Tactical MPT and Momentum: the Modern Asset Allocation (MAA)

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Abstract

Modern Portfolio Theory (MPT), as developed by Markowitz (1952) and others, is often described as a nice but impractical theory. The full MPT framework is very sensitive to parameters like the expected returns which are estimated with errors, resulting in allocations with even larger errors. This is known as the multiplication of errors. The same holds for the expected covariance matrix. In this paper we combine MPT with momentum, a simple covariance model and shrinkage estimators. First, we use historical estimates based on short (up to one year) lookback periods, in contrast to the traditional (multi-year) approach. Second, we use the "single-index model" of Elton (1976) to structure the covariance matrix and to arrive at an elegant analytical formula for the optimal allocations. Finally, we reduce estimation errors by partially "shrinking" all estimates for expected returns, volatilities and cross-correlations towards their means over assets. We call the resulting "tactical" (short-term) MPT model the "Modern Asset Allocation" (MAA). We illustrate the MAA model on nine universes (with 7 to 130 assets) over 1997-2013 and show that the MAA model beats the simple EW model consistently, proving the usefulness of MPT.

Keywords: Tactical Asset Allocation, Momentum, Markowitz, Elton, MPT, mean variance, minimum variance, Sharpe, EW, SIM

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1. Introduction

The Modern Portfolio Theory (MPT) goes back to a seminal article from Markowitz (1952). It's also known as the mean-variance framework. In principle, it's a strategic approach, or, in other words it aims at a long run (multi-year) optimal asset allocation.

The core of the MPT is the classical mean-variance framework for portfolio selection. This corresponds, under certain assumptions, to maximizing the Sharpe ratio. To compute the corresponding optimal portfolio, one needs to estimate the expected mean and covariance of assets returns, e.g. by their sample estimators from historical return data. These estimators often contain substantial estimation errors, especially for the mean return. As a result, mean-variance (or MPT) portfolios computed from sample estimators perform poorly.

In the strategic (multi-year) MPT framework, the expected mean and covariance of assets returns are often estimated over a multi-year historical window, say five to ten years (60-120 months). In a seminal article, DeMiguel (2007) have shown that the strategic sample-based MPT allocation is nearly always outperformed by a simple equal weight (EW) allocation[2]. They show that this also holds true for most of its extensions designed to reduce estimation errors, e.g. when shrinkage estimators are used. A similar conclusion was recently arrived at by Jacobs (2013). Both DeMiguel and Jacobs used multi-year windows (60-120 months) and therefore a strategic approach.

In this paper, we will take a more tactical approach to MPT and limit the estimation window to 12 months. We will call this approach tactical mean-variance optimization or *tactical* MPT. It relies heavily on the momentum anomaly (see eg. Jegadeesh, 1993, Hurst, 2012 and Keller, 2012).

Additionally, in order to cope with estimation errors for the (mean) return and covariances, we will use simple shrinkage estimators and a simple covariance model based on a single index (the market factor). The single index model (SIM) was first published by Elton (1976) and recently reinvented partly by Clark (2011).

The core of the SIM model is the distinction between the systematic effect, which relates the return of an asset to a market index (like the SP500 or EW index) through the "beta" coefficient on the one side and the residual (or idiosyncratic or non-systematic) effect on the other. By using this simple SIM model we are able to reduce the number of parameters in the NxN correlation/covariance matrix to the more manageable N beta's, where N is the number of assets in the universe.

We will call the resulting MPT model (with short lookback periods, shrinkage and the SIM assumption) the Modern Asset Allocation or MAA model. We will demonstrate that in such a tactical framework, MPT is easily capable of outperforming the equal weight (EW) solution, for many different universes.

[2] Kritzman (2010) found that EW can be beaten by "hand picking" the expected returns from even longer lookback periods than the 60 or 120 months of DeMiguel.

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In the next two sections we first introduce the SIM model and then derive the analytical formula for the optimal (MPT) allocation in this case. In section 4 we interpret this formula, while in the following section we focus on the estimation of expected returns etc. by using historical estimates over a certain lookback period. Shrinkage of all estimators is the topic of section 6. In section 7, 8 and 9 we look at the empirical verification of the model for various universes, starting with an "in-depth" investigation of the model for the same universe as in Keller (2012) in section 8. Section 10 concludes.

2. The single index model (SIM)

The basic assumption for our MAA model is the maximization of the Sharpe ratio (the "tangency solution") in the SIM model with no short sales (long only). The maximization of the Sharpe ratio in the SIM model is equivalent with the well-known mean-variance optimization in the Capital Asset Pricing Model (CAPM), which is based on the Modern Portfolio Theory (MPT) of Markowitz (1952) and Sharpe (1963). The SIM and CAPM model for returns r_i (i=1..N) can be expressed as a function of a market or index return r_m (independent of i) and a residual (random) effect. In formula:

(1) $r_i = a_i + b_i r_m + e_i$

where

- r_i is the (excess) return of asset i (i=1..N) in a universe of N assets[3],
- r_m is the market or index (excess) return,
- a_i is the "alfa" for asset i,
- b_i is the market (or index) "beta" for asset i, and
- e_i is the "idiosyncratic" residual for asset i.

In this model, a high beta b_i reflects a high systematic (or market driven) risk while a low beta reflects a low market (or systematic) risk. A negative beta reflects a market "hedge": if the market return goes up, the asset return is expected to go down. Notice that the residual (or stochastic) component e_i reflects some (independently distributed) noise.

Given these parameters and eq. (1), we can derive the optimal portfolio allocation which maximizes the Sharpe ratio. This is also called the mean-variance solution in the MPT literature. Instead of "single-index" or CAPM model, eq. (1) is also known as the "single-factor" model when r_m represents the single factor, not necessarily the market. We will use the term "single-index model" (SIM) in honor of Elton (1976).

[3] Notice that in this paper the number of assets in the universe "N" is similar to "U" in our previous paper (Keller, 2012).

The residuals e_i are assumed to be distributed independently with mean zero, so by combining assets into a portfolio, we can reduce the volatility of this part ("diversification"), while the first part represents the "systematic" (or market) risk which cannot be diversified away. This distinction between the non-diversifiable systematic and the diversifiable idiosyncratic part is crucial in what follows.

In the following we will assume that the risk-free rate r_f is zero, so that all excess returns r_i are equal to the total returns[4]. Since only r_i, r_m and e_i are assumed to be stochastic, we can easily find for the idiosyncratic (or residual) variance s_i of e_i:

(2) $s_i = s_{ii} - s\, b_i^2$

where

s_{ii} is the variance of the return of asset i,
s_i is the "idiosyncratic" or residual (e_i) variance of asset i (i =1..N),
s is the market (or systematic) variance (independent of i).

We will use the Equal Weight (EW) benchmark of the universe of N assets as the market index (also the benchmark in our FAA paper, see Keller 2012). Notice that this is only a proxy of a buy-and-hold portfolio, since it is rebalanced each month.

The market index has variance s and return r_m, which equals (in view of our EW assumption):

(3) $r_m = \sum r_i / N$

where the summation is over all assets (i=1..N). Note that theoretically the independence of the residuals is incorrect in this case (since r_m is dependent on r_i, see Fama, 1968), but the error goes to zero when N becomes large.

3. The MAA formula

We can now arrive at our MAA formula, which gives the long-only optimal MPT asset allocation, given the single index model (SIM) and the Maximization of the Sharpe ratio (MS).

This MAA formula was originally derived by us from a similar analytical formula for a long-only Minimum Variance (MV) portfolio, see also Scherer (2010), Clark (2011) and Jurczenko (2013).

[4] In other words, we prefer to include ETFs (like SHY) of low-volatility and low-maturity treasury notes in our universes instead of using excess returns over treasury bills. In practice the results are very similar. We focus here on simple investments, so no leverage, no shorts and no T-bills.

Later we learned that the same formula as our MAA formula appeared long ago in a classical paper from Elton (1976), who used the SIM (Single Index Model) moniker for this model.[5]

This MAA formula expresses the long-only optimal asset allocation shares w_i as a function of the expected returns r_i, the expected idiosyncratic variances s_i and the expected beta's b_i of the assets (i=1..N) for a given universe (see appendix A for a proof). In scalar form the **MAA formula** is:

(4) $w_i = s_p (1-t/t_i) \; r_i / s_i$ for $t_i > t$, else $w_i = 0$, i=1..N

where

$t_i = r_i/b_i$	is the "Treynor ratio" for asset i (see Treynor, 1966)
t	is the "Treynor threshold" (see Appendix A)
s_p	is the variance of the optimal (long-only) portfolio, independent of i

This is the main formula of this paper. It gives us the optimal MPT portfolio long-only allocation as an elegant analytical formula, given the SIM assumption.

From this eq. (4), we conclude that all assets i (=1..N) with a Treynor ratio t_i less than the threshold t are excluded (w_i=0), if we assume all beta b_i to be non-negative (which holds for nearly all assets). This condition is similar to the beta threshold in the MV portfolio of Clarke (2011) and Jurczenko (2013), where all assets with beta larger than this threshold are excluded.

Notice that (see Appendix A) both the variance of the optimal portfolio s_p and the Treynor threshold t are a function of the optimal weights w_i, and are therefore endogenous. So eq. (4) is not a "closed-form". Later on (see Appendix A) we see that this can be easily solved numerically. Notice that in case of a long-only solution ($w_i >= 0$) we can easily deduce the portfolio variance s_p from the normalization $\sum w_i = 1$.

4. Interpretation of the MAA formula

We can now interpret our MAA formula for the long-only maximum Sharpe portfolio allocation, eq. (4).

Let us first assume that there is no systematic part in eq. (1) by assuming that the market or index variance is zero (s=0), so there is only a diversification effect. Then, from eq. (2), we see that the ordinary variance equals the idiosyncratic variance ($s_{ii}=s_i$). We can also derive (see Appendix A)

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[5] One of the reasons for using this SIM model was that we were interested in an analytical formula and fast computation for large universes with singular covariance matrices. Recently, we understood that some quadratic programming solutions for MPT (including the Critical Line Algorithm from Markowitz himself) were also very fast even for large N and singular covariance matrices. See also Bailey (2012), Butler (2013), Kapler (2013), Kwan (2007), Nawrocki (1996), and Niedermayer (2006).

that when the market variance is zero (s=0), the Treynor threshold t is also zero (t=0). So the optimal weights are proportional to r_i/s_{ii}, the return/variance ratio, when $r_i>0$:

(5) $w_i = s_p\ r_i / s_{ii}$, for $r_i>0$, else $w_i=0$, i=1..N

It is now also clear that, without systematic component, all assets with non-positive expected return ($r_i<=0$) are excluded in the optimal portfolio ($w_i=0$). When we use past momentum as a proxy for the expected return ri (see next section), this corresponds to "absolute momentum"; see eg. Antonacci (2013) and Keller (2012).

Additionally, the non-zero weights ($w_i>0$) are proportionally to the (positive) expected returns r_i: the higher the expected returns, the higher the weight. This corresponds to some kind of "relative momentum" (see Keller, 2012).

At the same time, the weights w_i are inversely proportional to the asset return variance s_{ii} (=s_i when s=0). Notice that the variance s_{ii} is the square of the volatility v_i of asset i, so for each doubling of the volatility the weight reduces by a factor four. This special case of the Minimum Variance model (see Clarke, 2011) is similar to the "risk-parity" model but with variances instead of volatilities.

When the systematic part is non-zero (so s >0 and t>0), the long-only portfolio solution equals the solution without systematic component corrected with a "systematic" term ($1-t/t_i$) which reflects the impact of the market risk through the beta's b_i in the Treynor ratio's ($t_i=r_i/b_i$). While the Sharpe ratio (r_i/v_i) rules in the idiosyncratic world, in the systematic world the Treynor ratio (r_i/b_i) takes over, with beta b_i as systematic risk factor instead of volatility v_i .

The larger the market risk b_i of asset i is, the smaller the Treynor ratio t_i and therefore the smaller the systematic term ($1-t/t_i$), up to the point where this term becomes zero when the Treynor ratio t_i reaches the Treynor threshold t from above. Notice that when beta is negative, the factor ($1-t/t_i$) becomes larger than one: in this case, asset i has a negative systematic part and can therefore be used as a hedge.

5. Estimation and Momentum

In the theoretical SIM model, parameters like expected returns r_i etc. are assumed to be known, which is not realistic. In practice we have to use sample estimates of these parameters based on the past for the unknown future, which is the main problem. Given sample estimates of these parameters we can use eq. (4) to solve for the optimal max Sharpe portfolio.

It's important to emphasize that the quality of our optimal portfolio allocation model depends primary on the quality of these estimates. And since "prediction is very difficult, especially if it's about the future" (Niels Bohr), this is not trivial.

We will estimate all expected returns r_i by the rate of change (ROC) of the asset price over a certain lookback period, assuming some persistence over time. By using lookback periods of less than a year we will benefit from the well-known momentum anomaly for returns (see also Keller, 2012). We will assume that we rebalance monthly and that we have always historical daily total return (adjusted close) data for all N assets for the last months before rebalancing.

Besides returns, using daily historical data we can also arrive at sample estimates for expected volatilities and for expected correlations using historical estimates over similar lookback periods. Combining these will give the estimated expected (future) covariances. With e.g. 4 months lookback we can use around 84 days of daily data (total return, adjusted close) for computing historical volatilities and correlations.

Just like return momentum, we use the assumption of persistence ("momentum") to arrive at estimates for expected (ie. future) volatilities, variances, correlations and therefore covariances. This is the reason we sometimes speak of "generalized momentum" for the factors R (return), V (volatility) and C (correlation), see Keller (2012).

Notice that the lookback period can be different for all these three factors R, V, and C. In order to use the "tactical" term, we will assume all lookback periods to have a maximum length of 12 months. This is in contrast with most traditional MPT models where the lookback periods are often longer than one year.

In the following we will use the same symbols like r_i for the *estimated* (expected) returns, etc. Having estimates for the (expected) volatilities v_i and the (expected) correlations c_{ij} for asset i,j (i,j=1..N), we can estimate the covariances s_{ij}, the market variance s, and the beta's b_i from

(6) $s_{ij} = v_i v_j c_{ij}$, the covariance of asset i,j
(7) $s = \sum\sum s_{ij} / N^2$, the market variance
(8) $b_i = \sum s_{ij} / (sN)$, the beta of asset i

for i,j =1..N and use eq. (2) to estimate the idiosyncratic variance s_i.[6]

The computation of w_i and the endogenous Treynor threshold t (see Appendix A) can be done by sorting of the assets as described by Elton (1976) when all beta b_i are non-negative. When some beta's are negative Elton suggests a rather complex repeated sorting procedure. We found that simply iterating between t and w_i, (starting with t=0, calculating w_i, recalculating t, recalculating w_i, etc. until both converges) often works in a few iterations. This approach also works in case some assets have negative beta's.

[6] In practice, it is not necessary to compute the full NxN correlation matrix, since s_{ii} and b_i can easily be expressed as a function of the N correlations of each asset with the index EW.

6. Estimation and Shrinkage

We think that we can take advantage of the flexibility of short-term (max. one year lookback) momentum, not only for returns but also for volatilities, correlations and covariances. This is relevant since, even when volatilities and correlations are more stable over time than returns, they do change, in particular in times of crisis (see Chin 2013, Butler 2012, Schoen 2010 and Newfound, 2013). By reducing the NxN covariance structure to the N vector of beta's b_i, we don't run out of degrees of freedom as would happen with the estimation of the full covariance matrix and limited (short-term) historical data.

By using short-term momentum combined with the SIM assumption, changes in volatilities and correlations or beta's (like during the 2008 market crash) can be taken into account much better than in the traditional MPT model with lookback periods of years instead of months. That this flexibility might be relevant for asset allocation is also shown in the present discussion of low-volatility and low-beta indices and ETFs (see e.g. Blitz, 2012). The SIM model also gives us an elegant analytical solution, see eq. (4), while the classical MPT model (with unrestricted covariance matrix) needs numerical algorithms to solve for the optimal portfolio[7].

However, the disadvantage of using short lookback periods is that estimated expected returns, volatilities and beta's can change rapidly over time, leading to large errors and even to extreme weights in our optimal portfolio allocation. Therefore we will use shrinkage to reduce these errors. See also Ledoit (2004) and DeMiguel (2013). We will simply shrink all returns towards the average return, all volatilities towards the average volatility and all correlations towards the average (cross) correlation by introducing "weights" W_R, W_V, and W_C.

We will also shrink the market variance s (factor S) towards zero, in order to decrease the Treynor threshold t and therefore allow for more assets in our optimal portfolio. In other words, by shrinking s we have more weight on diversification and less weight on the systematic component. For this purpose we introduce the weight W_S. In the default MAA model we will therefore shrink the factor S simply to 50% to allow for more diversification, which improves our backtest results.

We use "weights" W_R, W_V, W_C and W_S for all factors R, V, C and S such that a weight W_R=1 implies no shrinkage of factor R and W_R=0 implies full (100%) shrinkage towards the mean, effectively disabling the variation in returns r_i. The same applies to factors V, C and S, with weights W_V, W_C and W_S between 1 (no shrinkage) and 0 (full shrinkage). Therefore the weight reflects the impact of each factor. The shrinkage formulas are

(9) $r_i = W_R\, r_i^s + (1-W_R)\, r_a^s$

(10) $v_i = W_V\, v_i^s + (1-W_V)\, v_a^s$

(11) $c_{ij} = W_C\, c_{ij}^s + (1-W_C)\, c_a^s \quad (i \neq j)$

(12) $s = W_S\, s^s$

[7] See also the references in footnote 5 for numerical algorithms.

where r_a^s, v_a^s and c_a^s are the average sample return, average sample volatility and the average sample (cross) correlation over all assets, respectively, and the superscript s refers to the sample estimators. The average return r_a^s is identical to the market return r_m, except for the exceptional case when $r_m<0$, then r_a^s equals a very small positive constant (eg. 1E-06)[8]. In what follows, we will for the sake of simplicity always speak of shrinkage to the market return r_m.

In the default **MAA model** in all our backtests (see section 8 and 9), we will set all shrinkage weights rather arbitrarily at 50% ($W_R=W_V=W_C=W_S=0.5$). Besides partially shrinking a factor to e.g. 50% to improve the (robustness of the) estimates of the expected value (eg. return, volatility, correlation or variance) we can also switch these these weights "on" ($W_X=1$) or "off" ($W_X=0$) to arrive at some special cases.

When eg. the factor S (market variance) is fully shrunk to zero ($W_S=0$, so $s=0$) the Treynor threshold becomes zero ($t=0$) implying there is no systematic component (no threshold) in eq. (4), and we arrive at eq. (5). Now all assets (with $r_i>0$) are part of the solution (full diversification) and the factor C (and therefore W_C) becomes irrelevant. When we only fully shrink factor C but not S ($W_C=0$, $W_S>0$) we arrive at the **Constant Correlation** model where all cross correlations are assumed to be equal.

A similar trick can be applied to returns and volatilities. When all weights are switched fully "on" ($W_R=W_V=W_C=W_S=1$) we arrive at the fully unshrunk **MAA-100** model. When r_i is fully shrunken ($W_R=0$) to r_m (independent of i) it can be shown that we arrive at the **Minimum Variance** (MV) model, since r_m disappears in the normalization constant. Then eq. 4 becomes (see Appendix A):

(13) $w_i = (1-b_i/b)\, s_p / s_i$ for $b_i < b$, else $w_i=0$, $i=1..N$.

where b is the beta threshold (see also Clarke, 2011). When we in addition fully shrink the factor V of volatility and the factor C of correlation (so $W_R=W_V=W_C=0$), we arrive at the **Equal Weight** (EW) model.[9]

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[8] This is done to preserve EW as a special case when all R, V, C weights are 0, even when $r_m<0$.

[9] In a follow-up paper, we will show that the MAA formula in eq. (4) also encompasses all members of the risk-parity family when we shrink asset returns to their volatilities. This way, we can arrive at the "naïve" Risk Parity (RP) model, the Equal Risk Contribution (ERC) model and Maximum Diversification (MD) model as members of the MAA family. See also Roncalli (2013), and Maillard (2009), Choueifaty (2011), Schoen (2012), Jurczenko (2013), and Hallerbach (2013).

Finally, in the MAA model we recognize the same factors R, V and C as in our **Flexible Asset Allocation** model (**FAA**, see Keller, 2012). However, instead of using (partial) shrinkage of r_i, v_i and c_{ij} as in the MAA model, we use a non-parametric model in case of the FAA model, based on (weighted) ranks. In the FAA model, we substitute ranks for r_i, v_i and b_i and choose the best TopX (out of N) assets on the general rank function:

(14) Rank asset i = W_R*rank(r_i) + W_V*rank(v_i) + W_C*rank(b_i), i=1..N

By adding the Absolute (A) momentum condition $r_i>0$ separately and depending on which W_R, W_V, W_C, or W_S is zero we can arrive at various FAA models with eg. A, AR, ARV and ARVC "generalized momentum" factors (see Keller, 2012).

The optimal FAA allocation is an *equal* weight allocation of the best ranking (TopX out of N) assets, where we also replace all TopX assets with $r_i<=0$ by cash in case of absolute (A) momentum. As such, FAA can be seen as a non-parametric (ranking) EW variant of our MAA model.

Notice that the MAA model uses *unequal* weights (see eq. 4) for *all* (instead of the TopX) assets, and no cash (except when all w_i=0).

7. MAA in practice

In the following sections we will apply our MAA model to various universes from 1997 to 2013, to demonstrate the superiority of "tactical MPT" over EW.

We will present examples of the MAA formula (eq. 4) in practice, showing backtests results for nine multi-asset universes ranging from seven to 130 assets, using the same default shrinkages (50%) and the same default lookback periods (4 months) for all universes. In section 8, we will also explore the MAA model in one universe (N=7) in more detail for other parameter values. As assets we will use broad ETF's and index funds, for equities, bonds, alternatives, etc., for both US and abroad (IM and EM). When historical data back to November 1997 is unavailable for ETFs we will extend them to the past by using similar index funds (see Appendix B for details).

To avoid datasnooping, we will, rather arbitrarily, set all lookback periods to four months, like we did in the FAA model Keller (2012). We will also assume for the default MAA model a shrinkage of 50% for all factors R, V, C and S for all universes. Notice that in addition to these default settings, we also restrict ourselves to the single-index-assumption (SIM) which reduces the NxN covariance matrix to the N-vector of market betas.

By shrinking the factor S (the estimated market variance) we are able to lower the Treynor threshold and therefore allow for more funds in the optimal solution. When factor S is not shrunk completely, only assets with Treynor ratio's above the threshold t are allocated. We will rather arbitrarily choose W_S=50% to arrive at solutions with somewhat more diversification than W_S=100% and therefore more assets in the solution, without disabling the C-factor completely.

Except for the special case when all w_i=0, MAA does not use a cash proxy, in contrast to the FAA model of Keller (2012) where cash is crucial. Another important difference is that FAA choses an equal weight (TopX) selection of assets while MAA has in principle variable weights for all assets, see eq. (4).

In order to enable comparison between different backtests, with different returns R, volatilities V and max drawdowns D, for every universe we will also include a version (called **MAA-TV**) where W_V is tuned such as to arrive at a target volatility V (eg. V≤10%) over the full backtest (1997-2013).

Notice that this volatility targeting is done over the backtest as a whole, instead of at each rebalancing as is usually the case. Therefore it is more a kind of volatility "calibration" to make backtests comparable. It also can be seen as a poor-man's form of leverage (since we can increase return R this way when volatility is below the target level), without paying interest. We will always present the results of both the default model (MAA with W_V=0.5) as well as the calibrated W_V model (MAA-TV), using a maximum volatility target of V≤10% for the latter.[10]

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[10] In an earlier draft version of this paper, in order to control for datasnooping, we used an "in-sample" tuning of W_V based on the first 4 years (1997-2001). It turns out that in most of the universes the "out-of-sample" (2001-

The daily data for the nine universes is from Yahoo (closed adjusted) and the timeframe for all backtest is November 1, 1997 to November 15, 2013 (16 year). Rebalancing is done on the first close of the new month, based on the data for the last close of the old month. For transaction costs we will use 10 bps. We assume there is no leverage possible and all trades are long only.

By using the same default weights (W=50% for factors R,V,C and S) and the same default lookback periods (four months for factors R, V and C) for all our universes for the MAA model, we try to avoid any datasnooping and show the robustness of the MAA model. In the next section we will also examine the robustness in more detail for other lookback and shrinkage parameters in case of the simplest (N=7) multi-asset model, like we did in Keller (2012) for the FAA model.

Legend for the various backtest statistics (see also Keller 2012):

R = CAGR, so annual Return (in %)
V = annual Volatility (in %)
D = max Drawdown over the full backtest 1997-2013 (in %)
T = annual Turnover
S = Sharpe ratio (with annual 2.5% risk-free rate)[11]
O = Omega ratio
Q5 = Calmar ratio (with 5% target return)

The Omega ratio O reflects the “gains to losses” ratio (around a target return of 0%). The Sharpe ratio S gives the annual return R above the average risk-free rate (here 2.5%) divided by the volatility V. The Calmar ratio Q5 gives the ratio of the return above an annual 5% target return and the max drawdown D. By using a 5% return target (instead of the risk-free rate of 2.5%) this ratio is more sensitive for higher returns R (than S and O). By using the max drawdown D (instead of the volatility), the Calman ratio Q5 is more sensitive for negative deviations than S.

We prefer therefore the Calmar ratio Q5 over S (and O) as the best metric to judge the return/risk performance of a backtest. This can be confirmed, in our opinion, by visual inspection of the equity graphs for different values of Q5.

2013) volatility was then often only slightly lower than the in-sample volatility. Therefore we decided to use the volatility tuning for the whole sample (1997-2013) as a simple calibration, similar to a poor-men’s leverage. We are aware that by doing so we introduce some look ahead biases. However, when we do this calibration for the N=7 universe in-sample over each of the four 4-year periods (1997-2001, 2001-2005, 2005-2009, 2009-2013) we arrive at similar in-sample weights (with W_V=7, 12, 30, 10%, resp.) and similar out-of-sample results as compared to W_V=18% (the optimal weight over 1997-2013); see also section 8 where we examine the robustness wrt. W_V.

[11] The 3-month T-Bill has an average annual return (CAGR) of approximately 2.5% over the period considered (1997 – 2013).

8. An in-depth exploration of the MAA model (N=7 universe)

Before we arrive at the empirical validation of MAA for various other universes, in this paragraph we explore the various corners of the MAA model applied to same global multi-asset universe (N=7) we introduced for the FAA model (Keller 2012). It consists of 7 index funds (VTSMX, FDIVX, VEIEX, VBMFX, VFISX, VGSIX, QRAAX) representing US, international (IM) and emerging market (EM) stocks, two US bonds, US REIT and a commodity index (corresponding ETFs: VTI, VEA, VWO, BND, IEI, VNQ, DBC).

Below we present the graph for the MAA-TV model (R=11.8% when W_V=18% and V≤10%) and for EW for this universe.

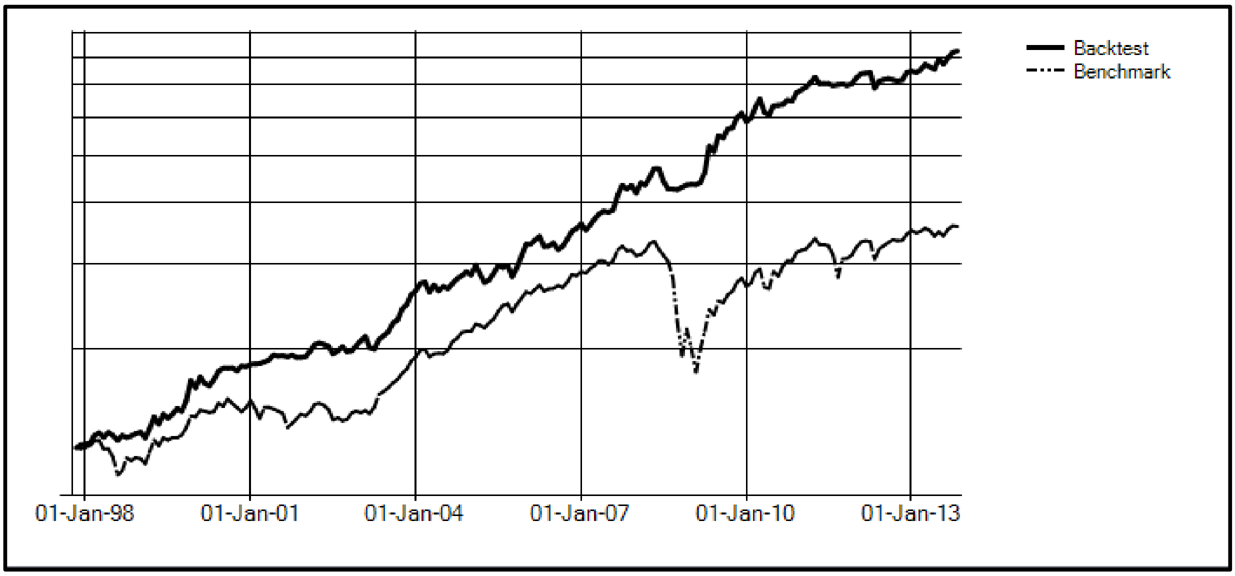


Figure 1. Equity line for MAA-TV and EW (N=7 universe)

In the table below we present the results for all the three models including the default MAA model (all W=50%), the MAA-TV model (W_V tuned to V≤10%) and the EW benchmark (all W=0). These results clearly prove the superiority of the MAA model over EW, in particular on R, D, S and Q5.

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	9.30	6.20	5.20	1.85	1.11	3.18	0.83
MAA-TV	50	18	50	50	11.80	10.00	9.60	2.60	0.93	2.53	0.71
EW	0	0	0	0	6.70	13.40	46.30	0.03	0.31	1.58	0.04

Table 1. Statistics for the N=7 universe

In the rest of this section we will examine this global multi-asset universe (N=7) in somewhat more detail to get a feeling of the robustness of the MAA model.

8.1 Decomposing the MAA model into factors

In this section we will examine the impact of the different factors R, V, C and S, by rebuilding the default MAA model step-by-step, starting at the EW benchmark and adding the various factors.

When we assume that factors R, V, and C are irrelevant, we can shrink these factors to their means by putting the weights W_R. W_V and W_C equal to zero. If all these weights are zero, we arrive at the benchmark, the **Equal Weight** (EW) model (W_R=W_V=W_C=0, W_S is then irrelevant).

When we only take into account the sign of the return r_i, such that w_i=0 when r_i<=0 (and equal weights w_i for all assets with r_i>0), we arrive at **model A** with only Absolute momentum (called factor A). Now, we replace r_i by sign r_i in eq. 5 and shrink s_{ii} to a constant (independent of i), since W_V=W_C=W_S=0. This absolute momentum overlay is very similar to the well-known SMA (eg. 200 days) overlay (see Faber, 2007) or to other absolute momentum overlays (see Antonacci, 2013).

For the absolute momentum model, see the table (model A) and graph below. It is clear that by simply adding absolute momentum to EW, all statistics (R, V, D, S, and Q) improves substantially with the A model. Of course, turnover T increases, as in all our non-EW models.

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
EW	0	0	0	0	6.70	13.40	46.30	0.03	0.31	1.58	0.04
A	Sign	0	0	0	9.70	9.40	21.30	2.62	0.78	2.32	0.22
AR-50	50	0	0	0	11.00	10.30	11.70	2.16	0.82	2.31	0.51
ARV-50	50	50	0	0	9.00	6.60	7.10	1.62	0.99	2.86	0.56
ARVS-50	50	50	0	50	9.60	6.80	6.10	2.05	1.04	2.94	0.75
MAA=ARVCS-50	50	50	50	50	9.30	6.20	5.20	1.85	1.11	3.18	0.83
MV=VCS-50	0	50	50	50	5.20	4.60	17.80	0.38	0.59	2.48	0.01
MAA-TV	50	18	50	50	11.80	10.00	9.60	2.60	0.93	2.53	0.71

Table 2. Statistics for N=7 universe

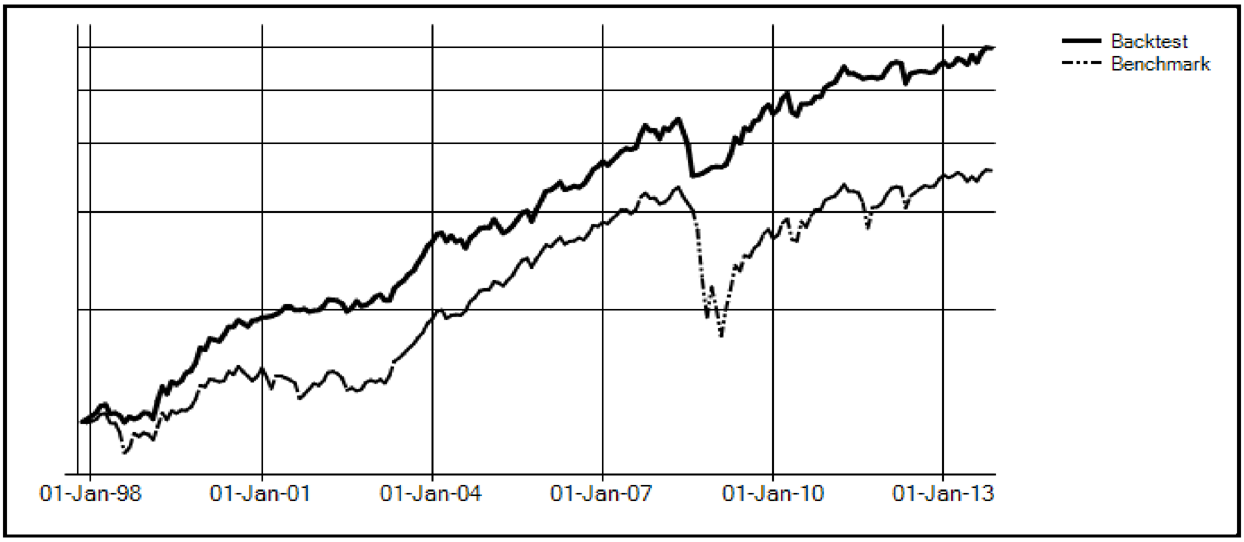


Figure 2. The A model (N=7 universe)

When we introduce (above factor A) the (shrunken) return factor R in the next step, we arrive at the **AR-50 model** with W_R=0.5 (and W_V=W_C=W_S=0). Then eq. 4 becomes w_i=s_p r_i if r_i>0 else w_i=0. Notice that absolute momentum (r_i > 0) is now implicit in the factor R instead of explicit as above (sign r_i) and the weights are variable instead of equal for assets i with r_i>0.

For the results of the AR-50 model, see table 2 above and the graph below. Again, most statistics improve upon the A model by adding the return factor R. All statistics (except V) are much improved, in particular return R is improved substantially, because of the introduced relative momentum. Even turnover T has improved.

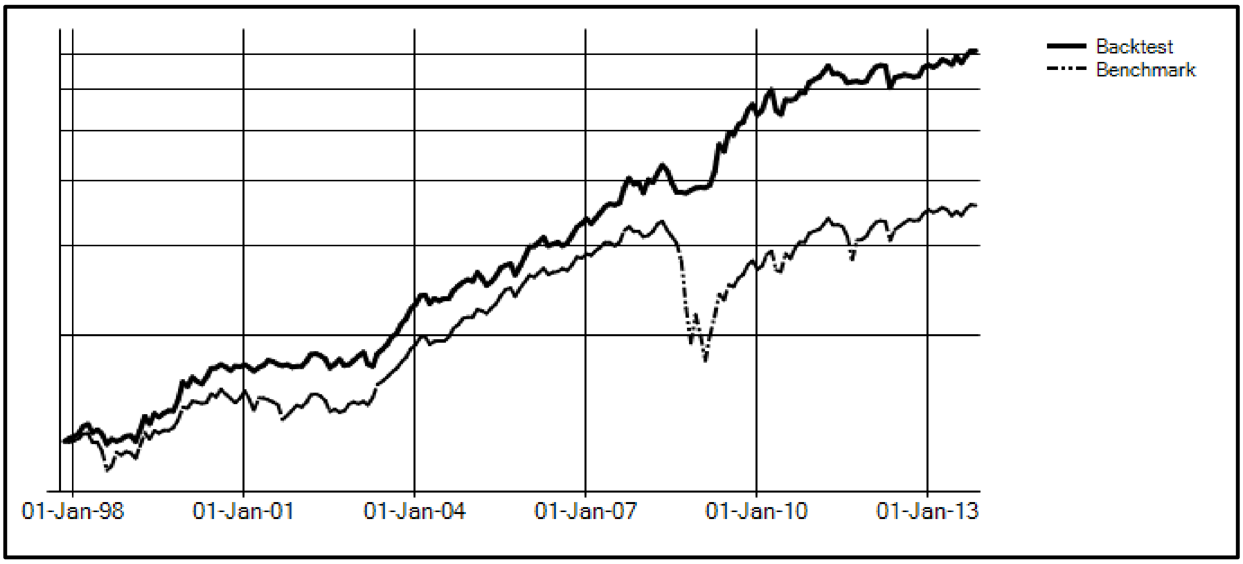


Figure 3. The AR model (N=7 universe)

In the next step we add the (shrunken) factor V to the AR model and arrive at the **ARV-50 model**, corresponding to eq. 5. Now W_R=W_V=0.5, W_C=W_S=0. See the table 2 above and the graph below. Again, all statistics improve upon the AR model by adding the volatility factor V, except the return R. As a result of the introduction of the factor V, both the backtest volatility V and the max drawdown D improve substantially (although at the expense of return R).

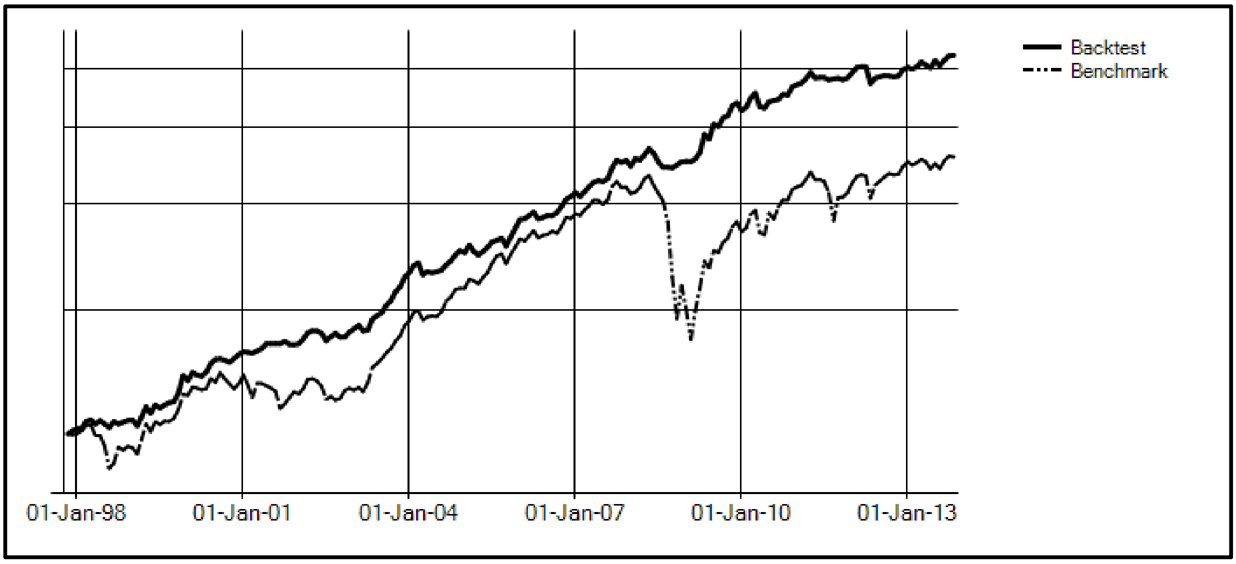


Figure 4 The ARV model (N=7 universe)

Now, we add the (shrunken) factors C and S to the ARV model in order to arrive at the final **ARVCS-50** model, with W_R=W_V=W_C=W_S=0.5 (the default MAA model), cf. eq. (4). See the above table and the graph below. Again, all statistics improve upon the ARV model with this ARVCS model, in particular V, D, S and Q5. Also the turnover T is improved.

From the above results, it is clear that with each additional factor (A, R, V, C and S) the return/risk performance improves compared to the previous model as shown by the Sharpe ratio S and the Calmar ratio Q5 (see Table 2), starting from EW.

In Table 2 above we also show the ARVS model, where we nullify the C factor while preserving the S factor, so W_R=W_V=W_S=50% and W_C=0. This is the **ARVS-50 model**, corresponding to a constant-correlation model, where all the cross-correlations are equal (by the shrinkage of C). From Table 2 we see that this model comes very close to the full MAA model (with free cross-correlations) but is still slightly worse on S and Q5.

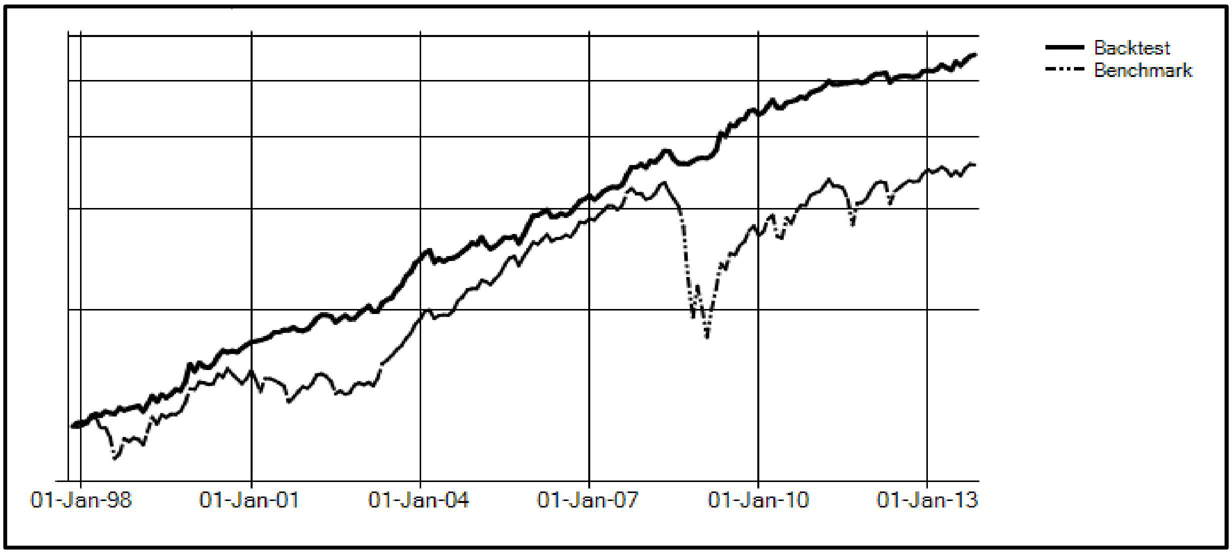


Figure 5. The full ARVCS (=default MAA) model (N=7 universe)

We can also calibrate the default MAA model by tuning W_V such that R is maximal over the backtest and V≤10% ("poor men's leverage"). This is the **MAA-TV** model. The optimal value turns out to be W_V=18%. See Table 2 above and the graph below. Notice that we now have the highest backtest return R even compared to the AR model but with much better return/risk statistics (see S, O and Q5).

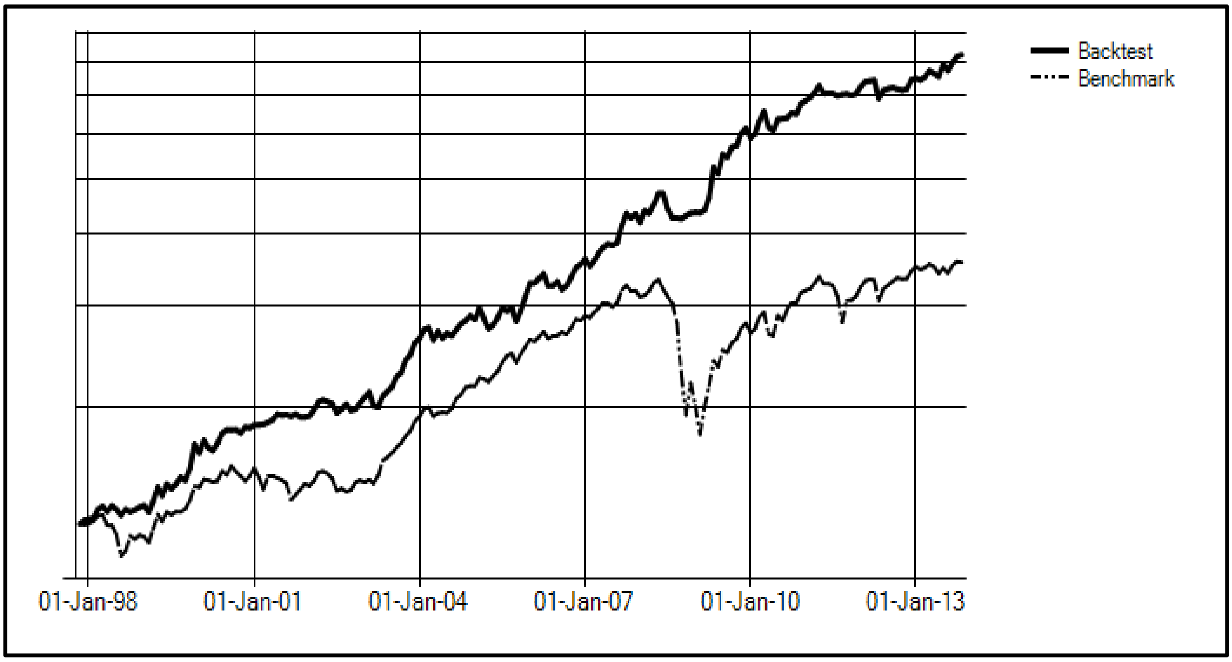


Figure 6. The MAA-TV (V=10%) model (N=7 universe)

Besides all these variations on the MAA model, we can also put the factor R "off" and arrive at the **Minimum Variance** model, with W_R=0 and W_V=W_C=W_S=0.5. This is the MV (or VCS-50) model. See also eq. 13. For performance, see Table 2 above and the graph below. The results are disappointing, with the exception of the backtest volatility V, which is the lowest of all, but also with one of the largest max drawdowns D and a very low return R! This MV model clearly shows what happens when MPT is not combined with (absolute and relative return) momentum. It beats EW on V, D, S, and O, but not on R, T, and Q5.

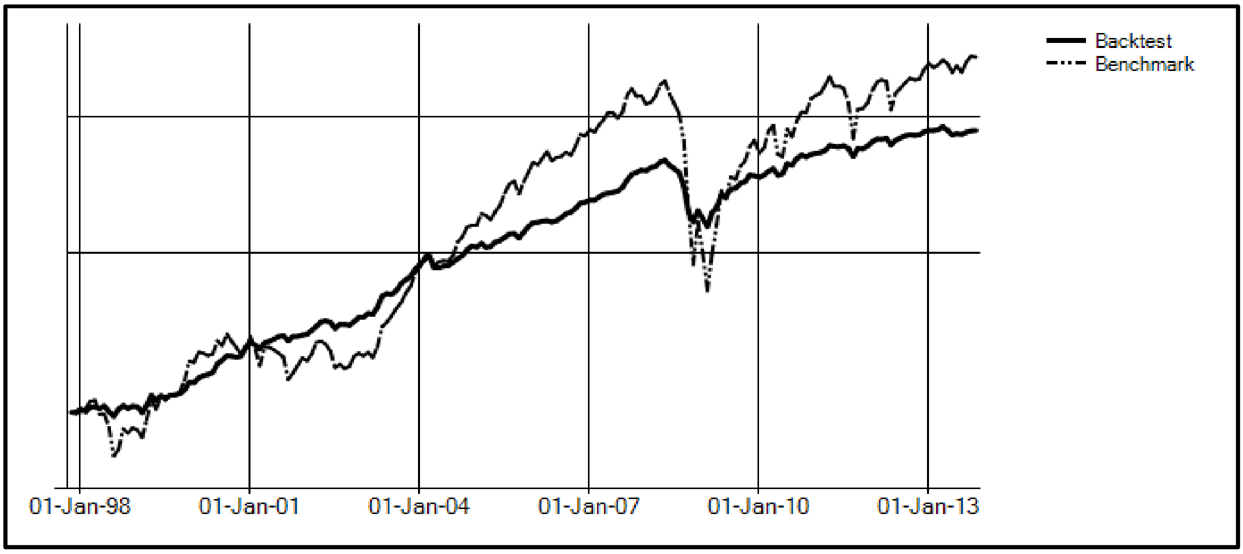


Figure 7. The MV (VCS-50) model (N=7 universe)

8.2 Robustness: the effect of no shrinkage

In table 3 we will show the effect of no shrinkage (W_R=W_V=W_C=W_S=100%), again step-by-step as in the previous section. The final model is the **ARVCS-100 or MAA-100** model. See the table and graph below.

If we compare the results of this final MAA-100 model with the default MAA model (with 50% shrinkage), it is clear that shrinkage helps: all statistics (R, V, D, S, O, Q5) are (much) better in the default MAA model (W=50%) than in the no-shrinkage case (W=100%), in particular D. This also holds for the partial models shown in Table 3, compared to the corresponding models in Table 2. Only the Minimum Variance (VCS-100) model has a D similar to the shrunken MAA model, but also a very small R (and negative Calmar ratio Q5).

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
AR-100	100	0	0	0	12.00	14.00	24.20	3.17	0.68	2.08	0.29
ARV-100	100	100	0	0	6.60	7.90	24.70	2.71	0.52	2.29	0.06
ARVS-100	100	100	0	100	7.50	10.10	24.30	3.49	0.49	2.08	0.10
ARVCS-100	100	100	100	100	7.10	9.00	24.30	2.97	0.51	2.32	0.08
MAA	50	50	50	50	9.30	6.20	5.20	1.85	1.11	3.18	0.83
VCS-100	0	100	100	100	4.10	2.10	3.40	0.36	0.76	4.53	-0.26

Table 3. The unshrunk model (N=7 universe)

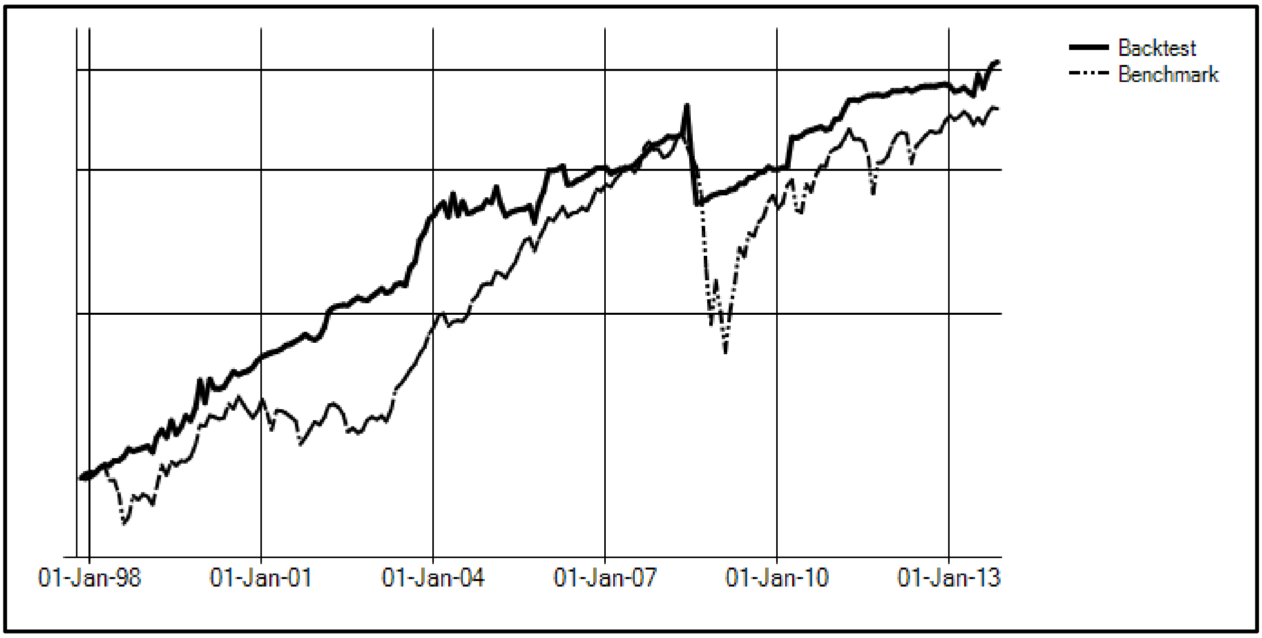


Figure 8. The MAA-100 (ARVCS-100) model (N=7 universe)

8.3 Robustness: the effect of different shrinkage weights

We also did some robustness test for this universe (N=7) by changing the various shrinkage weights for all factors (W_R, W_V, W_C and W_S) in the default MAA model.

In the next figure we present the Calmar ratio Q5 as a function of changes (W=0..100%) in the shrinkage weights W_R, W_V, W_C and W_S respectively around the MAA (W=50%) allocation. For example, the bars labeled W_R gives the values of Q5 when changing W_R from 0 to 100% while keeping all other weights at the default 50% of the MAA model. For comparison, we also show the Q5 score for the benchmark EW and for the fully "unshrunk" MAA-100 model (see 8.2).

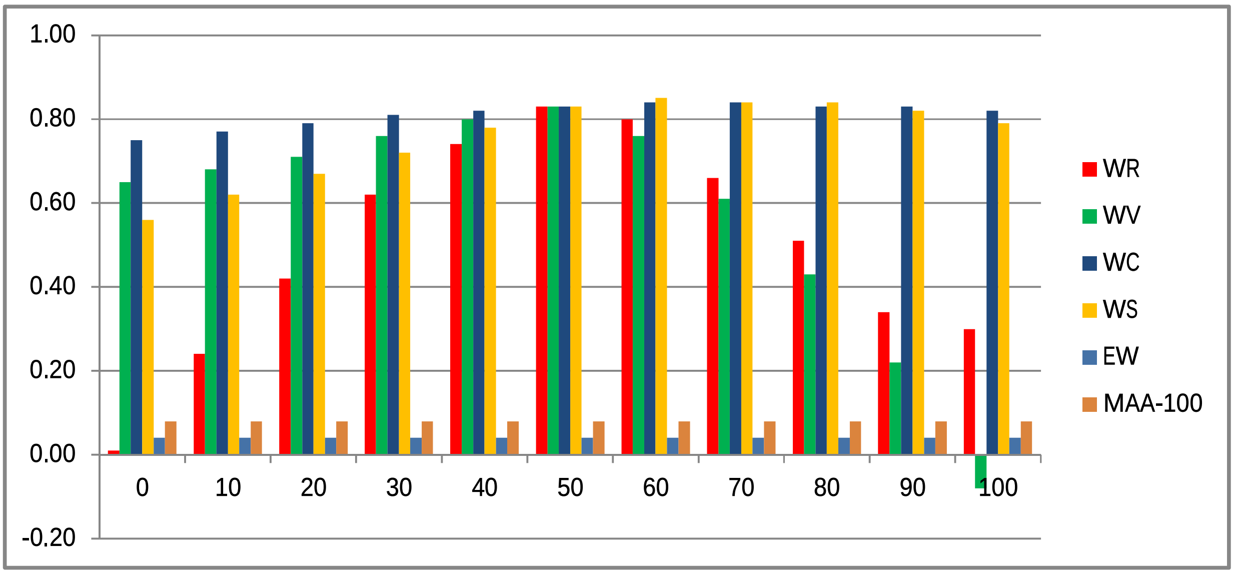


Figure 9. Calmar ratio Q5 for different shrinkage weights (MAA model) compared to EW and MAA-100 (N=7)

From the figure, it's clear that the biggest impact is by changing W_R, the shrinkage of returns: best Q5 is at W_R=50% and much less (symmetrical) when W_R moves to 0 or to 100%. To a lesser degree, also W_V has impact: best Q5 is at W_V=50% and much less (asymmetrical) for higher values of W_V.

The Q5 scores seems less variant to changes in shrinkage weights W_C and W_S, given the other MAA values (in particular W_R=50% and W_V=50%), with most (negative) impact for W_S<50% . Both W_C and W_S are slightly better when W>50%.

In comparison to EW, the Q5 scores for all these MAA variants are nearly always much better (except for MAA with W_R=0 or W_V=100%). The same also holds for the comparison to the fully unshrunk model MAA-100, for which Q5 is better than EW but still much worse than MAA for nearly all weights (except W_R=0 or W_V=100%).

8.4 Robustness: the effect of the length of the lookback period

Now we will check the robustness of the MAA model for different lookback periods for R, V and C (returns, volatilities and correlations) for the N=7 universe.

For the standard backtest model (MAA), we used a default lookback period of length 4 months for these three factors. Here we will change the length of the lookback period from 1 to 12 months. All lengths are the same for all three factors (R, V, and C). So when we use eg. a lookback period of 12 months, returns, volatilities and correlations are estimated on a historical lookback period of 12 months.

In the figure below we present the annual Return R, the Sharpe S and the Calmar ratio Q5 for different length of the lookback period (1,2,3,4,5,6,9 and 12 months) for all lookback factors R,V, and C. Because of the longest lookback period of 12 months, we had to start the backtests in July 1998, since we had data from July 1997.

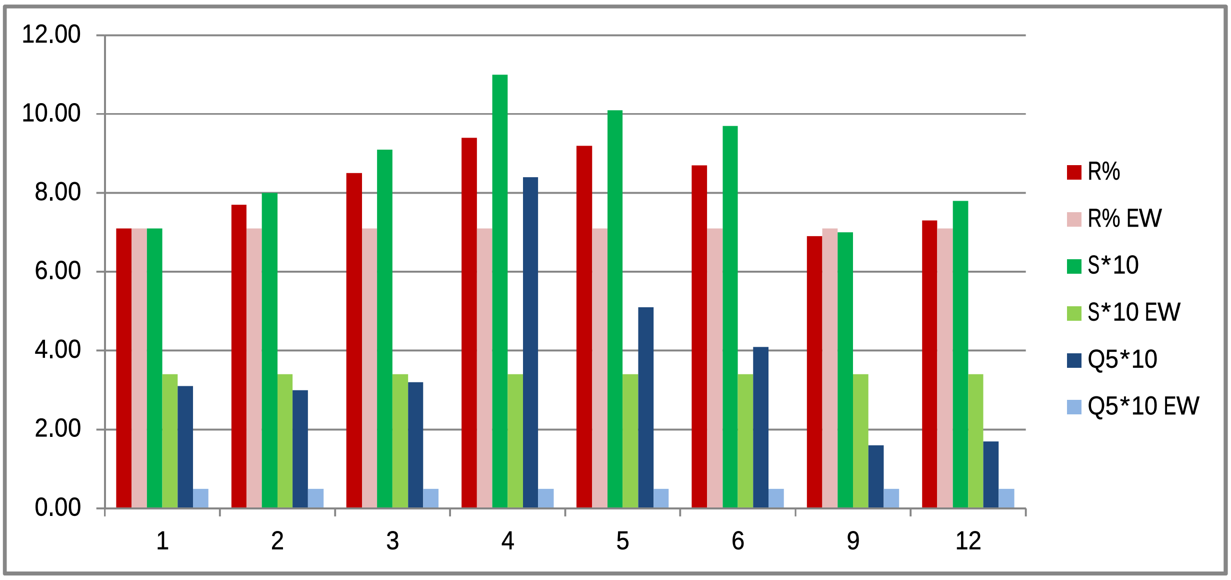


Figure 10. Annual return R, Sharpe S and Calmar Q5 by lookback period (months), 1998-2012 (MAA model, N=7 universe)

As shown, it is clear that a lookback period of length 4 months is optimal in terms of R, S and Q5. When we compare these results for the MAA model to the EW benchmark, it is also clear that they all are equal or better than the EW figures. In particular, the return/risk statistics S and Q5 are much better than EW for all the MAA models independent of the lookback lengths.

8.5 Robustness: the effect of transaction costs and out-of-sample behavior

We also looked at the influence of the **transactions costs** (including commission and slippage). As default we use a one-way cost of 10 bps (0.1%). What happens when we increase these costs? The graph below gives the answer for the default MAA model and the N=7 universe. For costs below 80 bps, both S and Q5 for the MAA model are still substantial above the same statistics for the EW model.

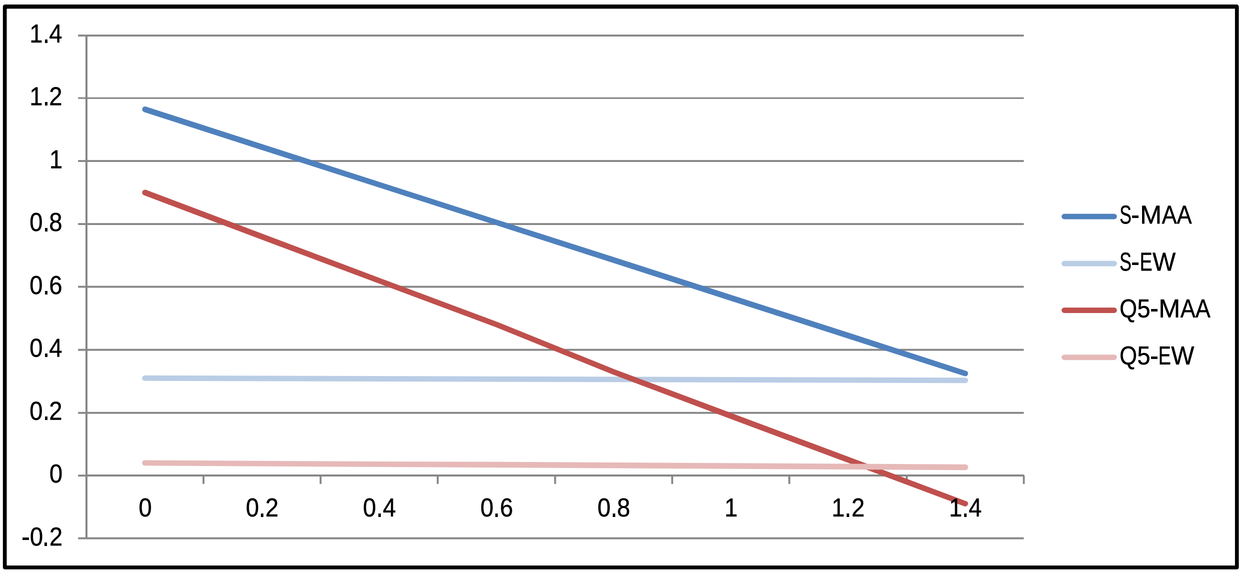


Figure 11. Sharpe S and Calmar Q5 as a function of transaction costs (%) in the MAA model (N=7 universe)

Finally, we will optimize all weights for the best **in-sample** backtest on early data (4 years from 1997, excluding the financial crisis in 2008) and see how that works out in the (much longer) **out-of-sample** (12 years: 2001-2013, including 2008). As optimization criteria we will maximize R for V≤10%, like the MAA-TV model, but now by tuning all weights W_R, W_V, W_C and W_S (with all lookback periods at 4 months).

When optimizing R this way over the first four years (in-sample: 1997-2001) we find as optimal values W_R=30%, W_V=0%, W_C=100%, W_S=100%. We call this the MAA-Opt model. When we run the full backtest (1997-2013) with these optimal weights we arrive at the results presented in the table and graph below. Notice that the results in the graph for the MAA-Opt model are now **out-of-sample** for the years 2001-2013.

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
EW	0	0	0	0	6.70	13.40	46.30	0.03	0.31	1.58	0.04
MAA	50	50	50	50	9.30	6.20	5.20	1.85	1.11	3.18	0.83
MAA-TV	50	18	50	50	11.80	10.00	9.60	2.60	0.93	2.53	0.71
MAA-Opt	30	0	100	100	12.70	9.80	8.50	2.51	1.05	2.74	0.91
MAA-TV1	50	7	50	50	12.40	11.20	11.00	2.80	0.89	2.41	0.67
MAA-TV2	50	30	50	50	11.00	8.60	8.00	2.35	1.00	2.71	0.76

Table 4. MAA-Opt and MAA–TV1 (in-sample 1997-2001) and MAA-TV2 (in-sample 2005-2009) over 1997-2013, N=7 universe

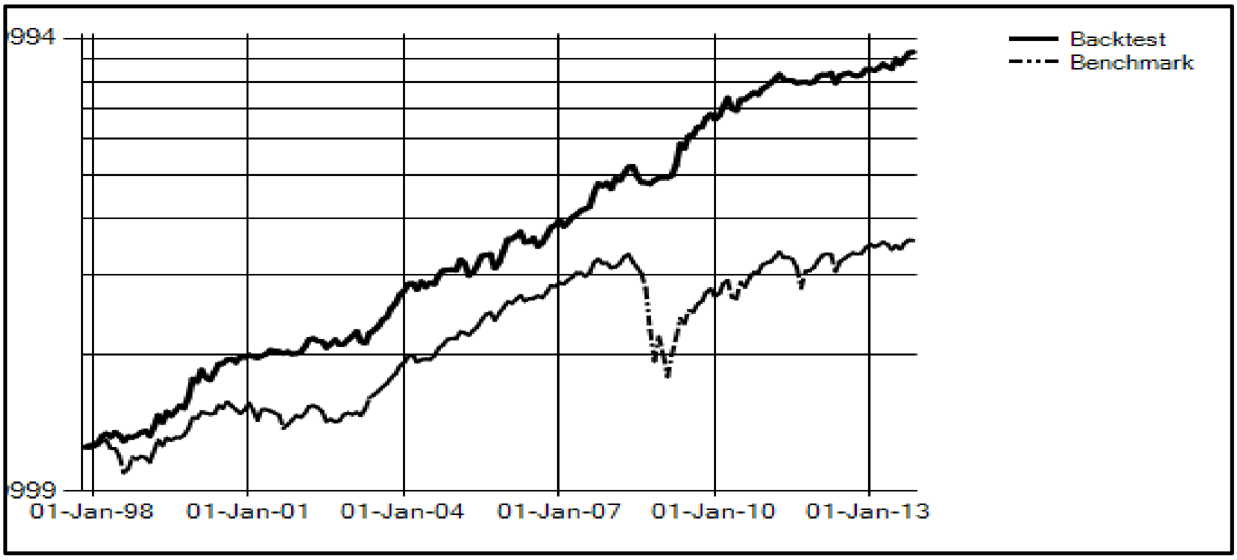


Figure 12. Out-of-sample test from 2001 (MAA-Opt, N=7 universe)

In the table we also give the results for the **MAA-TV** (again with $V \leq 10\%$) model tuned on different (in-sample) periods: on the full-backtest period of 1997-2013 (MAA-TV: W_V=18%), on early data 1997-2001 (MAA-TV1: W_V=7%) and on data from the financial crisis 2005-2009 (MAA-TV2: W_V=30%). Notice that over the four years from 1997, 2001, 2005, 2009, the optimal W_V ranges from 7% to 30% over these four in-sample periods, see also footnote 10.

All these (partly) out-of-sample results give similar return/risk results as the default MAA model (see Sharpe S and Calmar Q5) and all are much better than EW. Notice that even if we optimize the MAA-Opt model outside the financial crisis of 2008/2009, the out-of-sample backtest for 2008/2009 (see the graph) is much better than the benchmark EW in terms of the Sharpe and Calmar ratio's (S and Q5). The same message comes from the max drawdown D in Table 4 for the MAA-TV1 model (in-sample 1997-2001, out-of-sample after Nov. 2001).

9 Other universes

After examining the performance of the MAA model for the N=7 universe in detail, we will now examine the performance of our MAA (and MAA-TV) model for 8 additional universes, ranging from (N=) 8 to 130 assets. This way we can check for the robustness of our default MAA model (with 50% shrinkage and 4 month lookback) for other universes. All backtests are done on the same period of 16 years (Nov. 1, 1997- Nov. 15, 2013), like the first universe (N=7).

In Appendix B the detailed composition of each universe is given. Most universes consist of ETFs extended back to 1997 by using similar index funds if necessary.

9.1 Alternative investments (N=8)

The second universe consists mainly of “alternative” ETFs: two REIT ETFs (IYR and RWX), three commodity ETFs (GLD, DBC, DBE), and three Government bonds ETFs (IEI, IEF, TLT), all extended with similar index funds if necessary (see Appendix B for details).

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA over EW is again clear, especially on D and Q5.

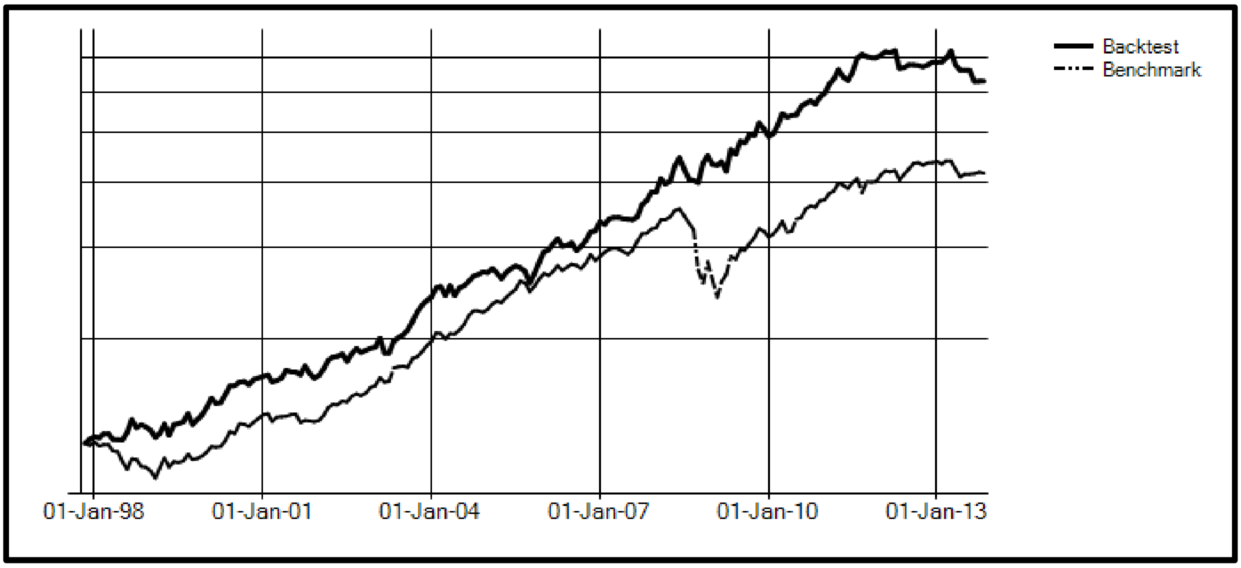


Figure 13. The MAA-TV for N=8 universe

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	9.80	8.30	11.60	2.58	0.88	2.54	0.41
MAA-TV	50	25	50	50	9.90	10.00	13.00	2.96	0.74	2.20	0.38
EW	0	0	0	0	7.60	10.00	32.70	0.03	0.62	1.88	0.08

Table 5. Statistics for N=8 universe

9.2 The global stock/bond universe (N=11)

The third universe is also a global multi-asset universe similar to the N=7 universe but now with 11 index funds mostly from Fidelity (SPY, FMCSX, FBGRX, FDIVX,FEMKX,FBNDX, FGOVX, FHIGX, FFXSX, FAGIX, FNMIX) representing US large/mid cap and growth stocks, international (IM) and emerging market (EM) stocks, US Corp/Gov/ Muni and junk bonds, and EM bonds (ETFs: SPY, IWM, VUG, VEA , VWO, LQD, IEF, MUN, SHY, JNK, EMB).

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on S, D and Q5.

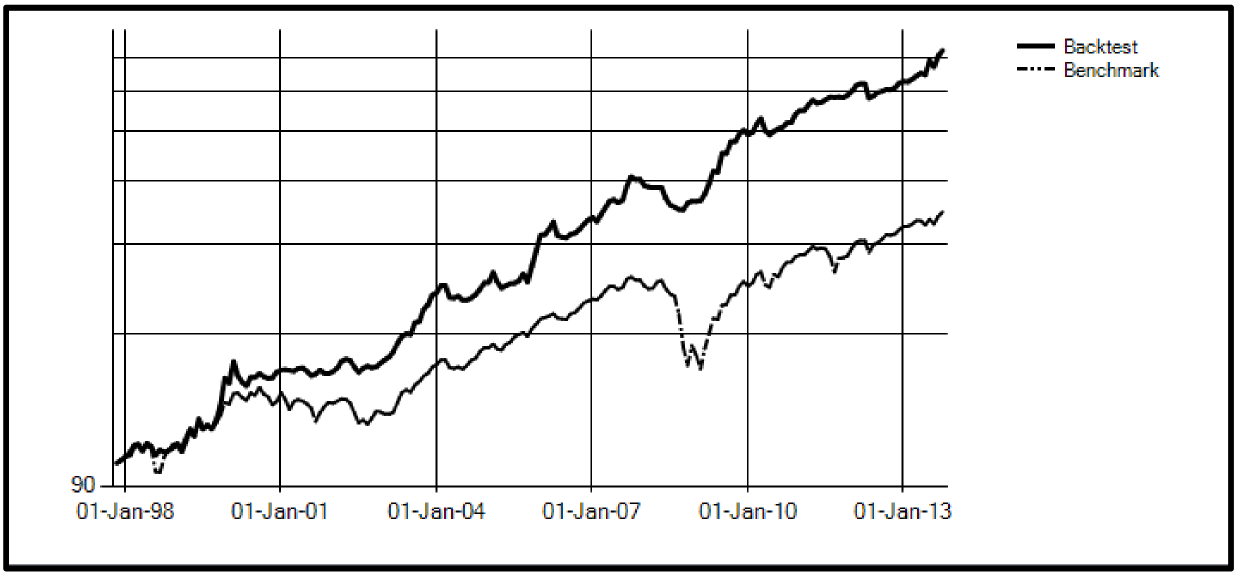


Figure 14. MAA-TV model for N-11 universe

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	8.80	5.50	5.10	2.13	1.16	3.73	0.76
MAA-TV	50	12	50	50	11.90	10.00	14.00	3.01	0.94	2.73	0.49
EW	0	0	0	0	7.40	10.80	34.00	0.03	0.45	1.76	0.07

Table 6. Statistics for N=11 universe

9.3 The US-sector universe (N=12)

The fourth universe consists of the 9 US sector ETFs from SPDR, and three bond ETFs: XLB, XLE, XLF, XLI, XLK, XLP, XLU, XLV, XLY, and IEI, IEF, TLT, all extended with similar index funds if necessarily (see Appendix B for details).

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on D and Q5.

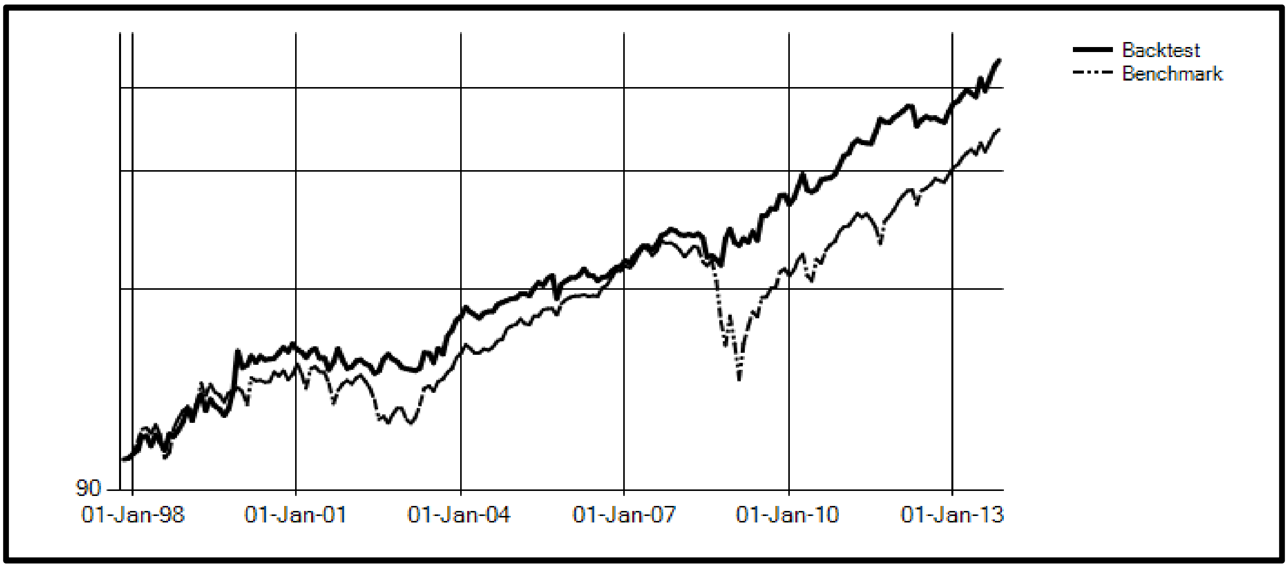


Figure 15. The MAA-TV model for N=12 universe

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	7.60	7.80	8.20	2.94	0.65	2.27	0.31
MAA-TV	50	18	50	50	8.30	10.00	11.70	3.46	0.58	2.04	0.28
EW	0	0	0	0	7.40	12.20	38.00	0.03	0.40	1.66	0.06

Table 7. Statistics for N=12 universe

9.4 The global bond universe (N=15)

The fifth universe consists of 15 bond ETFs, for US, IM and EM (Gov: SHY, IEI,IEF, TIP, TLT, muni: MBB, MUB, corp: CIU, LQD, high yield: JNK, HYG, EM bonds: PCY, EMB, and IM bonds: BWX, WIP). Again they are extended with index funds for the early years (see also appendix B for the full list).

The graph below is for the MAA-TV model and EW. For the MAA-TV model we had to tune also W_R in order to arrive at the largest volatility (V=7.90%) closest to V=10%, at W_V=0 and W_R=1. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on R, D and Q5.

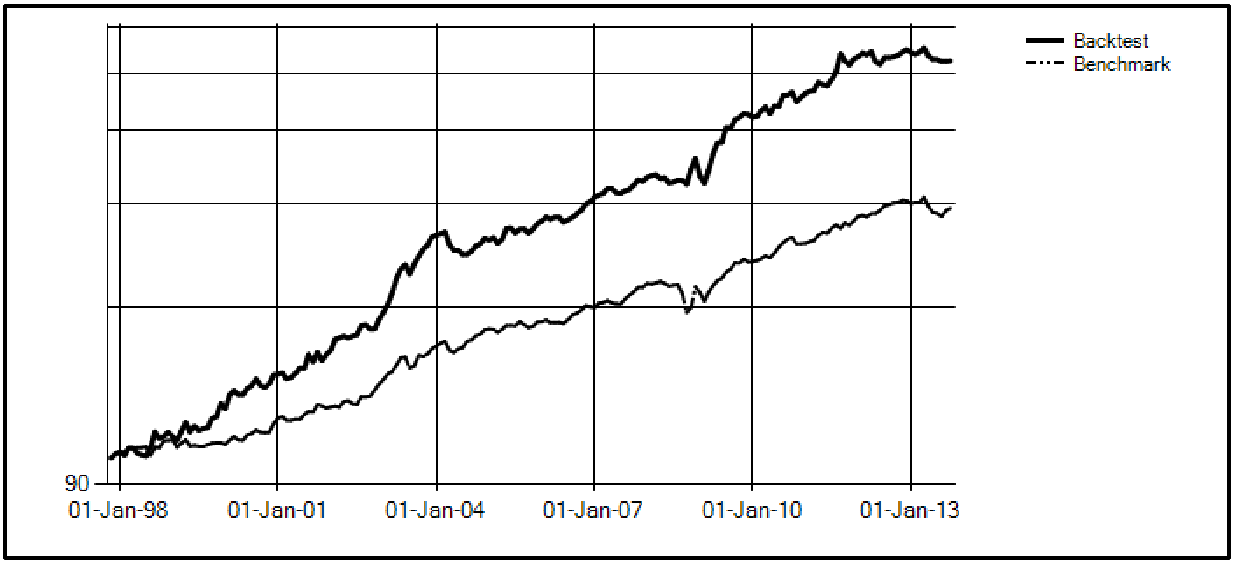


Figure 16. The MAA-TV model (N=15 universe)

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	7.20	5.20	7.30	3.68	0.91	3.10	0.30
MAA-TV	100	0	50	50	9.50	7.90	9.40	4.26	0.88	2.64	0.47
EW	0	0	0	0	6.30	5.60	11.30	0.03	0.68	2.48	0.12

Table 8. Statistics for N=15 universe

9.5 Countries (N=26c)

The sixth universe consists of 23 country ETFs (for US, IM and EM) and three bonds (IEI, IEF, and TLT). Again they are extended with index funds for the early years.

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on V, D, S and Q5.

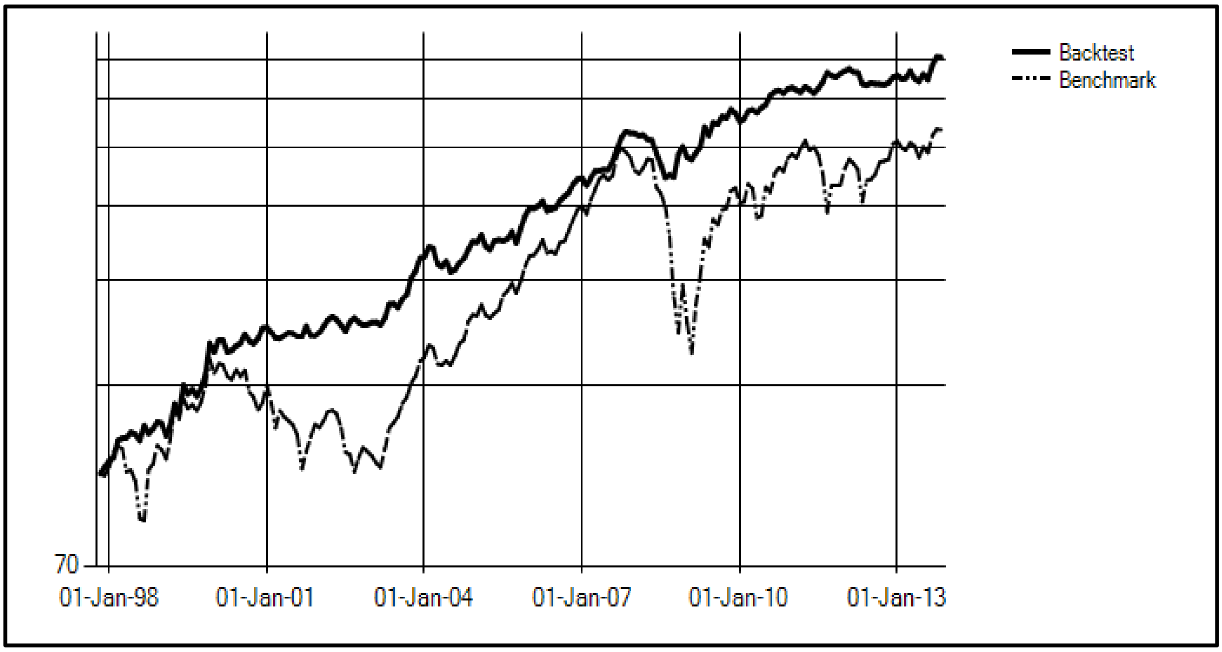


Figure 17. The MAA-TV model (N=26c universe)

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	10.30	11.30	18.70	3.40	0.70	2.13	0.28
MAA-TV	50	59	50	50	9.90	10.00	16.10	3.09	0.74	2.26	0.30
EW	0	0	0	0	8.40	20.80	54.50	0.03	0.16	1.47	0.06

Table 9. Statistics for N=26c universe

9.6 A global multi-asset universe (N=26)

The seventh universe consists of 7 global stock ETFs (US: VTI, IWM, QQQ, IM: VEA, SCZ, EM: VWO, EWX) and 14 bonds (see 8.4, except SHY), plus three commodity ETFs (DBA, DBE, DBP) and US and IM REITS (VNQ and RWX). Again they are all extended with index funds for the early years (see also appendix B for the full list).

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on S, D and Q5.

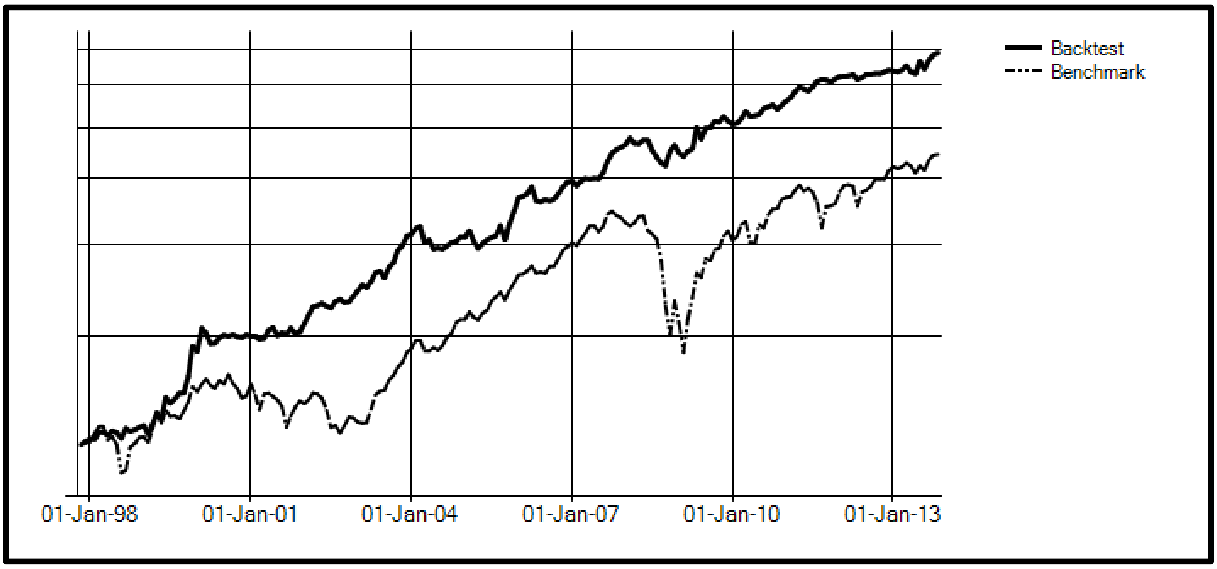


Figure 18. The MAA-TV model (N=26 universe)

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	9.3	7.2	7.3	3.03	0.95	2.87	0.59
MAA-TV	50	22	50	50	10.7	10.0	11.9	3.54	0.82	2.41	0.48
EW	0	0	0	0	7.2	10.3	30.7	0.03	0.46	1.81	0.07

Table 10. Statistics for N=26 universe

9.7 A global multi-asset universe (N=60)

The eighth universe consists of 9 US sector ETFs, 10 US style ETFs (large/mid/small, growth/value, etc.), 15 global bond ETFs (see section 9.4), 14 country/regional stock ETFs, plus three commodity ETFs (DBA, DBE, DBP) and four US and IM REITS. Again they are all extended with index funds for the early years (see also appendix B for the full list).

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on D, S and Q5.

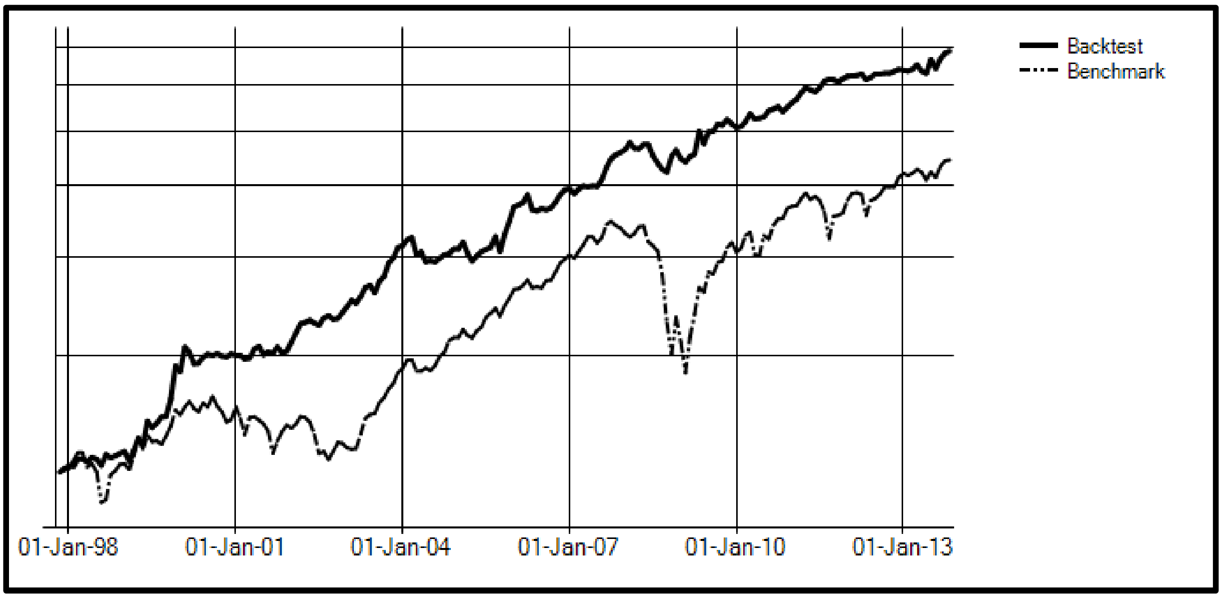


Figure 19. The MAA-TV model (N=60 universe)

Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	9.30	8.00	8.90	3.12	0.86	2.74	0.49
MAA-TV	50	36	50	50	10.40	9.90	11.30	3.60	0.80	2.47	0.48
EW	0	0	0	0	8.20	15.50	45.90	0.03	0.37	1.60	0.07

Table 11. Statistics for N=60 universe

9.8 A large global multi-asset universe (N=130)

The ninth universe consists of a rather arbitrary set of 130 index funds (US, IM and EM, stocks, bonds, etc.) mainly selected on the basis of available historical data (at Yahoo) and Assets under Management (the more the better). See also appendix B for the full list.

The graph below is for the MAA-TV model and EW. The table below has all the statistics, including those for the default MAA model. The superiority of MAA is again clear, especially on D, S and Q5.

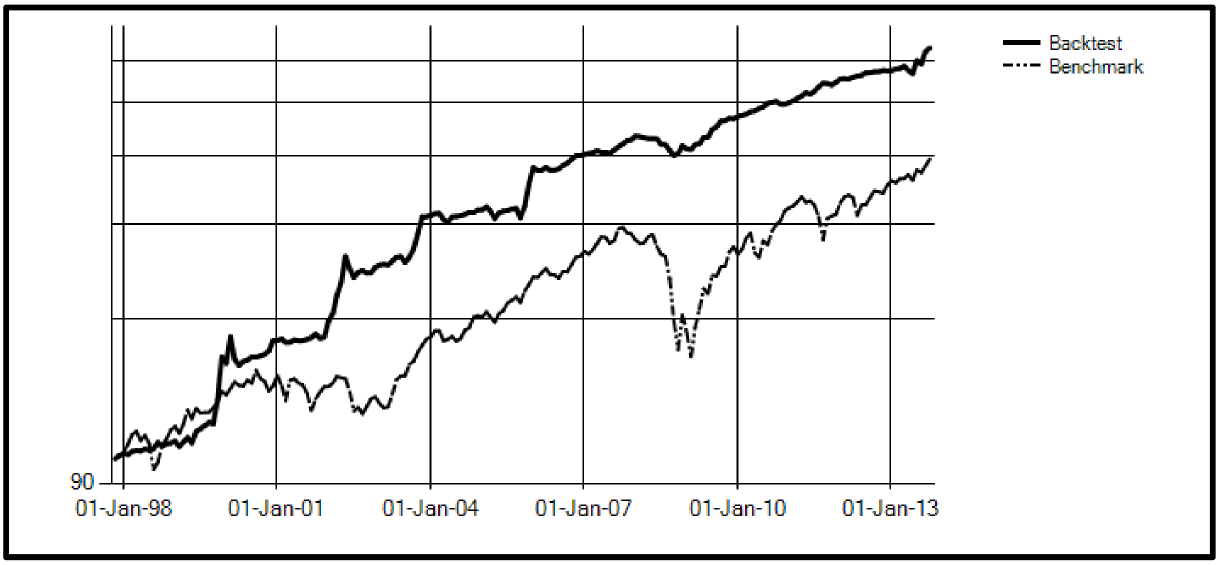


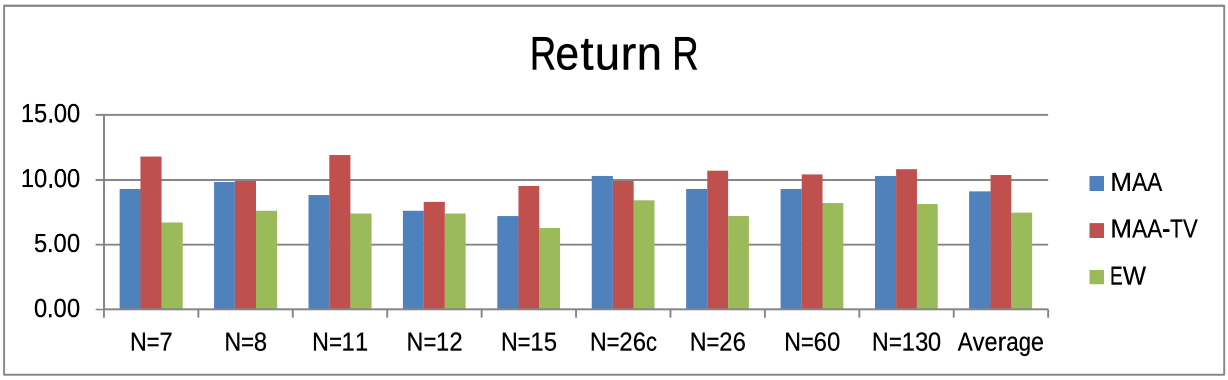
Figure 20. The MAA-TV model (N=130 universe)

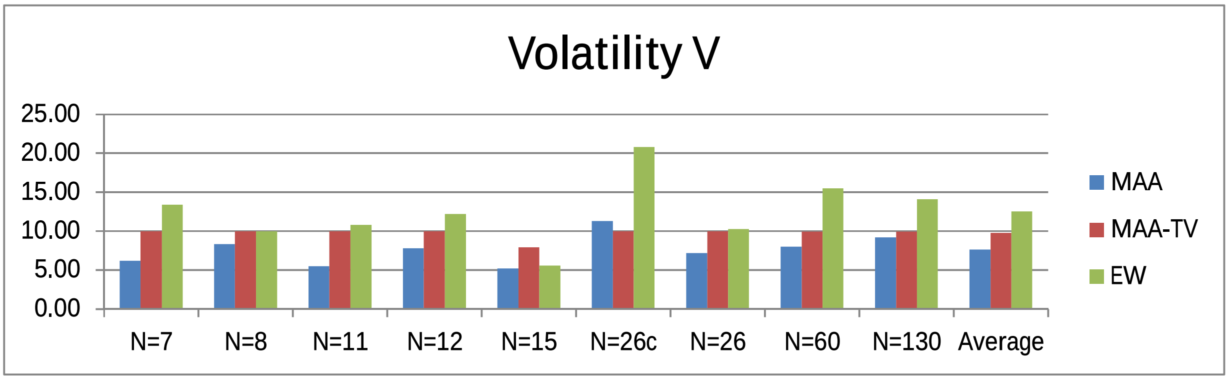
Model	W_R	W_V	W_C	W_S	R	V	D	T	S	O	Q5
MAA	50	50	50	50	10.30	9.20	10.30	2.85	0.85	3.56	0.52
MAA-TV	50	47.5	50	50	10.80	9.90	11.60	2.98	0.83	3.47	0.50
EW	0	0	0	0	8.10	14.10	42.20	0.03	0.40	1.63	0.07

Table 12. Statistics for N=130 universe

9.9 Summary (over nine universes)

Now we will summarize our findings for all the universes, from 7 to 130 assets. First we will give the backtest results for each model (MAA, MAA-TV and the EW) for all nine universes (1997-2013) in graphs (see also Appendix C for the tables).





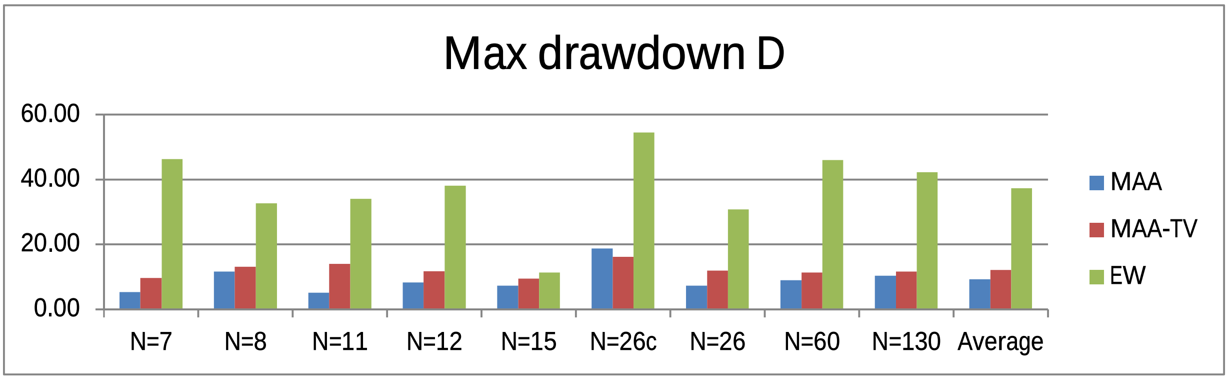
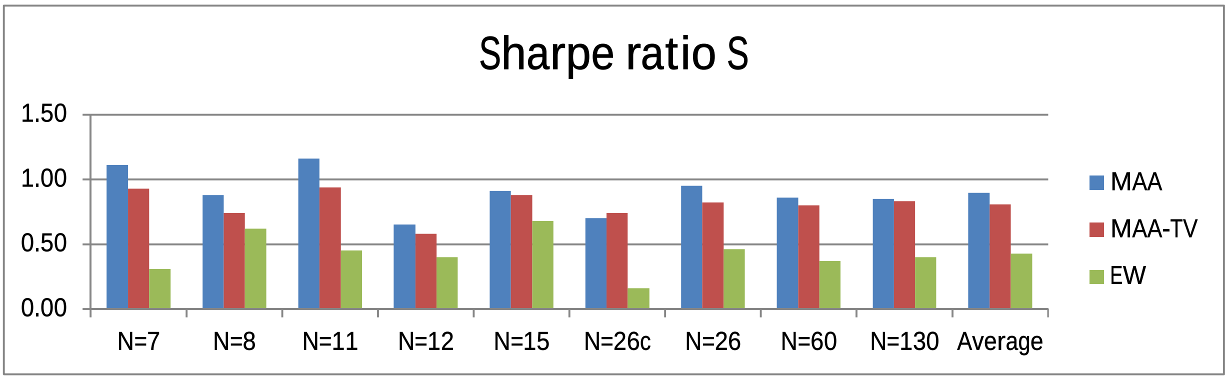


Figure 21. Basic statistics for all universes



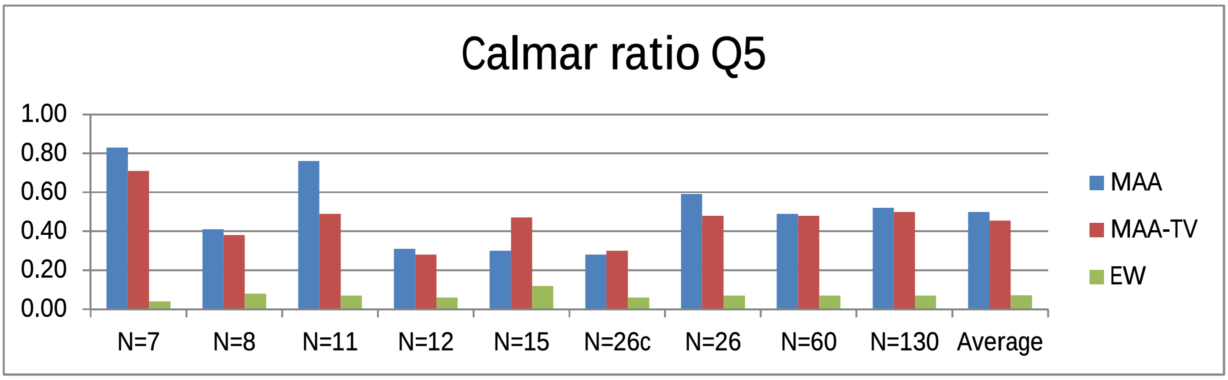


Figure 22. Return/risk statistics for all universes

From this figures (and Appendix C) we conclude:

- Returns R are around 9% (MAA) to 10.5% (MAA-TV) instead of 7.5% for EW
- Volatilities V are around 7.5% (MAA) to 10% (MAA-TV) instead of 12.5% for EW
- Max drawdowns D are around 10% for both MAA models instead of 40% for EW
- Sharpe ratio's S are around 0.8 (MAA-TV) to 0.9 (MAA) instead of 0.4 for EW
- Calmar ratio's Q5 are around 0.5 instead of 0.1 for EW

In addition to these results, notice that for *both* MAA models and for *each* universe, *each* statistic (R, V, D, T, S, O, Q5, see also Appendix C) is equal or better than the corresponding EW statistic with one exception: the turnover T of the EW model is smaller than the turnover of the MAA models, as is to be expected.

All in all, we think this summary proves the superiority of the tactical MPT (MAA) model over EW.

10 Conclusions

We have shown that the performance of a "tactical" (short-term) variant of the MPT model of Markowitz (1952) is superior to the EW model for various universes. In this tactical MPT model, we use short term lookback periods in order to have more flexibility and to benefit from the well-known momentum effect. Such a tactical MPT variant forms an appealing contrast to the classic or "strategic" MPT model with traditional lookback periods longer than one year, which has been shown to be beaten often by the EW model.

Additionally, by using the single-index (SIM) framework of Elton (1976) and some simple shrinkage estimators, we were able to arrive at a very elegant analytical and successful solution for the optimal long-only asset allocation, which we called the Modern Asset Allocation (MAA) model, honoring the Modern Portfolio Theory (MPT) of Markowitz (1952).

We have shown that under the SIM assumption, our general model encompasses special cases like Equal Weight (EW) and Minimum Variance (MV) besides other solutions, with and without constant cross-correlations. It all depends on the particular shrinkage of the factors R (return), V (volatility), C (correlation), and S (market variance).

By using default lookback periods (of 4 months) and shrinkages (50%) for expected returns, volatilities, and correlations and also 50% shrinkage for the market variance, we arrive at the MAA model which is easy to compute and to interpret. Since all parameters of the default MAA model are fixed a priori (all shrinkage weights at 50% and all lookback periods at 4 months, for all factors R, V, C, and S) over all nine universes tested, datasnooping seems not a great issue in view of the variation in universes. In addition, for one of the universes (N=7) we have given some detailed robustness results, which confirm this statement.

We also presented the "calibrated" MAA-TV model which enables us to set a maximum target volatility by tuning the weight W_V of the factor V (volatility) over a long history (here 16 years). This way we arrive at a higher return R when volatility is low, as an alternative for leverage, without paying interest. This also enables us to compare different models by simply comparing returns, given a target volatility of say, 10%.

We have demonstrated the superiority of the two MAA models over the EW model by running backtests for various multi-asset universes of ETFs and index funds (both globally, asset wise, and sector wise), ranging from seven to 130 funds. This shows that EW can be beaten by Markowitz's model, once we use a tactical instead of strategic approach to MPT.

For future research we would like to examine the risk parity family (see note 9) as a special case. Also we might extend the single-index model to the multi-index/group model where multiple indices (eg. one for bonds, one for equities, etc.) can be combined (see also Elton, 1977).

We also would like to consider more advanced (eg. EMA and GARCH like) models for estimating the expected R,V and C instead of using the simple sample statistics (and some shrinkage) over a lookback period of 4 months, like we did in this paper.

Finally, when we compare the Calmar ratio Q5 statistic of the default MAA model over all universes, the best results are for small universes (see eg. N=7 and N=11: Q5>0.75), while most other (larger) universes have Q5 scores below 0.6. So less (assets) is best? This is somewhat surprising. It might also suggest limitations of the SIM model for larger universes. Therefore, we would like to relax the SIM model to an unrestricted (semi-positive) covariance matrix, in order to see what happens when we increase the number of assets N in the tactical MPT model without the SIM restriction.

In conclusion, we think there are enough topics for future research when we take a more 'tactical' approach to good old MPT. So yes, the reports of the death of MPT have been greatly exaggerated!

Appendix A. Proofs of the SIM model

What follows is a matrix representation of the proof by Elton (1976). Let **S** be the expected (semi-positive definite and symmetrical) NxN covariance matrix, **w** the optimal weight vector and **r** the vector of expected returns, **l** a vector of one's, all of length N. Then the optimal Maximum Sharpe solution maximizes

(A.1) $\mathbf{r'l} / \mathrm{sqrt}(\mathbf{w'Sw}), \qquad \mathbf{w'l} = \mathbf{1}$

The solution is given by the vector of optimal weights

(A.2) $\mathbf{w} = s_p \mathbf{S}^{-1} \mathbf{r}$

where s_p is the normalization constant (and variance of the optimal portfolio), following from the constraint $\sum w_i = 1$. The NxN covariance matrix **S** equals, given the SIM assumptions, (see Clarke 2012):

(A.3) $\mathbf{S} = s\,\mathbf{bb}' + \mathrm{Diag}\,(\mathbf{s^e})$

where **b** equals the N-vector of beta's b_i, **S** equals the market variance, and $\mathbf{s^e}$ equals the N-vector of residual (or idiosyncratic) variances s_{ii}, i=1..N. The inverse of the matrix **S** equals (see Clarke, 2012)

(A.4) $\mathbf{S}^{-1} = \mathrm{Diag}\,(1/\mathbf{s^e}) - (\mathbf{b}/\mathbf{s}^e)(\mathbf{b}/\mathbf{s}^e)' / (1/s + (\mathbf{b}/\mathbf{s}^e)'\mathbf{b})$

Substitution of eq. (A.4) in eq. (A.2) gives eq. (4) with the "long-only" *Treynor threshold* t

(A.5) $t = (s \sum_p \mathbf{r_j b_j / s_j}) \;/\; (\mathbf{1} + s \sum_p \mathbf{b_j^2 / s_j})$

where $\sum_p$ equals the summation over all assets j in the portfolio (ie. with w_j>0) and $t_i = r_i / b_i$ equals the Treynor ratio for asset i (i=1,..,N). This can easily be implemented when b_i>0 by sorting all assets on their Treynor ratio (highest t_i first) and computing the Treynor threshold t for those assets already included until an asset t_i exceeds the Treynor threshold t (see Elton 1976). In practice one computes eq. 4 starting with t=0 and iterates then between t and w_i until convergence. This also works when b_i<0 for some i.

In the threshold t one recognizes in the summations the weighted returns r_i in the nominator and the weighted beta's b_i in the denominator, where in both cases the ratio of the systematic to the idiosyncratic risk b_i/s_i is used as weights. In that sense, the threshold t reflects the weighted Treynor ratio of the portfolio.

In the MV case (full shrinkage of all returns to r_m), we can compute the *beta threshold* b= 1/t assuming r_m=1, since r_m cancels in the quotient t_i/t in eq. (4) while the constant r_m is absorbed in the normalization constant s_p. Then eq. 4 becomes $w_i = (1-b_i/b)\ s_p / s_i$ for $b_i < b$ else $w_i=0$.

Appendix B. The nine universes

The parameters for all universes are similar and given here for documentation purposes, together with the optimal asset weights for 2008-2009 as example (for the case MAA-TV, N=7).

```
Backtester (v410)

03Nov1997-15Nov2013,cpf off,cash,close/close,lev=1,cost%j=0,tc=0.1%,d=0, 4m/4m/4m,
50/18/50/0/50%,TV=100%:

R=11.8%,V=10.0%,D=9.6%,W=68.4%,T=2.60,S0=1.19,S5=0.68,S=0.93,
O0=2.53,O2=0.41,H0=4.15,H5=2.40,Q0=1.23,Q5=0.71,KAA2

02-Jan-2008:  QRAAX (36.7%), VFISX (19.8%), VBMFX (18.0%), VEIEX (17.5%), FDIVX (8.0%)
01-Feb-2008:  QRAAX (34.6%), VFISX (33.7%), VBMFX (31.7%)
03-Mar-2008:  QRAAX (36.0%), VFISX (35.2%), VBMFX (28.8%)
01-Apr-2008:  QRAAX (43.7%), VFISX (31.0%), VBMFX (25.3%)
01-May-2008:  QRAAX (40.1%), VFISX (23.2%), VBMFX (20.4%), VGSIX (16.2%)
02-Jun-2008:  QRAAX (37.4%), VFISX (15.8%), VBMFX (13.4%), VEIEX (12.1%), VGSIX (9.0%), FDIVX (8.8%), VTSMX (3.6%)
01-Jul-2008:  QRAAX (42.8%), VFISX (20.7%), VBMFX (19.0%), VGSIX (7.5%), FDIVX (5.2%), VTSMX (4.8%)
01-Aug-2008:  QRAAX (32.8%), VFISX (24.5%), VBMFX (21.8%), VTSMX (9.1%), VGSIX (5.5%), FDIVX (4.6%), VEIEX (1.7%)
02-Sep-2008:  VFISX (69.0%), VBMFX (31.0%)
01-Oct-2008:  VFISX (71.8%), VBMFX (28.2%)
03-Nov-2008:  VFISX (100.0%)
01-Dec-2008:  VFISX (67.2%), VBMFX (32.8%)
02-Jan-2009:  VFISX (50.6%), VBMFX (49.4%)
02-Feb-2009:  VBMFX (53.9%), VFISX (46.1%)
02-Mar-2009:  VBMFX (62.1%), VFISX (37.9%)
01-Apr-2009:  VEIEX (38.1%), VBMFX (34.7%), VFISX (27.2%)
01-May-2009:  VEIEX (60.0%), VFISX (20.7%), VBMFX (19.4%)
01-Jun-2009:  VEIEX (46.2%), FDIVX (16.2%), VFISX (12.5%), VBMFX (11.9%), QRAAX (9.8%), VTSMX (3.4%)
01-Jul-2009:  VEIEX (34.2%), FDIVX (15.7%), VBMFX (12.7%), VFISX (11.1%), VTSMX (10.8%), QRAAX (10.8%), VGSIX (4.7%)
03-Aug-2009:  VEIEX (28.0%), FDIVX (16.8%), VBMFX (14.6%), VFISX (12.0%), VTSMX (11.5%), VGSIX (10.1%), QRAAX (7.0%)
01-Sep-2009:  VEIEX (21.8%), FDIVX (19.8%), VBMFX (14.8%), VTSMX (12.5%), QRAAX (11.1%), VFISX (10.4%), VGSIX (9.6%)
01-Oct-2009:  VGSIX (21.7%), VBMFX (18.9%), VTSMX (17.1%), VEIEX (16.6%), FDIVX (13.4%), VFISX (12.4%)
02-Nov-2009:  VGSIX (21.1%), VEIEX (19.2%), VBMFX (18.7%), VFISX (14.6%), FDIVX (13.9%), VTSMX (12.5%)
01-Dec-2009:  VGSIX (23.4%), VBMFX (20.1%), VFISX (16.6%), VEIEX (15.7%), VTSMX (14.0%), FDIVX (10.0%), QRAAX (0.2%)
```

Below we list all universes. A plus (+) means concatenation of funds, where preference is given to the most recent (left-handed) funds and the other funds are used to extend the history into the past.

B.1 Mixed Vanguard (N=7)

VTSMX
FDIVX
VEIEX
VBMFX
VFISX
VGSIX
QRAAX

B.2 Mixed ETFs (N=8)

IYR+IYRIDX+FRESX+STMDX+DREVX
RWX+RWXIDX+HAINX+VTRIX+VWIGX+SCINX+DREVX
GLD+GLDIDX+USAGX+FSAGX+FKRCX+DREVX
DBC+DBCIDX+FSENX+FSTEX+PRNEX+DREVX

DBE+GSP+USO+QRAAX+LEXMX
IEI+VFITX
IEF+VFITX
TLT+RYGBX

B.3 Mixed Fidelity (N=8)

SPY
FDIVX
FEMKX
FBGRX
FMCSX
FBNDX
FGOVX
FHIGX
FFXSX
FAGIX
FNMIX

B.4 Sector ETFs (N=12)

XLB+FSDPX
XLE+VGENX
XLF+FIDSX
XLI+FSCGX
XLK+FSPTX
XLP+FDFAX
XLU+FSUTX
XLV+VGHCX
XLY+FSRPX
IEI+VFITX
IEF+VFITX
TLT+RYGBX

B.5 Global Bonds ETFs (N=15)

IEI+VFITX
IEF+VFITX
SHY+FIGIX
TIP+VIPSX+PRRIX+LSGSX
TLT+RYGBX
CIU+TSWFX
LQD+TSWFX

MBB+VFIIX
MUB+BIV+FEPIX+NERYX
JNK+HYG+FAHDX+FAHYX
HYG+FAHDX+FAHYX
PCY+PEBAX+IHICX
EMB+PEBAX+PREMX
BWX+RPIBX+SCINX
WIP+BWX+MGGBX

B.6 Countries (N=26)

SPY
EWJ
EWG
EWC
EWU
EWH
EWA
EWS
EWL
EWI
EWQ
EWD
EWP
EWN
EWZ+FLATX
FXI+EWH
EWY+MAKOX
EWT+PRASX
EWW+FLATX
RSX+VEIEX
EWM+FEMKX
EPI+INP+PRASX
EZA+EEM+VEIEX
IEI+VFITX
IEF+VFITX
TLT+RYGBX

B.7 Mixed ETFs (N=26)

VEA+EFA+FDIVX
VTI+VTSMX
IWM+NAESX
QQQ+KTCAX

SCZ+EFA+HAINX
VWO+EEM+VEIEX
EWX+DGS+SDMGX
IEI+VFITX
IEF+VFITX
TIP+VIPSX+PRRIX+LSGSX
TLT+RYGBX
CIU+TSWFX
LQD+TSWFX
MBB+VFIIX
MUB+BIV+FEPIX+NERYX
JNK+HYG+FAHDX+FAHYX
HYG+FAHDX+FAHYX
PCY+PEBAX+IHICX
EMB+PEBAX+PREMX
BWX+RPIBX+SCINX
WIP+BWX+MGGBX
DBA+DJP+DBC+QRAAX
DBE+GSP+USO+QRAAX
DBP+GLD+CEF
RWX+IIRBX+FIREX+NIIAX
VNQ+VGSLX+VGSIX

B.8 Mixed ETFs and Index Funds (N=60)

BOND+IEI+VFIUX+VFITX+FSTGX+FGOVX+STVSX+FGMNX+FKUSX
SHY+FIGIX+TWUSX
IEF+PFGCX+PRULX+VUSTX+STVSX+PRCIX
TIP+VIPSX+PRRIX+LSGSX+PRCIX
TLT+RYGBX+PRULX+VUSTX+VWESX+PRCIX
CIU+TSWFX+VFICX+PRCIX
LQD+PRPIX+VWESX+PRCIX
MBB+VFIIX+FUSGX+FKUSX+DREVX
MUB+BIV+FEPIX+AGG+NERYX+SDFIX+PRCIX+DREVX
HYG+FAHEX+PSTKX+FAHYX+DREVX
JNK+FAHYX+NCINX
PCY+PAEMX+PEBIX+IHICX+FAEMX+FNMIX+NCINX
EMB+PEBCX+PEBAX+PREMX+SCEMX+FNMIX+NCINX
BWX+RPIBX+SCINX
WIP+BWX+MGGBX+BEGBX+RPIBX+SCINX
VPU+XLU+FSUTX+DREVX
VDC+XLP+FDFAX+DREVX
VHT+XLV+VGHCX+DREVX
VFH+XLF+CFIMX+SLASX+FSLSX+DREVX
VDE+XLE+RYEIX+VGENX+FSENX+PRNEX+DREVX

VGT+XLK+RYTIX+KTCBX+FTCHX+DREVX
VCR+XLY+FSCPX+FSRPX+FSLSX+DREVX
VIS+XLI+FCYIX+FSCGX+SOPFX+DREVX
VBK+IWO+VEXMX+NAESX+PENNX+FSLSX+DREVX
VBR+IWN+VSMAX+NAESX+PENNX+FSLSX+DREVX
VOE+IWS+VIMSX+FASPX+FSLSX+DREVX
VOT+IWP+VIMSX+SSVSX+DNLDX+SOPFX+SCDGX+DREVX
VTV+IWD+VIIIX+VINIX+VFINX+DREVX
VUG+IWF+VTCIX+VTSMX+RYNVX+VFINX+SCDGX+DREVX
QQQ+KTCAX+PRSCX+FTCHX
IWM+NAESX+PENNX+FSLSX
VIG+VDIGX+VFINX+OARDX+DREVX
VYM+VTV+IVV+RYNAX+RYNVX+VFINX+DREVX
PFF+VCVLX+FIDSX+FSLSX+DREVX
EWC+FICDX+PRNEX+DREVX
EWJ+VPACX+FPBFX+FOSFX+SCINX
EWA+FDIVX+FIGRX+HAINX+FOSFX+SCINX
EWG+VEURX+FIEUX+FOSFX+SCINX
EWU+VEURX+FIGRX+FOSFX+SCINX+DREVX
VGK+IEV+VEURX+HAINX+FIEUX+SCINX
SCZ+GWX+DLS+PRIDX+SCINX
EPP+EWA+FDIVX+FIGRX+HAINX+FOSFX+SCINX+DREVX
IDV+NIIAX+HAINX+SCINX+DREVX
FXI+EWH+VEIEX+FSEAX+PRASX+HAINX+FPBFX+FOSFX+SCDGX+DREVX
EWY+VEIEX+FPBFX+FOSFX+DREVX
EPI+INP+PRASX+HAINX+SLASX+PRIDX+FOSFX+DREVX
RSX+GUR+LETRX+VEIEX+FEMKX+HAINX+VGENX+DREVX
EWZ+FLATX+HAINX+PRNEX+DREVX
EWW+FLATX+ANCFX+GABAX+DREVX
AAXJ+VEMIX+VEIEX+FEMKX+FSEAX+PRASX+PRSGX+FPBFX+OARDX+DREVX
ILF+SLAOX+SLAFX+FLATX+ANCFX+HAINX+PRNEX+DREVX
EWX+DGS+SDMGX+VEIEX+FEMKX+FIGRX+SCINX
DEM+EEM+VEMIX+VEIEX+ANCFX+HAINX+OARDX+DREVX
DBA+DJP+DBC+QRABX+QRAAX+PSPFX+LEXMX
DBE+GSP+USO+QRAAX+MAGRX+LEXMX
DBP+GLD+CEF+USERX
VNQ+VGSLX+VGSIX+FRESX+STMDX
IYR+VGSIX+FRESX+STMDX
RWX+IIRBX+FIREX+NIIAX+HAINX+SCINX
VNQI+DRW+RWX+IIRAX+VGTSX+NIEQX+ACINX+SCINX+DREVX

B.9 Mixed Index Funds (N=130)

BGEIX BGNMX BTFTX BTTTX CFIMX CHTRX CPTNX DNLDX DREVX FAGIX
FBGRX FBIOX FBMPX FBNDX FCNTX FDCAX FDCPX FDEQX FDFAX FDFFX
FDGRX FDLSX FDVLX FEQIX FFIDX FGLDX FGMNX FGOVX FGRIX FHIGX
FHIIX FICDX FIDSX FIEUX FIGRX FIUIX FKRCX FKUSX FLPSX FMAGX FOCPX
FOSFX FPBFX FRESX FSAGX FSAIX FSAVX FSCGX FSCHX FSCSX FSDAX
FSDPX FSELX FSENX FSESX FSHOX FSLBX FSLEX FSLSX FSPCX FSPHX FSPTX
FSRBX FSRFX FSRPX FSTCX FSTEX FSUTX FSVLX FTCHX FTHRX FTRNX
FUSGX FWWFX GABAX GTAGX HAINX INEAX LEXMX NBSSX NCINX NSBIX
OARDX PENNX PRCIX PRESX PRHYX PRIDX PRNEX PRSCX RPIBX SCDGX
SCGDX SCINX SCSBX SEQUX SGINX SLASX SOPFX STMDX STRGX STVSX
TWUSX UNWPX USAGX USERX VAGIX VBMFX VEIPX VEURX VFIIX VFINX
VFSTX VGENX VGHCX VGPMX VMRGX VPACX VSGBX VTRIX VUSTX VWAHX
VWEHX VWESX VWIGX VWITX VWNDX VWNFX VWSTX VWUSX

Appendix C. Results per universe

MAA	R	V	D	T	S	O	Q5
N=7	9.30	6.20	5.20	1.85	1.11	3.18	0.83
N=8	9.80	8.30	11.60	2.58	0.88	2.54	0.41
N=11	8.80	5.50	5.10	2.13	1.16	3.73	0.76
N=12	7.60	7.80	8.20	2.94	0.65	2.27	0.31
N=15	7.20	5.20	7.30	3.68	0.91	3.10	0.30
N=26c	10.30	11.30	18.70	3.40	0.70	2.13	0.28
N=26	9.30	7.20	7.30	3.03	0.95	2.87	0.59
N=60	9.30	8.00	8.90	3.12	0.86	2.74	0.49
N=130	10.30	9.20	10.30	2.85	0.85	3.56	0.52
Average	9.10	7.63	9.18	2.84	0.90	2.90	0.50

Table 13. Statistics for the MAA model for all universes

MAA-TV	R	V	D	T	S	O	Q5
N=7	11.80	10.00	9.60	2.60	0.93	2.53	0.71
N=8	9.90	10.00	13.00	2.96	0.74	2.20	0.38
N=11	11.90	10.00	14.00	3.01	0.94	2.73	0.49
N=12	8.30	10.00	11.70	3.46	0.58	2.04	0.28
N=15	9.50	7.90	9.40	4.26	0.88	2.64	0.47
N=26c	9.90	10.00	16.10	3.09	0.74	2.26	0.30
N=26	10.70	10.00	11.90	3.54	0.82	2.41	0.48
N=60	10.40	9.90	11.30	3.60	0.80	2.47	0.48
N=130	10.80	9.90	11.60	2.98	0.83	3.47	0.50
Average	10.36	9.74	12.07	3.28	0.81	2.53	0.45

Table 14. . Statistics for the MAA-TV model for all universes

EW	R	V	D	T	S	O	Q5
N=7	6.70	13.40	46.30	0.03	0.31	1.58	0.04
N=8	7.60	10.00	32.70	0.03	0.62	1.88	0.08
N=11	7.40	10.80	34.00	0.03	0.45	1.76	0.07
N=12	7.40	12.20	38.00	0.03	0.40	1.66	0.06
N=15	6.30	5.60	11.30	0.03	0.68	2.48	0.12
N=26c	8.40	20.80	54.50	0.03	0.16	1.47	0.06
N=26	7.20	10.30	30.70	0.03	0.46	1.81	0.07
N=60	8.20	15.50	45.90	0.03	0.37	1.60	0.07
N=130	8.10	14.10	42.20	0.03	0.40	1.63	0.07
Average	7.48	12.52	37.29	0.03	0.43	1.76	0.07

Table 15. Statistics for the EW model for all universes

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