Scalar Field Dynamics In Steep Exponential Potential

A Dissertation submitted to

JAMIA MILLIA ISLAMIA

New Delhi

in partial fulfillment of the requirement for the degree of

Master of Science

in

Physics

by

Yedu Krishnan

Under the supervision of

Dr. Md. Wali Hossain

Department of Physics

Jamia Millia Islamia, New Delhi.

28th May, 2024

DECLARATION

I, Yedu Krishnan, hereby solemnly declare that the work presented in the thesis entitled **" Scalar Field Dynamics In Steep Exponential Potential."** has been done by me. I confirm that:

- The work is wholly done in candidature for a masters degree at **Jamia Millia Islamia** under the guidance of **Dr. Md. Wali Hossain.**
- I have consulted the published works of others and this is always clearly stated.
- Some of the results of the thesis may be reproduction of previous works done by others and the source of which are given as references.
- I have clearly acknowledged all main source of help.
- Where the thesis is based on works done by myself jointly with my supervisor, I have made clear what was done by others and what I have contributed myself.

Yedu Krishnan
Department of Physics
Jamia Millia Islamia
New Delhi-110025

ACKNOWLEDGEMENT

First and foremost, I would like to thank **Dr. Md. Wali Hossain**, for guiding me throughout the course. His devotion and efforts in guiding us was really valuable for me.

I also want to thank my teachers especially **Dr. Somasri Sen** and **Prof. Rathin Adhikari** for clearing my doubts and queries.

I also like to acknowledge my project group members :- namely **Shabnam Jahan** and **Sachin Kumar** for there help in my project, whether its related to data interpretation or constructive criticism. Similarly my thanks to my friends for providing me moral support.

I also want to thank **Dr. Mohammed Ahsanul Hoda Ahsan** and all the Physics department staffs whether its teaching or non-teaching. Last but not the least, my thanks to my parents for all the sacrifices they have made in there life for my sake and with God's blessing, I conclude my acknowledgement here.

Yedu Krishnan

ABSTRACT

In this project thesis, we are studying the cosmic expansion of the scalar field in steep-exponential potential. This potential also give us interesting dynamics during intermediate stages

We also shown that during late time we have shown that the scalar field, describing energy density follows and scales the matter energy density. So, the equation of state becomes approximately zero but during intermediate stages it can some interesting features and which can be useful in the early dark energy scenario.

First, we are calculating late time expansion parameters. After that we are adding Cosmological Constant to the exponential potential to observe the behaviour of the scalar field evolution in the steep exponential potential.

After that, we are observing evolution of energy density parameter for different components of energy densities.

Contents

Chapter 1

Introduction

1.1 History of the Universe.

" The most incomprehensible thing about the world, is that it is comprehensible. "
– Albert Einstein

The history of the universe can be understood as various epoch, ranging from the birth of the universe from Big Bang to the present expanding universe as represented in the Fig. 1.1. Because of our inability to establish an accepted theory of Quantum Gravity, the understanding of the universe below timescale of the order 10^{-43} s, so keeping this in mind, lets talk about the History of the universe.

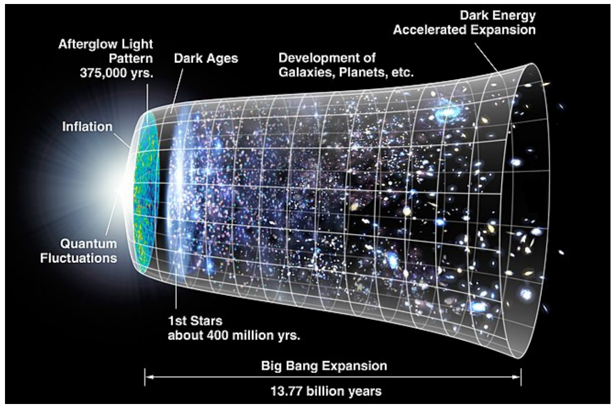

Figure 1.1: History of our Universe.**Source : Wikipedia**

1.1.1 Very Early universe

Around 13.8 billion years ago, universe came into the existence because of Big Bang (the reason, what cause the Big Bang is not known yet) and universe went under rapid expansion, faster than the speed of light. Although, this phase only happen for around 10^{-36} s, it plays very important role in resolving problems like :- Flatness problem, Horizon problem and many other issues that Big Bang theory failed to explain (discussing later). One important this which is important to be pointed out is the universe was very small and thus Inflation is a quantum mechanical phenomenon and thus an accepted Quantum Theory of Gravity become important.

When the cosmic inflation stopped, universe at this time was very hot with a temperature of 10 to 15 billion degrees Celsius. This was the time when all the fundamental forces of nature started to emerge. Soon minutes after the Big Bang, an era of Nucleosynthesis started which resulted in the formation of lighter elements like :- Hydrogen and Helium. This is also the reason why, the abundance of lighter elements in the universe is so large, compare to other elements because they were formed much much earlier during the few minutes after the birth of the universe.

1.1.2 Early Universe

As the universe keeps expanding, it began to cool down because of the cosmological red-shift and at the time roughly 3,80,000 years after Big Bang, it cooled enough to enable atomic nuclei to capture electrons and form atoms which previously was not been able to as, high energy photons collision not let atomic nuclei to form stable atom (**Era of Recombination**). It was during this time, universe started to become transparent and as a result it is the earliest point in our cosmic history that we cool see and **Cosmic Microwave Background Radiation** is the after glow which we can detect.

1.1.3 Dark Ages

But soon afterword, the universe again become dark as no source of light there except the photons released due to recombination and 21 cm radio emissions occasionally emitted by hydrogen atoms since no star formation started yet. At some point around 200 or 500 million years, the earliest generation of stars and galaxies started forming.

1.1.4 Present Era

Around 9.8 billion years later, the matter density soon started to fall below dark energy density (Physicist believe it is due to the vacuum energy density), as a result, universe again started to accelerate [10]. The main cause of this shift in dynamics can be interpreted due to a^{-3} dependence by matter distribution, whereas, density distribution of Cosmological constant, having no dependence of a. As, a result, with expansion of universe, due to significant red-shift, matter density falls which was represented by Fig. 1.2.

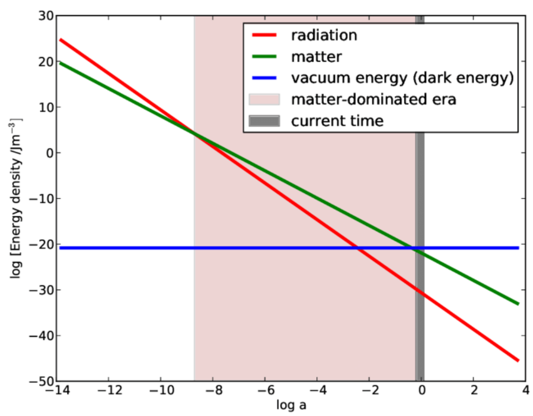

Figure 1.2: Evolution of energy distribution.**Source : Researchgate**

1.2 Elements of FLRW Cosmology

“Somewhere, something incredible is waiting to be known.” – Carl Sagan

Cosmology is the branch of physics, that deals with the study of universe and tries to answer some of the fundamental questions like :-

- **What universe is made up of?**
- **What is its structure?**
- **What is its origin?**
- **Where its heading in future and what its fate?**

Even though earlier civilisations tried to explain this but the modern aspects of the Cosmology only started in 1915 with Albert Einstein applying his theory of General Relativity to the structure of universe as a whole and theoretically came to the conclusion that universe is either contracting or expanding, which leads him to addition of an term, now known as the **"Cosmological Constant".**

Not only that, works done by **Lemaitre, Friedmann, Hubble** etc, also contributed immensely to Cosmology. Despite its initial success, it also faced some serious setbacks and failures like our inability to precisely determining value of Hubble constant, the true nature of dark energy and many other. Before that, lets discuss some important topic regarding this subject.

1.2.1 Basics of FLRW Cosmology

Just like any branch of science, Cosmology too starts with some basic assumptions:

- Universe can be assumed as **Homogeneous** and **Isotropic**, which means universe will appear to be same when observed from different places and different direction if distance is considered larger than few MPc.
- Einstein's theory of General relativity is the correct description for explaining the dynamics of objects under the influence of gravity.
- Gravity is the dominant force or governing force for this expanding universe.
- Matter distribution of the universe can be approximated as a perfect fluid, meaning that the off diagonal components of the Energy-momentum Tensor will be zero.

Based on this assumptions, we define a metric as :-

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1 - Kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right] \tag{1.1}$$

where t is time. r, θ and ϕ are the co-moving coordinates and $a(t)$ is the scale factor.

This metric is known as **FLRW Metric**. The constant K in this metric define the geometry of the spatial section of the spacetime, that is if universe is flat, open or closed universe, which are represent in Fig. 1.3. Based on that its associated K value will be $K = 0, -1, +1$, respectively.

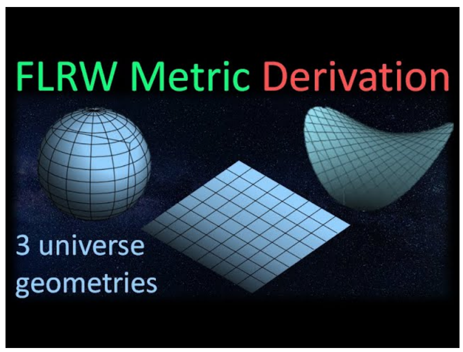

Figure 1.3: Geometries in FLRW Universe.**Source : Youtube.**

1.2.2 Geometries in FLRW Cosmology

Let us now understand, how metric defined in equation (1.1) have 3 sort of geometries.

Flat / Euclidean Geometry

This is the most trivial case where metric is defined as

$$dS^2 = dx^2 + dy^2 + dz^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1.2}$$

Closed / Spherical Geometry

Now, let us write the first non-trivial case, where metric is defined as follows

$$dS^2 = \frac{dr^2}{1-r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \tag{1.3}$$

To understand its geometry, let us do a transformation

$$dr' = \frac{dr}{\sqrt{1-r^2}}$$

Thus, $r' = \sin^{-1}(r)$ and using this, our metric becomes

$$dS^2 = (dr')^2 + \sin^2(r') \left[d\theta^2 + \sin^2\theta d\phi^2\right]$$

Now, to check what it looks like, let us consider a four dimensional flat space-time. Then in Cartesian co-ordinates it will be as

$$dS^2 = dw^2 + dx^2 + dy^2 + dz^2$$

Now, if we put a 3D surface here, then we will have following equation

$$w^2 + x^2 + y^2 + z^2 = a^2$$

This, 4 coordinates are not independent, if we now do another transformation as

$$w = a\cos r'$$

$$x = a\sin r' \sin\theta \cos\phi$$

$$y = a\sin r' \sin\theta \sin\phi$$

$$z = a\sin r' \cos\theta$$

Thus, we will obtain our metric as

$$dS^2 = a^2\left[dr'^2 + \sin^2 r'(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

This show that, the geometry is nothing but, when we put 4D flat euclidean surface in a 3D sphere.

Open / Hyperbolic Geometry

We final metric which we have is

$$dS^2 = \frac{dr^2}{1+r^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2 \tag{1.4}$$

Similar, approach will be made where we do transformation like

$$dr' = \frac{dr}{\sqrt{1 + r^2}}$$

Thus, $r = \sinh r'$. Now, instead of Euclidean space, we put a flat Minkowski space as

$$dS^2 = dw^2 - dx^2 - dy^2 - dz^2$$

Now, if we put a 3 hyperboloid, then we get $w^2 - x^2 - y^2 - z^2 = a^2$

With similar transformation but with hyperbolic trigonometric function, we obtain our metric as

$$dS^2 = a^2 \left[(dr'^2 + \sinh^2 r'(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

This is an example of a non-compact object. Thus, our geometry is a surface of 3 hyperboloid that is placed in 4 dimensional Minkowski space. For further information, visit [5]

1.2.3 The Friedmann's Equations

Now, if we apply Einstein's Field equation [2], which is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{1.5}$$

where $G_{\mu\nu}$ is the Einstein Tensor, $R_{\mu\nu}$ is the Ricci Tensor, R is the Ricci

Scalar and $T_{\mu\nu}$ is the energy-momentum tensor.

Now, in order for our Energy-Momentum Tensor to be consistent with our assumption that universe is homogeneous and isotropic, we insist on the ideal perfect fluid consideration, thus we can write our Energy-Momentum Tensor as

$$T_{\mu\nu} = (\rho + p)U_\mu U_\nu - p g_{\mu\nu} \tag{1.6}$$

Based on this, we can obtain the differential equations as

$$H^2 + \frac{K}{a^2} = \frac{8\pi G \rho_{\text{total}}}{3}\,, \tag{1.7}$$

$$\dot{H} - \frac{K}{a^2} = -4\pi G(\rho_{\text{total}} + p_{\text{total}})\,, \tag{1.8}$$

Where, $\rho_{\text{total}} = \sum_i \rho_i$ and $p_{\text{total}} = \sum_i p_i$ and $H = \frac{\dot{a}}{a}$. Here, i indicate each component of energy distribution. With the discovery done by Planck team that observed curvature of the universe is zero [11].

Equation (1.7) and (1.8) are known as **The Friedmann's Equations**.

Apart from this, there's also one another equation known as the **Continuity Equation** given as

$$\dot{\rho} + 3H(p + \rho) = 0 \tag{1.9}$$

The above equation is not an independent equation as one can derive (1.9) either using (1.7) and (1.8) or using the **Bianchi Identities**.

Thus, we have 3 unknowns (a, p and ρ) and 2 independent equations, so we in order to solve this equation we define a parameter :-

$$p = \omega\rho \tag{1.10}$$

This is known as **equation of state** parameter. Now lets understand how ω is used to determine the evolution of equations.

1.2.4 Equation of state parameter for different distributions

If we assume barotropic perfect fluid approximation then, we can consider the pressure will only depend on the matter distribution. Thus, equation(1.9) can be solvable and give us the result that

$$\rho \propto a^{-3(1+\omega)} \tag{1.11}$$

Now, imagine a cube where its edges are defined as one unit of scale factor (a). Inside this box contain the different types of distribution of energy namely :-

- **Non Relativistic Energy Distribution**
- **Relativistic Energy Distribution**
- **Vacuum Energy Distribution**

Non-Relativistic Energy Density

As universe is expanding, the edges of the cube with expand and because of this, there is a scaling of the edges by a factor of a(t). As, matter content won't be affected by the expansion thus the overall effect on matter density due to expansion will be :- $\rho_m \propto a^{-3}$. By comparing it with (1.11), we get that for matter distribution, $\omega = 0$.

Relativistic Energy Density

Considering the same situation now for relativistic energy distribution. However, this time, as the universe is expanding, the energy distribution of radiation will do affected due to the **Doppler red-shift.** As a result, as energy density have a^{-3} due of expansion, there also will have a^{-1} contribution. Thus, the overall effect on radiation density will be $\rho_r \propto a^{-4}$. Comparing this to, equation (1.11), we get that for radiation distribution, $\omega = \frac{1}{3}$.

Vacuum Energy Density

There's also one another energy distribution known as **Dark energy**. This is a peculiar type of energy distribution which is independent to the effect of cosmic expansion. As a result, it has constant energy density, that is,

$\rho_\Lambda \propto a^0$. Thus, we have $\omega = -1$. More on this, later.

1.3 Distances in Cosmology

Humans calculating distance since millennia, whether its calculating distances to prove earth is round by using basic knowledge of Trigonometry and imaginations or calculating the radius of the orbits of planet revolving around the Sun, long before Isaac Newton's Principia Mathematica where he wrote the laws of motions and Inverse square law of Gravitation. But, as we started to get familiarised with the vast scale of length, our universe encompasses, various techniques and modification in our prior knowledge been done. Lets deal with few methods and concepts, commonly used in Cosmic society to calculate the distances in cosmology.

1.3.1 Hubble's Law

Before stating Hubble's law, we define **co-moving observer** as those observer which are moving with the expansion of the universe, thus as far as expansion of the universe is concerned, this observer will remain constant or static. Since, there space-coordinates does not change. It will means that when they measure time, they will actually measuring the proper time. So, for all co-moving observer, there time will be the actual time.

So a relation can be deduced between **physical distance** and **co-moving distance** as following :

$$r_p = a(t)X_c \tag{1.12}$$

where, r_p is the physical distance, $a(t)$ is the scale factor, X_c is known as the co-moving distance. Now, as X_c does not be changing with time, then one can establish a relation called **Hubble's Law.**

$$v = \dot{r}_p = \dot{a}(t)X_c = H(t)r_p \tag{1.13}$$

where, $H(t)$ is the Hubble Parameter, v is the recession velocity.

Now, in general, if there is some small inhomogeneity at a particular place due to some extra mass or extra gravity, things will show tendency to go towards that due to gravitation attraction. Thus, our co-moving distance no longer be constant but will also undergo change. Due to this reason, an extra contribution will arise known as **peculiar velocity** which is an imprint of the inhomogeneity in the universe.

$$v = \dot{a}(t)X_c + a(t)\dot{X}_c \tag{1.14}$$

First term on right is our usual Hubble term while second term is our peculiar velocity contribution. Thus, if at some particular place, there are some local inhomogeneity, then they will produce extra effect on red-shift, while will be seen in our next section.

1.3.2 Redshift

From the theory of relativity, we know that if two points are located in far away position in space, then light or any radiation would take finite amount of time to reach there and this process won't be an instantaneous process which was earlier thought in Newtonian mechanics.

But, as the universe expands, it will suffer a shift in wavelength due to **Doppler Effect**. This phenomenon is known as the Cosmological red-shift as shown in Fig 1.4

$$1 + z = \frac{\lambda_0}{\lambda} = \frac{a_0}{a} \tag{1.15}$$

where, z is the red-shift parameter, and by convention, a_0 is set to 1 for present time.

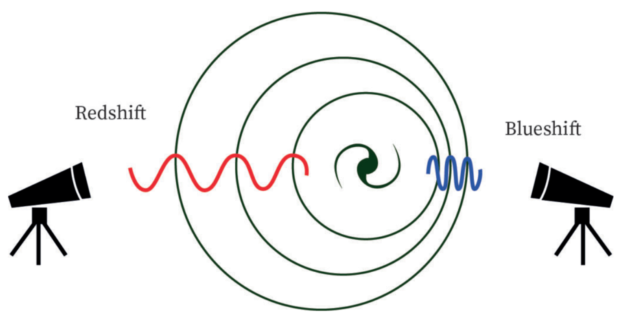

Figure 1.4: Example of red-shift and blue-shift.
Source : Bartleby.com

1.4 Λ-CDM Model in Cosmology

"In questions of science, the authority of a thousand is not worth the humble reasoning of a single individual." – Galileo Galilei

1.4.1 Non-Relativistic or Newton's Perspective

Let us consider, what would be the result which we should expect, if we apply classical laws (Newtonian Mechanics) in our homogeneous and isotropic universe. For this, let us consider a sphere, where a small mass, m (lets say, Newton himself) fixed at the centre of the sphere and a distant galaxy of mass M at a distance, R from the centre.

Now, from Newton's law of gravitation, we have

$$\ddot{a} = \frac{GM}{D^2} \tag{1.16}$$

where, D is the physical distance, now, from Hubble relation, $D = a(t)R$, we will end up with the relation

$$\frac{\ddot{a}}{a} = -\frac{GM}{a^3R^3} \tag{1.17}$$

since,

$$V = \frac{4\pi D^3}{3} = \frac{4\pi a^3 R^3}{3},$$

we get

$$\frac{\ddot{a}}{a} = -\frac{4\pi G\rho}{3} \tag{1.18}$$

Notice, equation (1.18) only points a decelerating universe and can't explain the accelerating universe. One can also conclude that a static model of the universe is impossible as it will require a universe which is empty, which is not the case [4].

1.4.2 Relativistic or Einstein's Perspective

Now, if we generalise our theory by adding relativistic corrections, then we can obtain that universe can be accelerating. This is because, pressure term is added when we consider general relativity in expansion of the universe. since, we can write

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(\rho + 3p)}{3} \tag{1.19}$$

In his original paper, Einstein pointed out the flaws in the dynamical universe and thus, he modeled a static universe, where $a(t) = \dot{a} = \ddot{a} = 0$. Based on this result, equation (1.19), this will indicate that, $(\rho + 3p) = 0$.

As, Einstein considered neither ρ nor p a negative quantity, thus, he added an extra term which is now known as **"Cosmological Constant"** which will act as a term that keeps the universe static.

1.4.3 Towards Λ-CDM Model

Addition of Cosmological Constant

In 1998, observations on Type Ia supernovae led by **Reiss, Perlmutter and Schmidt**, revealed that universe is undergoing an accelerated expansion and thus, an addition contribution due to Cosmological constant Λ must also be added in the Einstein's Field equation. So, our modified version of equation (1.5) will be

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}. \tag{1.20}$$

and as a result, our Friedmann's equations will also be modified as

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}, \tag{1.21}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{1.22}$$

Despite being a simple cosmological theory, addition of a constant Λ in our existing theory, it however faces some issues such as **Fine Tuning Problem** and **Hubble Tension** (which is still a big problem). Thus, we can conclude that something dynamical is necessary in our current theory which later emphasised to introduce scalar fields in cosmology.

Problems in Λ-CDM Model

- **Hubble Tension –** Determining Hubble constant plays a crucial role in study of Cosmology and for this reason, huge efforts have been made in order to precisely determine it. Main problem arises because of the fact that we have two accepted value of Hubble constant one came from the measurement done on the angular displacement and polarisation fluctuations done on the cosmic background radiation. Calculated value of Hubble constant by this method is 67.4 ± 0.5 km sec^{-1} Mpc^{-1}. While, measurement done on determining the velocity of recession and distances by galaxy give us the value of 73.0 ± 1.0 km sec^{-1} Mpc^{-1}. [9]

- **Fine Tuning Problem –** If we assume that the Cosmological constant arise form the vacuum energy density then there is disagreement of value associated if one considered observation data and theoretical data obtained from Quantum Field Theory. This data had a disagreement of 120 order of magnitude.

Chapter 2

Scalar Fields in Cosmological Background

2.1 Theory

Just like particle physics use scalar fields for dynamics, we also assumes a scalar field ϕ which is responsible for today's accelerating universe. If we assume that scalar field ϕ is minimally coupled to gravity, that is, there is no interaction between ϕ and curvature. However, its not necessary be the case as, there might be interaction between them. This kind of model is known as Non-Minimal Coupled Theory.

Now, let us define action [6], $\mathcal{S}$ associated with this as

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{2}R - \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right) - V(\phi)\right] + \mathcal{S}_m + \mathcal{S}_r \tag{2.1}$$

Now, if we do variation in our action with respect to ϕ, we get

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0. \tag{2.2}$$

This is our **Klein-Gordan equation in Zero Curvature Background Cosmology**. One thing worth mentioning here is that, our potential does not depend on time but our scalar field ϕ, can be a time dependent quantity.

Now, for energy-momentum tensor, we will variation in action with respect to $g^{\mu\nu}$, which give us

$$T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left[\frac{1}{2}g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + V(\phi)\right]. \tag{2.3}$$

Thus, we can have obtain pressure and energy density as

$$\begin{aligned} \rho &= -T_0^0 = \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= T_i^i = \frac{1}{2}\dot{\phi}^2 - V(\phi). \end{aligned} \tag{2.4}$$

As, a result we obtain Friedmann's equation as

$$\begin{aligned} 3H^2M_{Pl}^2 &= \rho_m + \rho_r + \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ (2\dot{H} + 3H^2)M_{Pl}^2 &= -\frac{1}{3}\rho_r - \frac{1}{2}\dot{\phi}^2 + V(\phi). \end{aligned} \tag{2.5}$$

The above equation is nothing but slight modification to already known equation (1.7) and (1.8) with addition contribution coming from our scalar field ϕ.

2.2 Calculations

If we consider potential of the following form

$$V = V_0 e^{-\lambda \phi^n / M_{Pl}^n} \tag{2.6}$$

For, the following potential, we will have

$$\mu = -M_{Pl} \frac{1}{V} \frac{dV}{d\phi} = \lambda n \left(\frac{\phi}{M_{Pl}} \right)^{n-1} \tag{2.7}$$

If, $n = 1$, then the above quantity will be a constant. equation (2.7), which give us the information about the slope. Similarly,

$$\Gamma = \frac{V''V}{V'^2} = 1 - \frac{(n-1)}{n\lambda} \left[\frac{M_{Pl}}{\phi} \right]^n \tag{2.8}$$

where, $V' = \dfrac{dV}{d\phi}$ and $V'' = \dfrac{d^2V}{d\phi^2}$

Now, this quantity will give us the nature of the potential and for an exponential function, the value will be $\Gamma = 1$.

On numerically solving three equations namely :- Friedmann's equations and Klein-Gordan equation in zero curvature background, we can obtain all the dynamical quantities and thus using scalar field model of the universe, underlining physics can completely be determined.

2.3 Results

For λ = 4 with n = 2.5

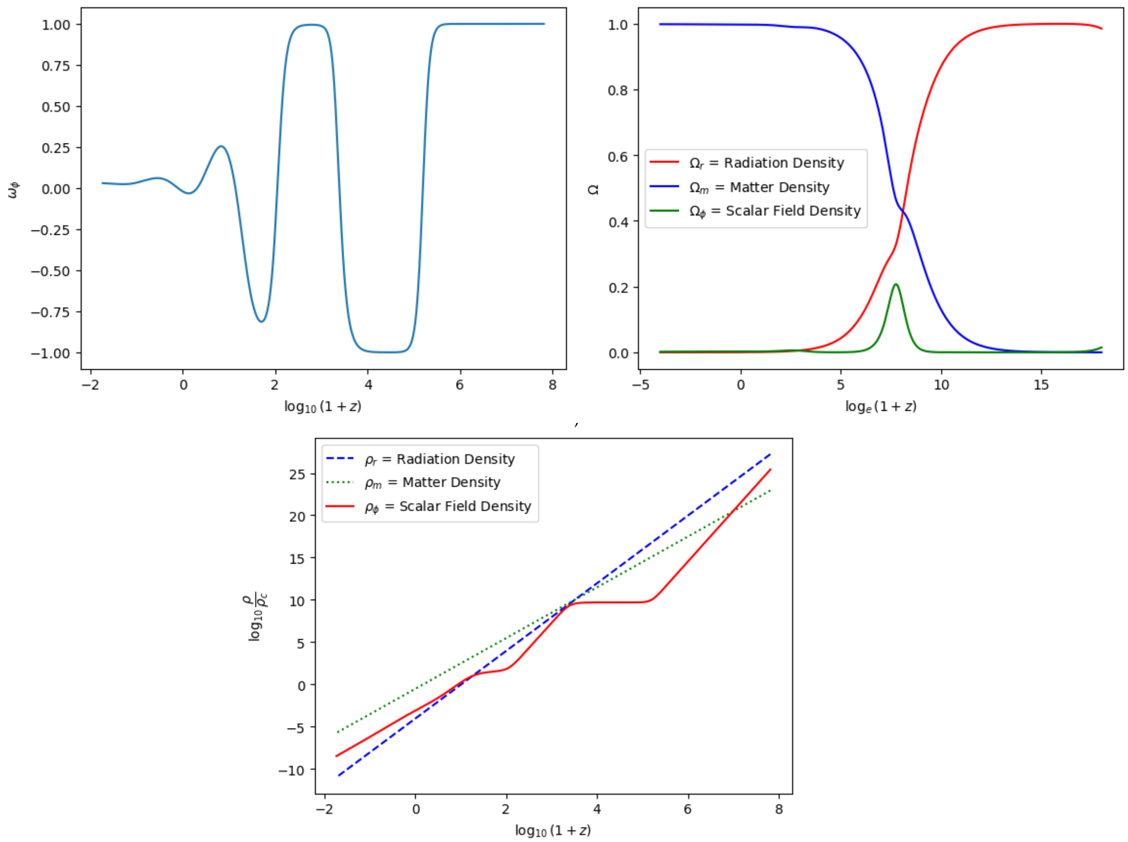

Figure 2.1: Evolution of the equation of state (top left), density parameters (top right) and energy densities (bottom) for λ = 4 and n = 2.5 for the potential (2.6).

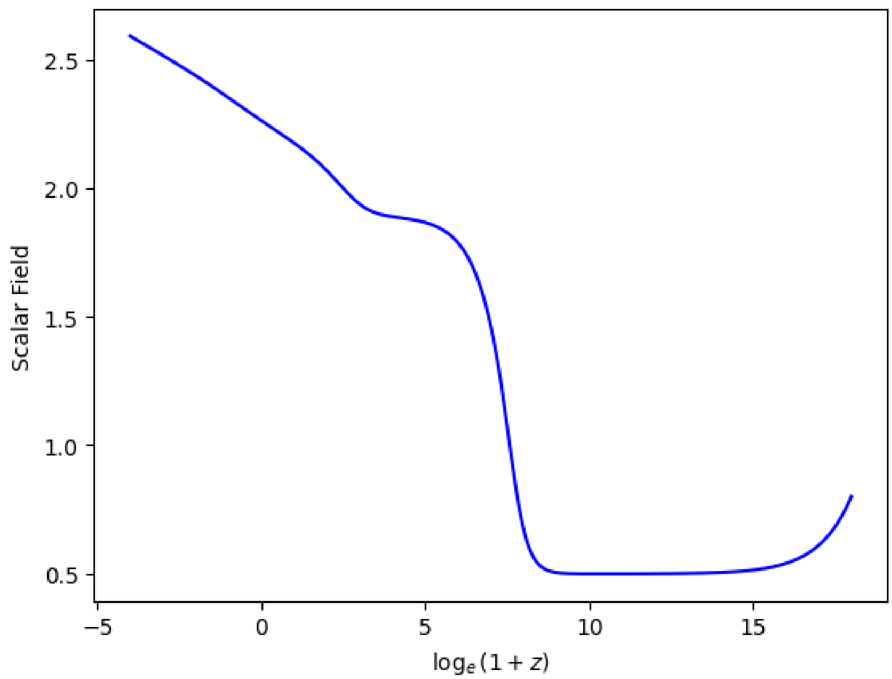

Figure 2.2: Scalar Field.

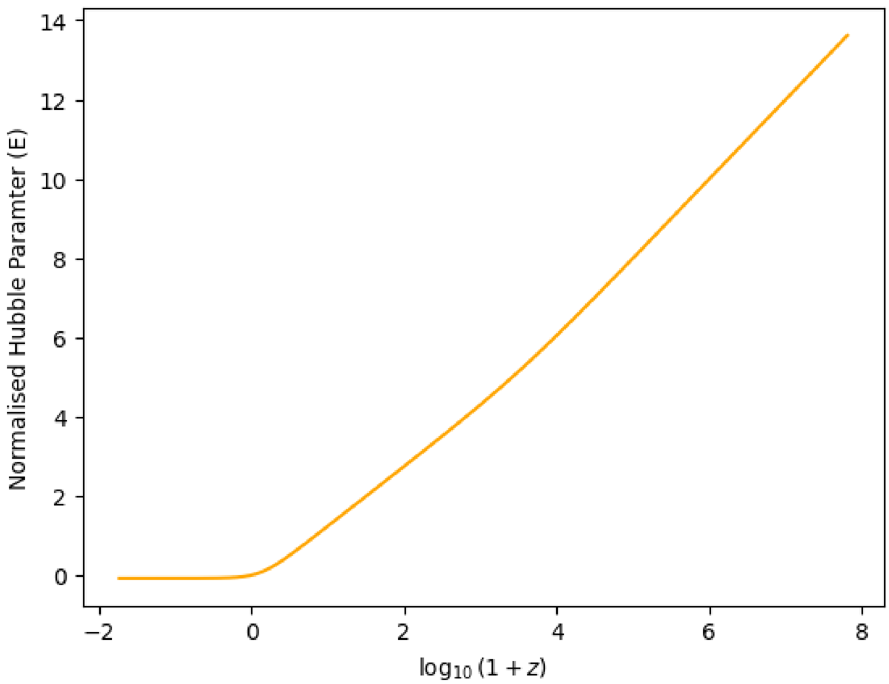

Figure 2.3: Evolution of Normalised Hubble Parameter.

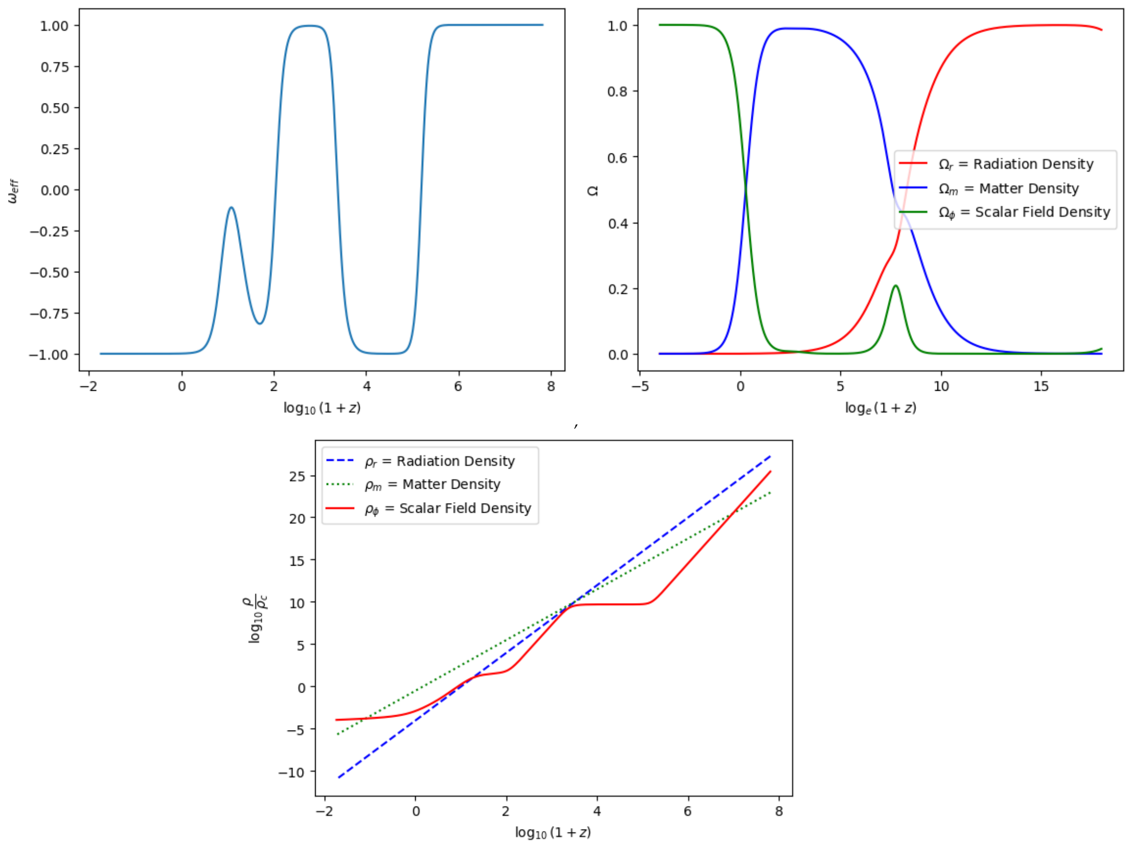

Figure 2.4: Evolution of the equation of state (top left), density parameters (top right) and energy densities (bottom) for $\lambda = 4$ and $n = 2.5$ for the potential (2.6) with a cosmological constant.

Chapter 3

Summary and Conclusion

Various model had been postulated to explain the accelerated expansion of the universe, whether its by modified theory of gravity, scalar fields, K-essence or any other models.

In our steep exponential potential, equation (2.6), for n = 1, our scalar field whose nature has been indicated in Fig 2.2 whose nature is evident from the effective equation of state, rolls down but after as it move down, due to Hubble Friction, $3H\dot{\phi}$ term, the scalar field slows down and eventually stops evolving. As, a result, our scalar field correspond to nature similar to cosmological constant due to our potential completely dominating to kinetic energy term.

From, 2.4 one can clearly observe that initially the kinetic part of the scalar field was dominant but due to Hubble friction the potential part starts dominating, during this period the scalar field behaves as the possible component of dark energy, then it again rolls down and the kinetic part again starts dominating then it again gets frozen, and then again the kinetic part starts dominating.

Few things can also be pointed out here as, the slope due to scalar field contribution is larger than the matter and radiation contribution, this indicates that the energy density scale of the scalar field is $\rho_\phi \propto a^{-6}$ [6]

Bibliography

[1] David H. Lyth, **The History of the Universe** (Springer, 2016). Available at https://www.amazon.in/History-Universe-Astronomers/dp/3319227432

[2] Jayant Vishnu Narlikar, **An Introduction to Cosmology** (Cambridge University Press, 2002). Available at https://www.amazon.in/Introduction-Cosmology-J-V-Narlikar/dp/0521793769

[3] Scott Dodelson, **Modern Cosmology** (Academic Press, 2003). Available at https://www.amazon.com/Modern-Cosmology-Scott-Dodelson/dp/0122191412

[4] Oliver Piattella, **Lecture Notes in Cosmology** (Springer, 2018). Available at https://www.amazon.in/Lecture-Notes-Cosmology-UNITEXT-Physics/dp/3319955691

[5] Edmund J. Copeland, M. Sami, and Shinji Tsujikawa.
Dynamics of dark energy.
Available at : https://arxiv.org/pdf/hep-th/0603057

[6] Chao-Qiang Geng, Md. Wali Hossain, R. Myrzakulov, M. Sami, and Emmanuel N. Saridakis.
Quintessential inflation with canonical and noncanonical scalar fields and Planck 2015 results.
Available at : https://arxiv.org/pdf/1502.03597

[7] David W. Hogg.
Distance measures in cosmology.
Available at : https://arxiv.org/pdf/astro-ph/9905116

[8] Sudipta Das.
Aspects of Quintessence Matter - The driver of the late time acceleration of the universe .
Available at : https://arxiv.org/pdf/0808.0826v1

[9] Marc Kamionkowski and Adam G. Reiss.
The Hubble Tension and Early Dark Energy.
Available at : https://arxiv.org/pdf/2211.04492

[10] Adam G. Riess, Alexei V. Filippenko, Peter Challis, Alejandro Clocchiattia, Alan Diercks, Peter M. Garnavich, Ron L. Gilliland, Craig J. Hogan, Saurabh Jha, Robert P. Kirshner, B. Leibundgut, M. M. Phillips, David Reiss, Brian P. Schmidt, Robert A. Schommer, R. Chris Smith, J. Spyromilio, Christopher Stubbs, Nicholas B. Suntzeff, John Tonry.
Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant.
Available at : https://arxiv.org/abs/astro-ph/9805201

[11] N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A. J. Banday, R. B. Barreiro, N. Bartolo, S. Basak, R. Battye, K. Benabed, J.-P. Bernard, M. Bersanelli, P. Bielewicz, J. J. Bock, J. R. Bond, J. Borrill, F. R. Bouchet, F. Boulanger, M. Bucher, C. Burigana, R. C. Butler, E. Calabrese, J.-F. Cardoso, J. Carron, A. Challinor, H. C. Chiang, J. Chluba, L. P. L. Colombo, C. Combet, D. Contreras, B. P. Crill, F. Cuttaia, P. de Bernardis, G. de Zotti, J. Delabrouille, J.-M. Delouis, E. Di Valentino, J. M. Diego, O. Doré, M. Douspis, A. Ducout, X. Dupac, S. Dusini, G. Efstathiou, F. Elsner, T. A. Enßlin, H. K. Eriksen, Y. Fantaye, M. Farhang, J. Fergusson, R. Fernandez-Cobos, F. Finelli, F. Forastieri, M. Frailis, A. A. Fraisse, E. Franceschi, A. Frolov, S. Galeotta, S. Galli, K. Ganga, R. T. Génova-Santos, M. Gerbino, T. Ghosh, J. González-Nuevo, K. M. Górski, S. Gratton, A. Gruppuso, J. E. Gudmundsson, J. Hamann, W. Handley, F. K. Hansen, D. Herranz, S. R. Hildebrandt, E. Hivon, Z. Huang, A. H. Jaffe, W. C. Jones, A. Karakci, E. Keihänen, R. Keskitalo, K. Kiiveri, J. Kim, T. S. Kisner, L. Knox, N. Krachmalnicoff, M. Kunz, H. Kurki-Suonio, G. Lagache, J.-M. Lamarre, A. Lasenby, M. Lattanzi, C. R. Lawrence, M. Le Jeune, P. Lemos, J. Lesgourgues, F. Levrier, A. Lewis , M. Liguori, P. B. Lilje, M. Lilley, V. Lindholm, M. López-Caniego, P. M. Lubin, Y.-Z. Ma, J. F. Macías-Pérez, G. Maggio, D. Maino, N. Mandolesi, A. Mangilli, A. Marcos-Caballero, M. Maris, P. G. Martin, M. Martinelli, E. Martínez-González, S. Matarrese, N. Mauri, J. D. McEwen, P. R. Meinhold, A. Melchiorri, A. Mennella, M. Migliaccio, M. Millea, S. Mitra, M.-A. Miville-Deschênes, D. Molinari, L. Montier, G. Morgante,

A. Moss, P. Natoli, H. U. Nørgaard-Nielsen, L. Pagano, D. Paoletti, B. Partridge, G. Patanchon, H. V. Peiris, F. Perrotta, V. Pettorino, F. Piacentini, L. Polastri, G. Polenta, J.-L. Puget, J. P. Rachen, M. Reinecke, M. Remazeilles, A. Renzi, G. Rocha, C. Rosset, G. Roudier, J. A. Rubiño-Martín, B. Ruiz-Granados, L. Salvati, M. Sandri, M. Savelainen, D. Scott, E. P. S. Shellard, C. Sirignano, G. Sirri, L. D. Spencer, R. Sunyaev, A.-S. Suur-Uski, J. A. Tauber, D. Tavagnacco, M. Tenti, L. Toffolatti, M. Tomasi, T. Trombetti, L. Valenziano, J. Valiviita, B. Van Tent, L. Vibert, P. Vielva, F. Villa, N. Vittorio, B. D. Wandelt, I. K. Wehus, M. White, S. D. M. White, A. Zacchei, A. Zonca.

Planck 2018 results. VI. Cosmological parameters.

Available at : https://arxiv.org/abs/1807.06209