Page No.:
Date:

YOUVA

* Maximum Likelihood Estimators for A and B in line-fitting Example.

$$x[n] = A + w[n] + Bn \; ; n = 0,1,2,\cdots,N-1$$
$$w[n] \sim \mathcal{N}(0,\sigma^2)$$

$A, B \rightarrow$ intercept, slope

$\Rightarrow$ likelihood function

$$p(x;A,B) = \left(\frac{1}{\sqrt{2\pi\sigma^2}}\right)^N e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A-Bn)^2}$$

since, $E[x[n]] = A+Bn \; ; \; var(x[n]) = \sigma^2$

$\Rightarrow$ Log-likelihood function

$$\ln p(x;A,B) = -\frac{N}{2}\ln 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A-Bn)^2$$

$\Rightarrow$ For MLE

Treating for A ;

$$\left.\frac{\partial \ln p(x;A,B)}{\partial A}\right|_{\substack{A=\hat{A}\\ B=\hat{B}}} = 0$$

$$\therefore \; 0 - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1} 2(x[n]-\hat{A}-\hat{B}n)(-1) = 0$$

$$\therefore \; \frac{1}{\sigma^2}\sum_{n=0}^{N-1}(x[n]-\hat{A}-\hat{B}n) = 0$$

Multiplying with σ^2 both the sides.

$$\therefore \; \sum_{n=0}^{N-1} x[n] - \hat{A}N - \hat{B}\frac{N(N-1)}{2} = 0$$

Multiplying by $\frac{1}{N}$ both the sides

$$\therefore \underbrace{\frac{1}{N}\sum_{n=0}^{N-1} x[n]}_{\text{I}} = \hat{A} + \hat{B}\underbrace{\frac{(N-1)}{2}}_{\text{II}}$$

I : Average of $x[n] = \bar{x}$

II : Average of $n = \bar{n}$

$$\therefore \bar{x} = \hat{A} + \hat{B}\bar{n} \longrightarrow (1)$$

Treating for B;

$$\left.\frac{\partial \ln p(x; A, B)}{\partial B}\right|_{\substack{A=\hat{A}\\ B=\hat{B}}} = 0$$

$$\therefore 0 - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1} 2(x[n] - \hat{A} - \hat{B}n)(-n) = 0$$

$$\therefore \frac{1}{\sigma^2}\sum_{n=0}^{N-1}\{[x[n] - \hat{A} - \hat{B}n]\cdot n\} = 0$$

Multiplying with σ^2 both the sides and substituting equation (1), we get

$$\therefore \sum_{n=0}^{N-1}(x[n] - (\bar{x} - \hat{B}\bar{n}) - \hat{B}n)\,n = 0$$

$$\therefore \sum_{n=0}^{N-1}(x[n] - \bar{x})\,n = \hat{B}\sum_{n=0}^{N-1}(n - \bar{n})\,n$$

$$\therefore \hat{B} = \frac{\sum_{n=0}^{N-1}(x[n] - \bar{x})\cdot n}{\sum_{n=0}^{N-1}(n - \bar{n})\cdot n}$$

$$\therefore \quad \hat{B} = \sum_{n=0}^{N-1} \frac{(x[n] - \bar{x})(n - \bar{n})}{(n - \bar{n})^2}$$

and $\hat{A} = \bar{x} - \hat{B}\,\bar{n}$