vector, the *distance* between two vectors, their *dot product*, etc.

Distance between two vectors

The distance from $\vec{a}$ to $\vec{b}$ in Figure 1 is defined as:

$$|| \vec{a}, \vec{b} || = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2}$$

This is the sum of their x and y-coordinates' differences squared (in any order). Finally, we take the square root of that sum. Substituting the values in the scenario description:

$$\cdots = \sqrt{(0.8 - 1.7)^2 + (1.6 - 2)^2}$$

$$\cdots = \sqrt{0.81 + 0.16} = 0.985$$

1.1 [1.1] Save $\vec{a}, \vec{b}, \vec{c}$ as numeric vectors `a`, `b` and `c` respectively. Compute the distances for each pair of vectors and save them accordingly: `dist_ab`, `dist_ac`, `dist_bc`. Hint: you can apply operations on all the coordinates at once: `(a-b)`.

(a) Make sure your result for `dist_ab` is the same as above.

In general, to get the distance between any two n-dimensional vectors

$$\vec{V} = \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}, \vec{U} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix}$$

we generalize the process with all their coordinates and take the square root:

$$|| \vec{V}, \vec{U} || = \sqrt{(v_1 - u_1)^2 + (v_2 - u_2)^2 + \cdots + (v_{n-1} - u_{n-1})^2 + (v_n - u_n)^2}$$

$$\cdots = \sqrt{\sum_{i=1}^{n} (v_i - u_i)^2}$$

1.2 [1.2] `aiwars_embeddings` contains a 1024-dimensional vector per post. Program a distance function `dist_function(V, U)` that takes any two equal-length, numerical vectors. It returns the distance between them. If you didn't in the previous exercise, make sure to use vectorized operations (e.g., `sum()`, squaring all elements at once as $(a-b)^2$) to generalize the computation.

(a) Verify your function is correct and reproduce the results in 1.1

Distance from one, to multiple vectors

Working with individual, separate objects such as `dist_ab`, `dist_ac`, and so on can be difficult and tedious. In R, we will often want all the distances with respect to $\vec{a}$ in a single "distances" vector or variable:

$$D_a = \begin{pmatrix} || \vec{a}, \vec{a} || \\ || \vec{a}, \vec{b} || \\ || \vec{a}, \vec{c} || \end{pmatrix}$$

We will now pass `dist_function()` through all the example vectors at a time to obtain D_a — the steps should be familiar from the previous Coding Lab.

1.3 [1.2.1] Create vector `D_a` with a *for* loop by following the steps below:

- Declare `D_a`, an empty vector.
- Use `rbind` to create `examples_matrix`, a matrix with 3 rows and 2 columns. One row for each example vector `a,b,c` and one column for each of their x, y-coordinates.

- Declare a `for(...)...` loop with as many iterations as rows in `examples_matrix`.
- Inside the *for* loop, code the following:
  - Subset `examples_matrix` to obtain vector `v`.
  - Obtain the `distance` between `a` and `v` with `dist_function`.
  - Use `append` to recursively add a `distance` on each iteration to vector `D_a`.

1.4 Verify your answers with those of 1.2. What should be the result for $||\vec{a}, \vec{v}||$when $\vec{v} = \vec{a}$? Why?

Finally, we can wrap the code we created for 1.3 in a function that directly returns D_a when given a vector and a matrix.

1.5 [1.2.2] Create a function `get_distances(u, M)` which takes a vector `u` of length n and a matrix `M` with an arbitrary number of rows k, and n columns. The function should `return` D_u, a vector of length k where each element is the distance between `vector` and a row of `matrix`.

1.6 Use `get_distances(_,_)` to reproduce 1.3. Verify it is the same vector.

The norm of a vector

The norm $|| \vec{V} ||$ (also called the *magnitude* or *length*) of a vector indicates how long the arrow is from the origin to the point it reaches. Since all embeddings are vectors at the origin (all their starting coordinates are 0), we can calculate this as the distance between a vector $\vec{V}$ and the n-dimensional zero: $||\vec{V}, \vec{0}||$.

$$|| \vec{V}, \vec{0} ||=|| \vec{V} ||= \sqrt{(v_1 - 0)^2 + \cdots + (v_n - 0)^2}$$

$$... = \sqrt{\sum_{i=1}^{n} v_i^2}$$

1.7 [1.3] Create `norm0_function`, a function that takes `V`, a numerical vector of length n as an input, and returns its norm from the origin. Hint: you can recycle `dist_function` inside `return()` with the appropriate `U`.

1.8 [1.3.1] Compute $|| \vec{a} ||, || \vec{b} ||$ and $|| \vec{c} ||$. Save them to `a_norm`, `b_norm`, `c_norm` respectively.

1.9 Similar to 1.3 and 1.5, create a function `get_norms0(M)` taking a matrix of n rows/vectors, all with the same number of dimensions/columns m, and returns a vector N_0 of length n with a norm from the origin for each row/vector in `M`. Input it the `examples_matrix` and verify your results in 1.8.

The dot product of two vectors

The dot or *scalar* product of two vectors $\vec{U} \cdot \vec{V}$ is defined as the sum of the product of each pair of their coordinates:

$$\vec{U} \cdot \vec{V} = u_1 * v_1 + u_2 * v_2 + \cdots + u_n * v_n$$

$$= \sum_{i=1}^{n} u_i v_i$$

You can use R's native vectorized multiplication: `sum(u*v)`.

1.10 [1.4] Create a function `dot_product(u,v)` that takes two numeric vectors of the same length as input and returns their dot or scalar product.

1.11 [1.4.1] As in previous exercises, we're interested in computing a vector $\mathbb{D}_a$ of dot products. Use a *for*-loop and wrap it in a function `get_dot_products(v, M)` taking a vector `v` of length n and a matrix `M` of as many rows as vectors and n columns and returns a dot product $\vec{V} \cdot \vec{m}$ for each row.

Part 2. Processing AIwars posts' text embeddings

We prepared the `aiwars_embeddings` dataset for you by using one of OpenAI's text embedding products[1]. For each post's `fulltext` in `aiwars`, there is a vector representation of length $n = 1024$ in `aiwars_embeddings`:

Variables

- `post_index`: a consecutive index **in the same order as** `aiwars`. A given post and its embedding share the same post index.
- `V1`, `V2`, ..., `V1024`: the coordinates $v_1, v_2, ..., v_{1024}$ of each vector embedding $\vec{V}$.

You will now apply the functions you previously programmed to measure the semantic similarity of real-world embeddings.

Semantic similarity measures: *Luddites* vs *Synthists*

To illustrate semantic similarity measures, we will take the embeddings from four select posts from the AIwars subreddit, each with distinct tones and topics:

2.1 [2.1] Filter `aiwars` for posts with the following `post_index` values: 456, 584, 2397, 2526. Select the `fulltext` and post index columns only. Assign the resulting dataframe to `tones`.

2.2 Skim the title and body of each of the posts in `tones`.

2.3 To keep track of them, give each embedding a label: `post_labels <- c("luddites", "review", "critique", "synthists")` and add the vector as a new variable `label`. Remember, `tones` should be in `post_index` ascending order for the labels to match.

2.4 [2.2] Retrieve the embeddings for `tones` in a 4×1024 matrix and save it as `tones_embeddings`:

(a) Filter `aiwars_embeddings` for the relevant `post_index`.

(b) Either drop `post_index` or tidy-`select` all variables `starts_with()` "V".

- Remember to keep track of the embeddings. We suggest you previously arrange by ascending post index.

(c) Coerce the resulting data frame into a matrix with `as.matrix()`

.

Distance-based semantic similarity

As exposed in the introduction, if two texts share style, vocabulary or topics, their embeddings will tend to be closer, and on the contrary, distinct semantics will produce embeddings that are farther apart.

2.5 [2.3]

Table 1: Pairwise distances between example embeddings (not the real values)

	luddites	review	critique	synthists
luddites	0.0000000	72.384921	18.532947	91.004327
review	72.384921	0.0000000	56.110239	8.231476
critique	18.532947	56.110239	0.0000000	44.873291
synthists	91.004327	8.231476	44.873291	0.0000000

Code a 2-D distance matrix like the one displayed above with the embeddings in `tones`. Hints:

- This is a matrix with `rownames` and `colnames`
- Use `rbind` to save each iteration on `tones`.
- Note: the matrix you get should be symmetrical. The distance between *critique* and *luddites* should be the same in (row 1 column 3) and (row 3 column 1).

2.6 Before you reflect on the results, consider what the bounds could be:

(a) If the embeddings could be of any magnitude, what would be the maximum distance between embeddings?

(b) Compute the norm about the origin for the embeddings in `tones`. What can you notice?

(c) Compute the norm for all of `aiwars_embeddings` by getting a matrix for all the embeddings as in 2.4.

(d) Based on this observation, what is the longest possible distance between two of these embeddings?

2.7 Which embedding pair is the furthest apart?

2.8 Which individual embedding seems to be further away from the rest?

2.9 Which two embeddings are the closest?

Direction-based measures: Cosine Similarity

An alternative approach is to compare the vectors' direction only, through the angle θ formed by two vectors $\vec{a}, \vec{b}$. The cosine of said angle θ is defined as:

$$cos(\theta) = \frac{\vec{a} \cdot \vec{b}}{|| \vec{a} || \ || \vec{b} ||}$$

- $\mathbf{cos}(\theta) = \mathbf{1}$ means the angle is $\theta = 0°$, indicating identical direction. These texts will be very close in meaning and topics.
- $\mathbf{cos}(\theta) = \mathbf{0}$, the angle is $\theta = 90°$, indicating the posts are orthogonal, and for the most part, semantically unrelated: different topics and intent.
- $\mathbf{cos}(\theta) = \mathbf{-1}$, with angle $\theta = 180°$, vectors point in opposite directions.

In our setup, cosine similarities in $[0.5, 0.7]$ usually indicate a non-trivial topical relation; values ≥ 0.8 often reflect very close texts (near-paraphrases). Scores in $[-0.3,\ 0.3]$ are not indicative of a strong semantic link. ≤ -0.3 cosines can occur but are uncommon and shouldn't be interpreted as "opposite views" on their own—you should inspect the post instead.

2.10 [2.4] Write a function `batch_cossim(u,M)` taking a vector of length n and a matrix with k row-vectors and n columns, returning the cosine of the angle θ between u and each row-vector. Keep in mind what you learned in 2.6b and consider that class of vectors only.

2.11 [2.5] Similar to 2.5, create a cosine similarity 2-D matrix for the `tones` embeddings. The diagonal elements must all be equal to 1.

AI-awareness and Simplicity measured in the *aiwars* posts

For the next case study, we shift our attention to general qualities within the posts. We're interested in vector directions capturing meaningful, human–interpretable features. For the AIwars subreddit, a predominant topical direction is that of **AI awareness**. It is also possible to track a stylistic direction that contrasts short, slogan-like, low-context statements with longer, hedged, evidence-seeking discussion; we refer to this as *anti-nuance* or **simplicity**.

The embeddings corresponding to said features are contained in `speech_anchors`.

3.1 [3.1] Create vectors `ai_awareness` and `simplicity`, the cosine similarities between the `aiwars_embeddings` dataset and each of the `speech_anchors` previously discussed. Add them as variables of `aiwars` for your next case study — remember to previously arrange both datasets by ascending `post_index`.

3.2 save the new dataset as `aiwars_updated` to use during this week's next session.

[0]These are usually very high-dimensional vectors with over 100 dimensions as in your dataset, but we will initially work with 2 to better illustrate the concepts.

[1]The coordinates for each embedding are outputs of a more complicated embedding model involving several neural network architectures and outside the scope of this course. For this week's case studies, we limit ourselves to the resulting embeddings.

Case study O. 3. 1

Instructions: Work in pairs or groups of three; solve the following exercises collaboratively, and put together the deliverable specified in Part 2.

Notebook: Open Case Study 1

Dataset: `aiwars_updated.csv`

One of you must share the case study's notebook (parts 1 and 2) and a fresh slides document (part 3) with the rest.

Part 1. Discrete and continuous methods

0.1 [0.1] Read in `aiwars_updated`; load/install the `tidyverse` packages.

Consider two continuous variables related to the semantics of text posted by redditors in r/Aiwars:

- **AI-awareness**: A, the extent to which a post's vocabulary and syntax depict artificial intelligence topics.
- **Simplicity**: S, a post's reliance on short, declarative phrasing and minimal argumentative or conceptual complexity.

1.1 Based on your previous knowledge, what are the sample spaces Ω_A, Ω_S ?

1.2 Provide a short explanation as to why these can't be discrete random variables.

1.3 [1.1] Use `ggplot2` to create A histogram plot for both A and S on the same x-axis.

(a) Declare a `ggplot()` on your data frame.

(b) Add a `geom_histogram()` layer with `ai_aware` as your x-`aes()`. Outside `aes()`, set the `fill` parameter to "blue", the `color` to "black", and the transparency (`alpha`) to 0.5.

(c) In the same plot, add another histogram layer analogously for `simplicity` but fill it with "red".

1.4 Examine the plot:

- What is the y-value?
- At what range of x-values does `ai_aware` have more observations?
- If we summed all y-values of the bins in the `simplicity` histogram, what should be the result?
- From the histogram, do we have an exact value for the following densities? $f_A(0.5)$, $f_A(0.499)$ $f_A(0.501)$? Why or why not?

1.5 More generally, which histogram seems more concentrated around a given range of values?

1.6 Are these histograms valid distributions?

1.7 We want the y-value to reflect a probability. Edit the `aes()` function in your histograms to include the following y-aesthetic: `y = after_stat(count/n)`. Compute and save `n` separately.

1.8 Similar to 1.4, what should be the sum of all the bins' y-values of any histogram?

1.9 Very roughly (no computation needed), what is x such that $F_A(x) \geq 0.5$?
1.10 Again roughly, what do you think $F_S(0.1)$ approximates to?
1.11 Histograms are discrete representations of a RVs. Change the `binwidth` of **just the red histogram** to 0.01. What can you observe? Did the distribution's relative shape change? Why?
1.12 Examine the new plot. What is the default bin width for the blue histogram then?

Recall the *Summarizing and Describing Data* lecture. We can get a **continuous** representation of our RV's distribution through kernel density estimation. Base R has a readily available function for this: `density()` — feel free to test it in your own time.

We will use `ggplot2`'s version: `geom_density()` to plot the estimated pdfs $\widehat{f_A(\cdot)}$ and $\widehat{f_S(\cdot)}$ for all the x-values in the domain of our data.

1.13 [1.1] Copy and paste your last histogram plot. Delete the `y` argument in `aes()` in both histograms. Substitute `geom_histogram` with `geom_density` in both layers.
1.14 Exclusively in terms of shape and position relative to each other, how do these distribution representations compare to those in 1.3?
1.15 Now examine the y-axis. How does it compare to your plot in 1.3?
1.16 Roughly, what is $max\ \widehat{f_S(x)}$? What is the minimum for both?
1.17 Are these valid distributions?
1.18 Roughly from the plot, what is $P(0.125 \leq S \leq 0.25)$? Now go back to 1.3 and compute the same probability.

Another way to compute the probability of an event for a continuous RV is through its CDF. Recall from the lecture that

$$P(a \leq X \leq b) = \int_a^b f_X(x)dx = F_X(b) - F_X(a)$$

Base R allows us to create a function for the empirical CDF of continuous variables with `ecdf()`. This function also has its `ggplot2` analog in `stat_ecdf()`, which you can use just as any other `geom`.

1.19 [1.3] Use `ecdf()` to create functions `F_A` and `F_S`, the empirical CDFs of A and S respectively. Make sure you have the correct object by quickly plotting the CDFs: `plot(F_A)`, `plot(F_S)`. Then answer the following questions:

(a) $F_S(0.25)$?
(b) $F_A(0.25)$?
(c) What is $s \in \Omega_S$ such that $P(S \leq s) = 0.8$?
(d) What is $a \in \Omega_A$ such that $1 - P(A \leq a) = 0.1$?

(e) Compute the *extremes* event probability for A: $P(S \leq 0.36 \cup S \geq 0.6)$
(f) Make a 95% inner interval for S. Find any $a, b \in \Omega_S$ such that $P(a \leq S \leq b) = 0.95$

1.20 [1.4] Use `stat_ecdf()` in `ggplot2` to jointly plot the CDFs for A and S. Make sure to `color` them blue and red, respectively.
1.21 Can you conclude there's first-order stochastic dominance of one of the random variables to the other?
1.22 In the lecture, height was the construct measured for women in both Bihar and the US. Are S and A measuring the same thing in the posts?
1.23 What, if any, could the implications of FOSD be in this case?

Part 2. Non-independence

2.1 Within the r/AIwars subreddit, conditional on learning posts tend to be long and argumentative, would their probability of being AI-related change?
2.2 [2.1] Let's begin by examining $f_{A,S}(a, s)$. Use `ggplot2`'s function `geom_bin_2d()` to plot a 2D-histogram with `ai_aware` in the `x`-axis and `simplicity` in the `y`-axis.
2.3 What does color represent in this plot? Note that the "minimal" unit colored is a rectangle tile.
2.4 Are the x and y-bandwitdths the same? What are they, approximately?
2.5 Similar to 1.8, modify the color to show a probability with `after_stat()`. This time change the `fill` aesthetic.
2.6 Generally describe the joint composition of the subreddit's posts in terms of their AI-awareness and their simplicity.
2.7 Do short, declarative post tend to discuss AI extensively?
2.8 Do relatively long and argumentative posts necessarily discuss AI deeply and technically?
2.9 What is the semantic region $\Omega_S \times \Omega_A$ most posts fall in?
2.10 **True** or **False**: if A and S were independent, we would have a perfectly squared 1×1 grid and all tiles would be of the same color, because $f_{A,S}(a, s) = f_A(a) \cdot f_B(b)$

Numerically, we can represent the joint pdf $f_{A,S}(a, s)$ of these two continuous RVs as plotted in 2.2 is a matrix with rows and columns corresponding to each x (`ai_aware`) and y-bin (`simplicity`), with a large number of bins. The next question's code will create such a matrix, `f_AS` for you. It should be 30×30.

2.11 [2.2] Run the pre-scripted code to get `f_AB`, the joint PDF $f_{A,S}(a, s)$. Then compute the marginals $h_A(a)$ and $h_S(s)$ and save them as `h_A` and `h_S`, respectively. Keep in mind these should be vectors of length 30.
2.12 Make quick base R plots of each marginal with `plot(y = h_A, x = as.factor(names(h_A)))` and similarly for h_S in a separate plot. Do they look familiar? Why?
2.13 Use `f_AS`, `h_A` or `h_S` — and their bin `names` — to calculate the following:
(a) $P(S \leq 0.36 \cap A \geq 0.628)$

(b) $P(0.15 \leq S \leq 0.36 \cap 0.628 \leq A \leq 0.859)$
(c) $P(S = 0.188 \cap A \leq 0.923)$
(d) $P(A \geq 0.628)$
(e) $P(A \geq 0.628 \cap 0.27 \leq S \leq 0.3)$
(f) $P(A \geq 0.628 \mid 0.27 \leq S \leq 0.3)$
(g) $P(A \geq 0.628 \mid 0.27 \leq S \leq 0.419)$
(h) $P(A \mid 0.27 \leq S \leq 0.3)$. Save as `A_cond` and plot this
(i) $P(S \mid A \in (0.795, 0.827])$. Save as `S_cond` and plot.

2.14 Based on your answers to 2.13f to 2.13i, are A and S independent?

2.15 Particularly on 2.13h and 2.13i, if A, S were independent, what should these plots look like? What differences can you notice?

2.16 Optionally, plot $f_{S|A'}(s|a = A')$ or $f_{A|S'}(a|s = S')$ alongside the curve they should approximate under independence.

2.17 [2.3] Finally, directly verify the claim in 2.10. Use `h_A` and `h_S` as well as a double (nested) loop to produce $g(a, s) = h_A(a) * h_S(s)$ for the same 30×30 grid. Then use `heatmap()` with the same arguments used in the previous code snippet to examine the joint density. Does this look like a homogeneous distribution? If not, why? If it does, at what regions of $\Omega_A \times \Omega_S$ do we have missing values, if any?

Part 3. Your own joint distributions

Your team will freely analyze the relationship between either AI-awareness or Simplicity and one other feature in the `aiwars` dataset.
Create a short deliverable following these instructions:

- Choose either `ai_aware` or `simplicity` and one other key variable.
- Your instructor will
  - Approve or change your choice based on their goals and what other students are working on.
  - Inform you of the format required for the deliverable. Usually a 5-slide eck or a show text document.
- Once your choice is approved:
- Once your choice is approved:
  1. Construct the joint distribution of the two variables (Depending on your choice of discrete or continuous, use a `geom_bin_2d`, a boxplot, or any of the plots reviewed in the lecture).
  2. Obtain and interpret the marginal distributions of each variable.
  3. Compute the conditional distributions (e.g. $f_{A|X}(a)$ or $f_{S|X}(s)$) for what you consider are relevant ranges of values. Evaluate how indpendent the analysis variables are.
  4. Include at least one visualization that helps highlight the relationship (histograms, heatmap, density plot, etc.).
  5. Provide 1–2 slides (or short paragraphs) summarizing the implications of the independence (or lack thereof) between variables plain language: What

does the relationship tell us about Redditor behavior?

– Submit as required. Your instructor will let you know if your team will briefly expose your deliverable to the class.

14.310x Flipped Classroom

Week 4 Instructions

Transforms of random variables
Moments of random variables
Introductory simulation in R

Checklist

- ☐ Complete Coding Lab 4 (Requirement)
- ☐ Solve Guided Case 3 (Session 1)
- ☐ Solve Guided Case 4 (Session 2)

Coding Lab L.4.4

Simulation

Instructions: Work individually. Answer all sections of the Lab.

Notebook: Coding Lab 4: Random Variables' simulation in R

Record your answers in your copy of the notebook, and if required, submit your work as specified by the instructor.

Guided Case G.4.3

Select transforms of random variables: theory and coding practice

Instructions: Work in pairs or teams of 3. Answer all questions 1-3 and submit your coursework as required by the instructor.

In the following questions, X will be a **known** random variable with pdf/pmf $f_X(x)$ (either provided or described for you to write down). Each question will propose a transform $Y = g(X)$ and you must derive $f_Y(y)$, the pdf/pmf associated with said transform of X. All exercises will ask you to follow a few analytical steps to then plot and perform simulations in R.

Question 1

$$X \sim U[-1, 1]$$
$$Y = e^X$$

1.1 What is the support Ω_x the support of X? Write down $f_X(x)$.

> $\Omega_X = [-1, 1]$, and $f_X(x) = \frac{1}{1-(-1)} = \frac{1}{2}$ for $-1 \leq x \leq 1$ and 0 otherwise.

1.2 What is Ω_Y, the support of Y?

1.3 As in the lecture, work out the CDF of Y first.

$$F_Y(y) \equiv P(Y \leq y)$$

(a) Substitute Y for $g(X)$.
(b) Get an expression in terms of the CDF of X: $P(X \leq g^{-1}(y))$.
(c) Evaluate the expression. $\int_{-\infty}^{g^{-1}(y)} f_X(x)dx$.

1.4 Get $f_Y(y)$. If the CDF is continuous you can take the derivative $\frac{\partial F_Y}{\partial y}$.

1.5 [1.1] Plot $f_Y(y)$ in R. Use `seq()` to create a vector `omega_y` that goes from $\frac{1}{e}$ to e in increments of 0.05. Then create `f_Y`, by applying your expression for the pdf $f\_Y(y)$ to each element in `omega_y`. Plot f_Y.

1.6 [1.2] Now implement a simulation for $f\_Y(y)$:
(a) `set.seed() to 100`
(b) Use `runif()` with the relevant parameters to draw a sample with 10,000 realizations of X. Save it as vector `X_sim`.
(c) Apply the $Y = g(X)$ transformation to your sample. Save it as `Y_sim`.
(d) Make a quick histogram plot of `Y_sim`. How well does it resemble the theoretical pdf you derived?

1.7 [1.3] Run the code provided to create a side-by-side plot of both the theoretical (blue) and the estimated (dashed red) pdfs. Make sure you followed the naming conventions so far.

Question 2

$$X \sim U[-1, 1]$$
$$Y = \frac{X^2}{1 + X^2}$$

2.1 What is Ω_y?
2.2 Get $F_Y(y)$. *Be very careful with the inequalities.*
  – Verify the following: $F_Y(0) = 0, \quad F_Y(\frac{1}{2}) = 1$
2.3 Get $f_Y(y)$ and plot it along the support of Y as in 1.5.
2.4 Similar to 1.6, simulate Y and plot its histogram with `breaks = 100`.
2.5 Plot the theoretical and estimated densities side by side like you did in 1.7. This time, reduce the bandwidth for the estimated density: set the `bw` argument in `density()` to 0.01.

Question 3: Probability integral transform

$$X \mid \ f_X(x) = \begin{cases} 4x & 0 \leq x \leq \sqrt{\frac{1}{2}} \\ 0 & \text{otherwise} \end{cases}$$

$$Y = F_(x)$$

3.1 Derive $F_X(x)$.
3.2 What is the support of any CDF? (What is Ω_y?)
3.3 According to the lecture, what should $f_Y(y)$ be? — Review the PIT slides if this is not clear.

In the lecture, Prof. Ellison mentioned that we can sample from any distribution by sampling from the distribution you got in 3.3 and transforming it by the inverse CDF of X:

$$Y = F_X(X)$$

$$\implies X = F^{-1}(Y)$$

3.4 From 3.1, solve for X as above —you should end up with a function in terms of Y.
3.5 [3.1] Confirm you get $f_X(x)$ back with a simulation:
  – Draw 10,000 observations from the distribution of Y: `Y_sim`.
  – Translate them to X draws by applying them function $F^{-1}(y)$ you found in the previous question: `X_sim`.

– Plot the histogram of `X_sim` with `breaks` set to 100. It should resemble $f_X(x) = 4x$.

[Harder] Question 4: Convolution

$$X_1 \sim U[0,1]$$

$$X_2 \mid \ f_{X_2}(x_2) = \begin{cases} 2x_2 & 0 \leq x_2 \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$Y = X_1 + X_2$$

$$X, Y \ \text{ independent.}$$

$$f_Y(y) = f_{X_1,X_2}(X_1, X_2) = \int_{R_1 \subset \Omega_{X_1}} \int_{R_2 \subset \Omega_{X_2}} f_{X_1}(x_1) \cdot f_{X_2}(x_2) dx_2 dx_1$$

4.1 Given that $\Omega_{X_1} = \Omega_{X_2} = [0,1]$ what is Ω_y?

Even in simple convolutions, finding the correct integration region is challenging. The following exercises are meant to guide you in setting up the correct bounds for f_{X_1} and f_{X_2}, in order to find $F_Z(z)$.

Given Y and X_2 below, write down the possible values of X_1:

4.2 $Y = 1.5$ and $X_2 = 0.9$

4.3 $Y \geq 0.5$, and $X_2 = 0.5$

4.4 Let $0 \leq Y < 1$ and $\frac{1}{4} \leq X_2 \leq \frac{1}{2}$.

(a) What values of Y are events with zero probability (i.e. they're **not** possible)?

(b) Write down the range of values X_1 can take.

(c) Now suppose we have $y \in Y$, a specific value in $[\frac{1}{4}, 1)$. Write down the possible values for X_1 **in terms of number** y and the bounds of X_2 as before.

4.5 Let $0 \leq Y < 1$ and $0 \leq X_2 \leq 1$.

(a) Consider numbers $y \in Y$ and $x_2 \in X_2$. Write down the bounds for X_1 in terms of these numbers (particularly, find the upper bound).

4.6 Let $1 \leq Y \leq 2$ and $0 \leq X_2 \leq 1 \mid \ y \in Y, \ x_2 \in X_2$. For the bounds on X_1 you now have to consider two events:

(a) When $y - x_2 \geq 1$.

(b) When $y - x_2 < 1$. *Hint: if x_2 is "too large", then x_1 can't be just any value, it will be restricted. And vice-versa.*

With your previous answers in mind, now derive the pdf of Y:

$$F_Y(y) \equiv P(Y \leq y) = P(X_1 + X_2 \leq y) = P(X_1 \leq y - X_2)$$

Case 1: $0 \leq y < 1$

$$... = \int_0^{\square} \int_0^{\square} f_{X_1}(x_1) \cdot f_{X_2}(x_2) dx_2 dx_1$$

4.7 Fill out the upper limits of the integrals and solve for $f_Y(y)$ when $0 \leq y < 1$

Case 2A: $1 \leq y \leq 2, \quad y - 1 \geq x_2$

$$... = \int_{\square}^{y-1} \int_{\square}^{\square} f_{X_1}(x_1) \cdot f_{X_2}(x_2) dx_2 dx_1 \tag{1}$$

4.8 Fill out the limits and solve for $F_Y(y)$.

Case 2B: $1 \leq y \leq 2, \quad y - 1 < x_2$

$$... = \int_{\square}^{\square} \int_{\square}^{\square} f_{X_1}(x_1) \cdot f_{X_2}(x_2) dx_2 dx_1 \tag{2}$$

4.9 Fill out the limits and solve for $F_Y(y)$.

4.10 Write down $F_Y(y)$ for all $1 \leq y \leq 2$, x_1 and x_2. *Hint: cases 2A and 2B are two (disjoint) partitions of* $1 \leq y \leq 2$.

4.11 Derive the pdf of Y for all $y \in \Omega_y$.

4.12 [4.1] Plot $f_Y(y)$ in R. Start by defining `omega_y` as in previous questions, then apply the correct pdf for each segment. Save the pdf as vector `f_Y` . *Suggestion: use* `ifelse()`

4.13 [4.2] Now perform a simulation. Draw 10,000 random observations for both X_1 (`X1_sim`) and X_2 (`X2_sim`; as you did for 3.4 and 3.5). Create the simulated convolution: `Y_sim` and recycle the code to plot the estimated density alongside the theoretical distribution.

Guided Case G.4.4

Moments of random variables: theoretical session

Instructions: Work on your own; answer parts 1-3 and only answer part 4 if required by your instructor. This case study is to be solved fully offline. Write down your answers in the format required by your instructor.

Part 1

1. Let $E(Y|X)$, where (Y, X) is a vector of random variables. Is this a function of X? If X is fixed ($X = x$), is $E(Y|X)$ a function of X?
2. Let $E(Y|X) = 3 + \frac{1}{2}X_1$, where $E(X_1) = 2$, $V(X_1) = 3$. What is $E(Y)$?
3. What is the correlation between Y and X_1?

   Let $\varepsilon \sim N(0, 9)$, and $E(Y|X) = 3 + \frac{1}{2}X_1 + \varepsilon$.
4. What is the correlation between Y and X_1?

   Let $E(Y|X) = 3 + \frac{1}{2}X_1 + X_2^2$, where $E(X_1) = 2$, $V(X_1) = 3$, $E(X_2) = 2$, $V(X_2) = 1$.
5. What is $E(Y)$?
6. If $Y|X = 3 + \frac{1}{2}X_1 + X_2^2$, what is the correlation between Y and X_1? Between Y and X_2?
7. If $Y|X = 3 + \frac{1}{2}X_1 + X_2^2 + \varepsilon$, what is the correlation between Y and X_1? Between Y and X_2?
8. Let $V(Y|X) = V[E(Y|X)] + E[V(Y|X)]$. Which component corresponds to the part explained by X? Which to the part that is unexplained by X? Answer in general terms.
9. What percent of $V(Y|X)$ is explained by $V[E(Y|X)]$ for the model in 7?

Part 2

A policy provides fixed grants for microentrepreneurs. Previous rounds of funded businesses have received on average \$5,000 in subsequent funding opportunities. Let X be subsequent funding.

1. What is the maximum share of the microentrepreneurs that receive \$100,000 or more?
2. What is the subsequent minimum funding amount over which possibly 25% of these microentrepreneurs were funded?
3. Based on previous measurements, an estimate for the standard deviation of X is 1,000. Using Chebyshev's inequality, calculate what is the maximum share of the microentrepreneurs that receive \$100,000 or more. Compare with your results from part 1. Interpret the results.

Part 3

Let $\{X_i\}_{i=1}^n \overset{\text{i.i.d.}}{\sim} U[1, 19]$.

1. What is the mean of X_i? What is its variance?
2. Is the sample mean $\bar{X}_n$ a random variable? Why?
3. What is the expectation of $\bar{X}_n$?
4. What is the probability that $\bar{X}_n$ is more than t away from the expectation of $\bar{X}_n$?
5. What happens to this probability if the sample size increases? What happens when $n \to \infty$?

Part 4

A set of 6 grants will be awarded at random to participating firms, 4 of which are specialty grants and 2 regular grants. There are 40 prospective firms, of which 14 are microenterprises, and 26 are SMEs. Among these firms, 8 microenterprises and 8 SMEs have an accreditation required for eligibility to receive a specialty grant.

1. What is the expected number of:
   - grants for microenterprises?
   - grants for SMEs?
   - grants for accredited firms?
   - grants for accredited microenterprises? Grants for accredited SMEs?
   - grants for any type of firm?
   - How does the number of expected grants for microenterprises change with the number of accredited microenterprises? Is there a minimum number of accredited microenterprises which makes the expected number of grants for microenterprises higher than the expected number of grants for SMEs (holding everything else constant)?

14.310x Flipped Classroom

Week 5 Instructions

Central Limit Theorem and the Law of Large Numbers
More advanced simulation in R

Checklist

- ☐ (**Optional**) Watch Youtube video 1, an informal introduction to the Law of Large Numbers.
- ☐ We will be working with the *Weak Law of Large Numbers*. Have a simple statement of the theorem such as this one readily available.(Requirement)
- ☐ Complete Open Case Study 2 (Sessions 1&2)

Exercise O.5.2

The law of large numbers (LLN)

Instructions: Work in pairs or groups of 3. Answer all questions and discuss your results with the class when appropriate.

Notebook: Scrap notebook (some comments)

There are multiple ways to go about many of the questions and coding tasks, and

you may solve them as you see fit. You will also need to provide open-ended answers to explain a number of the results you get. That said, some tasks will rely heavily on what we learned about *Simulation* in the Week 4 Coding Lab; your team may want to keep the notebook handy.

Scenario: We will be working with samples of five different continuous random variables as follows:

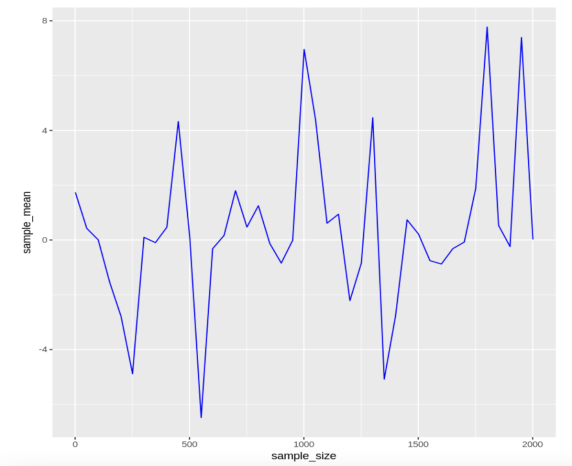
$$X_1 \sim N(5, 1)$$
$$X_2 \sim N(5, 9)$$
$$X_3 \sim U[2, 8]$$
$$X_4 \sim Exp(\frac{1}{5})$$
$$X_5 = \frac{X_a}{X_b} \quad | \quad X_a, X_b \overset{iid}{\sim} N(0, 1)$$

---

1. Get these distributions' expectations $\mu \equiv E(X)$ and variances $\sigma^2 \equiv Var(X)$.
2. For each RV, first run `set.seed(2)` **once**, then draw two samples: one of size $n = 5$ and another of size $n = 50$. Take their respective sample means:

$$\bar{X}_n = \frac{x_1 + x_2 + ... + x_n}{n}$$

   (a) Per the LLN, for which sample size would you expect $\bar{X}_n$ to be closer to μ?
   (b) Does this happen in all cases? If not, briefly discuss why.



3. For each RV, draw random samples of sizes $n = 1, 50, 100, 150, ..., 2000$ and create a plot similar to the one above — the plot must be reproducible.

   Suggestion: you can start by saving the sample means in a data frame like the one below

A data.frame: 5 × 3

RV_name	sample_size	sample_mean
<chr>	<dbl>	<dbl>
normal(0,1)	1	-0.5604756
normal(0,1)	10	0.2530814
normal(0,1)	20	-0.1478493
normal(0,1)	30	0.1767775
normal(0,1)	40	0.1003377

   (a) Is the LLN met for this RV? If not, discuss the reason.
   (b) As n increases, how does the sequence of sample means behave? Does the trajectory *steadily* converge to μ?
   (c) Discuss how your previous answer may explain what you observed in question 2..
   (d) Plot X_1 and X_2 together. Do they converge to the same μ? How are they different? Why?
   (e) From which RV did the example plot above likely originate?
      – Suppose the sample means keep oscillating in roughly the same way around μ as $n \to \infty$. Explain precisely what part of the Weak Law or Large numbers would not be met.

4. Now consider convolution $Y = X_1 + X_2$. Compute $E(Y)$ and $Var(Y)$.
5. `set.seed(123)` and simulate Y with a sample of size $n = 10,000$. Compute the sample's mean $\bar{X}_n$ and variance s^2. What is the difference between the s_y^2 and σ_y^2?

Independent, but non-identically distributed samples

Sometimes we cannot directly choose the population from which we draw. A common example of this situation is when sampling is done at the level of **clusters** or **groups**, such as schools or villages, rather than individuals. Each cluster may have its own distribution, so the resulting sample is not identically distributed. Note that the draws are still independent: no observation depends on the value of any other. Drawing a cluster only adds several random observations from a distribution "at once".

The following questions are meant to help you work out how the LLN applies in these cases.

Suppose we have 2 independent random variables:

$$Y \sim N(\mu_y, \sigma_y^2)$$
$$Z \sim N(\mu_z, \sigma_z^2)$$

We have $M = \{y_1, y_2, ..., y_{n_0}, z_1, z_2, ..., z_{n_1}\}$ a random sample of size n where n_0 observations come from Y, n_1 observations come from Z, and $n_0 + n_1 = n$.

6. Write down $\bar{M}$, the sample mean:

$$\bar{M} = \frac{(\quad) + (\quad)}{\square}$$

7. Compute $E(\bar{M})$ in terms of $w_y \equiv \frac{n_0}{n}, w_z \equiv \frac{n_1}{n}, \mu_y$, and μ_z. What type of average is this?
8. Compute $Var(\bar{M})$. You should get an expression $= \frac{\square}{n}$, where $\square$ is the same type of average as in the previous question.
9. With your previous answers in mind, suppose we now have sample R with the following mixing:
   - 30% from X_1
   - 20% from X_2
   - 40% from X_3
   - 10% from X_4

   (a) What are $E(\bar{R})$ and $Var(\bar{R})$?
10. Plot $\bar{R}_n$ for $n = 50, 100, ..., 2000$.
    (a) Does $\bar{R}_n$ exhibit the same type of convergence?
    (b) Discuss what would happen with $\bar{R}_n$ if we added X_5 draws to the mix.

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Week 6 Instructions

Estimator properties
Hypothesis testing and confidence intervals
Further simulation in R

Checklist

- ☐ Complete Coding Lab 5 (Session 1)
- ☐ Complete Guided Case Study 5 (Session 2)
  - ☐ Dataset: `consultations.csv`

Coding Lab L.6.5

Understanding Confidence Intervals Hypothesis Testing and Two Type Errors in R

Instructions: Work individually. Solve all exercises in the corresponding Colab notebook. Record your answers and/or code in your copy of the notebook.

Notebook: Coding Lab 5

Submit your work in the format required by the instructor.

Guided case study G.6.5

Examining *Clinicia*'s financial incentives for physicians

Instructions: Work in pairs or groups of three to solve all questions.

Notebook: Case Study 5

Dataset: consultations.csv

The team or pair may work on one notebook collaboratively. One team member needs to share editor access their the Colab Notebook with the rest.

Scenario:

Medical consultations take place in an environment with substantial **information asymmetry**. Patients are usually not qualified to determine whether diagnoses are accurate or if a prescribed treatment is correct. This creates incentives for health-care providers that can alter treatment intensity and the types of drugs prescribed, directly impacting expected **health outcomes** and total **treatment costs**. Especially when a provider stands to benefit financially from a certain treatment avenue over others.

Clinicia's hospital system is reported to have unusually high rates of **antibiotic prescription**, even for conditions where guidelines recommend more conservative treatment. Overuse of antibiotics not only raises treatment costs unnecessarily, but also accelerates **microbial resistance**, reducing the effectiveness of essential drugs. The country's Ministry of Health is aware of the reports and wants to test more formally for any differences in treatment cost (or prescription rates) in the presence of financial incentives. As part of the policy team, you are tasked with performing this testing.

Medical records in Clinicia include audio logs of consultations (the audio is altered to protect patient and provider privacy) along with administrative information on diagnoses, procedures, and prescriptions. The Ministry was granted access to a fully anonymized, random sample of these records. Your team processed all the data and produced the `consultations` dataset. For each medical consultation:

- `incentive =1`: Indicates the patient is buying the prescribed drugs at the hospital or at a commercially related pharmacy.
- `request =1`: Indicates patient requested antibiotics.
- `antibiotics =1`: Indicates the physician prescribed antibiotics.
- `totalprice`: The market price (in USD) of the drugs prescribed to the patient at the consultation.
- `multiple =1`: Indicates the physician prescribed two or more distinct drug categories (e.g., analgesic + antibiotic + expectorant)
- `grade2 = 1`: Indicates the physician prescribed a drug with a high potential

for abuse and dependence (e.g., oxycodone, metanphetamines, Aderall).

Exercises

1. Load or install the `dplyr` and `pwr` packages. Read in `consultations`.
2. Create `elsewhere`, which reflects the patient indicated they would buy the drugs in a place unrelated to the hospital.
3. Compute the average and standard deviation of `antibiotics`, `totalprice`, `multiple` and `grade2`, for the patients group that result from combining `elsewhere` and `request`.
4. Compute the number of observations of `antibiotics` by the combinations of per combination of `elsewhere` and `request`.
5. Test the hypotheses that there is no difference in antibiotic prescription between the patients that do not represent a financial interest (`incentive == 0 & request == 0`) and:
   - Those only requesting antibiotics.
   - Those only buying antibiotics from the hospital.
   - Those requesting antibiotics AND buying them from the hospital.

   Assume these 4 groups have the same variance. Report the value of the difference in means, the test statistic, the test's p-value and whether or not the null can be rejected with 95% *confidence*.
6. Are the observations for each patient group independent?
7. How sensible is it to assume our draws for the 4 patient groups are identically distributed (i.e., sampled from the same distribution)?
8. Perform the hypothesis tests in Question 5. again, but now assume each patient group has a different outcome variance.
9. Compute the 95% confidence interval for this difference. What is its interpretation?
10. Suppose now you implement a new experiment, where you want to be able to detect an effect with a certain probability, if present. How is this probability called?
11. Your colleagues at the policy team define an economically meaningful effect to be detected of x magnitude. What is x called?
12. Consider the distributions under H_0 (zero effect) and H_β (β effect, different from 0). What area of the curve of these distributions correspond to the power of the test?
13. Consider first that for this new experiment that for the control group we expect a total medication cost of \$100. We want to detect at least a change of \$40 in the total medication cost with 80% power and 5% significance, using a two-sided hypothesis test. Asume a pooled standard deviation of 60. Compute the sample size required for detecting at least a standardized change of \$40. Compute the sample sizes required also for detecting a change of \$10, \$20, \$80.
    Hint: Use the command `pwr.t.test()`
14. Compute the required sample size per group for detecting a difference of magnitude 0.05, 0.10, 0.20, and 0.40, with two-sided testing, with a significance of 0.05,

and power of 0.80.

15. Comment on your results. Explain the relation between sample size and effect size. Is this relation proportional?

Bibliography

Pop-Eleches, C., & Urquiola, M. (2013). Going to a better school: Effects and behavioral responses. *American Economic Review*, *103*(4), 1289–1324. https://doi.org/10.1257/aer.103.4.1289

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Week 7 Instructions

The potential outcomes framework
Experimentla design and randomized evaluation in economics

Checklist

- ☐ Set aside this Wikipedia article on testing two-proportions hypotheses, in case you need it during Session 2 (Requirement)
- ☐ Skim this Wikipedia article discussing the Local Average Treatment effect (LATE) , in advance of Session 2. (Requirement)
- ☐ Complete Guided Case Study 6 invidually (Session 1)
  - ☐ Retrieve the `students` dataset here
- ☐ Complete Open Case Study 3 in pairs or teams of 3. (Session 2)
  - ☐ You will continue to use `students`

Case study G.7.6

Evaluating AI-assisted learning on student outcomes

Instructions: Work on your own; read the scenario and answer the questions. Type your answers in the format required by the instructor.

To work on the `students` dataset you may use either a Colab notebook or your own

installation of R.

Dataset: students.csv

Your code will not be evaluated, but keep your R script or notebook tidy, as you may need to review some of your answers during Session 2.

Scenario:

You are the Government of *Novaria*'s new Minister of Education. The Prime Minister has tasked you with evaluating a primary education policy recommendation: the rollout of AI-assisted learning for mathematics curricula in grades 5-8.

The proposed program, *Project Mentor*, involves deploying a large language model (similar to ChatGPT) named *AlgebrAI* specifically trained and fine-tuned for elementary math tutoring. AlgebrAI's interface is tailored to deliver interactive, one-on-one tutoring sessions to students. The AI mentor adapts to each student's skill level and provides problem-solving guidance, hints, and feedback designed to help the students master their grade's math curriculum.

Each participating school receives a number of tablets with AlgebrAI pre-installed, configured for offline-first use and automatically synced with central servers when internet is available. Students selected for treatment attend 20-minute tutoring sessions per day under the supervision of a facilitator.

The Prime Minister believes Project Mentor can boost test scores nationwide, but political opponents have raised concerns over cost and long-term efficacy. You are now in charge of evaluating the impact of the program in grades. Your team provides you with:

1. A 6th-grade math test designed to perfectly measure domain of the curriculum in a scale from 0 to 100.
2. A list of 1,000 students enrolled in 6th grade across Novaria, picked at random — part of the `students` dataset. This list contains only the following variables:
   - `unit`: a consecutive number assigned to the student.
   - `W_school`: Indicates whether the student's school is managed by the government ($W_{school} = 1$) or if it is privately managed ($W_{school} = 0$)

You have authority to apply the exam to any 6th grader in Novarnia, and you can implement the program (tablet usage and monitor time) in all government-managed schools, but to include any students attending a private school to the program you must first obtain authorization from their school board.

Exercises

Consider $T_i \in \{0, 1\}$ the treatment status of student $i = 1, 2, ..., 1000$ — $T_i = 1$ if treated $T_i = 0$ if not treated. Potential outcomes $y_i(T_i)$ in `students`, measured in test results (grades 0 to 100) are defined:

- `y0`: vector $Y(0)$, assume we can't observe it unless specified.
- `y1`: vector $Y(1)$, assume we can't observe it unless specified.

1.1 What is the value of $y_3(1)$? Describe its meaning —in terms of the potential outcomes framework.
1.2 What is the value of $y_5(0)$? Describe its meaning.
1.3 Compute $\bar{Y}(1)$. Describe its meaning.
1.4 Suppose $T = 0$. What is the value of y_{20}^{obs} and y_{40}^{miss} ? Briefly explain why.
1.5 Suppose $T_i = 1$ for all $i = 1, 2, 3, ..., 1000$. What is $\bar{Y}^{obs}$?
1.6 Imagine you can observe both potential outcomes.
  (a) What is the causal effect of Project Mentor in student 245?
  (b) What is the estimated Average Treatment Effect (ATE) of Project Mentor?
  (c) Does the estimated ATE support the Prime Minister's claims?

In practice, only Y^{obs} will be available after applying the exam. You will observe **one test score per student**, as well as the students' treatment status: either treated or untreated.

After careful consideration, your team assigned N_0 students to control and N_1 students to treatment out of the $N_0 + N_1 \equiv N = 1000$ sampled. The assignment criteria included logistics, the school-year timeline, operation costs and potential political opposition. Treatment was allocated amongsts students per the rule

$$T = W_{school}$$

2.1 Explain the assignment rule in simple words
2.2 For each of the assignment criteria, provide a brief circumstance that likely motivated Novaria's government to conclude this was the best allocation.
2.3 What is the value of N_0?
2.4 What is the value of N_1?
2.5 Create a variable for $Y^{obs}(W)$, and name it `yobs_w`. With this variable:
  (a) Compute the value of $\bar{Y}^{obs}(1)$.
  (b) Compute the value of $\bar{Y}^{obs}(0)$.
2.6 Write down both expressions $\bar{Y}^{obs}(\cdot)$ more formally, in terms of summations.
2.7 Your team knows that $ATE = E(y_i^{obs}|W = 1) - E(y_i^{obs}|W = 0)$ but they don't know why, or how to estimate it from our sample.
  (a) What is $\widehat{ATE}$?
  (b) Justify your answer in terms of a famous mathematical theorem:
    i. $\square \xrightarrow{\square} E(y_i|W = 1)$
    ii. ...
    iii. Therefore the estimate ...
  (c) Compute $\widehat{ATE}$ using R.
  (d) **Reflect on the result**. How does it compare to your answers in 1.6b and 1.6c? At this point, do we have any way to diagnose the accuracy of this result?
2.8 Again, let's imagine we can observe potential outcomes. In the lecture, it was shown that the ATE can be decomposed in *treatment on the treated* and *selection*

bias. Write down that expression and estimate the values of:

(a) treatment of the treated
(b) selection bias
(c) each individual term in *selection bias*

2.9 What do these values imply for the experiment's design? Is the value for 2.8a a potentially good $\widehat{ATE}$? What would be omitted if we were to only consider this value?

Case study O.7.3

Evaluating AI-assisted learning on student outcomes (continued)

Instructions: Work in pairs or groups of 3; answer the questions as concisely as possible. Type your answers in a single shared document or in the format required by the instructor.

One of you must set up a blank Colab notebook to work on, and share it with the rest. Save any figures or output, and incorporate as required by the instructor.

This is a direct continuation of G.7.6, thus we will work under the context you already have. Continue to use `students` when necessary to answer the questions.

Scenario: You let the Prime Minister know the treatment allocation for the Project Mentor experiment you had previously agreed on is problematic. He hires a team of consultants to help you sort this design problem out, as well as polish other details of the experiment. The following exercises are contain some of the questions asked during the meetings held with the consulting team and the Prime Minister.

Meeting 1

1.1 Firstly, you are asked to explain generally why you cannot get a credible average treatment effect from the current treatment allocation. How would outcomes be different we were to scale up the program nationally? (use your "secret" knowledge of the potential outcomes)

1.2 You are asked to provide an alternative assignment that would create two groups equally representative of 6th-graders nationally. Create such assignment variable under the name `T`, and also create `yobs_t`, the outcome we would observe under this assignment. Calculate the $\widehat{ATE}$. How does this result compare to 2.8a in G.7.6? Without making any further calculations, what do you think the treatment effect among private schoolers will be?

1.3 The Prime Minister doesn't believe that your new assignment created comparable groups. One way to provide evidence of balance, is showing the groups have the

same composition of public and private schoolers. Formally show there is no evidence to reject the composition is the same. Be as conservative as possible with the variance.

1.4 Imagine everyone can observe potential outcomes. From the definition of ATE, show treatment and control are comparable more decisively.

Meeting 2

While more convinced of your new assignment, the Pime Minister still insists asking for permission to private schools is impractical and will delay matters. Conveniently, the consulting team asks two questions:

- Whether attending a private/public school in Novartia actually creates systematic differences between students; particularly, differences related to the outcome. This has not been formally shown.
- Whether there may be differences in treatment effects between public and private students (e.g. the mean effect is larger for any), as these differences would justify different rollouts.

To provide evidence in favor or against these questions, bear in mind we have to start from the following assumption:

$$
\begin{aligned}
y_i(0)|W=0 &\sim Distr(\mu_0,\ \sigma^2_{0,0}) \\
y_i(1)|W=0 &\sim Distr(\mu_1,\ \sigma^2_{1,0}) \\
y_i(0)|W=1 &\sim Distr(\nu_0,\ \sigma^2_{0,1}) \\
y_i(1)|W=1 &\sim Distr(\nu_1,\ \sigma^2_{1,1})
\end{aligned}
$$

Where $\sigma^2_{0,0} \neq \sigma^2_{0,1} \neq \sigma^2_{1,0} \neq \sigma^2_{1,1}$

2.1 In your own words what do these assumptions mean? What do they entail when it comes to testing hypotheses? According to the lecture, what should we assume about the correlation among these random variables if we want to be conservative?

2.2 Answer the question about systematic differences in outcomes between public and private schools with the evidence you have.

(a) State the appropriate null hypotheses and their alternates.
(b) Test them with the appropriate statistic (estimate any parameters you don't know).
(c) Tie your conclusions directly to the question with 95% confidence.

2.3 Answer the question about differences in treatment effects.

(a) State the appropriate null hypotheses (test equality).
(b) Make inference on $\hat{\nu}_1, \hat{\nu}_0, \hat{\mu}_1, \hat{\mu}_0$ accordingly.
(c) Tie your conclusions directly to the question with 95% confidence.

Meeting 3

Satisfied with your answers, the Prime Minister and consultants approach you with some final questions about the program's broader implications:

3.1 **SUTVA violations** In your own words, briefly explain the Stable Unit Treatment Value Assumption (SUTVA). Could implementing Project Mentor violate this assumption in Novaria? Provide a specific scenario illustrating such a violation clearly. How might these externalities affect the accuracy of your estimates?

3.2 **Alternative policies with proven outcomes (e.g. TaRL)** The consultants suggest evaluating cheaper alternatives, like Teaching at the Right Level — targeting teaching to each student's current skill level without advanced technology; human mentors, complementary material, smaller traditional groups (more teachers), among others. These alternatives currently have more robust evidence of their effects.

(a) What, if any, are some differences between the theory of change underlying TaRL and that of Project Mentor?

(b) If differences exist, briefly discuss how you would test them within an experimental design, clearly describing treatments, assignment and measured outcome(s).

(c) Apart from the treatment effects, what else is necessary if we wanted to fairly compare any known TaRL intervention to Project Mentor in terms of efficiency?

(d) In this comparison, how important do you think scale would be? Briefly describe how costs for one and the other would behave.

3.3 **Non-compliance** Not every school or student may strictly follow the treatment assignment. Describe one realistic scenario of non-compliance in this project. Explain briefly how such non-compliance might bias the estimated treatment effect. Suggest one practical strategy to reduce or mitigate non-compliance.

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Week 8 Instructions

The linear regression model
Linear models in R: `lm` class, parsing outputs with `broom`

Checklist

- ☐ Watch the linear models with R: `lm` tutorial[4] (Requirement)
- ☐ Complete Guided Case Study 7 (Session 1)
  - ☐ Get the `bike_rentals` dataset here
- ☐ Complete Guided Case Study 8 (Session 2)
  - ☐ Get the `student_performance` dataset here
- ☐ Complete Guided Case Study 9 (Session 2)
  - ☐ Get the `kc_house_data` dataset here
- ☐ (Optional) Explore **Review Notes** and further **Readings** available.

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[4] **Module 8** > **Introduction to the Class lm** in the online component of the course.

Guided case study G.8.7

Part A - Bike rentals

Instructions: Work in pairs or groups of three. Solve the exercises for Part A of *Linear Regression* in Colab. Work collaboratively in a single Notebook.

Notebook: Part A

Dataset: bike_rentals.csv

Type your answers in the notebook, and submit your work per the instructor's requirements.

Guided case study G.8.8

Part B - Student performance

Instructions: Work in pairs or groups of three. Solve the exercises for Part B of *Linear Regression* in Colab. Work collaboratively in a single Notebook.

Notebook: Part B

Dataset: student_performance.csv

Type your answers in the notebook, and submit your work per the instructor's requirements.

Guided case study G.8.9

Part C - KC Housing Data

Instructions: Work in pairs or groups of three. Solve the exercises for Part C of *Linear Regression* in Colab. Work collaboratively in a single Notebook.

Notebook: Part C

Dataset: kc_house_data.csv

Type your answers in the notebook, and submit your work per the instructor's requirements.

Additional resources

- Drive: Review Notes
- Drive: Readings

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Week 9 Instructions

Regression Discontinuity Design (RDD)
RDD modeling in R

Checklist

- ☐ Complete Coding Lab 6 (Session 1)
- ☐ Complete Guided Case 10 (Session 2)
- ☐ Review the reference material as needed (Optional)

Coding Lab C.9.6

Coding Lab 6— Further regression tools

Instructions: Work individually. Complete all the coding lab's exercises.

Notebook: Further regression tools

Upon completion, submit your work in the format required by the instructor.

Guided Case Study G.9.10

Regression Discontinuity: Replicating *Going to a better school* (Pop-Eleches & Urquiola, 2013)

Instructions: Work in pairs or teams of three. We will go over a section of *Going to a better school: Effects and Behavioral Responses* to examine outcomes differences of children attending higher achievement schools.

Notebook: Guided Case 10— RDD paper replication

Dataset: Schools.dta

Paper: (Pop-Eleches & Urquiola, 2013)

Scenario: In 2002, the Romanian Ministry of Education centralized the high school admissions system, ranking students by their standardized test scores and matching them to schools in descending order of achievement. This reform generated a compelling natural experiment: some students who barely made it into higher-achieving schools were nearly identical, academically, to those who just missed the cutoff. In this guided case study, you will analyze data from this setting to investigate whether attending a better school causally affects student outcomes. You'll explore the design of the regression discontinuity strategy used by Pop-Eleches and Urquiola (2013), and replicate parts of their analysis using real admissions and outcomes data from Romanian high schools.

A Colab notebook has been set up with the full case directions therein. Submit your coursework in the format required by your instructor.

Additional materials and references

References:

Drive:

- (Pop-Eleches & Urquiola, 2013)
- Causal inference textbook
- Econometrics textbook

Additional materials:

Drive:

- Review notes week 9

Bibliography

Pop-Eleches, C., & Urquiola, M. (2013). Going to a better school: Effects and behavioral responses. *American Economic Review*, *103*(4), 1289–1324. https://doi.org/10.1257/aer.103.4.1289

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Week 10 Instructions

Random assignment and Instrumental Variable (IV) models
More on IV: multiple instruments, multiple stages.

Checklist

- ☐ Read the *details* on `ivreg()` in the the `AER` package. Make sure you learn how to enter a 2-stage specification. (Requirement)
- ☐ Complete Open Case 4 (Sessions 1 & 2)
  - ☐ `worms.csv`

Case Study O.10.4

RCTs as an IV problem: (wormsKenya)

Instructions: Work in pairs or groups of 3. Answer the following questions collaboratively.

Notebook: Case O.4

Dataset: worms.csv

Make sure you understand how to specify IV models with function `ivreg()` in the `AER` package.

Scenario: In rural western Kenya, intestinal worms (hookworm, roundworm, schistosomiasis) are highly prevalent and infect a large share of primary school children. Worm infections can lead to iron-deficiency anemia, malnutrition or abdominal pain; all important factors in school absenteeism and inattention. Non-severe cases may present as asymptomatic, with more subtle impairments in children's educational attainment and health outcomes. Deworming treatment with Albendazole and Praziquantel pills is cheap and effective, but rarely provided at scale.

(**wormsKenya**) designed an intervention to distribute deworming pills to schools in a region surrounding Victoria Lake. They partnered with an NGO (ICS Africa) to implement an RCT where schools were phased to treatment as follows:

- **Group 1** began treatment on 1998 while groups 2 and 3 remained controls.
- **Group 2** phased in treatment on 1999.
- **Group 3** began on 2001 — after the study window for these published results.

For simplicity, we will focus only on the short-term outcomes of the first intervention. That means only schools in Group 1 schools will be considered treated in our dataset.

Glossary for `worms`

- `pupid` — pupil ID (unique child identifier).
- `baseline_school` — ID of the child's school.
- `baseline_grade` — The student's academic grade. Encoded 1 to 11.
- `assigned_98` — Indicator:=1 the student's school is part of Group 1.
- `treated_98` — Indicator:=1 the student received **any** deworming treatment in 1998 (Albendazole or Praziquantel).
- `attendance_98` — The student's observed attendance rate (proportion of times present during surveyor's visits 1 through 8)
- `testscores_98` — Reported average test scores mid-1998 (standardized by grade).
- `sex` — Pupil's gender. Encoded 1 for male and 0 for female.
- `house_floor` — Material of household floor. Encoded 0 for mud, and 1 for cement.
- `weight` – Pupil's weight in kg.

Out of practicality, categorical variables contain a level `"N/A"`, as opposed to R's `NA`. And a small number of missing values in continuous covariates are mean-imputed.

Part 1. Random assignment as an instrument

Suppose the researchers don't provide the deworming treatment. Instead, they decide to widely disseminate deworming information across all schools, including where to purchase the drugs at low prices:

1.1 If a survey was conducted at the end of 1998, how would you expect covariates such as `house_floor`, or `weight` compare between those students with (`treated_98 == 1`) and those in the (`treated_98 == 0`) group?

1.2 As previously noted, children with heavy worm infections are prone to missing school. Search the paper's introduction for evidence on the effectiveness of Albendazole and Praziquantel in abating worm infections.

(a) Suppose students' absenteeism is 100% due to infection symptoms. What should be the mean of `attendance_98` among treated children?

(b) Empirically, not all school absenteeism is worm-related. Describe the covariates of the **treated** students you expect to miss school the most.

1.3 Back to (**wormsKenya**)'s randomized assignment. How would your answers to 1.1 and 1.2 change if each value in `treated_98` were drawn iid from a Bernoulli with $p = 0.5$?

1.4 [1.1] But did every student assigned to treatment receive any? What is the average compliance for assigned and non-assigned students?

Suppose we now have the model

$$X_i = \pi_0 + \pi_1 Z_i + v_i, \quad i = 1, 2, .., N \tag{3}$$

Where $X_i = \mathbf{1}$(i treated) and $Z_i = \mathbf{1}$(i's school assigned to treatment). π_0, π_1 are coefficients, and $v_i \sim N(0, \sigma_v^2)$

1.5 [1.2] Use `lm()` to get the OLS estimates $\hat{\pi}_0$, $\hat{\pi}_1$ and answer the following:

(a) Interpret the model: which variable does this model explain in terms of which other variable?
(b) What does $\hat{\pi}_0$ represent?
(c) $\hat{\pi}_1$ can be expressed as a difference of means. Write it down.
(d) Test the hypothesis that $H_0 : \pi_0 = 0$ and $H_1 \neq 0$ with 95% confidence. What does this result imply? Do you think the data-generating process warrants it?

1.6 What percentage of students were treated? What is the attendance rate among those treated and untreated?
1.7 What is the percentage points difference in attendance between compliers and non-compliers [hint: first create `complied` ...]

We have the following model

$$Y_i = \alpha + \beta X_i + \epsilon_i, \tag{4}$$

Along with the IV assumptions:

$$\mathbb{E}[Z_i u_i] = 0, \quad \text{and} \quad \text{Cov}(Z_i, X_i) \neq 0, \tag{5}$$

$$\hat{\beta}_{\text{Wald}} = \frac{\mathbb{E}[Y \mid Z = 1] - \mathbb{E}[Y \mid Z = 0]}{\mathbb{E}[X \mid Z = 1] - \mathbb{E}[X \mid Z = 0]}. \tag{6}$$

Again, X_i is a treatment dummy. Y_i is the attendance rate for student i, and the error $u_i \sim N(0, \sigma_\epsilon^2)$ Constants α, β are the model's coefficients.

8. [1.4] Run the regression
9. Interpret $\hat{\beta}$.
10. Do you believe $\mathbb{E}[u_i \mid X_i] = 0$ here? Use the story about who chooses (or is able) to take the drugs.
11. In which direction would you *expect* the bias to go: would OLS overestimate or underestimate the causal effect? (Justify with a short causal story.)

1. [1.5] Wald estimator. Compute $\hat{\beta}_{\text{Wald}} = \frac{\mathbb{E}[Y|Z=1]-\mathbb{E}[Y|Z=0]}{\mathbb{E}[X|Z=1]-\mathbb{E}[X|Z=0]}$
   (a) Compute the numerator and denominator separately using group means.
   (b) Compare $\hat{\beta}_{\text{Wald}}$ to your na"ive OLS estimate from 8.. Are they similar? Which one is larger?
   (c) Explain (in words) what variation identifies $\hat{\beta}_{\text{Wald}}$.
2. [1.6] 2SLS using `ivreg()`.Run `ivreg(attendance_98 treated_98 | assigned_98, data=worms)`.

1. Verify it matches your Wald estimate (up to rounding).
2. Extract and report the first-stage coefficient $\hat{\pi}_1$ and interpret it.
3. What does the 2SLS estimate measure conceptually? (Hint: "Local" average treatment effect.)

1. [1.7] Heterogeneity teaser (no extra theory required). Split the sample by `baseline_grade` into `lower` (1–5) vs `upper` (6–11), and re-run the reduced form in each subsample.
   (a) Where do you see larger ITT effects? Provide a short hypothesis (e.g., older children, opportunity cost, etc.).

Part 2. Covariates

In a randomized experiment, adding covariates is not required for identification. However, it can (i) improve precision, and (ii) help us detect problems with data quality (e.g., missingness patterns).

2.1 [2.1] Balance checks (quick).For each covariate below, compare means between `assigned_98==1` and `assigned_98==0`. Use a difference-in-means test: `sex`, `house_floor`, `weight`.
2.2 Which covariates look well-balanced? Which look suspicious?
2.3 Explain why balance is expected under random assignment (in expectation) but not guaranteed in finite samples.

1. [2.2] Missingness patterns. For each covariate above, compute the fraction equal to `"N/A"` (for categoricals) or missing/imputed (for continuous).
   (a) Is missingness correlated with `assigned_98`? Run a regression of a missing dummy on `assigned_98`.
   (b) Why is missingness correlated with treatment assignment a problem even in an RCT?

1. [2.3] First stage with covariates.Estimate

$$X_i = \pi_0 + \pi_1 Z_i + \delta' W_i + v_i$$

   where W_i includes the covariates above.
   (a) Does $\hat{\pi}_1$ change a lot? Why might it (or might it not)?
   (b) Report the R^2. Does adding covariates make X more predictable?

1. [2.4] 2SLS with covariates. Run

$$Y_i = \alpha + \beta X_i + \theta' W_i + u_i$$

   instrumenting X_i with Z_i (and keeping W_i in both stages).
   (a) Compare your IV estimate $\hat{\beta}$ with and without covariates. Should it change much? Why?
   (b) Compare standard errors. Did precision improve?

1. [2.5] A small specification ladder. Create a table with the IV estimate $\hat{\beta}$ under:

(a) no covariates;
(b) plus demographics (`sex`, `baseline_grade`);
(c) plus SES proxies (`shoes`, `uniform`, `house_floor`);
(d) plus health (`weight`).

What pattern do you see? Provide one sentence interpreting the stability (or lack of stability).

Bibliography

Pop-Eleches, C., & Urquiola, M. (2013). Going to a better school: Effects and behavioral responses. *American Economic Review*, *103*(4), 1289–1324. https://doi.org/10.1257/aer.103.4.1289