Supplementary Content

1 Introduction

This document provides extra content that complements the paper "P. Serna-Torre, P. Hidalgo-Gonzalez, Frequency dynamics with inverters: proof of stabilizability and existence of Nash equilibrium". The content of this report is also motivated by the reviewers' feedback we received in the review round of the mentioned paper. Finally, we were not able to include this report in the mentioned paper because of space limitations.

2 Optimal gain of integral controller in Conventional Frequency Regulation (CFR)

2.1 Motivation

One of the contributions of our paper is to demonstrate that our proposed control scheme outperforms Conventional Frequency Regulation (CFR) strategy. In this strategy, we consider that Primary Frequency Regulation (PFR) and Secondary Frequency Regulation (SFR) are control schemes in charge of drive frequency deviation ω to zero (P. Kundur, Balu, & Lauby, 1994; Milano, Dörfler, Hug, Hill, & Verbič, 2018).

PFR is the Power-frequency droop function that is placed in each synchronous generator (SG) to control its input mechanical power. The droop function exerts a variation of the input mechanical power when frequency deviates. PFR usually actuates in the first five to ten seconds when a frequency deviation arises in the power system. However, the PFR is not able to restore the frequency to its nominal value because of its proportional action nature. Then, SFR kicks in to restore ω to zero.

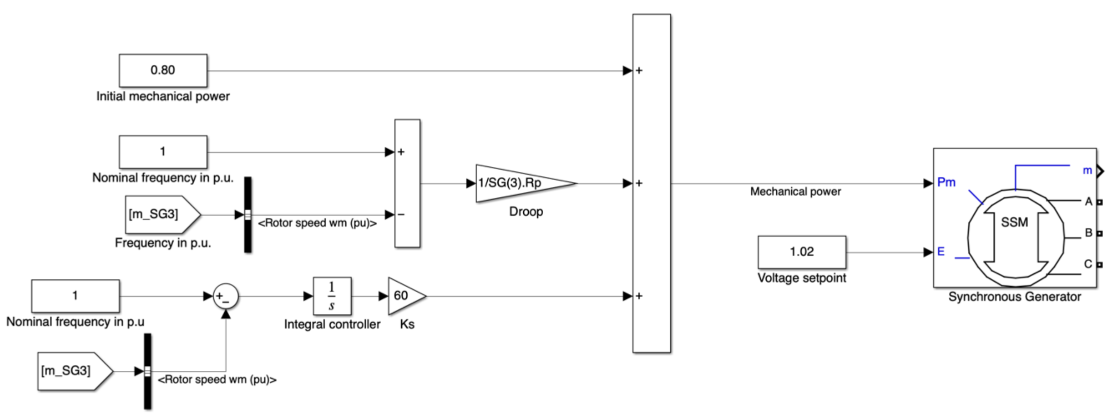

Figure 1: Primary Frequency Regulation and Secondary Frequency Regulation schemes in synchronous generator

SFR is a control scheme that basically consists of an integral controller that is set up in some SGs. The integral controller actuates the input mechanical power to drive ω to zero. The selection of the SGs depends on the areas where they are located, and in practice, is assigned to SGs with capability to vary its power fast. We refer the reader to more detailed information of SFR in the works (P. Kundur et al., 1994; Saadat, 2009). In Fig. 1, we explicitly show how we implement PFR and SFR for a SG in the CFR.

2.2 Mathematical formulation

In Section III-A, we consider that each of SGs 1, 3, and 5 has the integral controller with gain K_s that simulates the SFR in CFR. In what follows, we describe the modeling and computation to obtain an optimal gain K_s^*.

First, we model the frequency dynamics of the system as one-bus system in which an equivalent generator and load are connected. The equivalent generator has a constant inertia m_{eq} and an constant damping d_{eq} of the system. The parameters m_{eq} and d_{eq} can computed using the inertia and damping of the generators of the system as (Prabha Kundur, 1994) illustrates. Then, the frequency dynamics model of the one-bus system is

$$\dot{\omega} = -m_{eq}^{-1} d_{eq} \omega + m_{eq}^{-1}(P_m - \delta), \tag{1}$$

where $\omega \in \mathbb{R}$ is the frequency deviation, $P_m \in \mathbb{R}$ is the total input mechanical power of the generators and $\delta \in \mathbb{R}$ is the total load disturbance. We propose the compensator with dynamics

$$\dot{v} = -\omega, \tag{2}$$

where $v \in \mathbb{R}$ is the state of the compensator. Then, the augmented state-space representation is

$$\begin{bmatrix} \dot{\omega} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} -m_{eq}^{-1} d_{eq} & 0 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} m_{eq}^{-1} \\ 0 \end{bmatrix} (P_m - \delta) \tag{3}$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \omega \\ v \end{bmatrix} \tag{4}$$

Using the theory of optimal output-feedback regulator given in (Rodrigues, 2022), we first calculate the stabilizing Riccati solution P of the state-feedback problem of the system with dynamics given by (3). Then we formulate the optimal output-feedback regulator problem, in which we aim to find static controller K_s^* for the control action $u = K_s^* y$:

$$\min\ J(x_0, u) = \int_{t_0}^{\infty} \left\{ x^\top Q x + u^\top R u \right\} dt, \ , \tag{5}$$

$$\text{subject to: } (3), (4) \tag{6}$$

where $x = \begin{bmatrix} \omega & v \end{bmatrix}^\top$, $Q = Q^\top \succeq 0$, $R = R^\top \succeq 0$. Second, the work in (Rodrigues, 2022) proposes that the optimal controller, that in our case is K_s^*, can be calculated solving the Semidefinite Programming (SDP) problem

$$\max_{K_s, \alpha} \alpha \tag{7}$$

$$\text{s.t.: } \begin{bmatrix} Q - PBK_sC - C^\top K_s B^\top P & PB \\ B^\top P & R \end{bmatrix} \succeq \alpha I \tag{8}$$

where $B = \begin{bmatrix} m_{eq}^{-1} \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$, and P is the stabilizing Riccati solution of the associated state-feedback problem. We have also set $Q = 10^3 I_2$, $R = 1$. Note that P is input data in the SDP problem (7), (8). We solve it using YALMIP and setting MOSEK as solver. We obtain the optimal gain $K_s^* = 68.6$. Note that the control action is $u = K_s^* y = K_s^* C x = K_s^* v = K_s^* \int -\omega dt$. Thus, K_s^* is the optimal gain of the integral controller that simulates the Secondary Frequency Regulation of CFR.

2.3 Computational implementation

First, install YALMIP and Mosek in the Matlab installation. The Matlab code to compute the optimal gain K_s^* is

```
% Input data
Stotal =  100*6; % Total apparent power (MVA)
Heq = 4*4*100/Stotal + 2*0.6*100/Stotal; % Equivalent inertia (s)
Deq = 1; % Equivalent damping
Req = 0.05; % Equivalent droop
Meq = 2*Heq;

Grid.ssr.A = -(Deq + 1/Req)/(Meq);
Grid.ssr.B = 1/Meq;
Grid.ssr.C = eye(size(Grid.ssr.A,1));
Grid.ssr.D = zeros(size(Grid.ssr.C,1), size(Grid.ssr.B,2));

% Definition of matrices
A = [Grid.ssr.A 0;-1 0];
B = [1/Meq; 0];
C = [0 1];
Q = 1e3*eye(2);
R = 1;

% Get state-feedback controller (K) and solution of Algebraic
   Riccati
% Equation (P)
[K,P,~] = lqr(A,B,Q,R);

% Get output-feedback controller (F) via SDP.
yalmip('clear')
n = size(A,1);
m = size(B,2);
F = sdpvar(1,1, 'full');
alpha = sdpvar(1,1, 'full');

constraints = [[Q - P*B*F*C-C'*F'*B'*P P*B;
                B'*P                   R]>=alpha*eye(n+m)];
objective_function = -alpha;
```

```
% Set some options for YALMIP and solver
options = sdpsettings('verbose',1,'solver', 'mosek');

% Solve the problem
model = optimize(constraints,objective_function,options);

F_sol = value(F) % Optimal gain Ks
alpha_sol = value(alpha);
eig(A+ B*F_sol*C) % Check that eigenvalues are placed on the left-
   half of the complex plane
```

References

Kundur, P. [P.], Balu, N., & Lauby, M. (1994). *Power system stability and control*. McGraw-Hill Education. Retrieved from https://books.google.com/books?id=2cbvyf8Ly4AC

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Milano, F., Dörfler, F., Hug, G., Hill, D. J., & Verbič, G. (2018). Foundations and challenges of low-inertia systems (invited paper). In *2018 power systems computation conference (pscc)* (pp. 1–25). doi:10.23919/PSCC.2018.8450880

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